1. Filters (3 pts)

Image: I

$$\begin{matrix} 1 & 5 & 9 & 13 \\ 2 & 6 & 10 & 14 \\ 3 & 7 & 11 & 15 \\ 4 & 8 & 12 & 16 \end{matrix} \qquad 10 \rightarrow I(1,2)$$

Design a 3×3 filter (Right) to carry out the given calculation (Left) at I(1, 2)

Example

$$6 + 10 + 7 \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

a.

$$\frac{5+13+7+15}{4} \Rightarrow \begin{bmatrix} 1/4 & 0 & 1/4 \\ 0 & 0 & 0 \\ 1/4 & 0 & 1/4 \end{bmatrix}$$

b.

$$-5-9-13+\frac{7+11+15}{3} \Rightarrow \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$$

c.

$$5+6+7+10+13+14+15 \Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

2. Cross-Correlation ⊗ (2 pts)

$$G[i,j] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} H[u,v] F[i+u, j+v]$$

$F[i+u, j+v]$ $\qquad$ $H[u,v]$ $\qquad$ 0-based Index

$$F: \begin{array}{c|cccc} & & & & \xrightarrow{j} \\ \hline & 0 & 3 & 6 & 9 \\ & 1 & 4 & 7 & 10 \\ \downarrow i & 2 & 5 & 8 & 11 \end{array} \qquad H: \begin{array}{ccc} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & -1 \end{array}$$

Caculate G and show your work

Example:

$$G[1,1] = 0\cdot0 + 1\cdot3 + 0\cdot6 + 1\cdot1 + 0\cdot4 + -1\cdot7 + 0\cdot2 + 0\cdot5 + -1\cdot8 = \underline{-11}$$

a. $G[1,2]$ (i.e. 4) $= 0\cdot3 + 1\cdot6 + 0\cdot9 + 1\cdot4 + 0\cdot7 + (-1)\cdot10 + 0\cdot5 + 0\cdot8 + (-1)\cdot11 = \underline{-11}$

b. $G[2,1]$ (ie. 2) (zero padding) $= 0\cdot1 + 1\cdot4 + 0\cdot7 + 1\cdot2 + 0\cdot5 + (-1)\cdot8 + 0\cdot0 + 0\cdot0 + (-1)\cdot0 = \underline{-2}$

3. Convolution * (2 pts)

$$G[i,j] = \sum_{u=-K}^{K} \sum_{v=-K}^{K} H[u,v]\,F[i-u, j-v]$$

$$G[1,1] = \begin{aligned} &-1\cdot 0 + 0\cdot 3 + 0\cdot 6 \\ &+ -1\cdot 1 + 0\cdot 4 + 1\cdot 7 \\ &+ 0\cdot 2 + 1\cdot 5 + 0\cdot 8 \end{aligned} = 11$$

Hint:

$$\left(H^T = \begin{bmatrix} -1 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right)$$

0	3	6	9
1	4	7	10
2	5	8	11

a. $G[1,2] = \begin{aligned} &(-1)\cdot 3 + 0\cdot 6 + 0\cdot 9 \\ &+ (-1)\cdot 4 + 0\cdot 7 + 1\cdot 10 \\ &+ 0\cdot 5 + 1\cdot 8 + 0\cdot 11 \end{aligned} = \underline{11}$

b. $G[2,1] = \begin{aligned} &(-1)\cdot 1 + 0\cdot 4 + 0\cdot 7 \\ &+ (-1)\cdot 2 + 0\cdot 5 + 1\cdot 8 \\ &+ 0\cdot 0 + 1\cdot 0 + 0\cdot 0 \end{aligned} = \underline{5}$

(zero-padding)

4. Mean Filter (2 pts)

IMAGE / FILTER

$$F: \begin{matrix} \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi & \phi \\ \phi & \phi & \phi & 70 & 140 & 210 & 70 & \phi & \phi \\ \phi & \phi & 70 & 140 & \phi & \phi & 140 & 70 & \phi \\ \phi & \phi & \phi & \phi & 70 & 70 & \phi & \phi & \phi \end{matrix} \qquad H: \begin{bmatrix} \phi & 1/7 & 1/7 \\ \phi & 1/7 & 1/7 \\ 1/7 & 1/7 & -1/7 \end{bmatrix}$$

a. Fill in the values in the blank cells.

$G = H \otimes F =$

i \ j	0	1	2	3	4	5	6	7	8
0									
1		-10	0	60	70	30	20	30	
2		10	40	40	50	80	50	10	

b. Show your work for two cells.

$G[1,1] = 70 \cdot (-1/7) = -10$

$$G[1,3] = 0\cdot 0 + \tfrac{1}{7}\cdot 0 + \tfrac{1}{7}\cdot 0 + 0\cdot 70 + \tfrac{1}{7}\cdot 70 + \tfrac{1}{7}\cdot 140 + \tfrac{1}{7}\cdot 70 + \tfrac{1}{7}\cdot 140 + (-\tfrac{1}{7})\cdot 0 = \underline{60}$$

$$G[2,5] = 0\cdot 140 + \tfrac{1}{7}\cdot 210 + \tfrac{1}{7}\cdot 70 + 0\cdot 0 + \tfrac{1}{7}\cdot 0 + \tfrac{1}{7}\cdot 140 + \tfrac{1}{7}\cdot 70 + \tfrac{1}{7}\cdot 70 + (-\tfrac{1}{7})\cdot 0 = \underline{80}$$

5. $\otimes$ vs $*$ (3 pts)

$H \otimes F = H * F$ ——— ①

$\mathrm{trace}(H) = 12$ ——— ②

$\mathrm{sum}(H) = 0$ ——— ③

Give an example of H that can, where H's size is 3×3

a. satisfy ①, ②, ③

$$\begin{bmatrix} 3 & -5 & 4 \\ -5 & 6 & -5 \\ 4 & -5 & 3 \end{bmatrix}$$

b. satisfy ① but not ②, ③

$$\begin{bmatrix} 4 & -6 & 5 \\ -6 & 7 & -6 \\ 5 & -6 & 4 \end{bmatrix}$$

c. satisfy ②, ③, but not ①

$$\begin{bmatrix} 2 & 2 & -3 \\ -1 & 9 & 4 \\ -4 & -10 & 1 \end{bmatrix}$$

6. Write a function using only Numpy that can generate an image of a grid. (4 pts)

Input :

k	size of each cell
t	thickness of grid lines
(m, n)	dimensions of the grid
v	intensity value of grid lines
w	Intensity value inside each cell

Example

$k = 2$, $t = 1$, $(m, n) = (1, 5)$, $v = 9$, $w = 1$

$\Rightarrow$

```
9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9
9 1 1 9 1 1 9 1 1 9 1 1 9 1 1 9
9 1 1 9 1 1 9 1 1 9 1 1 9 1 1 9
9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9
```

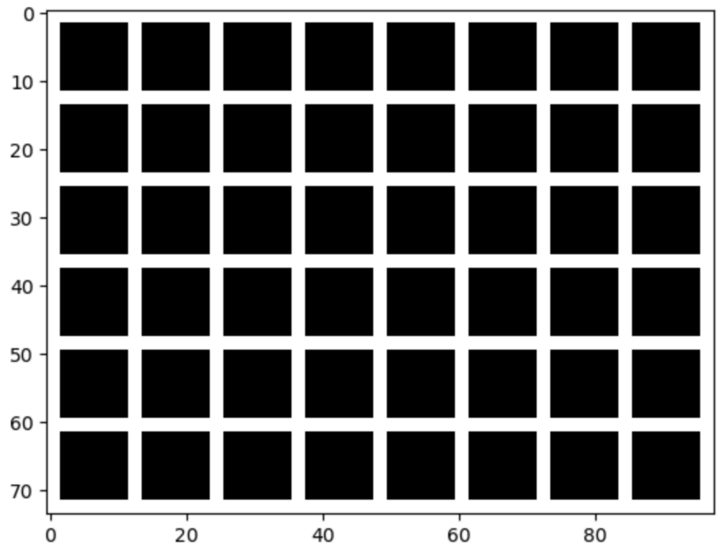
a. $k = 1$, $t = 2$, $(m, n) = (2, 2)$, $V = 3$, $w = 0$

(Draw the output by hand)

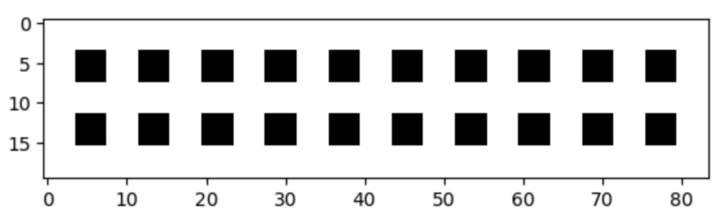
$\Rightarrow$

```
3 3 3 3 3 3 3 3
3 3 3 3 3 3 3 3
3 3 0 3 3 0 3 3
3 3 3 3 3 3 3 3
3 3 3 3 3 3 3 3
3 3 0 3 3 0 3 3
3 3 3 3 3 3 3 3
3 3 3 3 3 3 3 3
```

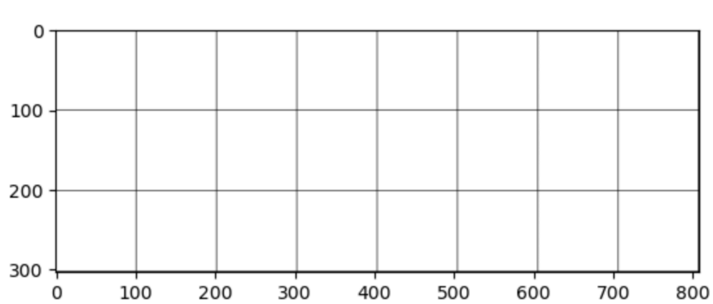
b. $k=10,\ t=2,\ (m,n)=(6,8),\ V=255,\ W=128$



c. $k=4,\ t=4,\ (m,n)=(2,10),\ V=255,\ W=0$



d. $k=100,\ t=1,\ (m,n)=(3,8),\ V=0,\ W=255$



7. Use scipy.ndimage to compute (2 pts)

(Hint: convolve(), correlate(), "constant" mode)

$$F = \begin{bmatrix} 5 & 2 & 0 & 1 & 8 \\ 9 & 1 & 4 & 3 & 2 \\ 3 & 4 & 0 & 5 & 1 \\ 3 & 1 & 2 & 2 & 1 \end{bmatrix}, \quad H = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

a. $F \otimes H = \begin{bmatrix} 31 & 14 & 7 & 15 & 35 \\ 45 & 23 & 20 & 24 & 20 \\ 28 & 21 & 15 & 26 & 12 \\ 16 & 13 & 11 & 16 & 7 \end{bmatrix}$ (0.5 pt)

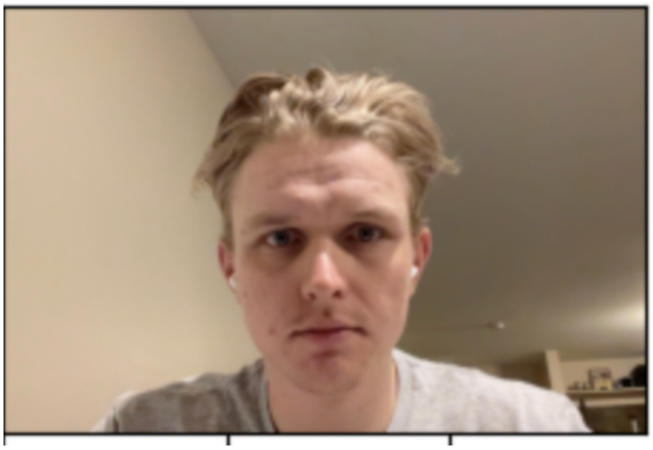
b. $F * H = \begin{bmatrix} 31 & 14 & 7 & 15 & 35 \\ 45 & 23 & 20 & 24 & 20 \\ 28 & 21 & 15 & 26 & 12 \\ 16 & 13 & 11 & 16 & 7 \end{bmatrix}$ (0.5 pt)

c. $F * H * H * H = \begin{bmatrix} 1200 & 872 & 488 & 872 & 1136 \\ 1656 & 1368 & 960 & 1224 & 1040 \\ 1280 & 1160 & 1008 & 1112 & 680 \\ 656 & 688 & 608 & 696 & 400 \end{bmatrix}$

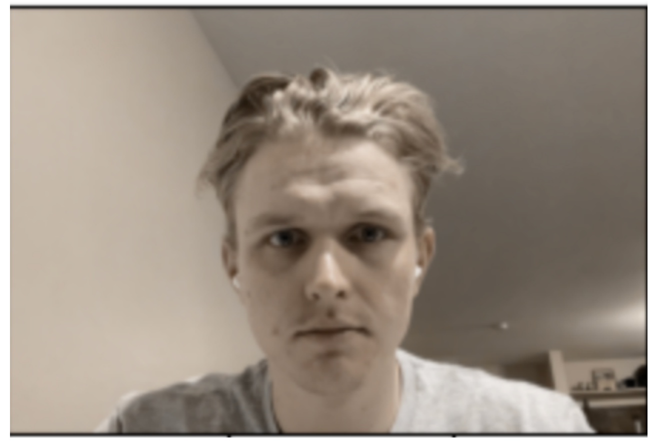
(1 pt)

8. Apply scipy.ndimage.guassian_filter (2pts) to a photo of your face with different σ's

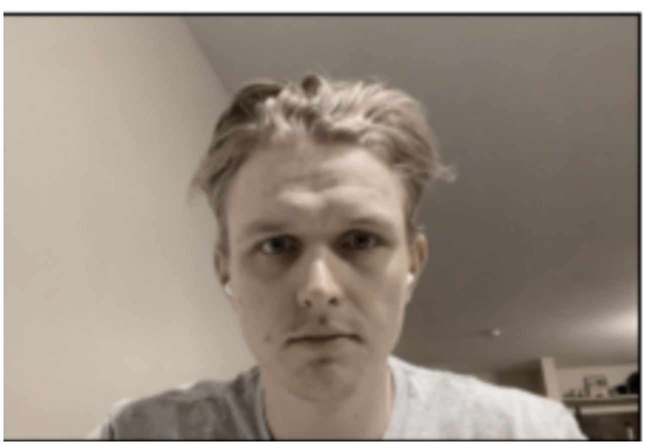
a. original



b. $\sigma = 1$

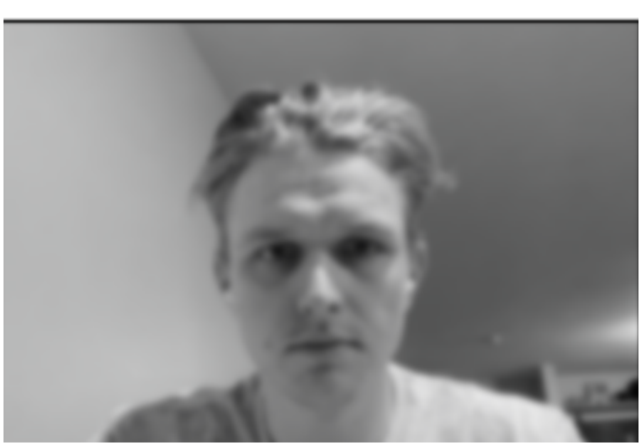


c. $\sigma = 3$



d. $\sigma = 5$

e. $\sigma = 7$



f. $\sigma = 9$