Clustering with Reaction-Diffusion equations

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Outline

Initial task

$$\frac{\partial u_1}{\partial t} = \mathrm{div}(D_1(u_1, u_2) \cdot \mathrm{grad}(u_1))$$

$$\frac{\partial u_2}{\partial t} = \mathrm{div}(D_2(u_1, u_2) \cdot \mathrm{grad}(u_2))$$

where

$$D_1 = \begin{cases} 1 & \text{if } u_1 > u_2 \\ 0 & \text{otherwise} \end{cases}$$

$$D_2 = \begin{cases} 1 & \text{if } u_1 < u_2 \\ 0 & \text{otherwise} \end{cases}$$

Why not diffusion only?

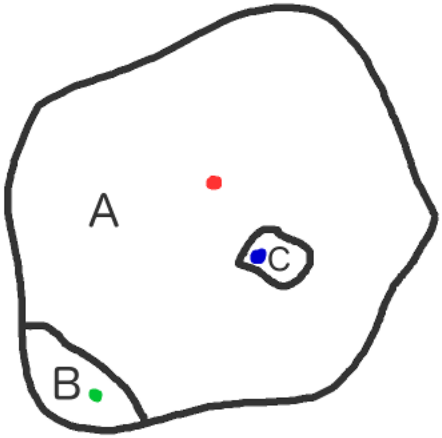

Figure 1: Diffusion only

Reaction term

$$\frac{\partial u}{\partial t} = \mathrm{div}(D(u) \cdot \mathrm{grad}(u)) + f(u, x, y)$$

"Sources and sinks"

The term $f(u, x, y)$ describes "sources and sinks" of a matter letting us to control "termination" and "producing" of the matter in each point.

Competitive Lottka-Volterra equations

$$\frac{\partial u}{\partial t} = \gamma \cdot u \cdot (1 - \alpha \cdot u)$$

Coefficients

The coefficient α represents the capacity of environment. It defines the maximum possible amount of species in a point (as soon as $\alpha \cdot u = 1$, $\frac{\partial u}{\partial t} = 0$ and there is no growth in this point). The γ is a growth rate for the specie.

Competitive Lottka-Volterra equations

$$\frac{\partial u_1}{\partial t} = \gamma_1 \cdot u_1 \cdot (1 - \alpha_{1,1} \cdot u_1 - \alpha_{1,2} \cdot u_2)$$
$$\frac{\partial u_2}{\partial t} = \gamma_2 \cdot u_2 \cdot (1 - \alpha_{2,1} \cdot u_1 - \alpha_{2,2} \cdot u_2)$$

Coefficients

Here the γ_i is a growth rate for species i and $\alpha_{i,j}$ is an element of community matrix. Coefficients of this matrix reflect influence of a species j on a species i.

Competitive Lottka-Volterra equations with dfiffusion

$$\frac{\partial u_i}{\partial t} = \mathrm{div}(D_i \cdot \mathrm{grad}(u_i)) + \gamma_i \cdot u_i \cdot (1 - A \cdot u)$$

where A is the community matrix. Now the species can move in space according to diffusion tensor D.

Attacker and defender

The "defender" should be strong and fast within the object area but weak and slow outside of the object. The attacker should have more homogeneous distribution of "attacking" force and be relative fast but should lose within the object area.

The equation system

$$\frac{\partial u_1}{\partial t} = \mathrm{div}(D_1 \cdot \mathrm{grad}(u_1)) + f_1 \cdot u_1 \cdot (1 - u_1 - b(f_2) \cdot u_2) \tag{1}$$

$$\frac{\partial u_2}{\partial t} = \mathrm{div}(D_2 \cdot \mathrm{grad}(u_2)) + f_2 \cdot u_2 \cdot (1 - u_2 - f_1 \cdot u_1) \tag{2}$$

where

$$b(x) = \begin{cases} 1 & \text{if } x > C_{boost} \\ \left(\frac{x}{C_{boost}}\right)^{C_{supp}} & \text{otherwise} \end{cases}$$

f_1 - feature map for the attacker, f_2 - feature map for the defender, $b(f_2)$ - boosting / suppressing function for the defender, u_1 - attacker population and u_2 - defender population.

Image features

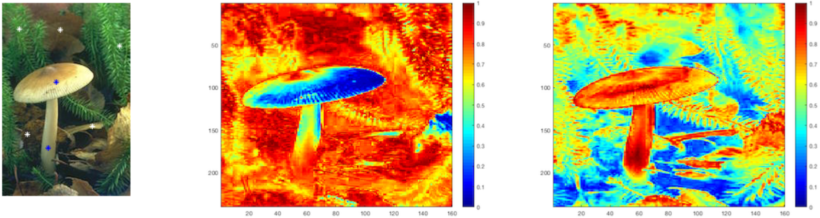

Figure 2: Merged feature maps. From left to right: original image with seed points, background feature map, object feature map.

Boosting / suppressing function

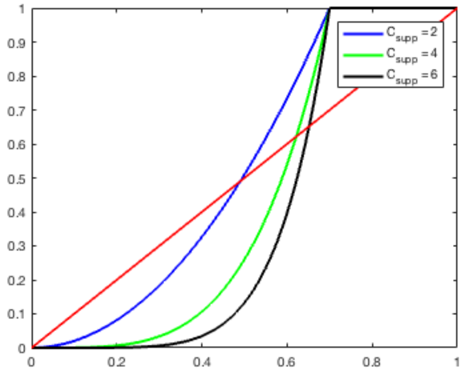

Figure 3: Boosting / suppressing function

Influence of the boosting / suppressing function

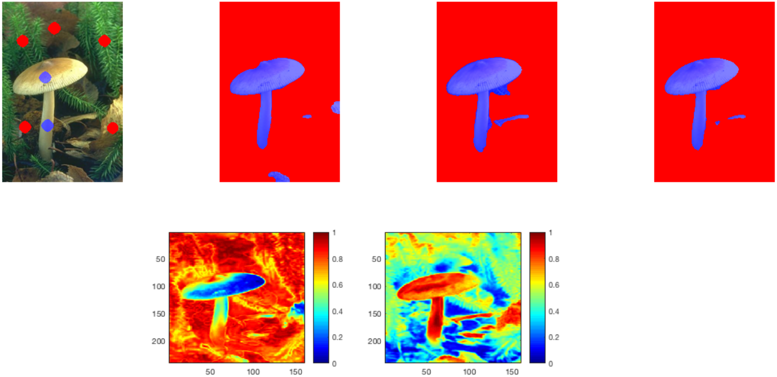

Figure 4: From left to right, top to bottom: original image with seed points, no boosting, clustering with $C_{supp} = 2$, $C_{supp} = 6$, feature map of the background, feature map of the object.

Anisotropic diffusion

$$g(|\nabla u|^2) = \frac{1}{1 + |\nabla u|^2 \cdot \lambda^2}$$

$$v_1 \parallel \nabla u, v_2 \perp \nabla u$$

$$k_1 = f \cdot g(|\nabla f|^2), k_2 = f + f \cdot (1 - g(|\nabla f|^2))$$

$$D = V \cdot L \cdot V^T$$

Intuition behind anisotropic diffusion

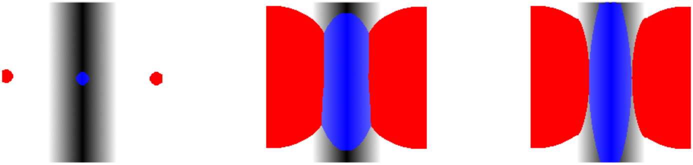

Figure 5: Influence of anisotropy. From left to right: original image, isotropic nonlinear diffusion, anisotropic nonlinear diffusion

Intuition behind anisotropic diffusion

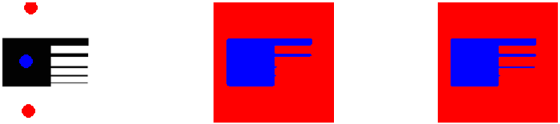

Figure 6: Different anisotropy ratio. Top row from left to right: initial state, evaluation result without anisotropy, evaluation result with anisotropy ($\lambda = 100$).

Anisotropic diffusion illustration

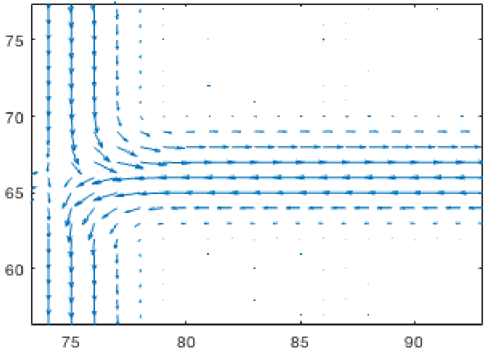

Figure 7: Main direction of anisotropic diffusion

Anisotropic diffusion illustration

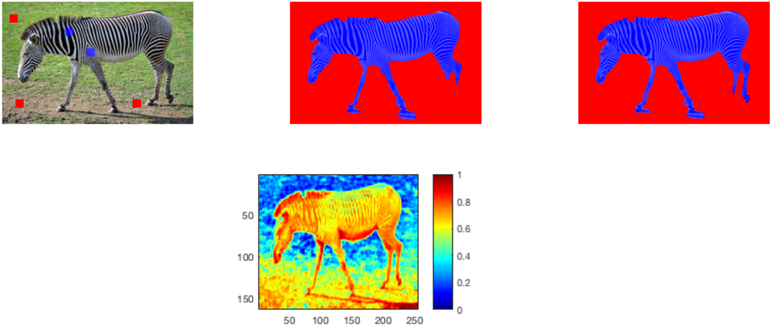

Figure 8: Influence of anisotropy

Potential advantages and disadvantages of the method

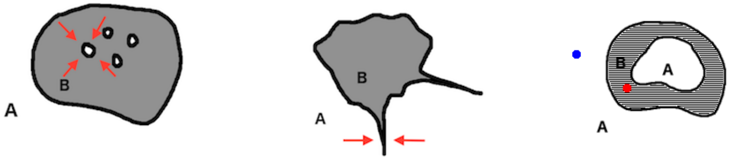

Figure 9: A small region within bigger region. A complex border with thin elements. Barrier.

Important advantage: a possibility to use different features for diffusion and reaction term.

Potential advantages and disadvantages of the method. Illustration.

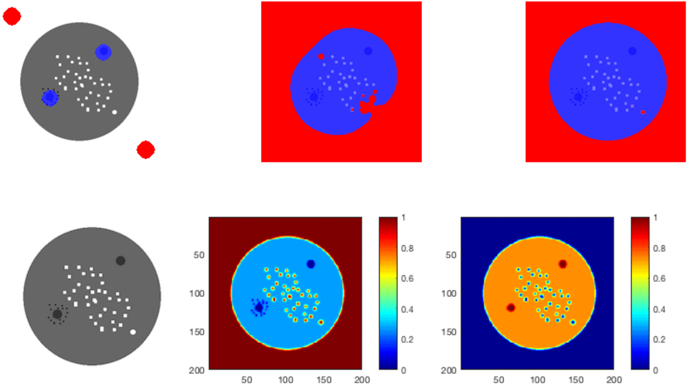

Figure 10: Influence of the defender boosting.

More examples

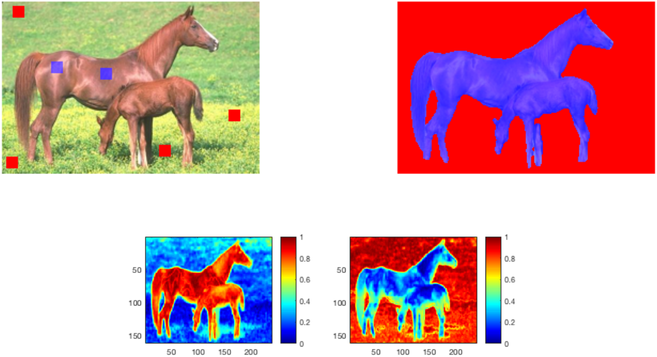

Figure 11: Horses. Top: the original image. Middle: clustering result. Bottom left: object feature. Bottom right: background feature. $\lambda = 100$, $C_{boost} = 0.7$, $C_{supp} = 4$, $t = 100$, Delta E only

Examples

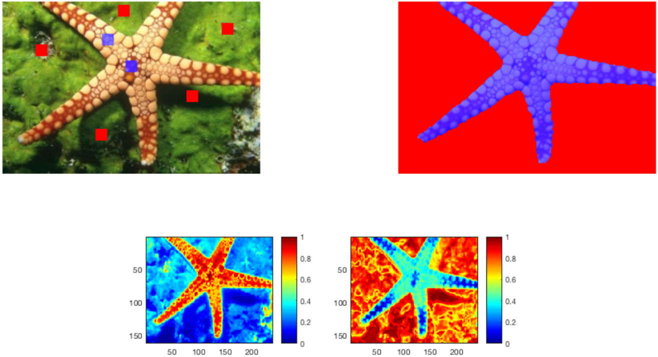

Figure 12: Starfish. Top: the original image. Middle: clustering result. Bottom left: object feature. Bottom right: background feature. $\lambda = 100$, $C_{boost} = 0.7$, $C_{supp} = 4$, $t = 90$, Delta E only

Examples

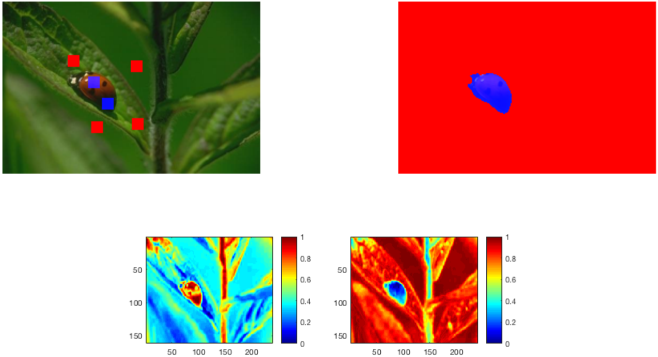

Figure 13: Ladybug. Top: the original image. Middle: clustering result. Bottom left: object feature. Bottom right: background feature. $\lambda = 100$, $C_{boost} = 0.7$, $C_{supp} = 4$, $t = 90$, Delta E only

Comparison with Chan-Vese

Figure 14: Chan-Vese segmentation. Top left: original image with initial contour. Top right: clustering with high contour smoothness. Bottom: clustering with low contour smoothness.

Comparison with Chan-Vese

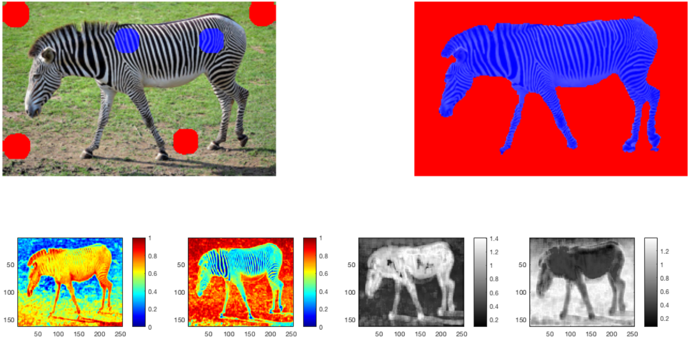

Figure 15: Mutiple features.

Comparison with Chan-Vese

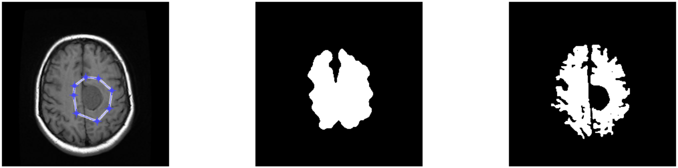

Figure 16: Chan-Vese segmentation. Top left: original image with initialization contour. Top right: clustering result with high smoothing . Bottom: clustering results with low smoothing.

Comparison with Chan-Vese

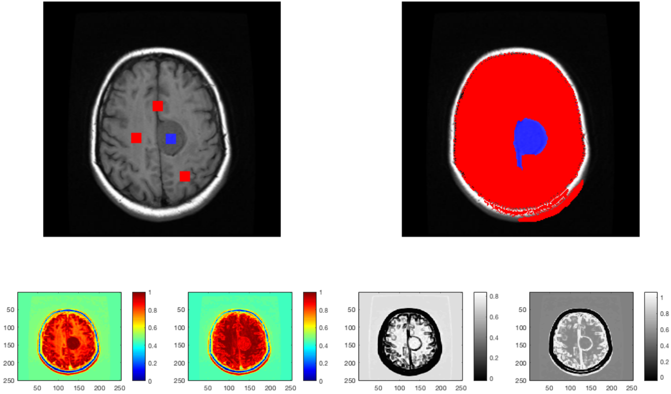

Figure 17: Mutiple features.

Robustness against noise.

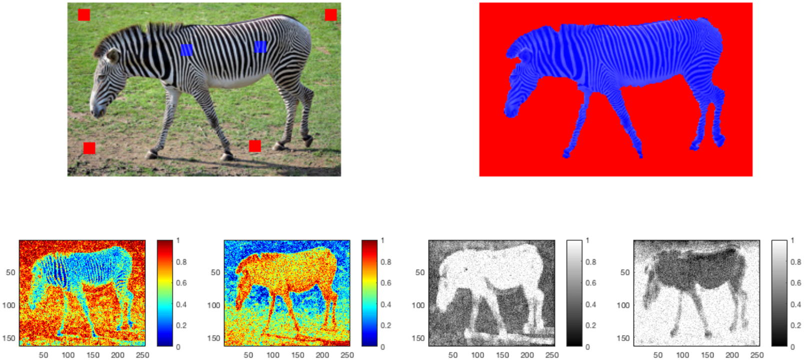

Figure 18: Noise tests.

Conclusion. Future work.

Advantages:

- Interactivity
- Flexibility / Adaptability
- Different features for diffusion and reaction terms
- Robustness against the noise

Drawbacks:

- Complexity
- Amount of parameters
- No automatical stop point

Future work: simplification, automatisation of the stop point, performance improvement

Simplification

$$\frac{\partial u_1}{\partial t} = \mathrm{div}(D_1 \cdot \mathrm{grad}(u_1)) + f_1 \cdot u_1 \cdot (1 - u_1 - b(f_2) \cdot u_2)$$
$$\frac{\partial u_2}{\partial t} = \mathrm{div}(D_2 \cdot \mathrm{grad}(u_2)) + f_2 \cdot u_2 \cdot (1 - u_2 - f_1 \cdot u_1)$$

$$\frac{\partial u_1}{\partial t} = \mathrm{div}(D_1 \cdot \mathrm{grad}(u_1)) + u_1 \cdot (f_1 - u_1 - u_2)$$
$$\frac{\partial u_2}{\partial t} = \mathrm{div}(D_2 \cdot \mathrm{grad}(u_2)) + u_2 \cdot (b(f_2) - u_2 - u_1)$$

Different featuremaps

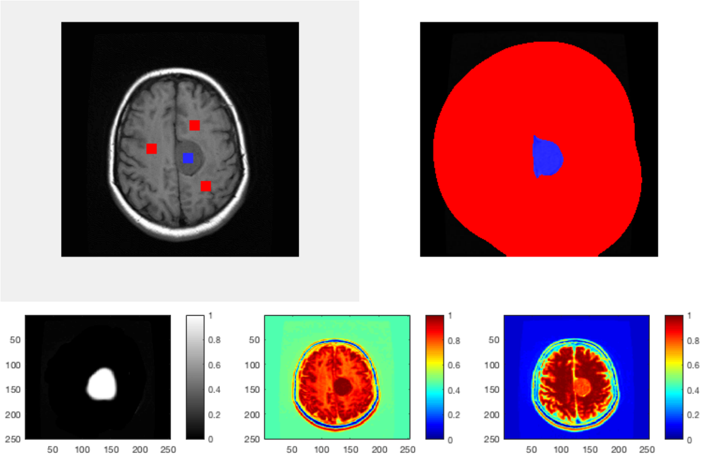

Figure 19: Mutiple features.