Name: $\boxed{\text{Nikolaas Bender}}$
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CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**
Problem Set 4a (11 points) **Fall 2019, CU-Boulder**

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.
- Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.
- For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.
- You may work with other students. However, **all solutions must be written independently and in your own words.** Referencing solutions of any sort is strictly prohibited. You must explicitly cite any sources, as well as any collaborators.

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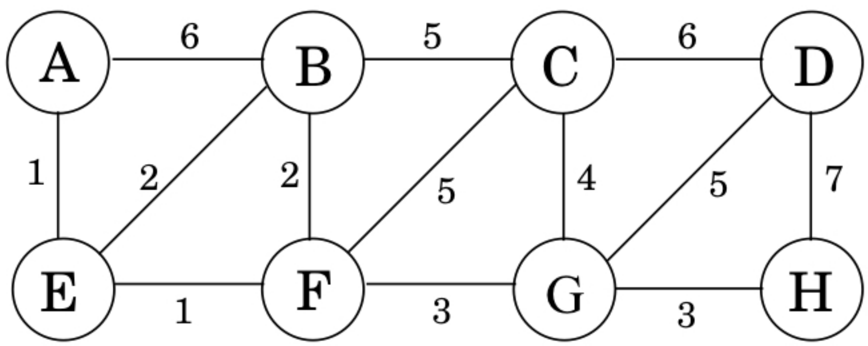
1. (1 pt) What is the definition of a Minimum Spanning Tree (MST)?

   Solution. A minimum spanning tree is a sub graph that connects all of the vertices together without any cycles and the minimum edge weight.

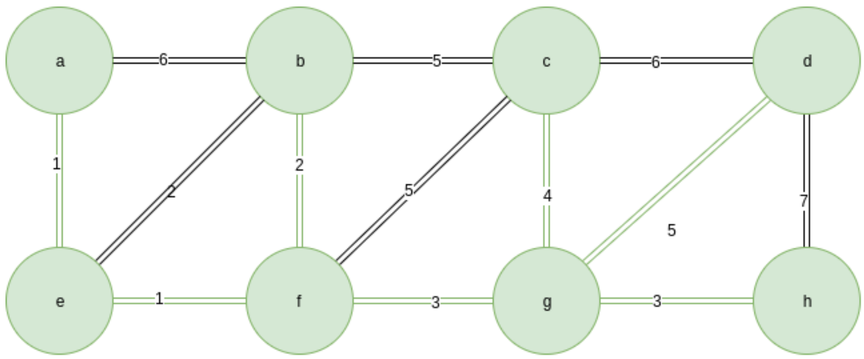
2. (1 pt) Describe in one or two sentences, a greedy rule for constructing an MST.

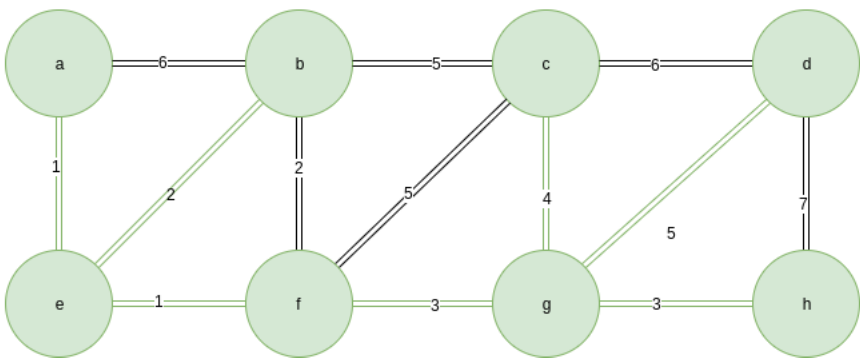
   Solution. At each vertex select the lowest weight edge that has not been visited yet.

3. (3 pts) How many unique MSTs does the following graph have :



   Solution. an MST of 19 is the optimal and there are two solutions that will give the total edge weight of 19.





4. (3 pts) Suppose that you have calculated the MST of an undirected graph $G = (V, E)$ with positive edge weights.
   If you increase each edge weight by 2, will the MST change? Prove that it cannot change or give a counterexample if it changes. (Note: Your proof, if there is one, can be a simple logical argument.)

   Solution. The way this reads, we are adding 2 to EVERY EDGE WEIGHT IN THE TOTAL GRAPH, doing so would not change the mst. The weight of the mst would change but because everything is scaled up by the same amount then the MST configuration won't change as the mst has been scaled up by the same amount as the rest of the graph.

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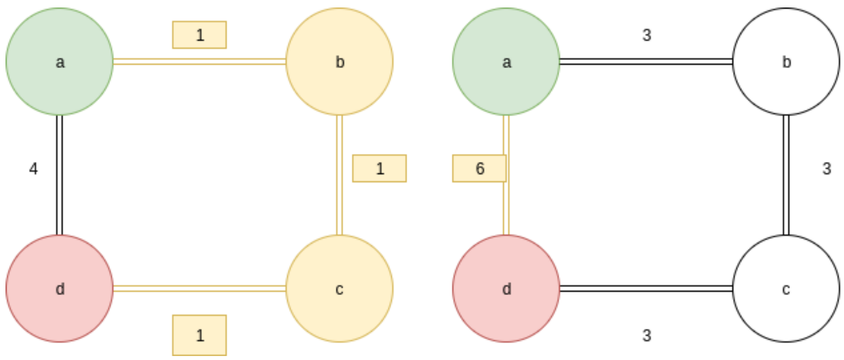
5. (3 pts) Suppose that you have calculated the shortest paths to all vertices from a fixed vertex $s \in V$ of an undirected graph $G = (V, E)$ with positive edge weights.
   If you increase each edge weight by 2, will the shortest paths from s change? Prove that it cannot change or give a counterexample if it changes.(Note: Just as in Part a, your proof can be a simple logical argument.)

   Solution.



   This is a counter example showing that the fastest path between two vertices can change when all of the edge weights are scaled up.