CMIS Hand-in

Nikolai Plambech Nielsen
lpk331@alumni.ku.dk
Niels Bohr Institute, University of Copenhagen

1 INTRODUCTION

When solving differential equations on a computer, one has several options. In this hand-in the finite difference method (FDM) is explored. In general one represents

2 L1A

The idea behind the finite difference method is to replace all derivatives in the equations with differences instead. This can be seen as not letting the spacing h in the differential quotient go all the way to zero, but just to some small value. Since we are working with regular grids, a convenient value is the spacing between points Δx:

$$\frac{df(x_i)}{dx} = \lim_{h \to 0} \frac{f(x_i + h) - f(x_i)}{(x_i + h) - x_i} \to \frac{f(x_i + \Delta x) - f(x_i)}{\Delta x} \quad (1)$$

In this particular case, the finite difference is called the "forward difference" (FD) since it uses the point of interest x_i and the one next to it $x_{i+1} = x_i + \Delta x$. In the usual notation we have

$$\frac{df(x_i)}{dx} \approx \frac{f_{i+1} - f_i}{\Delta x} \quad (2)$$

There is also the "backwards difference" (BD):

$$\frac{df(x_i)}{dx} \approx \frac{f_i - f_{i-1}}{\Delta x} \quad (3)$$

And the "central difference" (CD):

$$\frac{df(x_i)}{dx} \approx \frac{f_{i+1} - f_{i-1}}{2\Delta x} \quad (4)$$

These can be shown explicitly from the Taylor Polynomials for $f(x)$ around x_i. There are of course also higher order differences. In particular we use the second order central difference when studying the heat equation, as this includes a second derivative:

$$\frac{d^2 f_i}{dx^2} \approx \frac{f_{i-1} - 2f_i + f_{i+1}}{\Delta x^2} \quad (5)$$

These