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CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**
Problem Set 6a (10 points) **Fall 2019, CU-Boulder**

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.
- Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.
- For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.
- You may work with other students. However, **all solutions must be written independently and in your own words.** Referencing solutions of any sort is strictly prohibited. You must explicitly cite any sources, as well as any collaborators.

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1. (1 pt) What do the edge weights of a graph G in a maximum-flow network represent?

   Solution. The maximum throughput of the edge. (capacity)

2. (2 pts) What are the two conditions that must be met for network flow?

   Solution. Each edge must have a non negative flow less than or equal to the capacity of the edge.
   Each vertex same flow going into and out of it.

3. (2 pts) What do the edge weights in the residual graph G_f represent? Include both forward and backward edges.

   Solution. The weights indicate how much flow each edge will allow. Forward is the remaining capacity. Backwards is the total capacity - forward, the amount that can be pushed backwards through it.

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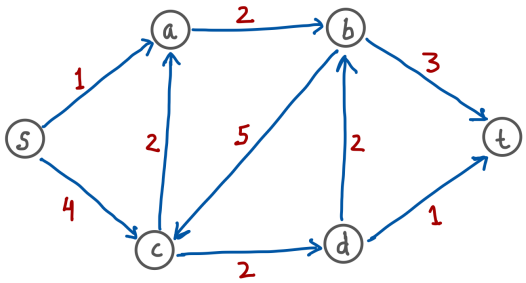
4. (5 pts) Based on the following network and the given edge capacities answer the following.



(a) (1 pts) Can the max flow be 5 (capacity(e_{sa}) + capacity(e_{sc}))? Justify your answer in one sentence.

Solution. No, because the max flow into the sink is 4.

(b) (2 pts) For the graph, identify one simple $s-t$ path and the bottleneck edge value on that path. Also report the maximum allowed flow on this $s-t$ path.

Solution. $s \to a \to b \to t$ the bottleneck is $s \to a$ and the maximum of just this path is 1.

(c) (2 pts) Assuming all $f(e)$ are initially 0 where f represents flow, what are the residual capacities on the forward and backward edges of G_f after one iteration of the Ford-Fulkerson algorithm. Use the simple path you identified in Part b.

Solution. If I understand the question correctly, I am supposed to use the path I found in part b.

EDGE	FORWARD	BACKWARD
$s \to a$	0	1
$a \to b$	1	1
$b \to t$	2	1
$s \to c$	4	0
$c \to a$	2	0
$c \to d$	2	0
$b \to c$	5	0
$d \to b$	2	0
$d \to t$	1	0