Calibrating MEMS Accelerometers

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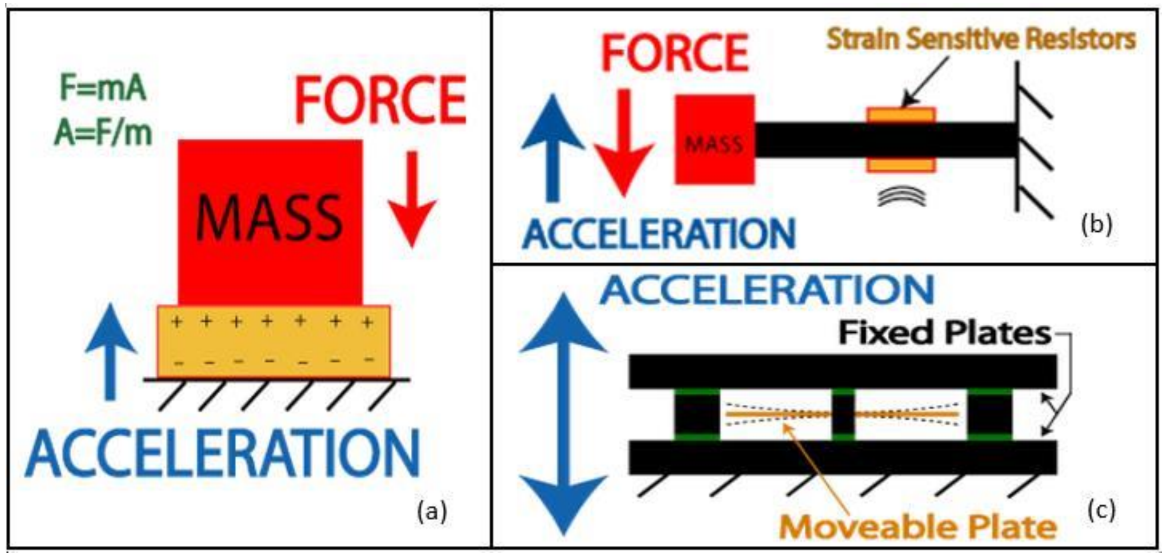
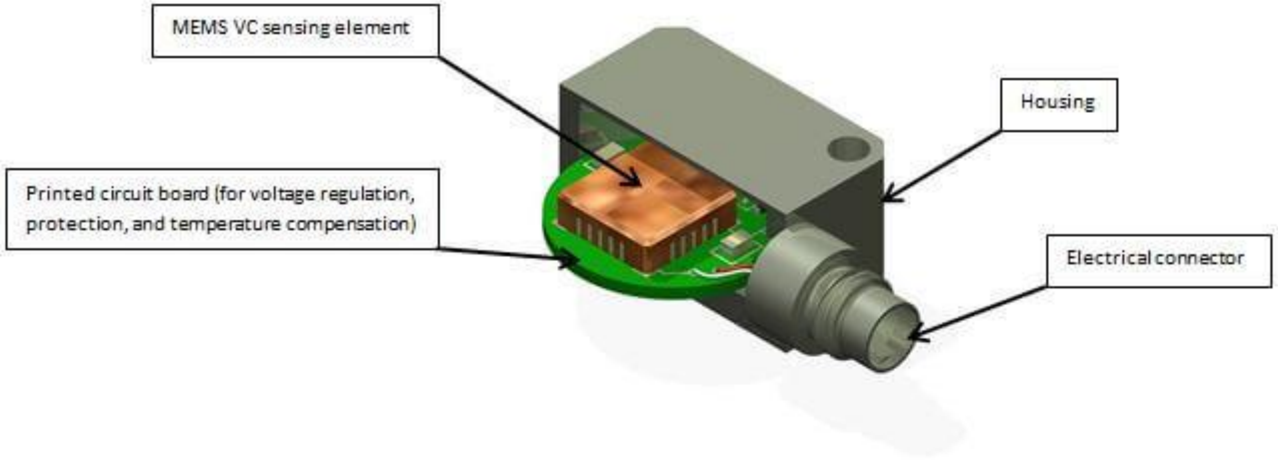
for
Application of mathematics for modelling real processes,
Sofia University
June 2020

- Introduction to accelerometers
- MEMS Accelerometers
- Calibration Errors
- Mathematical Model
- Numerical Results

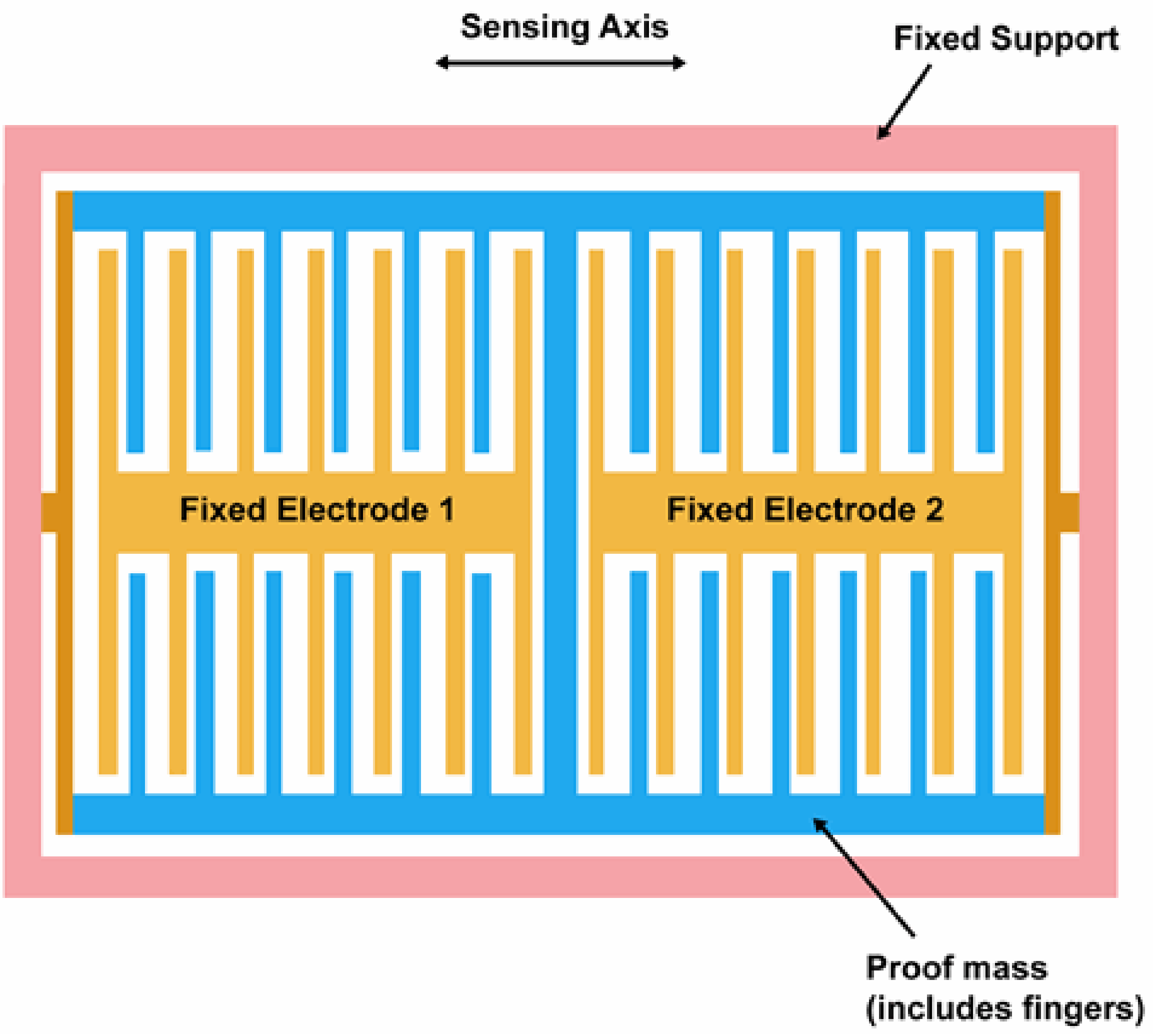
Introduction to accelerometers

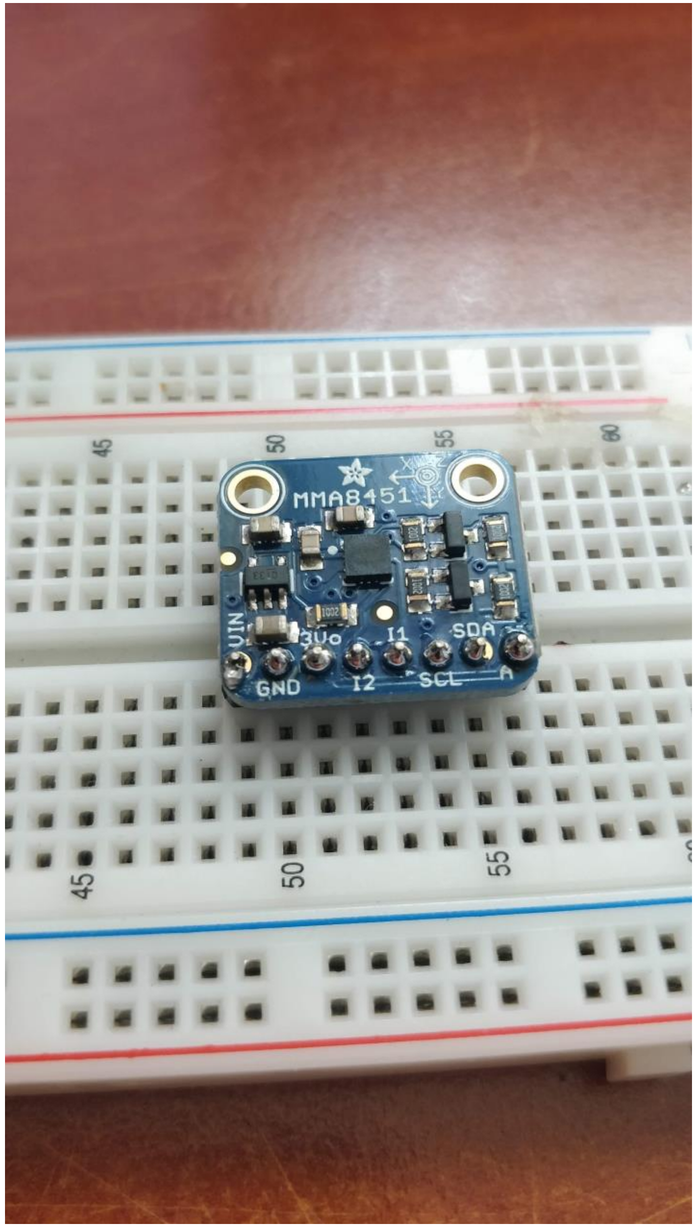
- **MEMS**
- Piezoelectric
- Piezoresistive
- Capacitive





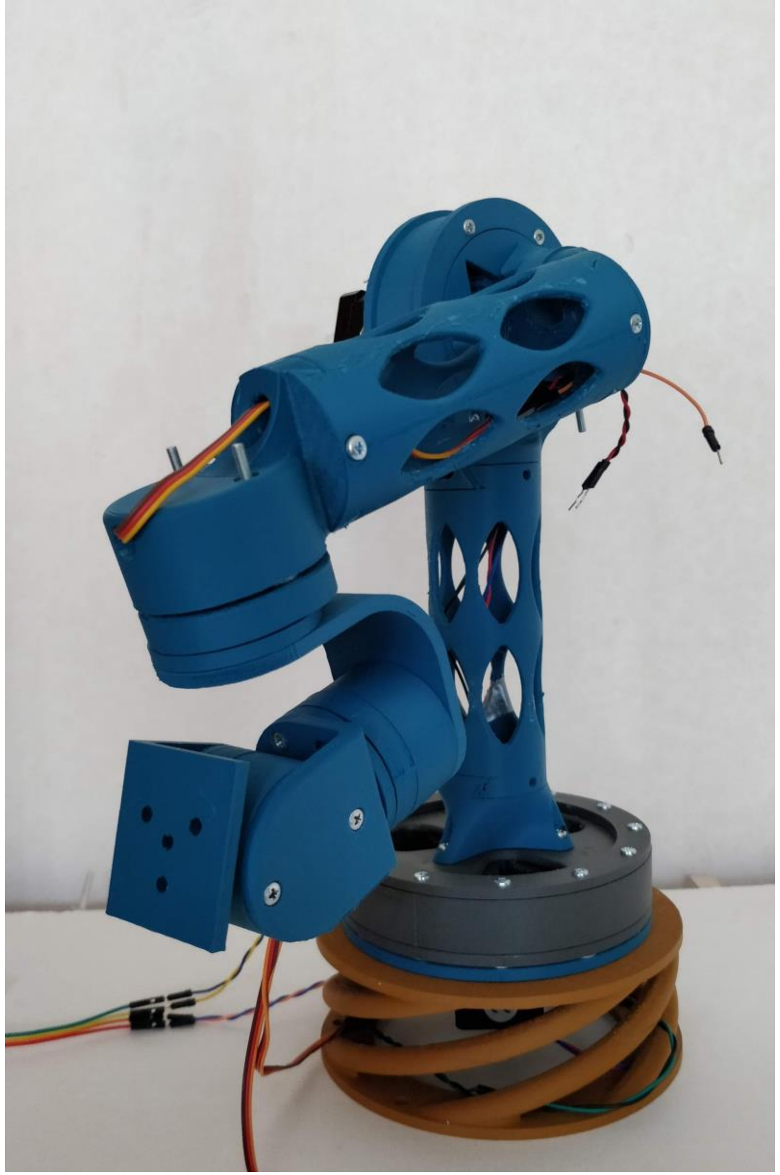
MEMS Accelerometers





Practical Applications

- Sensing applications in industrial settings.
- Robotics
- Drones/Aircraft



Calibration Errors

- Constant Bias
- Scaling Errors
- Errors due to the non-orthogonality of the axes
- Thermo-Mechanical White Noise / Velocity Random Walk
- Flicker Noise / Bias Stability
- Temperature Effects

Input Sensor Data

Uncalibrated		
X	Y	Z
0.686143985	9.693013241	0.146230973
0.307313184	-9.555131822	0.121707371
10.20588166	0.146627372	0.293913142
-9.235730337	0.149835656	-0.153514714

Norms Before Calibration
9.71837
9.56085
10.2112
9.23822
9.72837

Mathematical Model

We use a linear relationship between the raw the data and the calibrated data

$$\underbrace{\begin{pmatrix} x \\ y \\ z \end{pmatrix}}_{v} = \underbrace{\begin{pmatrix} M_{xx} & M_{xy} & M_{xz} \\ M_{yx} & M_{yy} & M_{yz} \\ M_{zx} & M_{zy} & M_{zz} \end{pmatrix}}_{M} \cdot \begin{pmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{pmatrix} + \underbrace{\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix}}_{B}$$

where the parameters

- M_{xx} ,M_{yy} ,M_{zz} are the scale coefficients
- B_x ,B_y, B_z are the constant offsets
- M_{zx} , M_{zy}, M_{yx} , M_{xy}, M_{xz} , M_{yz} take care of the non-orthogonalities of the axes.

We can say that the sensor data is calibrated when $|v| - g \approx 0$.

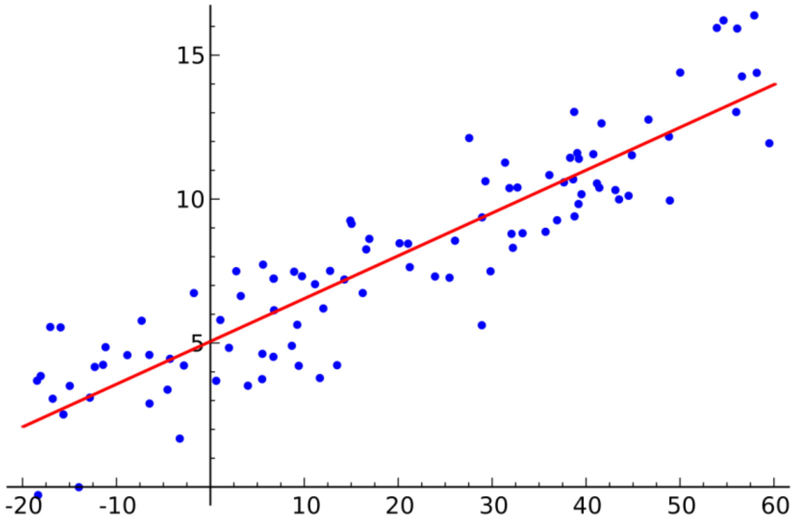
Data calibration methods

In order to calibrate the raw measurements, we need to minimize the error

$$Err(M, B) = \sum_{i=1}^{n} \left(M_{xx}x_i + M_{xy}y_i + M_{xz}\, z_i + B_x\right)^2 + \left(M_{yx}x_i + M_{yy}y_i + M_{yz}\, z_i + B_y\right)^2 + \left(M_{zx}x_i + M_{zy}y_i + M_{xz}\, z_i + B_z\right)^2 - g^2$$

with respect to the parameters

M_{xx}, M_{xy}, M_{xz} , M_{yx} , M_{yy} , M_{yz}, M_{zx} , M_{zy}, M_{zz} and B_x, B_y,B_z.



Minimizing the error

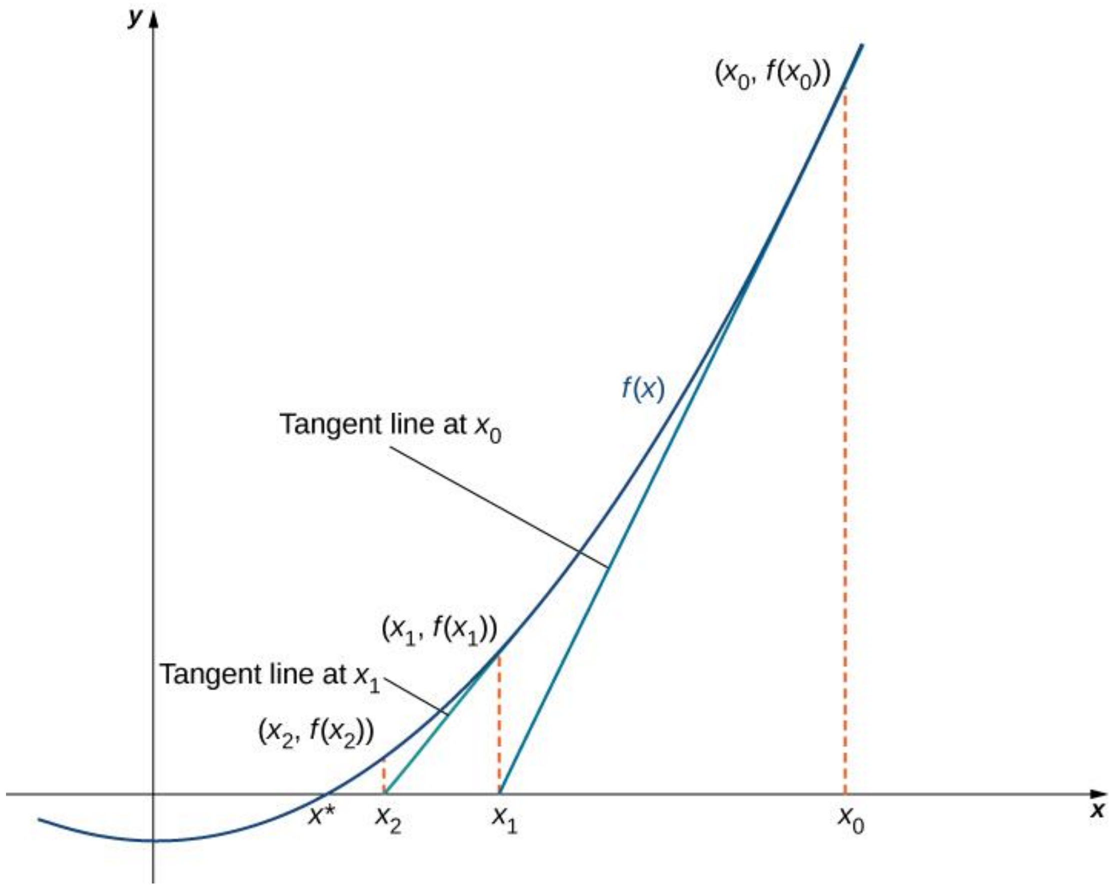
- Using Partial Derivatives for minimizing the error.
- Solving a system of 12 equations
- Method for solving equations.
- Initial Guess:

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad B = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Newton’s Method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



M Matrix & B Vector Comparison

M Matrix Values (BI)		
1.00432	-0.0247	-0.0738
0.01773	1.01322	-0.0892
0.07967	0.08645	0.99275

M Matrix Values (NM)		
0.22671	0.07337	0.97174
0.60662	0.7872	-0.1967
-0.772	0.64003	0.12634

B Matrix Values (BI)
-0.488037
-0.0814727
0.0155627

B Matrix Values (NM)
-0.0558736
-0.35741
0.337929

Calibrated Data Comparison

Calibrated (BI)		
X	Y	Z
-0.0487887	9.73885	1.05333
0.0473013	-9.76837	-0.665145
9.73665	0.22181	1.13312
-9.75604	-0.0796934	-0.859694

Calibrated (NM)		
X	Y	Z
0.952926	7.66039	6.03053
-0.568964	-7.71673	-5.9995
2.55426	5.89126	-7.40974
-2.28789	-5.81181	7.54415

Norms After Calibration (BI)
9.79577
9.7911
9.80487
9.79417
9.7825

Norms After Calibration (NM)
9.79577
9.7911
9.80487
9.79417
9.7825

Thank you for the attention!