Bias and variance

$$Err(\boldsymbol{x}) = \mathbb{E}_D\left[\left(f(\boldsymbol{x};D) - \bar{f}\,(\boldsymbol{x})\right)^2\right] + \left(\bar{f}(\boldsymbol{x}) - y\right)^2 + \mathbb{E}_D\left[(y_D - y)^2\right]$$

- variance
- bias^2
- noise

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$$
\begin{aligned}
E(f;D) =& \mathbb{E}_D\left[\left(f(\boldsymbol{x};D) - y_D\right)^2\right] \\
=& \mathbb{E}_D\left[\left(f(\boldsymbol{x};D) - \bar{f}(\boldsymbol{x}) + \bar{f}(\boldsymbol{x}) - y_D\right)^2\right] \\
=& \mathbb{E}_D\left[\left(f(\boldsymbol{x};D) - \bar{f}(\boldsymbol{x})\right)^2\right] + \mathbb{E}_D\left[\left(\bar{f}(\boldsymbol{x}) - y_D\right)^2\right] \\
& + \mathbb{E}_D\left[2\left(f(\boldsymbol{x};D) - \bar{f}(\boldsymbol{x})\right)\left(\bar{f}(\boldsymbol{x}) - y_D\right)\right] \\
=& \mathbb{E}_D\left[\left(f(\boldsymbol{x};D) - \bar{f}(\boldsymbol{x})\right)^2\right] + \mathbb{E}_D\left[\left(\bar{f}(\boldsymbol{x}) - y_D\right)^2\right] \\
=& \mathbb{E}_D\left[\left(f(\boldsymbol{x};D) - \bar{f}(\boldsymbol{x})\right)^2\right] + \mathbb{E}_D\left[\left(\bar{f}(\boldsymbol{x}) - y + y - y_D\right)^2\right] \\
=& \mathbb{E}_D\left[\left(f(\boldsymbol{x};D) - \bar{f}(\boldsymbol{x})\right)^2\right] + \mathbb{E}_D\left[\left(\bar{f}(\boldsymbol{x}) - y\right)^2\right] + \mathbb{E}_D\left[\left(y - y_D\right)^2\right] \\
& + 2\mathbb{E}_D\left[\left(\bar{f}(\boldsymbol{x}) - y\right)\left(y - y_D\right)\right] \\
=& \mathbb{E}_D\left[\left(f(\boldsymbol{x};D) - \bar{f}(\boldsymbol{x})\right)^2\right] + \left(\bar{f}(\boldsymbol{x}) - y\right)^2 + \mathbb{E}_D\left[\left(y - y_D\right)^2\right]
\end{aligned}
$$