MA 266 Practice Problems

Problem 1): Use the definition of the Laplace transform to show that

$$\mathscr{L}\{\delta(t)\} = 1$$

Hint: $\int_{-\infty}^{\infty} \delta(t-c) f(t)\, dt = f(c)$.

Problem 2): Find the Laplace transforms of the following functions

a) $f(t) = \sin(2t)\cos(2t)$ (Hint: pull out an old trig identity)

b) $f(t) = e^{5t} t^5$

Problem 3): Find the inverse Laplace transform of the following functions (for a), write your solution in terms of convolution integrals).

a) $F(s) = \dfrac{s-2}{(s^2+3)(s^2-2s+2)}$

b) $F(s) = \displaystyle\sum_{n=1}^{N} \frac{n!}{ns^n}$

Problem 4): Find the Laplace transform $F(s)$ of the given function, and determine for which *real* values of s $F(s)$ is valid.

$$f(t) = \sum_{n=0}^{\infty} \delta(t-n)$$

Problem 5): Find the general solution for the given differential equation.

$$y^{(6)} - 2y^{(3)} + y = 0$$

Problem 6): Determine a homogeneous differential equation that has the following general solution.

$$y(t) = c_1 e^{2t} + c_2 t e^{2t} + c_3 \sin\left(\sqrt{2}t\right) + c_4 \cos\left(\sqrt{2}t\right)$$

Problem 7): Given the following first-order system, what can you say about the stability of the origin assuming that $|\alpha| \neq |\beta|$?

$$\mathbf{x}'(t) = \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix} \mathbf{x}(t)$$

Problem 8): Consider a damped spring-mass system with an external force given by $\cos(\eta t)$.

a) Argue/justify why we don't have to worry about resonance in this case.

b) If the general solution to the system is of the form

$$y(t) = Re^{-kt}\cos(\omega t - \delta) + y_p(t, \eta)$$

where $k > 0$, what value of η will result in the solution with the greatest maximum modulus? (i.e, $|y(t)|$)

Problem 9): Consider the following function:

$$f(t) = u_0(\sin(\pi t)), \quad t \geq 0$$

Write $f(t)$ in terms of $u_k(t)$'s. That is, step functions with only t as the argument.

Problem 10):