1

3) Er ikke et vektorrom. Vanlig multiplikasjon er ikke likt skalar multiplikasjon.

4) Er et vektorrom.

5) —— " ——

6) —— " ——.

2

a) True b) True c) False

f) True g) False h) True

3

a) $\begin{bmatrix} a_1 & 0 \\ b_1 & 0 \end{bmatrix} + \begin{bmatrix} a_2 & .. \\ .. & ... \end{bmatrix} = \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix}$ ✓

$\Rightarrow k \cdot 0 = 0$

$\boxed{\text{alle matriser på denne formen er et subspace.}}$

b) $k \cdot A = \begin{bmatrix} ak & 1 \cdot k \\ bk & 1 \cdot k \end{bmatrix} \neq \begin{bmatrix} a & 1 \\ b & 1 \end{bmatrix}$ når $k \neq 1$

$\boxed{\text{Er ikke et subspace.}}$

Svar b.

c) Gitt en A slik at,

$$A \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$2A \begin{bmatrix} 1 \\ -1 \end{bmatrix} \neq \begin{bmatrix} 2 \\ 0 \end{bmatrix}.$$

$\boxed{4}$ a) $M_a = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

Finn,

$$M_a = c_1 A + c_2 B + c_3 C + c_4 D$$

$$= c_1 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c_4 \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$$

$$1 = c_1 + 2c_4 \qquad 1 = c_1 + 4 \Rightarrow c_1 = -3$$

$$2 = -c_1 + c_2 + c_3$$

$$\longrightarrow \quad 2 = 3 + 12 + c_3 \Rightarrow c_3 = -13$$

$$2 = c_4$$

$$4 = 2c_1 + c_2 - c_4 \qquad 4 = -6 + c_2 - 2 \Rightarrow c_2 = 12$$

Slik at,

$$\boxed{M_a = -3A + 12B - 13C + 2D}$$

Svar a.

b)

$$M_b = \begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

$$M_b = c_1 A + c_2 B + c_3 C + c_4 D$$

$$= c_1 \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} + c_3 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c_4 \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix}$$

$$3 = c_1 + 2c_4 \qquad c_1 = 3-2\cdot 1 = 1$$
$$1 = -c_1 + c_2 + c_3$$
$$\rightarrow \qquad c_3 = 1+1-1 = 1$$
$$1 = c_4$$
$$2 = 2c_1 + c_2 - c_4 \qquad c_2 = 2+1-2\cdot 1 = 1$$

$$\boxed{M_b = A + B + C + D}$$

Svar b.

[5]

a) Sjekk om matrisene er linært uavhengige.

Husk: $c_1 A + c_2 B + c_3 C + c_4 D = 0$

Hvor ikke alle $c_n = 0$, gir linær avhengighet. For oss betyr det at vi må se om det <u>ikke</u> finnes en slik c.

$$c_1\begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix} + c_2\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix} + c_3\begin{bmatrix}0 & 1\\ 0 & 1\end{bmatrix} + c_4\begin{bmatrix}0 & 0\\ 1 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$\begin{aligned} c_1 + c_2 &= 0 \\ c_2 + c_3 &= 0 \\ c_1 + c_3 &= 0 \\ c_3 + c_4 &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} c_1 &= -c_2 \\ c_3 &= -c_2 \\ c_1 &= c_3 \\ c_3 &= -c_4 \end{aligned}$$

Matrisene er lineært avhengige.

Det vil si,

$$\boxed{\text{Matrisene spenner ikke over } M_{22}}$$

Svar a.

b) $\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Test om de er lineært uavhengige.

$$c_1 \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c_3 \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} + c_4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} c_1 + c_3 + c_4 &= 0 \\ -c_1 + c_2 + c_3 &= 0 \\ c_3 &= 0 \\ c_1 + c_4 &= 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} c_3 &= 0 \\ c_1 &= -c_4 \\ c_1 &= c_2 \end{aligned}$$

De er lineært avhengige, så,

$\boxed{\text{Matrisene spenner ikke over } M_{22}.}$

Svar.

c)

$$c_1\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix} + c_2\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix} + c_3\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix} + c_4\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$c_1 + c_2 + c_3 + c_4 = 0$$

$$\boxed{\text{Matrisene spenner over } M_{22}.}$$

$\boxed{6}$ La $Ax = b$,

$$\begin{bmatrix}1 & 1 & 2\\ 1 & 0 & 1\\ 2 & 1 & 3\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} = \begin{bmatrix}-1\\ 0\\ 2\end{bmatrix}$$

(1) Gauss eliminasjon.

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & x_1 \\ 1 & 0 & 1 & x_2 \\ 2 & 1 & 3 & x_3 \end{array}\right] \quad (-1) \quad (-2)$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & x_1 \\ 0 & 0 & 1 & x_2 \\ 0 & -1 & -1 & x_3 - 2x_1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 2 & x_1 \\ 0 & -1 & -1 & x_3 - 2x_1 \\ 0 & 0 & 1 & x_2 \end{array}\right]$$

$$\Rightarrow \quad x_2 = 1, \quad x_1 = 4, \quad x_3 = -2 + 2 \cdot 4 = 6$$

(2) Sum kolumene,

$$4\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + 6\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \neq \begin{bmatrix} 17 \\ 10 \\ \dots \end{bmatrix}$$

$$\boxed{b \text{ er ikke i kollonne-rommet til } A}$$

Svara.

b) $\begin{bmatrix} 1 & -1 & 1 \\ 9 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \overset{?}{=} \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$

(1) Gauss eliminasjon,

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 5 \\ 9 & 3 & 1 & 1 \\ 1 & 1 & 1 & -1 \end{array}\right] \begin{matrix} (-9) \\ \\ (-1) \end{matrix}$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 5 \\ 0 & 12 & -8 & -44 \\ 0 & 2 & 0 & -6 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 5 \\ 0 & 2 & 0 & -6 \\ 0 & 12 & -8 & -44 \end{array}\right] \leftarrow \tfrac{1}{2}$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 12 & -8 & -44 \end{array}\right] \quad (-12)$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & -8 & -8 \end{array}\right] \leftarrow -\tfrac{1}{8}$$

$$= \left[\begin{array}{ccc|c} 1 & -1 & 1 & 5 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\Rightarrow \quad x_1 - x_2 + x_3 = 5 \quad \rightarrow \quad x_1 = 1$$

$$x_2 = -3$$

$$x_3 = 1$$

Slik at,

$$\begin{bmatrix} 1 \\ 9 \\ 1 \end{bmatrix} - 3 \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix} \quad \checkmark$$

$\boxed{\text{b er i kollonnerommet til A.}}$

Svar b.

c) $$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 2 \\ -1 & 1 & -1 & 0 \\ -1 & -1 & 1 & 0 \end{array}\right]$$

$$x_3 - x_2 - x_1 = 0 \rightarrow x_3 = x_2 + x_1$$

$$x_2 - x_3 - x_1 = 0 \rightarrow x_2 = x_1 + x_3$$

$$x_1 - x_2 + x_3 = 2$$

$$x_1 - \cancel{x_2} + \cancel{x_2} + x_1 = 2 \Rightarrow \underline{x_1 = 1}$$

$$x_2 = 1 + x_2 + 1 \Rightarrow \underline{x_2 = 1}$$

$$\Rightarrow \underline{x_3 = 2}$$

$$\Rightarrow \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} + 2\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \neq b$$

$\boxed{\text{b er ikke i kollonnerommet til A.}}$

Svar c.

d) $$\left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 4 \\ 0 & 1 & 2 & 1 & 3 \\ 1 & 2 & 1 & 3 & 5 \\ 0 & 1 & 2 & 2 & 7 \end{array}\right] \quad (-1)$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 4 \\ 0 & 1 & 2 & 1 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & 2 & 7 \end{array}\right] \quad (-1)$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 4 \\ 0 & 1 & 2 & 1 & 3 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right] \quad \begin{array}{c} (-1) \\ (-1) \\ (-2) \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 1 & 0 & -7 \\ 0 & 0 & 0 & 1 & 4 \end{array}\right]$$

$\Rightarrow \quad x_1 = 2x_2 \qquad x_1 = \underline{12}$

$x_3 = -7 \qquad \& \quad x_2 = -1 + 7 = \underline{6}$

$\underline{x_4 = 4}$

$$\Rightarrow 12\begin{bmatrix}1\\0\\1\\0\end{bmatrix} + 6\begin{bmatrix}2\\1\\2\\1\end{bmatrix} - 7\begin{bmatrix}0\\2\\1\\2\end{bmatrix} + 4\begin{bmatrix}1\\1\\3\\2\end{bmatrix} = \begin{bmatrix}28\\ \vdots\\ \vdots\\ \vdots\end{bmatrix} \quad \times$$

$$\boxed{b \text{ er ikke i kollonnerommet til } A.}$$

Svar d.

$\boxed{7}$

a) $$\left[\begin{array}{ccc|c} 3 & 2 & -1 & 0 \\ 6 & 4 & -2 & 0 \\ -3 & -2 & 1 & 0 \end{array}\right] \begin{array}{l} \\ (-2) \\ (1) \end{array}$$

$$\left[\begin{array}{ccc|c} 3 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\underline{3x + 2y - z = 0}$$

$$t_1 = x = \frac{1}{3}(z - 2y), \quad t_2 = y = \frac{1}{2}(z - 3x), \quad t_3 = z = 2y + 3x$$

$$\boxed{t = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{3}(z - 2y) \\ \frac{1}{2}(z - 3x) \\ 2y + 3x \end{bmatrix}}$$

Svar a.

b) $\Rightarrow \begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix} + C + \begin{bmatrix} -1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}$ ✓

Svar b.

c)/d) Jeg vet ikke hva jeg skal gjøre her.