Numerical Methods in Physics Week 1

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1 Nuclear Decay

We have two radioactive isotopes, A and B, with populations $N_A(t)$ and $N_B(t)$, and decay times τ_A and τ_B. Type A decays into type B, while B decays into something else we don't track. The relevant differential equations are

$$\frac{\mathrm{d}N_A}{\mathrm{d}t} = -\frac{N_A}{\tau_A}, \quad \frac{\mathrm{d}N_B}{\mathrm{d}t} = \frac{N_A}{\tau_A} - \frac{N_B}{\tau_B}. \tag{1.1}$$

The purpose of this assignment is to solve this problem numerically for a number of different conditions, using Euler integration. This method is used for its simplicity and ease of implementation.

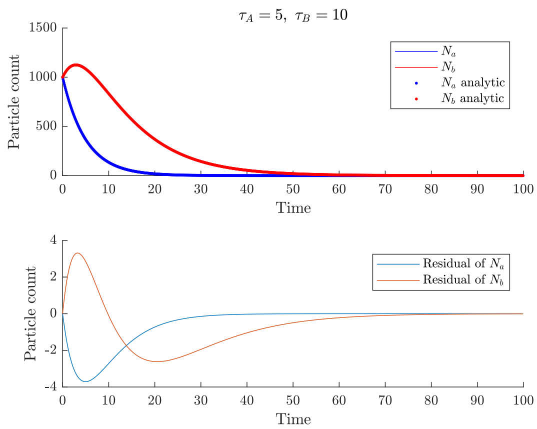
1.1 Compare the numerical and analytical solutions

The analytical solutions are given in the assignment, and are as follows:

$$N_A(t) = N_A(0)\exp\left(-\frac{t}{\tau_A}\right), \tag{1.2}$$

$$N_B(t) = \begin{cases} N_B(0)\exp\left(-\frac{t}{\tau_A}\right) + t\frac{N_A(0)}{\tau_A}\exp\left(-\frac{t}{\tau_A}\right), & \tau_A = \tau_B, \\ N_B(0)\exp\left(-\frac{t}{\tau_B}\right) + \frac{N_A(0)}{\frac{\tau_A}{\tau_B}-1}\left[\exp\left(-\frac{t}{\tau_A}\right) - \exp\left(-\frac{t}{\tau_B}\right)\right], & \tau_A \neq \tau_B. \end{cases} \tag{1.3}$$

The results, along with residual plots, for $N_A(0) = N_B(0) = 1000$ and $\tau_A = 5, \tau_B = 10$ are shown below:



$\tau_A = 5,\ \tau_B = 10$
Particle count
Time
N_a
N_b
N_a analytic
N_b analytic
Residual of N_a
Residual of N_b