Hidden Markov and Other Models for Discrete-valued Time Series

Iain L. MacDonald

University of Cape Town
South Africa

and

Walter Zucchini

University of Göttingen
Germany

CHAPMAN & HALL

London · Weinheim · New York · Tokyo · Melbourne · Madras

Contents

Preface xiii

PART ONE Survey of models 1

1 A survey of models for discrete-valued time series 3
1.1 Introduction: the need for discrete-valued time series models 3
1.2 Markov chains 6
1.2.1 Saturated Markov chains 6
1.2.2 A nonhomogeneous Markov chain model for binary time series 11
1.3 Higher-order Markov chains 12
1.4 The DARMA models of Jacobs and Lewis 17
1.5 Models based on thinning 21
1.5.1 Models with geometric marginal 21
1.5.2 Models with negative binomial marginal 23
1.5.3 Models with Poisson marginal 25
1.5.4 Models with binomial marginal 29
1.5.5 Results not based on any explicit distributional assumption 30
1.6 The bivariate geometric models of Block, Langberg and Stoffer 32
1.6.1 Moving average models with bivariate geometric distribution 32
1.6.2 Autoregressive and autoregressive moving average models with bivariate geometric distribution 34
1.7 Markov regression models 37
1.8 Parameter-driven models 42
1.9 State-space models 45

1.10 Miscellaneous models 47
1.11 Discussion 49

PART TWO Hidden Markov models 53

2 The basic models 55
2.1 Introduction 55
2.2 Some theoretical aspects of hidden Markov models in speech processing 57
2.3 Hidden Markov time series models: definition and notation 65
2.4 Correlation properties 69
2.4.1 The autocorrelation function of a Poisson-hidden Markov model 70
2.4.2 The autocorrelation function of a binomial-hidden Markov model 74
2.4.3 The partial autocorrelation function 77
2.5 Evaluation of the likelihood function 77
2.6 Distributional properties 80
2.6.1 Marginal, joint and conditional distributions of the observations 80
2.6.2 The Markov chain conditioned on the observations 84
2.6.3 Runlength distributions for binary hidden Markov models 86
2.7 Parameter estimation 90
2.7.1 Computing maximum likelihood estimates 90
2.7.2 Asymptotic properties of maximum likelihood estimators 95
2.7.3 Use of the parametric bootstrap 96
2.8 Identification of outliers 97
2.9 Reversibility 101
2.10 Discussion 105

3 Extensions and modifications 109
3.1 Introduction 109
3.2 Models based on a second-order Markov chain 110
3.3 Multinomial-hidden Markov models 115
3.3.1 The likelihood 116
3.3.2 Marginal properties and cross-correlations 117
3.3.3 A model for categorical time series 119
3.4 Multivariate models 121

3.4.1 The likelihood function for multivariate models 122
3.4.2 Cross-correlations of models assuming contemporaneous conditional independence 122
3.4.3 Cross-correlations of models not assuming contemporaneous conditional independence 124
3.4.4 Multivariate models with time lags 125
3.4.5 Multivariate models in which some variables are discrete and others continuous 126
3.5 Models with state-dependent probabilities depending on covariates 128
3.6 Models in which the Markov chain is homogeneous but not assumed stationary 129
3.7 Models in which the Markov chain is nonhomogeneous 130
3.8 Joint models for the numbers of trials and the numbers of successes in those trials 133
3.9 Discussion 135

4 Applications 137
4.1 Introduction 137
4.2 The durations of successive eruptions of the Old Faithful geyser 138
4.2.1 Markov chain models 138
4.2.2 Hidden Markov models 140
4.2.3 Comparison of models 144
4.2.4 Forecast distributions 146
4.3 Epileptic seizure counts 146
4.4 Births at Edendale hospital 152
4.4.1 Models for the proportion Caesarean 152
4.4.2 Models for the total number of deliveries 159
4.4.3 Conclusion 161
4.5 Locomotory behaviour of *Locusta migratoria* 162
4.5.1 Multivariate models 163
4.5.2 Univariate models 166
4.5.3 Conclusion 167
4.6 Wind direction at Koeberg 168
4.6.1 Three hidden Markov models for hourly averages of wind direction 168
4.6.2 Model comparisons and other possible models 173
4.6.3 Conclusion 176
4.7 Evapotranspiration 177

4.8 Thinly traded shares on the Johannesburg Stock Exchange 178
4.8.1 Univariate models 179
4.8.2 Multivariate models 180
4.8.3 Discussion 182
4.9 Daily rainfall at Durban 184
4.10 Homicides and suicides, Cape Town, 1986–1991 191
4.10.1 Models for firearm homicides as a proportion of all homicides, suicides and legal intervention homicides 191
4.10.2 Models for the number of firearm homicides 194
4.10.3 Firearm homicides as a proportion of all homicides, and firearm suicides as a proportion of all suicides 195
4.10.4 Models for the proportions in each of the five categories of death 200
4.11 Conclusion 201

Appendices **203**
A Proofs of results used in the derivation of the Baum–Welch algorithm 203
B Data 207

References **217**

Author index **227**

Subject index **231**