2a)

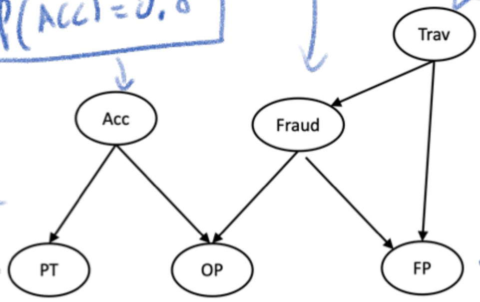
- Acc – card holder has an account with the merchant for current transaction.
- $Fraud$ – current transaction is fraudulent.
- $Trav$ – card holder is currently traveling.
- FP – current transaction is a foreign purchase.
- OP – current purchase is an online purchase.
- PT – previous transaction with the same merchant was made in the past week.

Tr	P(Fr | Tr)
T	0.01
F	0.004

P(Trav) = 0.05

P(Acc) = 0.8

Acc	P(PT | Acc)
T	0.1
F	0.01



Fr	Tr	P(FP | Fr, Tr)
T	F	0.1
F	F	0.01
T	T	0.9
F	T	0.9

Fr	Acc	P(OP | Acc, Fr)
T	T	0.8
T	F	0.1
F	T	0.6
F	F	0.3

2b) $P(\text{fraud}) = P(\text{fraud}, \text{trav}) + P(\text{fraud}, \neg\text{trav})$

Prior Probability:

$$= P(\text{fraud} \mid \text{trav})\, P(\text{trav}) + P(\text{fraud} \mid \neg\text{trav})\, P(\neg\text{trav})$$

$$= 0.01 \cdot 0.05 + 0.004 \cdot 0.95$$

$$\boxed{= 0.0043}$$

$P(\text{Fraud} \mid fp, \neg op, pt)$ calculations:

WAS NOT IN THE MOOD TO CODE ... OH WELL ENJOY :)

Trav	$f_1(\text{Trav}) = P(\text{Trav})$
T	0.05
F	0.95

Fraud	Trav	$f_2(\text{Fraud}, \text{Trav}) = P(\text{Fraud} \mid \text{Trav})$
T	T	0.01
T	F	0.004
F	T	0.99
F	F	0.996

Fraud	Trav	$f_3(\text{Fraud}, \text{Trav}) = P(fp \mid \text{Fraud}, \text{Trav})$
T	T	0.9
T	F	0.1
F	T	0.9
F	F	0.001

Fraud	Acc	$f_4(\text{Fraud}, \text{Acc}) = P(\neg op \mid \text{Fraud}, \text{Acc})$
T	T	0.2
T	F	0.9
F	T	0.4
F	F	0.7

Acc	$f_5(\text{Acc}) = P(\text{Acc})$
T	0.8
F	0.2

Acc	$f_6(\text{Acc}) = P(pt \mid \text{Acc})$
T	0.1
F	0.01

$$f_7(\text{Fraud}) = \sum_{\text{Trav}} f_1(\text{Trav}) \cdot f_2(\text{Fraud}, \text{Trav}) \cdot f_3(\text{Fraud}, \text{Trav})$$

$$\begin{aligned} f_7(\text{Fraud} = T) &= f_1(\text{trav}) \cdot f_2(\text{fraud}, \text{trav}) \cdot f_3(\text{fraud}, \text{trav}) \\ &+ f_1(\neg\text{trav}) \cdot f_2(\text{fraud}, \neg\text{trav}) \cdot f_3(\text{fraud}, \neg\text{trav}) \end{aligned}$$

$$\begin{aligned} &= 0{,}05 \cdot 0{,}01 \cdot 0{,}9 \\ &+ 0{,}95 \cdot 0{,}004 \cdot 0{,}1 \\ &= 0{,}00083 \end{aligned}$$

$$\begin{aligned} f_7(\text{Fraud} = F) &= f_1(\text{trav}) \cdot f_2(\neg\text{fraud}, \text{trav}) \cdot f_3(\neg\text{fraud}, \text{trav}) \\ &+ f_1(\neg\text{trav}) \cdot f_2(\neg\text{fraud}, \neg\text{trav}) \cdot f_3(\neg\text{fraud}, \neg\text{trav}) \end{aligned}$$

$$\begin{aligned} &= 0{,}05 \cdot 0{,}99 \cdot 0{,}9 \\ &+ 0.95 \cdot 0{,}996 \cdot 0.01 \\ &= 0{,}054012 \end{aligned}$$

$$f_8(\text{Fraud}) = \sum_{\text{ACC}} f_4(\text{Fraud}, \text{ACC}) \cdot f_5(\text{ACC}) \cdot f_6(\text{ACC})$$

$$\begin{aligned} f_8(\text{Fraud} = T) &= f_4(\text{fraud}, \text{acc}) \cdot f_5(\text{acc}) \cdot f_6(\text{acc}) \\ &+ f_4(\text{fraud}, \neg\text{acc}) \cdot f_5(\neg\text{acc}) \cdot f_6(\neg\text{acc}) \\ &= [0.2 \cdot 0{,}8 \cdot 0.1] + [0{,}9 \cdot 0{,}2 \cdot 0{,}01] \\ &= 0.0178 \end{aligned}$$

$$f_8(\text{Fraud}=F) = f_4(\neg\text{fraud}, \text{acc}) \cdot f_5(\text{acc}) \cdot f_6(\text{acc}) + f_4(\neg\text{fraud}, \neg\text{acc}) \cdot f_5(\neg\text{acc}) \cdot f_6(\neg\text{acc})$$

$$= [0{,}4 \cdot 0{,}8 \cdot 0{,}1] + [0{,}7 \cdot 0{,}2 \cdot 0{,}01)$$

$$= 0{,}0334$$

$$\Pr(\text{fraud} \mid fp, \neg op, pt) = \frac{f_7(\text{fraud}) \cdot f_8(\text{fraud})}{f_7(\text{fraud}) \cdot f_8(\text{fraud}) + f_7(\neg\text{fraud}) \cdot f_8(\neg\text{fraud})}$$

$$\boxed{\approx 0{,}008123}$$

$$\Pr(\neg\text{fraud} \mid fp, \neg op, pt) = 0.991877$$

2c) $Pr(Fraud \mid fp, \neg op, pt, trav)$ calculations:

$f_1() = P(trav)$

0.05

Fraud	$f_2(Fraud) = P(Fraud \mid Trav)$
T	0.01
F	0.99

Fraud	$f_3(Fraud) = P(fp \mid Fraud, Trav)$
T	0.9
F	0.9

Fraud	Acc	$f_4(Fraud, Acc) = P(\neg op \mid Fraud, Acc)$
T	T	0.2
T	F	0.9
F	T	0.4
F	F	0.7

Acc	$f_5(Acc) = P(Acc)$
T	0.8
F	0.2

Acc	$f_6(Acc) = P(pt \mid Acc)$
T	0.1
F	0.01

$$f_7(\text{Fraud}) = \sum_{Acc} f_4(\text{Fraud} \mid Acc) \cdot f_5(Acc) \cdot f_6(Acc)$$

$$f_7(\text{Fraud} = T) = \left[f_4(\text{fraud} \mid acc) f_5(acc) \cdot f_6(acc) \right] + \left[f_4(\text{fraud} \mid \neg acc) f(\neg acc) \cdot f_6(\neg acc) \right]$$

$$= [0{,}2 \cdot 0{,}8 \cdot 0{,}1] + [0{,}9 \cdot 0{,}2 \cdot 0{,}01]$$

$$= 0{,}0178$$

$$f_7(\text{Fraud} = F) = \left[f_4(\neg\text{fraud} \mid acc) f_5(acc) \cdot f_6(acc) \right] + \left[f_4(\neg\text{fraud} \mid \neg acc) f(\neg acc) \cdot f_6(\neg acc) \right]$$

$$= [0{,}4 \cdot 0{,}8 \cdot 0{,}1] + [0{,}7 \cdot 0{,}2 + 0{,}01]$$

$$= 0{,}0334$$

$$P(\text{fraud} \mid fp, \neg op, pt, \text{trav})$$

$$= \frac{f_1() \cdot f_2(\text{fraud}) \cdot f_3(\text{fraud}) \cdot f_7(\text{fraud})}{f_1() \cdot f_2(\text{fraud}) \cdot f_3(\text{fraud}) \cdot f_7(\text{fraud}) + f_1() \cdot f_2(\neg\text{fraud}) \cdot f_3(\neg\text{fraud}) \cdot f_7(\neg\text{fraud})}$$

$$= \frac{0{,}05 \cdot 0{,}01 \cdot 0{,}9 \cdot 0{,}0178}{0{,}05 \cdot 0{,}01 \cdot 0{,}9 \cdot 0{,}0178 + 0{,}05 \cdot 0{,}99 \cdot 0{,}9 \cdot 0{,}0334}$$

$$\boxed{= 0{,}00535}$$

$$p(\neg\text{fraud} \mid fr, \neg op, pt, \text{trav}) = 0{,}99465$$

2d)

(d) **[20 pts]** Suppose you are not a very honest employee of the credit card company and you just stole a credit card. You know that the fraud detection system uses the Bayes net designed earlier, but you still want to make an important online purchase. What can you do prior to your online purchase to reduce the risk that the transaction will be rejected as a possible fraud?

What to hand in: Tell me the action taken and indicate by how much the probability of a fraud gets reduced. Follow the same instructions as for Question 2b.

When an online purchase is made the likelihood of it being a fradulent is:

$P(\text{fraud} \mid op) = 0.006$

However, the likelihood decreases if the credit card has an account with the merchant

$P(\text{fraud} \mid op, acc) = 0.0057$

In summary, to reduce the risk that the online transaction will be flagged, you shoud only make transactions with a merchant that the card holder has an acount with. This will reduce the probability of a flaged fraud by $\approx 0.0003 = 0.006 - 0.0057$

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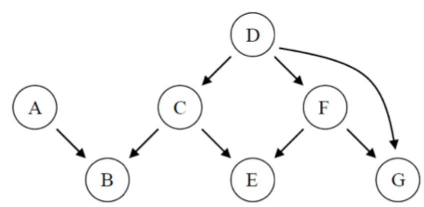
i. Are D and G independent?
ii. Are D and G independent given F?
iii. Are A and G independent?
iv. Are A and G independent given B?
v. Are A and G independent given B and C?
vi. Are A and G independent given B and D?
vii. Are A and G independent given B, D and E?



3a)

i) FALSE: No D-seperation rules apply for $D \to G$.

ii) FALSE: $D \to F \to G$ are D-seperated by rule (1).
However, NO D-seperation rules apply for $D \to G$.

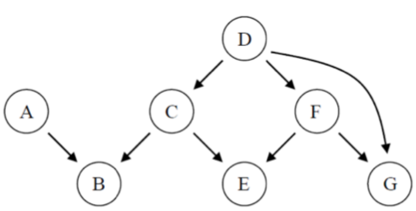
iii) TRUE: Since B is not in evidence the d-seperation rule (3) applies to all undirected paths between A and G => conditionally independent

iv) FALSE: Since B is now in evidence rule (3) no longer applies and (1) & (2) are not applicable (no evidence) => not conditionally independent

v) TRUE: Notice that since C is in evidence, rule (1) applies to
$A \to B \leftarrow C \leftarrow D$ and rule (2) applies to $A \to B \leftarrow C \to E$. All undirected paths from A to G pass through D or E
=> all undirected paths are D-seperated
=> conditionally independent

vi) TRUE: The D-separation rule (3) applies to $C \to E \leftarrow F$ and rule (2) applies to $C \leftarrow D \to F$ or $C \leftarrow D \to G$. All undirected paths from A to G include these options. All are blocked $\Rightarrow$ conditionally independent

vii) FALSE: There is no D-separation rules that apply to the path: $A \to B \leftarrow C \to E \leftarrow F \to G$ since E is now in evidence rule (3) doesn't apply anymore.
$\Rightarrow$ NOT conditionally independent



- Can restrict attention to *relevant* variables. Given query Q, evidence **E**:
  - Q is relevant
  - if any node Z is relevant, its parents are relevant
  - if E∈**E** is a descendent of a relevant node, then E is relevant

3b

(b) **[14 pts]** Suppose a mechanics would like to know $\Pr(C|A = true, E = false)$. What is the subset of relevant variables that is sufficient to answer this query? Give a brief justification based on the rules to identify relevant variables.

- C is relevant given its the query
- D is relevant as its a parent to C
- E is relevant as $E \in \mathbf{E}$ and is a decendant of C
- F is relevant as its a parent to E

NO more rules apply. $\boxed{\text{Relevant: } C, D, E, F}$