EXERCISE 11
Model-Free Control with tabular and linear methods

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Objective: Value-function based methods can often converge faster than Monte-Carlo methods and form the basis of several recent successes of reinforcement learning. We will start by investigating two important control methods which use action-value functions: Sarsa learning, which is on-policy, and Q-learning which is off policy. In the later part of the exercise we will investigate n-step TD learning, which combines advantages of MC and TD(0) learning, a subject which will lead to eligibility traces next week.

In the last part of todays exercise we will look at value function approximations which are essential for scaling up reinforcement learning. These introduce a number of complications and design choices we will return to in two weeks, but to avoid much of that discussion we will focus on the important case of linear approximators.

(22 lines of code)

Material: Obtain exercise material from our gitlab repository at
`https://gitlab.gbar.dtu.dk/02465material/02465students`

Contents

1 Tabular control methods

In this section we will consider the basic (tabular) basic versions of Q and Sarsa learning, and later extend sarsa with linear function approximators (we will return to Q learning

when we discuss deep reinforcement learning). Note that while these two methods appear quite similar implementation wise, they share the distinction that Sarsa is trained on-policy while Q-learning is off-policy, and the reader is advised to consider how (in the code) it is apparent Q-learning is off-policy (hint: we use a variable in Sarsa learning we do not use in Q-learning).

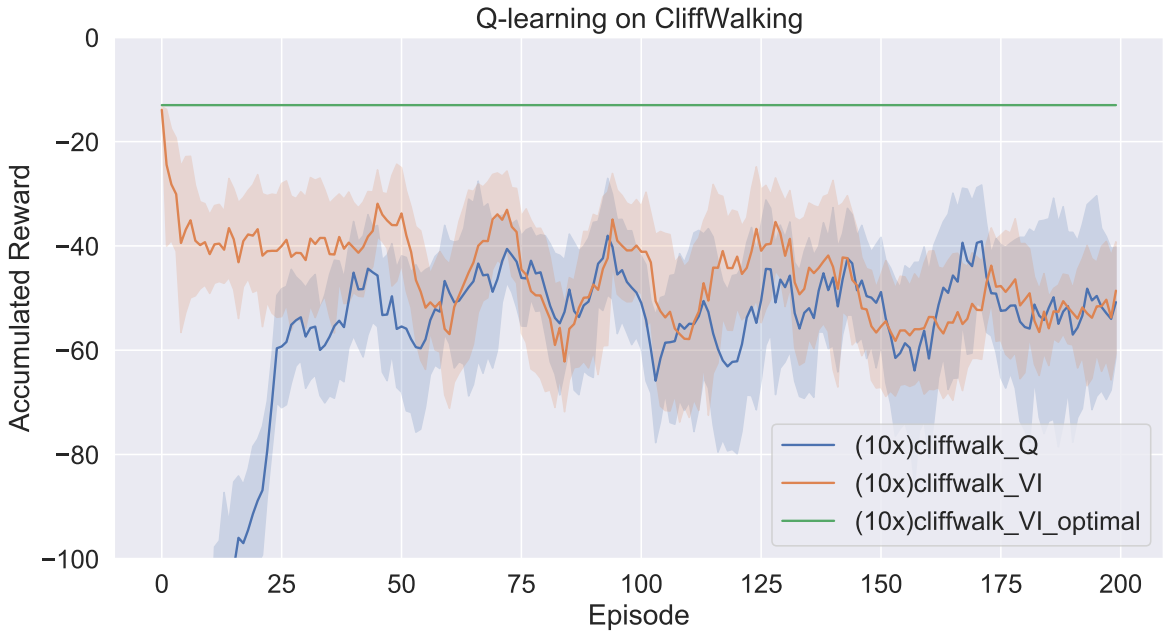
1.1 Q-Learning (`q_agent.py`)

Our implementation will be based on [SB18, Section 6.5], i.e. we update one Q-value at a time and use ε-greedy exploration. Note we will be extending the `Agent`-class we worked with last week and I encourage the reader to look at the code for this class for ideas, for instance the `Agent.pi_eps`-function.

Problem 1 Q-learning

Complete the implementation of the Q-learning agent and test it on the cliff walking task. The obtained result should be comparable to the figure in [SB18, Example 6.6], however I think that this figure is averaged over quite a few runs. I am not entirely sure what α should be, so I selected $\alpha = 0.5$ because this was the value one page back; are other values better?

Info: Writing the Q-learning is about 2-lines of code, however it takes a little work to make it robust to the various use cases we can imagine such as action spaces which depend on state etc. I strongly recommend using the build-in `Agent.Q`-data-structure to store the Q-values as this will make the code significantly more robust.



1.2 Sarsa (`sarsa_agent.py`)

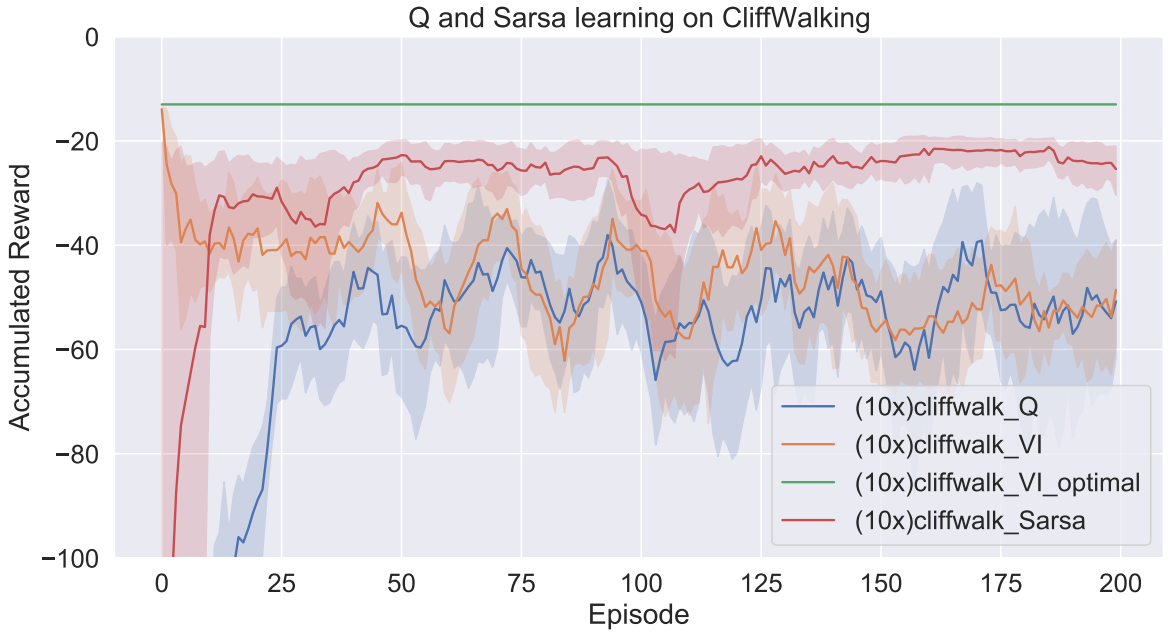
The next method we will consider will be Sarsa for on-policy control as in [SB18, Section 6.4]. Be warned Sarsa is a bit more tricky to implement as it requires us to know the future actions (here, A') "*in advance*" compared to Q-learning.

In practice, this means we must generate the A' values in the `Agent.train` function (where we know S'), store the action, and then simply return the action when π is called. This works well for all steps except the first ($t = 0$) where `agent.train` has not yet been called, and so in the case where $t = 0$ we must actually generate the action similar to Q-learning when the policy is called.

Problem 2 *Sarsa-control*

Complete the implementation of the Sarsa agent and reproduce the result in the figure in [SB18, Example 6.6]; however note this is with the caveat that I am not entirely sure about the parameters/runs used to generate the figure.

Info:



2 n-step Sarsa (`nstep_sarsa_agent.py`) ★★

The next method we will consider will be Sarsa for on-policy control as in [SB18, Section 7.2].

Be warned Sarsa is a bit more tricky to implement as it requires us to know the future actions (here, A_{t+k}) "*in advance*" relatively to Q-learning in order to compute the return, however it is in my opinion very worthwhile to implement this method in order to properly understand eligibility traces which will be the subject next week.

In our implementation, we store the previous $n+1$ values of S, A, R in lists as a buffer so we can compute the return. Then, once that buffer has enough elements (the check is already in the code as given), we can perform the update of the Q-value for the *past* time step $\tau = t - n + 1$ (as in the pseudo code). This requires us to compute the return, starting in τ and counting n steps into the future, but is identical to the pseudo code.

Note that when the environment terminates (i.e. `done=True`) we still have n missing updates we must perform on the buffer elements, however these updates should be similar to that for a given τ and thus the code is re-used.

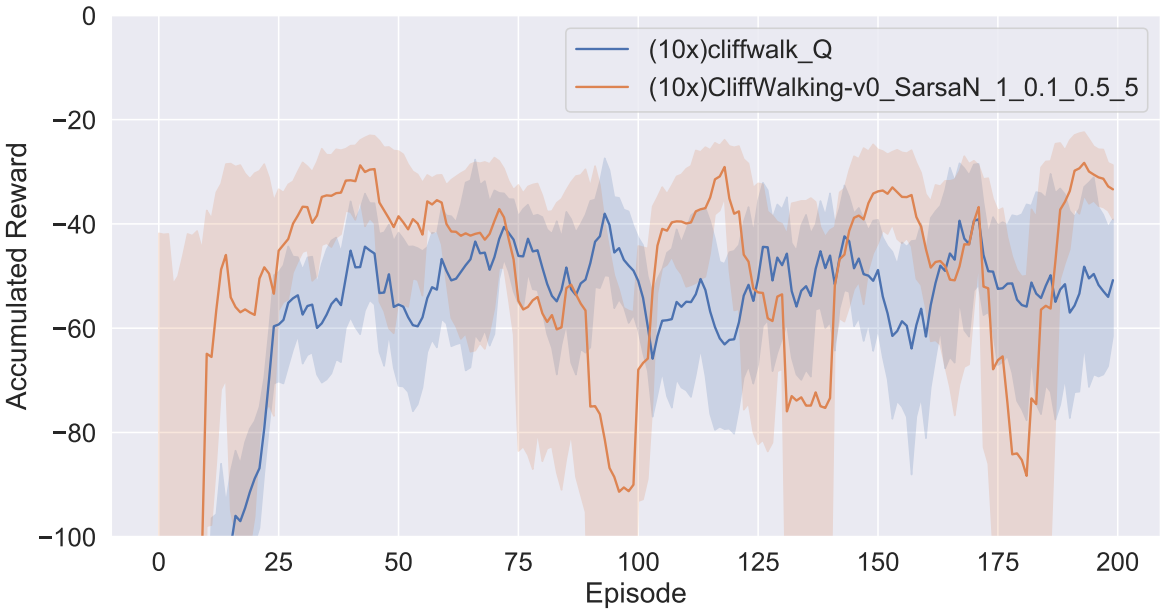
This is complicated, and you are likely to make a bug along the way. However noting that $n = 1$ corresponds to regular Sarsa, we can check that we compute the same updates in this case. I have inserted such a check and I strongly recommend debugging the code for the $n = 1$ case to ensure it works before moving on to $n > 1$.

Note some methods are abstracted out to make the semi-gradient easier to implement, but if you want to follow this path is up to you.

Problem 3 *n-step Sarsa-control*

Complete the implementation of the n-step Sarsa agent and reproduce figures comparable to [SB18, Example 6.6] using $n = 5$; Note I am not sure about the settings of the parameters and you might be able to find better parameters or a nicer example.

Info: With the caveat I am not sure about the settings for the simulations I get the following results. I think what they indicate is that n-step Sarsa perhaps learn a bit faster but with more variance relative to Sarsa (which is quite natural considering what we know about MC learning)



3 Linear feature encoding

In this section we will consider approximations of the value or action-value function. This means that we represent the action-value function $q(s, a)$ using an approximation

$$q(s, a) \approx \hat{q}(s, a, \mathbf{w})$$

depending on a weight-vector $\boldsymbol{w}$ which we have to learn. There are two potential benefits from learning an approximation rather than using the tabular case:

- The state space $\mathcal{S}$ might be continuous and therefore unsuitable for tabular methods
- There might be an unreasonably large number of action-value pairs

As discussed in [SB18, Chapter 11] there are many choices available for the function appropriator, and since we do not wish these choices to get in the way of the learning algorithms we will as a first step consider the case where $\hat{q}(s, a, \mathbf{w})$ is linear; as a benefit, this also allows us to be consistent with the methods in [SB18, Chapter 12]. In other words we assume:

$$q(s, a) \approx \hat{q}(s, a, \mathbf{w}) = \mathbf{x}(s, a)^\top \boldsymbol{w} \tag{1}$$

here, $\boldsymbol{x} : \mathcal{S} \times \mathcal{A} \mapsto \mathbb{R}^d$ is a function which maps each state, action pair into a d-dimensional feature vector. We have to choose this function based on one of the methods described in [SB18, Section 9.5]. Note some of the algorithms we will see momentarily are based on the gradient, but this is easy to compute in a linear representation:

$$\nabla \hat{q}(s, a, \mathbf{w}) = \mathbf{x}(s, a)$$

There are a multitude of ways to construct the function $\hat{q}$ or even $\boldsymbol{x}$, and likely the more sensible way to handle this would be to build a class system which has to be specified externally to the agent. I feel this creates unnecessary complications and have therefore chosen to handle this functionality with a single class `...` which selects $\boldsymbol{x}$ depending on user input and the observation space – from a software engineering perspective this is not very nice, but it simplifies things a great deal.

3.1 Episodic semi-gradient Q-learning (`semi_grad_q.py`)

Even though the book focuses on Sarsa (on policy) learning with approximations, we will first discuss Q-learning as it is simpler and we can re-use our Q-learning code when we later build the Sarsa implementation.

the ideas are easier introduced in the context of Q-learning. The actual change to the basic Q-learning method is one line, namely that instead of doing this update:

$$Q(S, A) \leftarrow Q(S, A) + \alpha\delta, \quad \delta = \left[R + \gamma \max_a Q\left(S', a\right) - Q(S, A)\right]$$

we do this:

$$\mathbf{w} \leftarrow \mathbf{w} + \alpha\delta\nabla\hat{q}\left(S, A, \mathbf{w}\right)$$

To implement this, we need an actual feature approximator. For that purpose I have made a small tile encoder using the same tile-encoding scheme as Sutton. This means that $\boldsymbol{x}(s, a)$ will be binary. Using the encoder is very easy, the important parts are these:

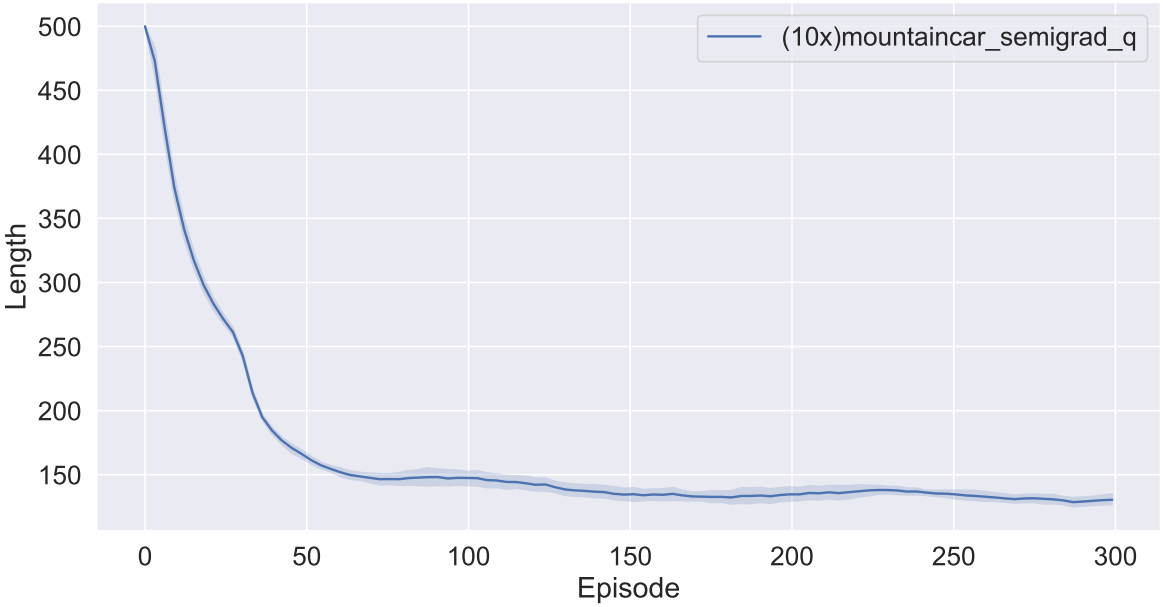
```python
# semi_grad_q.py
class LinearSemiGradQAgent(QAgent):
    def __init__(self, env, gamma=1.0, alpha=0.5, epsilon=0.1, q_encoder=None):
        """ The Q-values, as implemented using a function approximator, can now be
        ↪  accessed as follows:

        >> self.Q(s,a) # Compute q-value
        >> self.Q.x(s,a) # Compute gradient of the above expression wrt. w
        >> self.Q.w # get weight-vector.

        I would recommend inserting a breakpoint and investigating the above
↪   expressions yourself;
        you can of course al check the class LinearQEncoder if you want to see how it
↪   is done in practice.
        """
        super().__init__(env, gamma, epsilon=epsilon, alpha=alpha)
        self.Q = LinearQEncoder(env, tilings=8) if q_encoder is None else q_encoder
```

Note in particular the example in the comments: this will be how we access the $\boldsymbol{x}$-feature vector and compute the q-values.

Problem 4 *Semi gradient Q-learning*

Complete the implementation of the linear semi-gradient Q-learning agent. It might be of help comparing the semi-gradient version of Sarsa ([SB18, Section 10.1]) to the tabular version to better understand the new notation.

Info: The script tries to solve the MountainCar example (same setup as [SB18, Figure 10.2]), and you might consider reducing the number of runs while you debug your code. I obtain the following result:



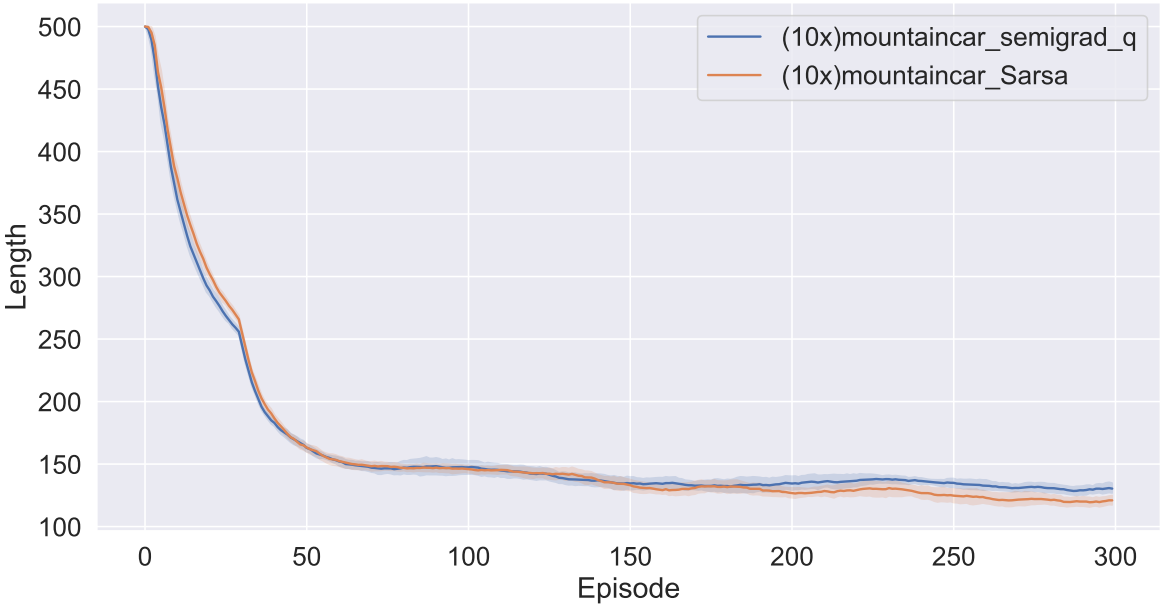
3.2 Episodic semi-gradient Sarsa (`semi_grad_sarsa.py`)

Our second task will be episodic semi-gradient Sarsa [SB18, Section 10.1]. The code is nearly identical to the basic Sarsa method we just implemented with the feature-approximator idea added back in, and I would therefore recommend starting with the tabular Sarsa solution.

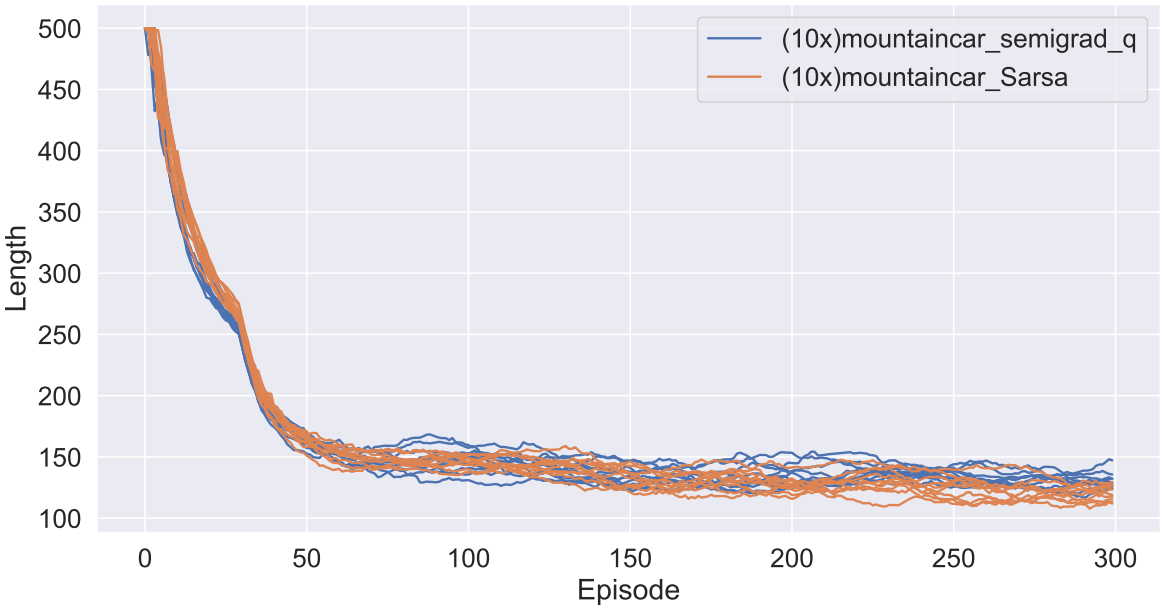
Problem 5 *Semi gradient Sarsa-learning*

Complete the implementation of the linear semi-gradient Sarsa-learning agent from [SB18, Section 10.1]).

Info: I have added a check to see if the weights diverge which happened to me in some of the problems. It might also be of use to add a regularization term, however I have not tested this idea.



Often, the averaging can hide important details about the individual runs, such as whether the average performance is highly driven by outliers. I have included options to turn off averaging and get the individual runs as shown below:



References

[SB18] Richard S. Sutton and Andrew G. Barto. *Reinforcement Learning: An Introduction*. The MIT Press, second edition, 2018. (See **sutton2018.pdf**).