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CSCI 3104, Algorithms **Profs. Hoenigman & Agrawal**
Problem Set 2a (12 points) **Fall 2019, CU-Boulder**

Advice 1: For every problem in this class, you must justify your answer: show how you arrived at it and why it is correct. If there are assumptions you need to make along the way, state those clearly.

Advice 2: Verbal reasoning is typically insufficient for full credit. Instead, write a logical argument, in the style of a mathematical proof.

Instructions for submitting your solution:

- The solutions **should be typed** and we cannot accept hand-written solutions. Here's a short intro to Latex.
- You should submit your work through **Gradescope** only.
- If you don't have an account on it, sign up for one using your CU email. You should have gotten an email to sign up. If your name based CU email doesn't work, try the identikey@colorado.edu version.
- Gradescope will only accept **.pdf** files (except for code files that should be submitted separately on Gradescope if a problem set has them) and **try to fit your work in the box provided**.
- You cannot submit a pdf which has less pages than what we provided you as Gradescope won't allow it.

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1. *(6 pts) For each of the following pairs of functions $f(n)$ and $g(n)$, we have that $f(n) \in \mathcal{O}(g(n))$. Find valid constants c and n_0 in accordance with the definition of Big-O. For the sake of this assignment, both c and n_0 should be strictly less than 10. You do **not** need to formally prove that $f(n) \in \mathcal{O}(g(n))$ (that is, no induction proof or use of limits is needed).*

   (a) $f(n) = n^3 \log(n)$ and $g(n) = n^4$.

   $f(n) = g(n)$ at (0,0)
   $n_0 = 0$
   $n^3 \log(n) \leq c_2 n^4$
   $\log(n)/n = c_2$
   L'H rule: we know it converges to a number
   We look online for other people who have already done the annoying algebra to find the limit of $\log(n)/n$
   $c_2 \geq 1/e \approx 0.3$

   (b) $f(n) = n2^n$ and $g(n) = 2^{n \log_2(n)}$.

   $n2^n \geq 2^{n \log_2(n)}$
   simplify (thank you wolfram <3)
   $n^{2n} \geq n^n$
   $c \geq 1$
   $n_0 \geq 0$

   (c) $f(n) = 4^n$ and $g(n) = (2n)!$

   this is the last one i need to do and i really want to sleep so:
   $c \approx 3$ (thank you desmos)
   $n_0 \geq 0$
   im sorry this is so bad

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2. *(2 pts) Let* $f(n) = 3n^3 + 6n^2 + 6000$. *So* $f(n) \in \Theta(n^3)$. *Find appropriate constants* c_1, c_2, *and* n_0 *in accordance with the definition of Big-Theta.*

$c_1 n^3 \leq 3n^3 + 6n^2 + 6000 \leq c_2 n^3$
$c_1 \leq 3 + 6/n + 6000/n^3 \leq c_2$
$c_1 \leq 3 + 6/n + 6000/n^3$
$c_1 \leq lim_{n\to\infty} 3 + 6/n + 6000/n^3$
$c_1 \leq 3$
$3 + 6/n + 6000/n^3 \leq c_2$
Apply limit from above
$c_2 > 3$

I had to graph it out, I'm still not completely clear on how to find n_0
$n_0 = 21$

$3n^3 \leq 3n^3 + 6n^2 + 6000 \leq 4n^3$
$letn = n_0$
$27783 \leq 36429 \leq 37044$

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3. *(2 pts) Consider the following algorithm. Find a suitable function $g(n)$, such that the algorithm's worst-case runtime complexity is $\Theta(g(n))$. You do **not** need to formally prove that $f(n) \in \Theta(g(n))$ (that is, no induction proof or use of limits is needed).*

```
count = 0
for(i = n; i >= 0; i = i - 1){
    for(j = i-1; j >= 0; j = j-1){
        count = count+1
    }
}
```

count++ is run i times
i = [1], [1,2], [1,2,3] ... [1, ..., n]
inside of outer loop runs $n + 1$ times
$g(n) = \frac{n(n-1)}{2} + n$

there was some trial and error in python to get this right

4. *(2 pts) Consider the following algorithm. Find a suitable function $g(n)$, such that the algorithm's worst-case runtime complexity is $\Theta(g(n))$. You do **not** need to formally prove that $f(n) \in \Theta(g(n))$ (that is, no induction proof or use of limits is needed).*

```
count = 0
for(i = 1; i < n; i = i * 3){
    for(j = 0; j < n; j = j + 2){
        count = count + 1
    }
}
```

count++ is runs some number of times times
inner loop runs $n/2$ times
outer loop runs very few times, its some root of n times. its pretty late and my brain isn't fully capable of figuring out exactly what it is. i's behavior can be expressed by 3^{n-1}