A PDE Optimisation of Retinex Theory

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Outline

- Land's Retinex theory.
- Morel's formalisation and its interpretation.
- Extension of the Morel's equation.
- Experemental results.
- Conclusions.

Retinex theory. Properties of HVS.

- **Adapts itself to the properties of environment illumination.**
- **Wide dynamic range** (details in shadows and on the light simultaneously).
- **Color constancacy** (we can see colors even under non white external illumination).

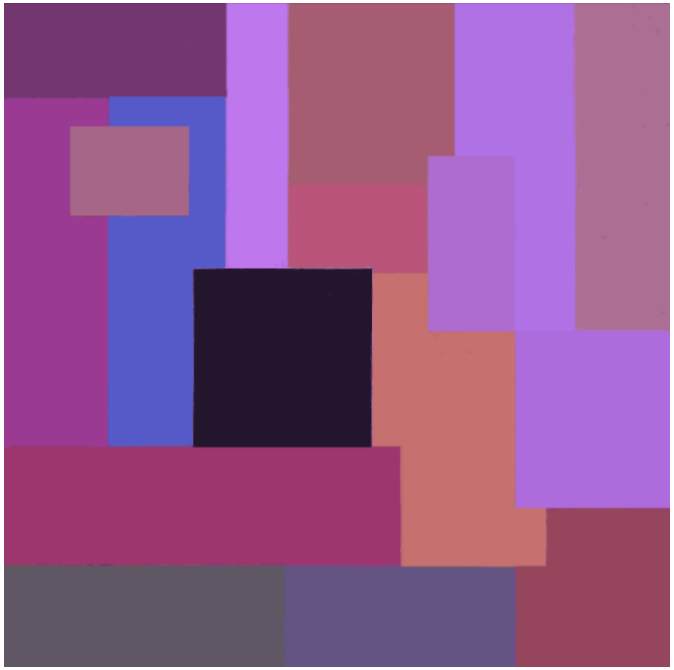
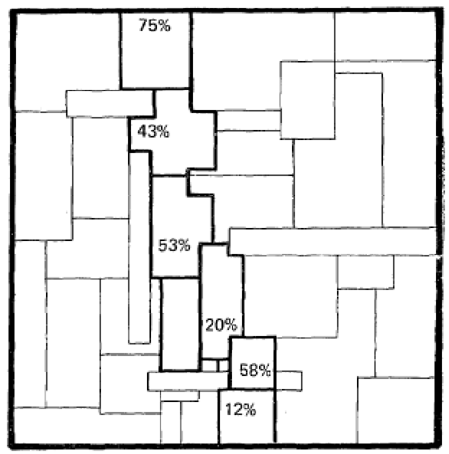
 

Image source: E. H. Land, “The retinex theory of color vision,” Scientific American, vol. 237, no. 6, pp. 108– 128, Dec 1977.

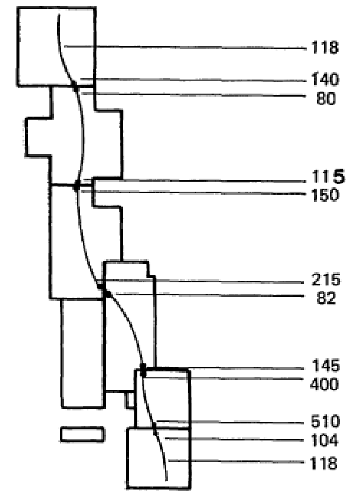
Retinex theory. Motivation.

- **Object recognition** (remove environmental effects)
- **Digital photography** (color balancing, dynamic range extension)
- **Texture processing** (clear a texture from shadows, reflexes and other effects of external illumination)

Retinex theory. The Idea of E. Land.



$$\frac{75}{43} \times \frac{43}{53} \times \frac{53}{20} \times \frac{20}{58} \times \frac{58}{12} = \frac{75}{12} = \frac{6.25}{1}$$



$$\frac{140}{80} \times \frac{115}{150} \times \frac{215}{82} \times \frac{145}{400} \times \frac{510}{104} = \frac{6.25}{1}$$

$$I(x_1) = R(x_1) \cdot B(x_1)$$

Image source: E. Land and J. McCann, "Lightness and retinex theory," J. Opt. Soc. Amer., vol. 61, no. 1, pp. 1–11, Jan 1971.

Morel's formalization.

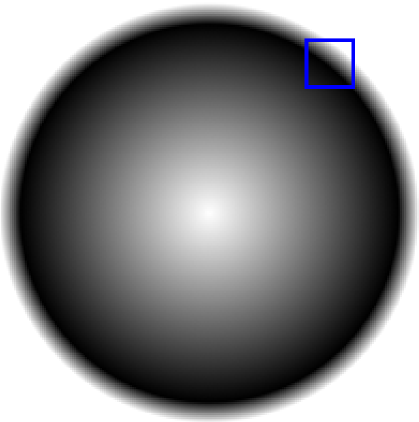
$$L(I_i) \;=\; \frac{1}{N}\sum_{k=1}^{N} \log(\frac{I_i}{I_k})$$

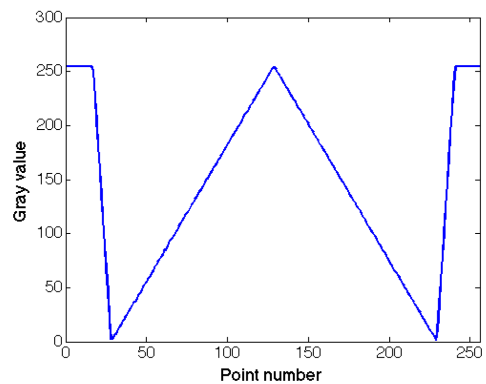
$$-\Delta L(x) \;=\; \sum_{A=\mathrm{up,\,down,\,left,\,right}} \delta\left(\log\left(\frac{I(x)}{I(x_A)}\right)\right) = F(x)$$

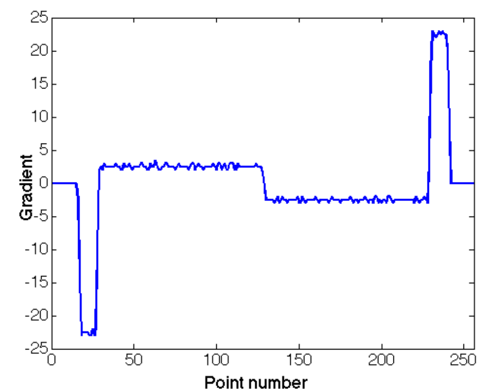
$$\frac{\partial L(x)}{\partial n} \;=\; 0, \quad \forall\, x \in \partial D$$

Morel's formalization. Interpretation.

$$L(x,y) = \underset{L}{argmin}(\|\nabla L(x,y) - \delta(\nabla \tilde{I}(x,y))\|_2^2)$$







Morel's formalization.

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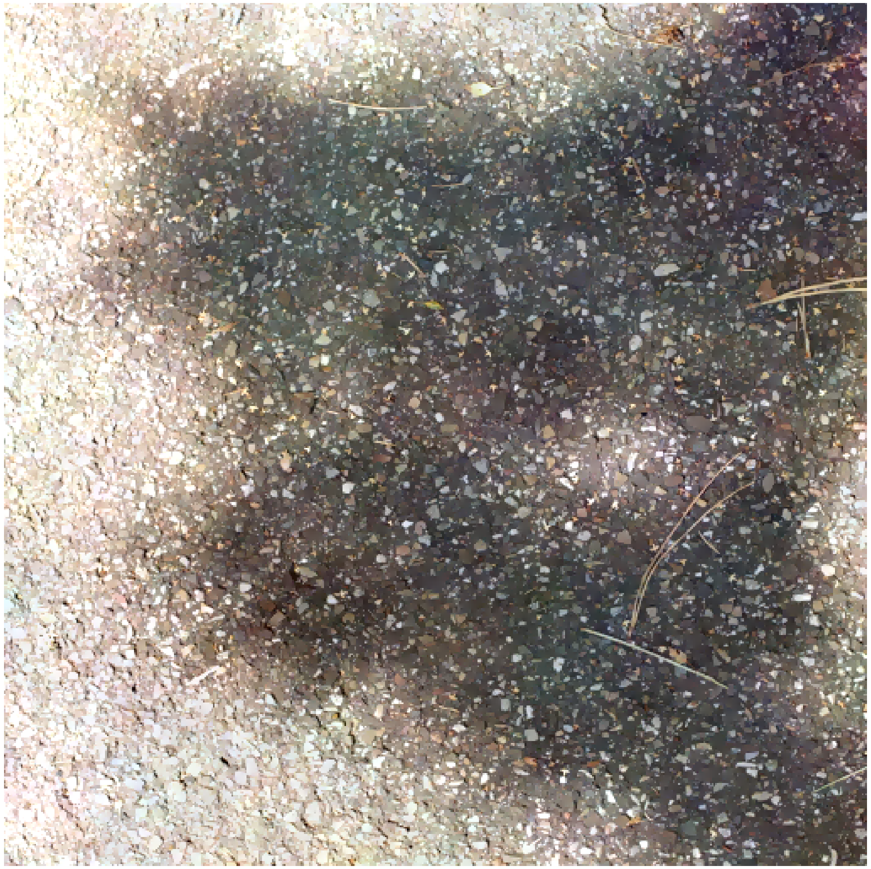


Morel's formalization.

Morel's formalization. Extension.

$$L(x, y) = \underset{L}{argmin}(\|\nabla L(x, y) - \delta(\nabla \tilde{I}(x, y))\|_2^2)$$

$$\delta_{ext}(\nabla \tilde{I}(x, y)) = \nabla \tilde{I}(x, y) - \alpha \hat{\delta}(\nabla \tilde{I}_b(x, y))$$

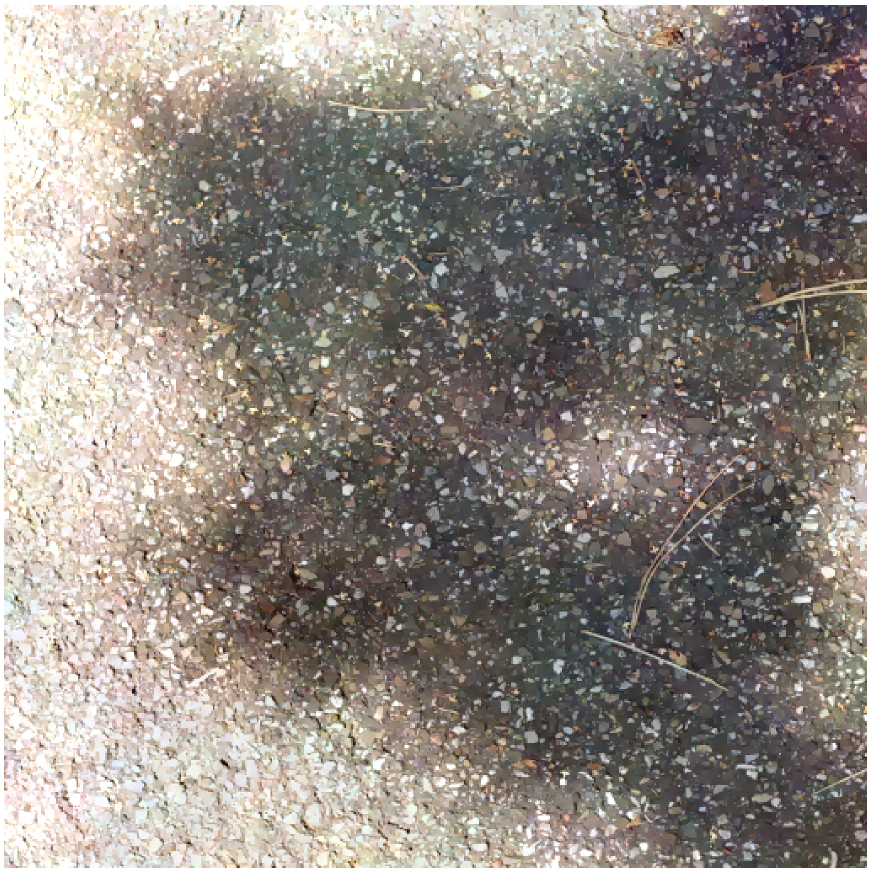
$$\hat{\delta}(x) = \begin{Bmatrix} 0, & for \quad |x| \geq threshold \\ x, & otherwise \end{Bmatrix}$$

$$-\Delta L(x) \; = \sum_{A = \mathrm{up}, \mathrm{down}, \mathrm{left}, \mathrm{right}} \delta\left(\log\left(\frac{I(x)}{I(x_A)}\right)\right) = F(x)$$

$$\Delta L_{ext} = \sum_{A = \mathrm{up}, \mathrm{down}, \mathrm{left}, \mathrm{right}} \log\left(\frac{I(x)}{I(x_A)}\right) - \alpha \hat{\delta}\left(\log\left(\frac{I_b(x)}{I_b(x_A)}\right)\right) = -F_{ext}$$

Experimental results.

Experimental results.



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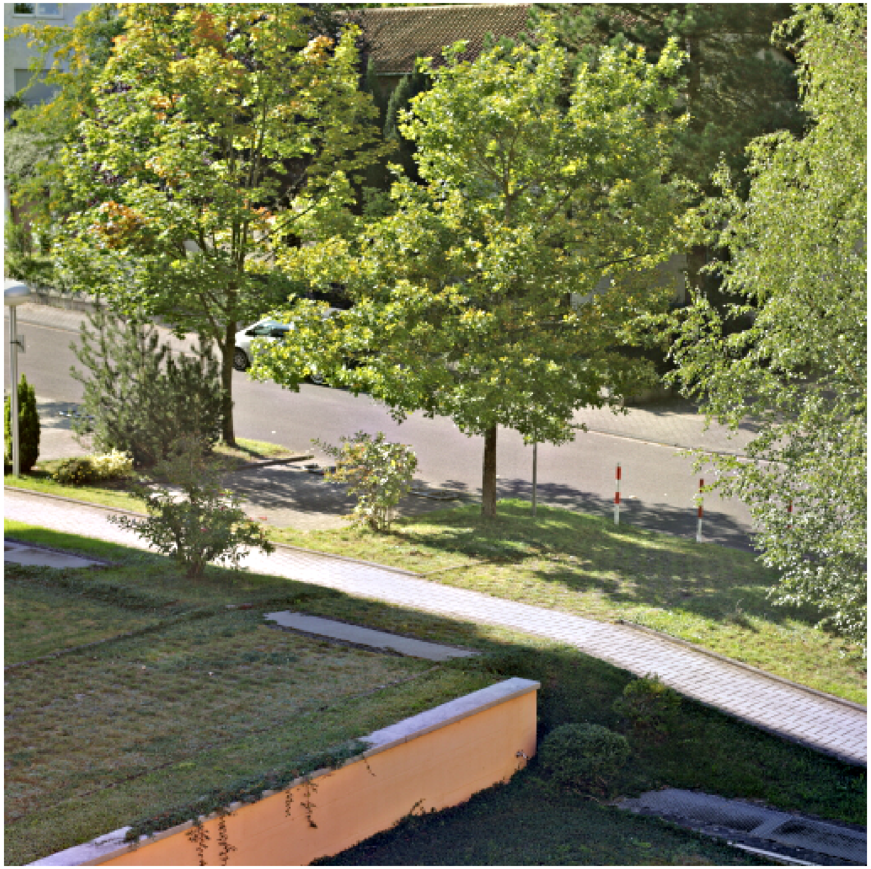
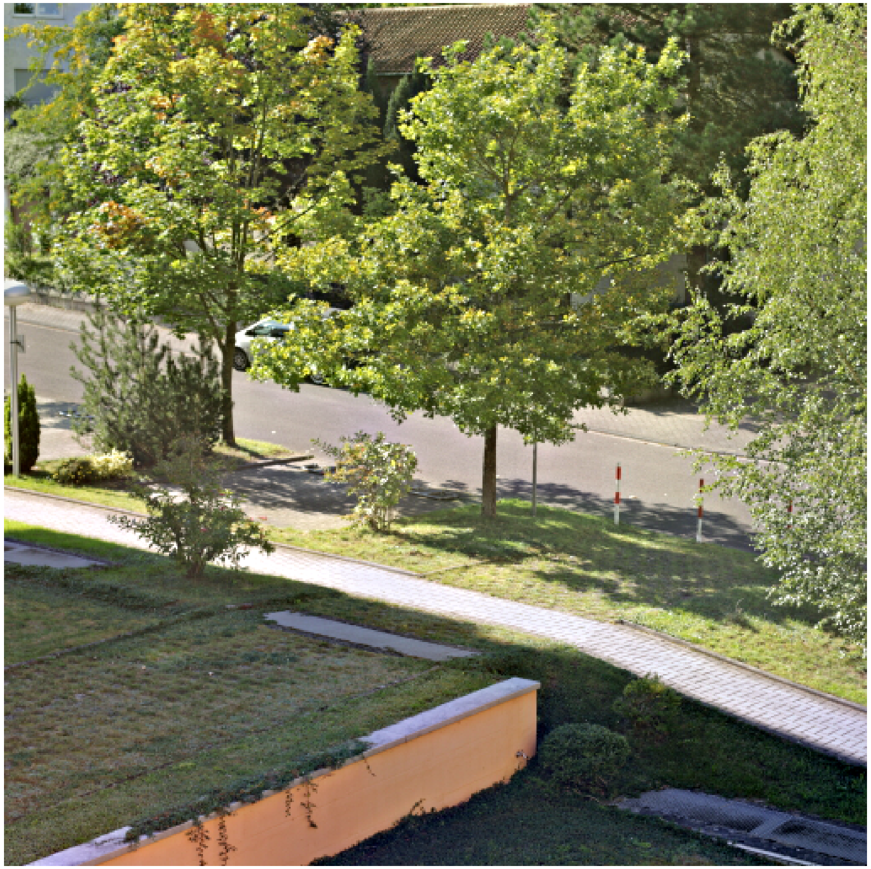


Experimental results.



Conclusions

- The original Morel's equation works not so good in case of natural textures.
- The extended version of the equation gives better results.
- The extended version works worse with objects of different scale.
- Possible improvements: multi-scale retinex, edge detectors etc.