Name: Nikolaas Bender
ID: 106977096

CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**
Problem Set 5a (11 points) **Fall 2019, CU-Boulder**

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.
- Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.
- For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.
- You may work with other students. However, **all solutions must be written independently and in your own words.** Referencing solutions of any sort is strictly prohibited. You must explicitly cite any sources, as well as any collaborators.

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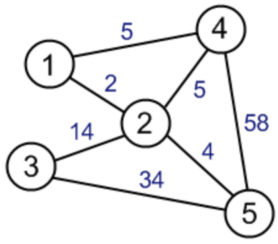
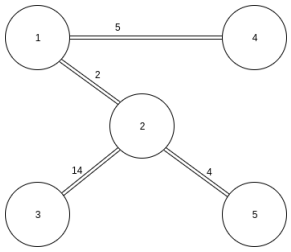
1. (7 pts) Consider the following weighted graph.



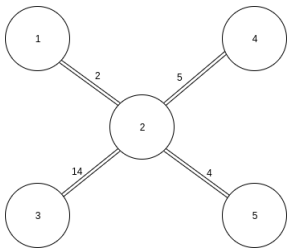
   (a) (2 pts) Run Dijkstra's algorithm on this graph to obtain a tree of shortest paths. Use vertex 1 as the source vertex.

Solution.



(b) (2 pts) Run Kruskal's algorithm on this graph to obtain a minimum spanning tree.

Solution.



(c) (1 pts) Is the tree of shortest paths produced by Dijkstra's algorithm a minimum spanning tree? Justify your answer.

Solution. Dijkstra's algorithm minimizes the path weight to a specific node where an MST triest to minimize the weight to all nodes in a graph.

(d) (2 pts) Find two vertices u and v, where the $u - v$ path in the Kruskal tree is not a shortest $u - v$ path.

Solution. $1 \rightarrow 4$

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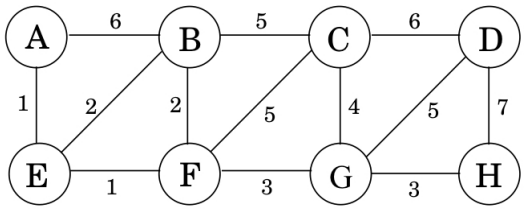
2. (1 pt) Provide a brief description of what the *find(v)* and *union(A,B)* features of the union-find algorithm produce.

   Solution. $find(v)$ gets the group that v is a part of.
   $union(A, B)$ combines group A and B along the shortest possible edge, then it changes what group the smaller of the two sets belongs to.

3. (3 pts) Identify three edges in the following graph G that won't be included in any MST of G. Provide a 3-4 sentence explanation of your answer.



   Solution. These edges will ever be included in an MST because shorter paths between the each of the two points than the edge that directly connects the two vertices more directly. Each of these edges are the heaviest edge of a cycle.
   $A \to B$
   $B \to C$
   $C \to D$
   $H \to D$