03_02_Algortmo_Expectation_Maximization

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Passo: M

```
y <- matrix(c(10,15, 17, 22, 23, NA),2,3,byrow=TRUE); y
```

```
##      [,1] [,2] [,3]
## [1,]   10   15   17
## [2,]   22   23   NA
```

```
em1 <- function(y23, y){
ystar <- y
ystar[2,3] <- y23
mu.hat <- mean(ystar)
alpha.hat <- apply(ystar, MAR = 1,
mean) - mean(ystar)
beta.hat <- apply(ystar, MAR = 2,
mean) - mean(ystar)
y23 <- mu.hat + alpha.hat[2] + beta.hat[3]
return(c(mu = mu.hat,
alpha = alpha.hat,
beta = beta.hat,
y23 = y23))
}

em1(21,y)
```

```
##     mu alpha1 alpha2  beta1  beta2  beta3    y23
##     18     -4      4     -2      1      1     23
```

```
set.seed(1832)
em.step <- function(y, epsilon= 1e-8){
trace <- NULL
convergenza <- FALSE
trace <- t(em1(y23 = mean(y, na.rm = TRUE), y = y))
y23id <- grep("y23", colnames(trace))
h <- 0
while(!convergenza){
h <- h + 1
trace <- rbind(trace,
em1(y23 = trace[h, "y23"],
```

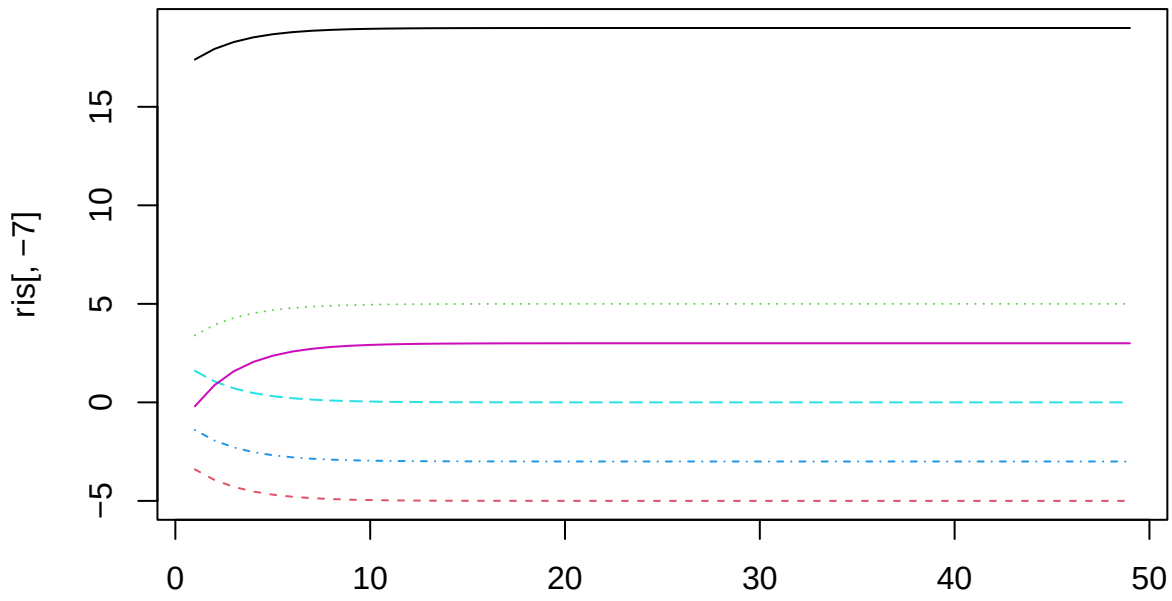
```
y = y))
convergenza <- (dist(trace[h:(h+1), -y23id]) < epsilon)
}
return(trace)
}

em.step(y)
```

```
##             mu    alpha1   alpha2     beta1        beta2      beta3      y23
##  [1,] 17.40000 -3.400000 3.400000 -1.400000 1.600000e+00 -0.2000000 20.60000
##  [2,] 17.93333 -3.933333 3.933333 -1.933333 1.066667e+00  0.8666667 22.73333
##  [3,] 18.28889 -4.288889 4.288889 -2.288889 7.111111e-01  1.5777778 24.15556
##  [4,] 18.52593 -4.525926 4.525926 -2.525926 4.740741e-01  2.0518519 25.10370
##  [5,] 18.68395 -4.683951 4.683951 -2.683951 3.160494e-01  2.3679012 25.73580
##  [6,] 18.78930 -4.789300 4.789300 -2.789300 2.106996e-01  2.5786008 26.15720
##  [7,] 18.85953 -4.859534 4.859534 -2.859534 1.404664e-01  2.7190672 26.43813
##  [8,] 18.90636 -4.906356 4.906356 -2.906356 9.364426e-02  2.8127115 26.62542
##  [9,] 18.93757 -4.937570 4.937570 -2.937570 6.242951e-02  2.8751410 26.75028
## [10,] 18.95838 -4.958380 4.958380 -2.958380 4.161967e-02  2.9167607 26.83352
## [11,] 18.97225 -4.972254 4.972254 -2.972254 2.774645e-02  2.9445071 26.88901
## [12,] 18.98150 -4.981502 4.981502 -2.981502 1.849763e-02  2.9630047 26.92601
## [13,] 18.98767 -4.987668 4.987668 -2.987668 1.233175e-02  2.9753365 26.95067
## [14,] 18.99178 -4.991779 4.991779 -2.991779 8.221170e-03  2.9835577 26.96712
## [15,] 18.99452 -4.994519 4.994519 -2.994519 5.480780e-03  2.9890384 26.97808
## [16,] 18.99635 -4.996346 4.996346 -2.996346 3.653853e-03  2.9926923 26.98538
## [17,] 18.99756 -4.997564 4.997564 -2.997564 2.435902e-03  2.9951282 26.99026
## [18,] 18.99838 -4.998376 4.998376 -2.998376 1.623935e-03  2.9967521 26.99350
## [19,] 18.99892 -4.998917 4.998917 -2.998917 1.082623e-03  2.9978348 26.99567
## [20,] 18.99928 -4.999278 4.999278 -2.999278 7.217488e-04  2.9985565 26.99711
## [21,] 18.99952 -4.999519 4.999519 -2.999519 4.811659e-04  2.9990377 26.99808
## [22,] 18.99968 -4.999679 4.999679 -2.999679 3.207772e-04  2.9993584 26.99872
## [23,] 18.99979 -4.999786 4.999786 -2.999786 2.138515e-04  2.9995723 26.99914
## [24,] 18.99986 -4.999857 4.999857 -2.999857 1.425677e-04  2.9997149 26.99943
## [25,] 18.99990 -4.999905 4.999905 -2.999905 9.504511e-05  2.9998099 26.99962
## [26,] 18.99994 -4.999937 4.999937 -2.999937 6.336340e-05  2.9998733 26.99975
## [27,] 18.99996 -4.999958 4.999958 -2.999958 4.224227e-05  2.9999155 26.99983
## [28,] 18.99997 -4.999972 4.999972 -2.999972 2.816151e-05  2.9999437 26.99989
## [29,] 18.99998 -4.999981 4.999981 -2.999981 1.877434e-05  2.9999625 26.99992
## [30,] 18.99999 -4.999987 4.999987 -2.999987 1.251623e-05  2.9999750 26.99995
## [31,] 18.99999 -4.999992 4.999992 -2.999992 8.344152e-06  2.9999833 26.99997
## [32,] 18.99999 -4.999994 4.999994 -2.999994 5.562768e-06  2.9999889 26.99998
## [33,] 19.00000 -4.999996 4.999996 -2.999996 3.708512e-06  2.9999926 26.99999
## [34,] 19.00000 -4.999998 4.999998 -2.999998 2.472341e-06  2.9999951 26.99999
## [35,] 19.00000 -4.999998 4.999998 -2.999998 1.648228e-06  2.9999967 26.99999
## [36,] 19.00000 -4.999999 4.999999 -2.999999 1.098818e-06  2.9999978 27.00000
## [37,] 19.00000 -4.999999 4.999999 -2.999999 7.325456e-07  2.9999985 27.00000
## [38,] 19.00000 -5.000000 5.000000 -3.000000 4.883637e-07  2.9999990 27.00000
## [39,] 19.00000 -5.000000 5.000000 -3.000000 3.255758e-07  2.9999993 27.00000
## [40,] 19.00000 -5.000000 5.000000 -3.000000 2.170505e-07  2.9999996 27.00000
## [41,] 19.00000 -5.000000 5.000000 -3.000000 1.447004e-07  2.9999997 27.00000
## [42,] 19.00000 -5.000000 5.000000 -3.000000 9.646691e-08  2.9999998 27.00000
## [43,] 19.00000 -5.000000 5.000000 -3.000000 6.431127e-08  2.9999999 27.00000
## [44,] 19.00000 -5.000000 5.000000 -3.000000 4.287418e-08  2.9999999 27.00000
```

```
## [45,] 19.00000 -5.000000 5.000000 -3.000000 2.858279e-08  2.9999999 27.00000
## [46,] 19.00000 -5.000000 5.000000 -3.000000 1.905519e-08  3.0000000 27.00000
## [47,] 19.00000 -5.000000 5.000000 -3.000000 1.270346e-08  3.0000000 27.00000
## [48,] 19.00000 -5.000000 5.000000 -3.000000 8.468973e-09  3.0000000 27.00000
## [49,] 19.00000 -5.000000 5.000000 -3.000000 5.645983e-09  3.0000000 27.00000
```

Trace Plot:

```
ris<- em.step(y)
matplot(ris[,-7], type = "l")
```




```
names1 <- expression(mu,
alpha[1],
alpha[2],
beta[1],
beta[2],
beta[3])
pal1<- c("red", "yellow", "green", "violet", "blue", "orange")
matplot(ris[,-7],
type = "l",
col = pal1,
lwd = 2,
lty = 1,
xlab = "Iterazioni dell'algoritmo EM",
```

```
ylab = "Stime dei parametri del modello")
legend(x = 0,
y = 15,
legend = names1,
lwd = 2 ,
col = pal1,
lty = 1,
horiz=TRUE,
cex=0.8)
```