The Iterated Prisoner's Dilemma

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September 2016

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A dissertation submitted in partial fulfilment of the requirements for MSc (in Operational Research and Applied Statistics) by taught programme.

Executive Summary

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Chapter 1

Introduction

Game theory is a set of analytical tools and solution concepts, which provide explanatory and predicting power in interactive decision situations, when the aims, goals and preferences of the participating players are potentially in conflict, Szabo and Fath (2007) [28]. The Prisoner's Dilemma(PD) is a well known example in Game Theory and in recent years has become the gold standard of understanding evolution of co-operative behavior [17]. Thus, it has been a topic of focus in various fields, such as biology, sociology, ecology and psychology.

In the example of the Prisoner's Dilemma(PD) two criminals have been arrested and interrogated, with no way of communicating, by the police. They are given only two choices, to either cooperate with each other or to defect. Now let us consider that the prisoners would be put back in their cells and would be asked the same question tomorrow. Furthermore, let this happen repeatably. This is referred to as the Iterated Prisoner's Dilemma(IPD) an example that has been a rich source of research material since the 1950s but has earned much interest in the 1980s due to the work done by the political scientist Robert Axelrod [3, 4, 5].

In 1980, Axelrod held the first ever IPD computer tournaments [3, 4], he invited academics from various fields to submit their strategies in computer code. The tournaments were of a round robin topology, the first competition included thirteen strategies, while the second one sixty-four. In both tournaments the strategy Tit for Tat was announced the winner and for many years it was consider to be the most successful strategy. Tit for Tat is a deterministic strategy that will always cooperate in the first round and afterwards it copies the opponents last move.

A large volume of literature emerged on the topic following this, including some criticism about these initial tournaments. Scientists questioned whether the conditions that the first tournament took place favored tit for tat. An argument was that the initial tournaments though they included a 1% chance of players misunderstanding their opponent's move in any round they did not examine noise. Noise is the probability that the player will submit the wrong move. David and Vivian Kraines [13] stated that TFT performed rather poorly when noise was introduced in the tournament. Another aspect, is the payoff matrix which according to Kretz [14] the precise choice of the payoff matrix is relevant to the results.

Furthermore, another aspect needed to be taken into account was the network topology underlying the tournament. In 1992 Nowak's and May's paper [21] spatial tournament are introduced. In which the players are placed on an two-dimensional spatial and allowed to play a game with only the immediate neighbors. Thus, squares that are adjoin. An example of this is shown in Figure ...

Their tournament considered the PD and the players could only defect or co-operate. They provided proof that cooperative behavior can emerge from a PD tournament in spatial topology. Many works on the IPD and spatial tournament were held due to their original paper. Such as [11, 22, 18, 21, 8, 20, 16]. These tournaments use either the PD or IPD and simple to complex strategies.

One can argue that the real life interactions are better represented by spatial tournament because in real life not all players interact with all opponents. Additionally, an interesting aspect of the spatial topology are the results compared to those of a round robin tournament. This dissertation will be focusing on reproducing a spatial tournament with some of the most successful strategies of various tournaments that have been held. For the spatial topology it will use various graphs compared to other works done only using lattices. Concluding how spatial topology, with any given graph, affects the effectiveness of these strategies.

1.1 The Prisoner's Dilemma

The PD was originally formulated in Merril [**M. Flood and Melvin Dresher**], who were working on the Flood-Desher Experiment at the RAND cooperation in 1950. Later in 1950, the mathematician Albert W. Tucker presented the first formal representation of the PD, titled A Two-Person Dilemma in a seminar at Stanford University [10].

A description of the PD, found in [15] is as follows: There are two players that simultaneously have to decide to whether Cooperate (C) or Defect (D) with each other, without exchanging information.

- If both players choose to cooperate they will both receive a reward (R)
- If a player defects and the other cooperates then the defector receives a temptation payoff (T) and the cooperator a sucker payoff (S)
- If both players defect they will both receive a penalty (P)

Figure 1.1 illustrates the payoffs matrix.

		Player II	
		Cooperate	Defect
Player I	Cooperate	***R*=3** / ***R*=3**	***T*=5** / ***S*=0**
	Defect	***S*=0** / ***T*=5**	***P*=1** / ***P*=1**

Figure 1.1: The payoff matrix for the Prisoners Dilemma

Taking into account the assumptions that both players are rational and that there is no way of communication between them. No matter what the other player does, defecting will be their dominant choice as it yields a higher payoff than cooperation. Thus a pure Nash Equilibrium exists when both players defect. Even though, both players would do better if they were to cooperate.Thus creating the dilemma.

Furthermore, for this to hold there are some extra assumptions for the relationship of the four outcomes. The T temptation to defect has to offer the highest payoff for a player and the worst he could get has to be the sucker S. Likewise, the reward for mutual cooperation should exceed that of mutual defection P. Thus the next condition is 1.1:

$$T > R > P > S \tag{1.1}$$

Moreover, it is assumed that the average of T and S is less than the reward for mutual cooperation 1.2 :

$$R > (T + S)/2 \tag{1.2}$$

Same conditions such as rationality, no communication and (1.1), (1.2), apply for the IPD. An IPD is nothing more than a PD were the players interact for an infinite number of times.

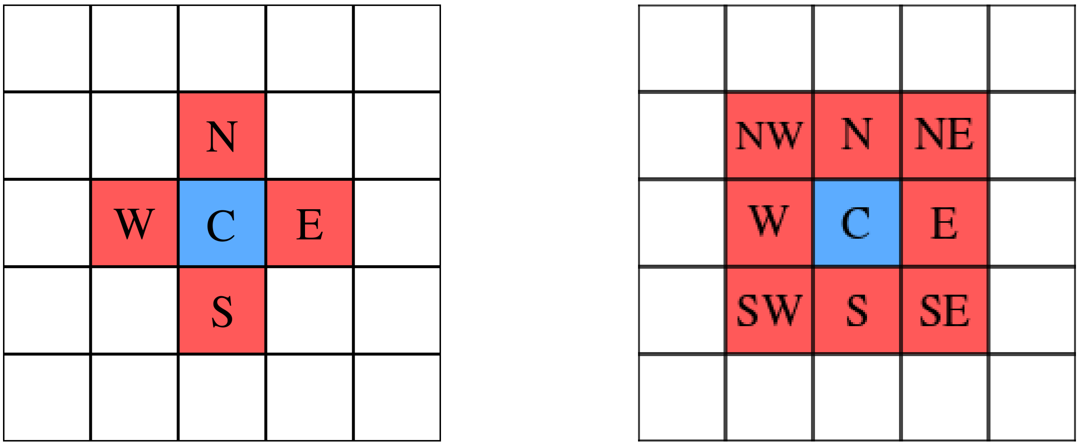
1.2 Problem Description

Axelrod's tournaments set a seed for generations of tournaments in the Prisoner's using computer modeling. Research has shown that by altering the environment of a tournament the effectiveness of some strategies can change radically. An aspect that has been investigated as to how the tournament results can bee affected was the topology. Nowak and May [21] introduced the spatial topology only to set yet another seed in the PD tournaments. Even so, spatial topology still has not been fully explored with only a small number of papers focusing on this specific topology. A goal of this dissertation is to understand the current state of the art in spatial prisoners dilemma tournaments.

As described in [**Maschler**] the contemporary world is full of networks. That is one of the main reasons this dissertation will be focusing on such topology. Work done at this point consider spatial topology to be that of a square lattice were each edge represents a player which interacting/playing with only his neighbors in their von Neuman or Moore's neighborhood. A von Neuman neighborhood comprises the four cells orthogonally surrounding a central cell where the Moore's neighborhood eight cells. As shown in Figure 1.2.

Some further work address the spatial topology in a variate of graphs [19, 28]. This dissertation will also follow this approach. Szabo and Fth dealt with numerous graphs such as, lattices, small- world, scale free graphs and evolving networks. We will consider a spatial topology to be any given graph where the players are the nodes and only play other nodes that are linked to by an edge.

Another disadvantage of the aforementioned work is that it lacked in terms of best practice of reproducibility [3, 4, 25, 9, 27] . Due the work done by Axelrod library [29]. An open source python Package which allows for the easy reproducibility of experiments. As it allow to reproduce an IPD tournament and chose between



(a) von Neuman's neighborhood (b) Moore's neighborhood

Figure 1.2: Possible neighborhoods. (a) A von Neuman's where each node has four neighbors. (b) Moore's where each node has eight neighbors.

131 strategies already given by the library. Code written for the purpose of this dissertation has been contributed to the library: see `https://github.com/Axelrod-Python`.

1.3 Structure of Dissertation

This dissertation is organized into 7 Chapters. Proceeding this introduction:

- In Chapter 2 we review previous literature dedicated to the PD/IPD and tournaments that have been conducted, different topologies and evolution.

Chapter 2

Literature Review

Following the initial work done by Axelrod, there are many other papers that have tried to tackle the PD and make their conclusions on cooperation in both a theoretical and real life setting. In this chapter a review of some of this work done in the IPD competitions, in spatial and evolutionary game theory will be carried out.

2.1 Tournaments

In order to identify the condition under which cooperation could emerge in the game of the Prisoner's Dilemma better, Robert Axelrod held a tournament in 1980. He invited a number of well-known game theorists to submit strategies for a computer tournament. Each strategy has to specify whether to cooperate or defect based on the history of previous moves made by both players. Strategies played again each other as well as a further Random strategy, that would randomly choose between C and D and with its own twin (same strategy). The tournament was a round robin with the payoff matrix 1.1. All entries knew the exact length (200 moves) of each game. To improve the reliability of the scores the entire round robin tournament was repeated five times. Fourteen strategies were submitted and by the end Tit for Tat was announced the winner. Surprisingly in the second tournament held where 64 strategies competed and all submitters had full knowledge of what have happened in the first tournament, Tit for Tat managed to get first place again[3].

As explained in [4], Tit for Tat, a simple strategy was able to beat sophisticated and more complex strategies thanks to three specific characteristics of the strategy:

- Niceness: A strategy is categorized as nice if it was not the first to defect, or at least, it will not do this until the last few moves.
- Forgiveness: The propensity to cooperate in the moves after the opponent defected.
- Clarity: After opponents identified that they were playing Tit for Tat choose to cooperate for the rest of the game.

The first tournaments were an innovation in combining computer modeling and Game Theory and in providing insights in the behavior emerging from simple dynamics. Moreover, Axelrod was the first to speak about niceness, forgiveness and gave an illustration that cooperation can be a victorious and advantageous strategy.

Another concept that had been developed was the Evolutionary Game Theory (EGT). EGT is an application of game theory to biological contexts, arising from the realization that frequency dependent fitness introduces a strategic aspect to evolution. In 1973, Maynard and Price introduced the concept of Evolutionary Stable Strategy (ESS), which is an extension of a Nash Equilibrium. If a population of the same strategies cannot be invaded by any alternative strategy that is initially rare then that strategy is an ESS. In his third tournament Axelrod [5] using the same set of strategies (63), the tournaments introduced a dynamical rule that mimics Darwinian selection. In this evolutionary computer tournament after a round robin game the score for each player was evaluated, and the strategies with high score would be adapted while the lowest ones ones would diminish. In most of these simulations, the success of Tit-for-Tat was confirmed because the population would end up with some mutually cooperating strategies prevailed by Tit-for-Tat.

There have been other tournaments, based off of Axelrods, exploring different environments and submitting new strategies. Boyd & Lorderbaum [17] state that no pure strategy is evolutionary stable because each can be invaded by the joint effect of two invading strategies when long term interaction occurs in th repeated game and future moves are discounted. In 1991 Bendor, Kramer and Stout [6] introduced noise to the IPD. Where noisy randomly flip the choice made by a strategy. The results of their tournament was that the strategies that were more generous, cooperated more than their opponents did, were more effective than Tit for Tat. Moreover, Kerts 2011 conducted a tournament where the payoff matrix was altered though satisfying the conditions 1.1, 1.2.

Furthermore, two more notable tournaments took place in 2005 and 2012. In the 2005 IPD competition a team from the University of Southampton participated using a group of strategies which won the top three propositions. These strategies were designed in such why that thought a predetermined sequence of five to ten moves would recognize each other. Once the two Southampton players recognized each other they would take up the roles of a ruler and a slave. The ruler would always cooperate where the slave would defect in order to maximize the payoff of the ruler. If the opponent was recognized to not being one of the team then the Southampton player would always choose to defect to minimize the score of the opponent [15]. Lastly, the Stewart- Plotkin [26] tournament which consisted of nineteen strategies, including a new set of strategies; the Zero- determinant(ZD) strategies. The ZD are strategies for the stochastic iterated prisoner's dilemma, discovered by Press and Dyson in 2012 [24]. The ZD apply a linear relationship between their own payoff and that of the opponent. Some review tournaments are listed on the Table 2.1:

Year	Reference	Number of Strategies	Type
1979	[3]	13	Standard
1979	[4]	64	Standard
1984	[5]	64	Evolutionary
1991	[6]	13	Noisy
2005	[9]	223	Varied
2012	[26]	13	Standard

Table 2.1: An overview of a selection of published tournaments.

In this section the work done for tournaments in the IPD has been cited and analyzed. Starting with the the work of Axelrod, the reasons the tournaments where organized and what where the fundings. Moreover, some research that has been generated the following years are stated. Below, a specialized case of these research will be studied. That case is that of the spatial structure tournaments.

2.2 Spatial Structure Tournaments

Further research was spawn in 1992 as to how the Prisoner's Dilemma could shade some insight into physics and biology. Where Nowak and May believed exist potential dynamics of spatially extended systems. Their tournament was a simple and purely deterministic spatial version of the PD in a two dimensional lattice. With players having no memory of the previous rounds and no strategical elaboration. Thus, the players could either always cooperate or defect. In each round each player interact with the immediate neighbor. [1] They used an evolution rule that after each round round the nodes with the lowest score in their neighborhood would copy the strategy of the player with the highest score. This was done to study which behavior, defection or cooperation , would last. The conclusion was that co-operational behavior is possible in the PD by using a spatial topology. Nowak produced more work on the topic on papers of his such as [22] (Nowak 1994). In his subsequent papers Nowak et al. (1994a,b) different spatial structures where studied. Including triangular and cubic lattices and a random grid. It turned out that cooperation can be maintained in spatial models even for some randomness.

On the other hand, in [16] players were allowed to have memory and therefore added complex strategies to the tournament such as Tit for Tat and Anti Tit for Tat. This was followed by the work of [8] which introduced even more complex strategies. Brauchli et all compares the spatial model with a randomly mixed model. A more complex strategy that they have tested was PAVLOV. A win-stay, lose-switch strategy. According to their fundings, there is more cooperative behavior in a spatial structure tournament and evolution is more less chaotic than in unstructured populations. Also as stated, generous variants of PAVLOV are found to be very successful strategies in playing the Iterated Prisoner's Dilemma.

Spatial topology has been defined by most scholars as a square lattice where the nodes - players only interact with their neighborhoods. Including connections between four or eight nearest neighbor sites, Neuman's or Moore's, according to Figure 1.2. A square lattice is a graph and one could argue that a round robin tournament itself is the complete graph on all players [7]. But in the above papers no authors defined the topology as a graph, apart from [20].

[1] In Nowak's and May's experiment, the result's hold for all three cases that the player interacts with 4, 6 and 8 neighbors.

In [20], an interesting approach was used. They presented a new spatial prisoner's dilemma game model in which the neighborhood size was increased onto two interdependent lattices. They implement the utility by integrating the payoff correlations between two lattices. A player would mimic a random player in his next move, base on a function that consider the utility of the player. It was characterized as a most realistic scenario.

Real life interactions are more likely to be like any given graph depending on the industry than a complete graph. Fatha et all [28], have considered a numerous of graphs, such as :

- Lattice, the interaction network is defined by the sites of a lattice. the distance between a pair does not exceed a given value. The most frequently used structure is the square lattice with von Neumann neighborhood and Moore neighborhood.
- Small word, a graph that is created from a square lattice by randomly rewiring a fraction of connections in a way that conserve the degree for each site.
- Scale-free graphs, a network that has a power-law degree distribution, regardless of any other structure.
- Evolving networks, networks that change as a function of time (this will not be considered in this dissertation).

The major theme of their review was how the graph structure of interactions could modify long term behavioral patterns emerging in evolutionary games. These graphs compose only a small fraction of graphs that exist. In this dissertation we will consider a list of graphs.

2.3 Axelrod Python Library

The Axelrod library [29] is an open source Python package that allows for reproducible game theoretic research into the Iterated Prisoner's Dilemma `https://github.com/Axelrod-Python`. For many of the tournaments aforementioned the original source code is almost never available and in no cases is the available code well-documented, easily modified or released with significant test suites. Due to that reproducing the results has not been an easy task.

However, Axelrod library manages to provide such a resource, with facilities for the design of new strategies and interactions between them, as well as conducting tournaments and ecological simulations for populations of strategies.

Strategies are implemented as classes which have a single method, `strategy()`. It only takes one argument, which is the opponent's previous moves and returns an action. These actions can be either to cooperate C or to defect D. At thins moment the Axelrod library consists of 131 strategies.Can be found in the Appendix. As an example we can see in Figure the source code for the famous strategy Tit for Tat.

```
class TitForTat(Player):
    """
    A player starts by cooperating and then mimics
    the previous action of the opponent.

    Note that the code for this strategy is written
    in a fairly verbose way. This is done so that it
    can serve as an example strategy for those who
    might be new to Python.
    """

    # These are various properties for the strategy
    name = 'Tit For Tat'
    classifier = {
        'memory_depth': 1,  # Four-Vector = (1.,0.,1.,0.)
        'stochastic': False,
        'makes_use_of': set(),
        'inspects_source': False,
        'manipulates_source': False,
        'manipulates_state': False
    }

    def strategy(self, opponent):
        """This is the actual strategy"""
        # First move
        if len(self.history) == 0:
            return C
        # React to the opponent's last move
        if opponent.history[-1] == D:
            return D
        return C
```

Listing 2.1: Tit for Tat source code.

Additionally, tournament is a class responsible for coordinating the play of generated matches. It achieves that by calling a match generator class which returns all the single match parameters, such as turns, the game and the noise. Axelrod has the capability to write out the results into a csv file and also out put plots with the ranks of the strategies.

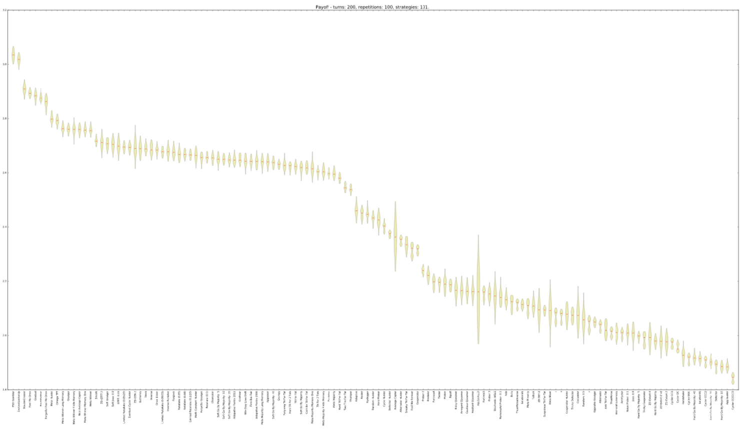
Furthermore, a basic tournament of 200 turns, 100 repetitions and the 131 strategies that exist in the library is being produced continuously. The current winner is called PSO gambler and it is a look up strategy. It uses a lookup table with probability numbers generated using a Particle Swarm Optimisation (PSO) algorithm, the sourse code can be found here : `http://axelrod.readthedocs.io/en/latest/_modules/axelrod/strategies/gambler.html?highlight=Gambler`) and a description of how this strategy was trained is given here: `https://gist.github.com/GDKO/60c3d0fd423598f3c4e4`. It uses a 64-key lookup table (keys are 3-tuples consisting of the opponent's starting actions, the opponent's recent actions, and our recent action) to decide whether to cooperate (C) or defect (D). The actions for each key were generated using an evolutionary algorithm.

To reproduce a basic tournament with the 131 strategies using Axelrod :

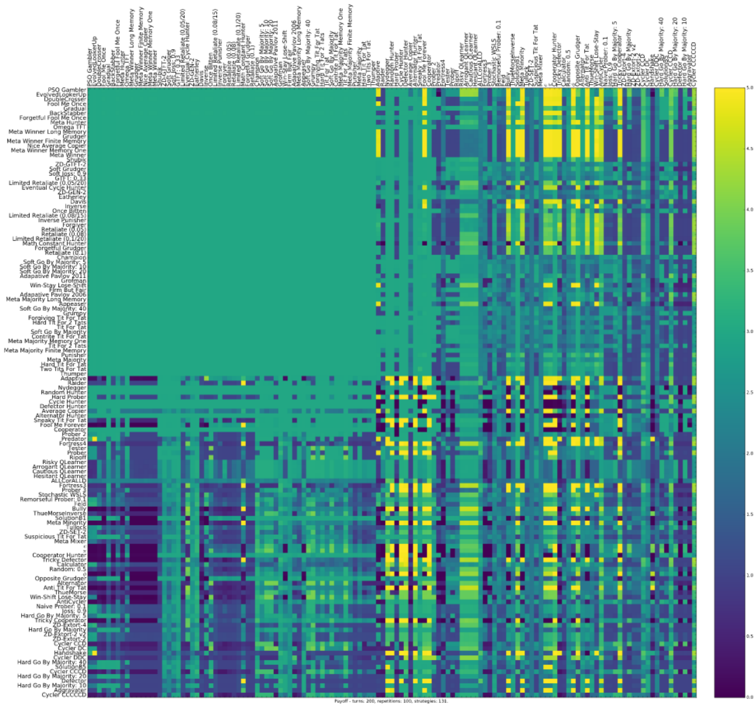
```
>>> import axelrod
>>> strategies = [s() for s in axelrod.ordinary_strategies]
>>> tournament = axelrod.Tournament(strategies)
>>> results = tournament.play()
>>> plot = axelrod.Plot(results)
>>> p = plot.payoff()
>>> p.show()
```

Listing 2.2: A simple set of commands to create a basic tournament. The output is shown in Figure 2.1.

Here are illustrated the results of the last tournament :



(a) Ranked violin plot



(b) Payoffs

Figure 2.1: Result Plots. (a) Ranked violin plot, the mean utility of each player. (b) Payoffs, the pair wise utilities of each player.

More details for the documentation of the library can be found here : `https://axelrod.readthedocs.io/en/latest/index.html`. Because is an open source library it makes it easy to contribute to it and make modifications needed for this dissertation.

Chapter 3

Implementation of spatial tournaments

3.1 Introduction

In this chapter we will discuss some initial experiments and their results. These experiments will be performed using three chosen topologies and two different sizes of tournaments. Emphasis will be given in the analysis of the results and identification of strategies behaviors. In addition, the source code committed to the Axelrod-Python library to implement the spatial topology will be discussed. As well as two important aspects of developing test driven development and version control.

3.1.1 Code Discussion

As analyzed in chapter 2, the Axelrod library uses a `Tournament` class to run any given tournament. The `Tournament` class itself calls upon another class the `Match Generator` which is responsible for generating matches. In the case of a standard round robin, there is a `RoundRobinTournament` class and a `RoundRobinMatches` class that generates matche parameters for each 2 player tuple. The parameters and the indices of the pair are used by the `build_single_match` method. A generator that lives within the match generator class. For a round robin tournament the structure of the code is illustrated in 3.1.

In order for to implement a Spatial topology tournament a similar approach is needed. Firstly a new `Match Generator` class was written. The `SpatialMatches`

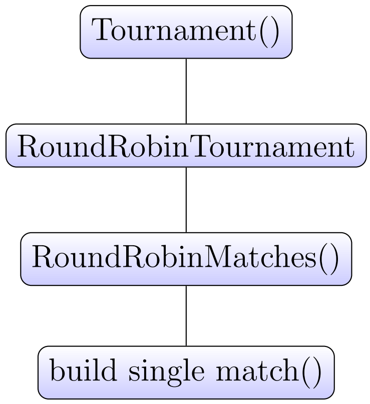


Figure 3.1: Code structure for a Round Robin tournament.

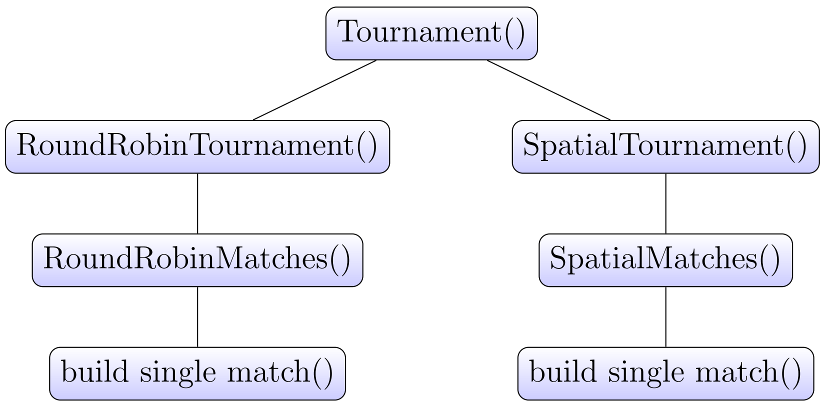


Figure 3.2: Code structure for when Round Robin and Spatial tournaments are implemented.

is a class that generates spatially-structured matches. In these matches, players interact only with their neighbors rather than the entire population. According to [1] graphs can be represented in many different ways, one of which is by lists of edges. Due to a various number of python packages that are used for graph manipulation, we want to keep a more generalized representation of the edges. Thus they will be passed as a list argument and `SpatialMatches` will only create matches between the ending nodes of these edges. Finally the class `SpatialTournament` runs the spatial tournament. A representation of the code structure now, that the spatial tournaments have been added, can be seen in Figure 3.2

The Axelrod-Python library is a Test Driven Development (TDD) software. All the components are automatically tested using a combination of unit, property and integration tests (using `travis-ci.org`). Once a new feature is added to the library, corresponding test must also be written. The test are used to ensure compatibility and ensure that we get the expected results. In [23] Percival explains from his personal experiences the importance of TDD and having tests for every single line of code. For the spatial tournament new unit test have been added. Unit tests are used to invoke a unit of work and check that the behavior is expected. A unit of work can be any single logical functional use in the system. In summary, unit tests help to write clean and bug free code [23]. The unit tests for the `SpatialTournament` can be found below 3.4.

```
class TestSpatialTournament(unittest.TestCase):

    @classmethod
    def setUpClass(cls):
        cls.game = axelrod.Game()
        cls.players = [s() for s in test_strategies]
        cls.test_name = 'test'
        cls.test_repetitions = test_repetitions
        cls.test_turns = test_turns
        cls.test_edges = test_edges

    def test_init(self):
        tournament = axelrod.SpatialTournament(
            name=self.test_name,
            players=self.players,
            game=self.game,
            turns=self.test_turns,
            edges=self.test_edges,
            noise=0.2)
        self.assertEqual(tournament.match_generator.edges, tournament.edges)
        self.assertEqual(len(tournament.players), len(test_strategies))
        self.assertEqual(tournament.game.score(('C', 'C')), (3, 3))
        self.assertEqual(tournament.turns, 100)
        self.assertEqual(tournament.repetitions, 10)
        self.assertEqual(tournament.name, 'test')
        self.assertTrue(tournament._with_morality)
        self.assertIsInstance(tournament._logger, logging.Logger)
        self.assertEqual(tournament.noise, 0.2)
        anonymous_tournament = axelrod.Tournament(players=self.players)
        self.assertEqual(anonymous_tournament.name, 'axelrod')
```

Listing 3.3: Source code for TestSpatialTournament class, which is for testing the spatial tournaments.

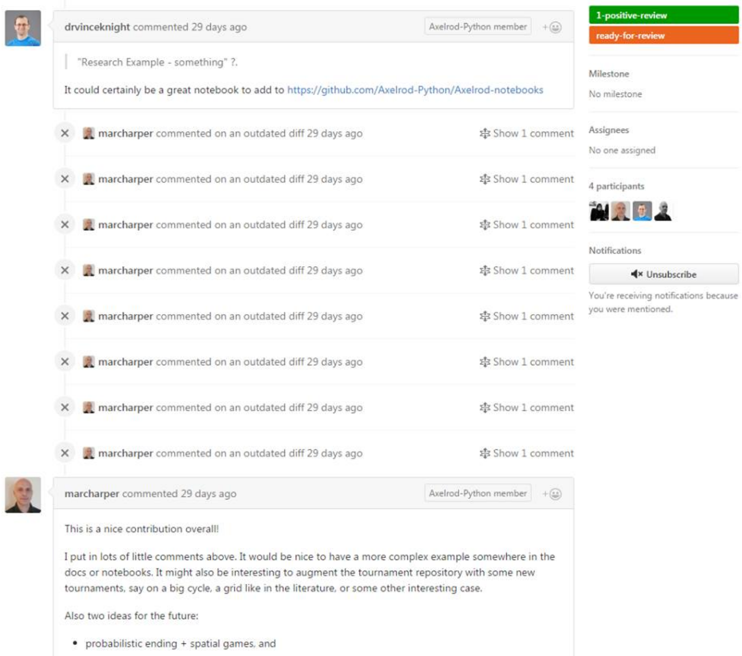
`TestSpatialTournament()` it a simple class for a unittests written for the spatial tournaments. In 3.4, whether the values of the attributes were passed correctly is being tested. Also whether the scores are as anticipated. In 3.3, another test is shown. Here the results of a complete spatial tournament are compared to those of a round robin one.

```
@given(strategies=strategy_lists(strategies=deterministic_strategies,
                                 min_size=2, max_size=2),
           turns=integers(min_value=1, max_value=20))
    def test_complete_tournament(self, strategies, turns):
        """
        A test to check that a spatial tournament on the complete multigraph
        gives the same results as the round robin.
        """
        players = [s() for s in strategies]
        # edges
        edges=[]
        for i in range(0, len(players)) :
            for j in range(i, len(players)) :
                edges.append((i, j))
        # create a round robin tournament
        tournament = axelrod.Tournament(players, turns=turns)
        results = tournament.play()
        # create a complete spatial tournament
        spatial_tournament = axelrod.SpatialTournament(players, turns=turns,
                                                       edges=edges)
        spatial_results =  spatial_tournament.play()
        self.assertEqual(results.ranked_names, spatial_results.ranked_names)
        self.assertEqual(results.nplayers, spatial_results.nplayers)
        self.assertEqual(results.nrepetitions, spatial_results.nrepetitions)
        self.assertEqual(results.payoff_diffs_means,
                                         spatial_results.payoff_diffs_means)
        self.assertEqual(results.payoff_matrix,
                                              spatial_results.payoff_matrix)
        self.assertEqual(results.payoff_stddevs,
                                             spatial_results.payoff_stddevs)
        self.assertEqual(results.payoffs, spatial_results.payoffs)
        self.assertEqual(results.cooperating_rating,
                                         spatial_results.cooperating_rating)
        self.assertEqual(results.cooperation, spatial_results.cooperation)
        self.assertEqual(results.normalised_cooperation,
                                     spatial_results.normalised_cooperation)
        self.assertEqual(results.normalised_scores,
                                          spatial_results.normalised_scores)
        self.assertEqual(results.good_partner_matrix,
                                        spatial_results.good_partner_matrix)
        self.assertEqual(results.good_partner_rating,
                                        spatial_results.good_partner_rating)
```

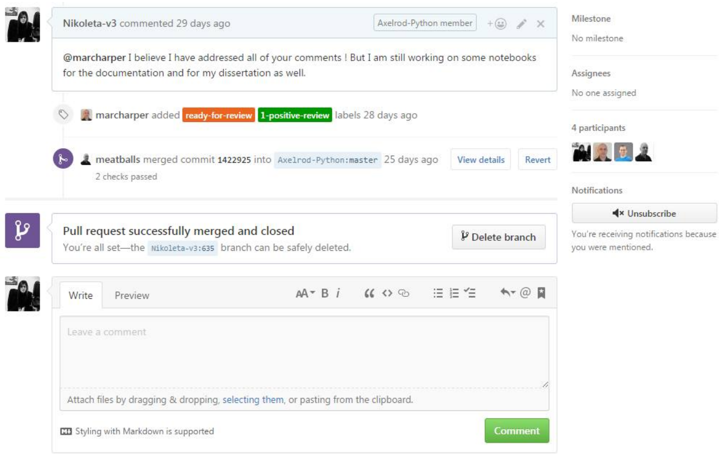
Listing 3.4: Source code for testing a complete spatial tournament.

The source code for the spatial tournaments and the tests can be found in the Axelrod-Python library. The library is available at `https://github.com/Axelrod-Python`, which is a hosted git repository. This introduce us to another important aspect of software development, the version control system (VCS). [23] stated that TDD and VS go hand in hand. VCS, is a tool that manages and tracks different versions of software [30]. Software code is expanding continuously, thus it can be an enormous complicated system. Lines of codes are deleted, implementing errors occur and programmers forget. Thus, having the ability to go back to any given point can be very important. Git is a specific powerful and famous VSC. It was invented by Linus Torvalds and was span to to life in April 2005.

The Axelrod-Python library is a project which includes numerous contributors. Any given addition to the library has to be submitted by a pull request. Each submission is then reviewed by the members. This applies to the spatial tournament as well. In Figure conversation containing comments and corrections can be seen.



(a) Comments and corrections.



(b) Merging a pull request after reviews.

Figure 3.3: On line communication and reviews, through GitHub, with members of the Axelrod-Python library.

O.Campbell and M.Harper are the main contributors of the library, alongside V.Knight. Their work, in company of other contributors included myself, had an impact on the Game Theory society. The project involves various research topics of the IPD, such as Noise tournaments, Probabilistic ending and Spatial tournaments. Strategies from different works and authors and the capability of reproducing any of these given tournaments. A publication was achieved earlier in 2016 [12].

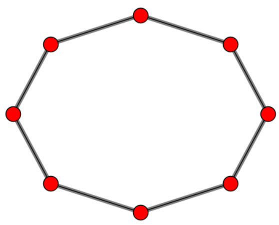
In the next section an overview and a summary of the initial experiments performed is given.

3.1.2 Experiment and the three topologies

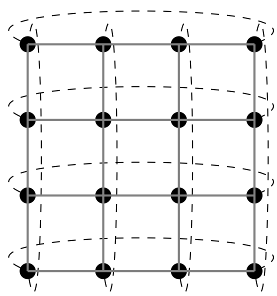
In this chapter three simple spatial topologies are considered, all with deterministic neighborhood size. These will be used to begin to understand how topology can affect the outcome of tournaments and which strategies tend to perform well. The three topologies considered are well represented in the literature [?]:

- A cyclic network: the neighbourhood size is 2.
- A periodic lattice: the neighbourhood size is 4.
- A complete graph: the neighbourhood size if $N-1$ (where N is the number of total strategies. This corresponds to a round robin tournament.

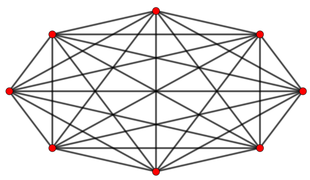
Figure 3.4, shows an example of all the aforementioned topologies.



(a) Cyclic network.



(b) Periodic lattice with degree 4 network.



(c) A complete or round robin network.

Figure 3.4: Network topologies.

For each topology, a fixed number of strategies out of the 132 of Axelrod-Python library are chosen randomly. The number of strategies define the size of the tournament. In the experiments the tournament size can be either five or fifty.

Subsequently, the strategies are allocated on the graph, based on the topology, and they compete with their neighbors on a IPD tournament. For the cyclic and lattice topology, once the first game is complete, the strategies are randomly shuffled and allocated on the graph again. This aims to ensure that their particular position on the network is taken in to account. This shuffle is repeated 10 times. The selection of strategies is repeated 100 times and each tournament of an IPD consists of 200 turns and 10 repetitions. 1 illustrates these rules.

Algorithm 1 Simple Experiments Rules

```
procedure RUNNING EXPERIMENT
loop:
    for i ← 0 to 100 do
        player ← random.strategies.
        for i ← 0 to 10 do
            G ← create.graph.
            edges ← G.egdes
            results ← play.tournament. loop.
```

The Axelrod tournaments themselves make usage of match memory and CPU power and by adding these additional rules to the tournaments was only increasing in usage. For that reason all the tournaments and their results are run in Raven. Raven is the super computer of Cardiff University. All the scripts and pbs file, for communicating with Raven, can be found on github : `https://github.com/Nikoleta-v3`.

For the data preparations and analysis the following libraries have been used:

- Numpy 1.11.1:
- Pandas 0.18.1:
- Matplotlib 1.5.1:
- Statsmodels 0.6.1:
- Networkx 1.11:

Furthermore, some simple functions for measures such as neighborhood scores were written.

In the next section, an initial analysis of the experiments aforementioned is held. Moving on in section ?? to a more intense analysis. The concept of winning ratio and normalized average score are introduced. Finally, summarizing the results in 3.3, offering further avenues of research.

3.1.3 Initial Analysis

For each of the tournament run, the following parameters are recorded:

- players list
- seed
- parameter
- player index
- player name
- cooperating ratio
- degree
- neighbors
- neighborhood size
- ranking
- scores
- normalized scores
- average score
- R
- P
- S
- T
- connectivity
- clustering
- cliques
- neighbors scores
- normalized neighbors score
- normalized average neighborhood score

This section will describe the findings of a basic statistical analysis of these records.

For the spatial tournaments, for both tournament sizes (5 and 50) 5Also, we have so far used the we achieved 1000 tournaments. Containing 100 different strategy sets, where each the exists for 10 tournaments.

Moreover, for the cyclic experiment we can see in Table 3.1, that degree is fixed at 2 and the payoffs are fixed to $R = 3, P = 1, S = 0, T = 5$ for both tournament sizes. For tournaments of size 5, the mean average score is 2.45, with a minimum value of 0.0175 and a maximum value of 4.95. The mean average score of the neighbors score is 980.95 with a standard deviation of 219.87. Moreover the minimum value is set at 141.55 and the maximum at 1756.50. The mean average score does not seem to differ for size equal 5, which is at 2.39. Though, in the experiment with a size of fifty a strategy achieved a score of 0. Additionally the average score of the neighbors ranges from 19.30 to 1884.5. Concerning the graph measures, as explained in [chapter 2], clustering coefficient is zero and connectivity fixed to 2.

For the lattice topologies a table that summarizes the data set is shown in Table 3.2. For both size values the payoffs are the same ($R = 3, P = 1, S = 0, T = 5$) and the degree is fixed at 4. For size 5 the mean average score varies between 0.018 and 4.97. The average score of the neighbors varies between 832.67 and 2895.42.

Cyclic	tournament size 5 and 50			tournament size 5			tournament size 50		
	(R,P,S,T)	degree	connectivity	average score	average neighbors score	clustering	average score	average neighbors score	clustering
mean	(3,1,0,5)	2.0	2.0	2.45	980.95	0.00	2.39	957.23	0.00
std	(0,0,0,0)	0.0	0.0	0.74	219.87	0.00	0.77	231.32	0.00
min	(3,1,0,5)	2.0	2.0	0.01	141.55	0.00	0.00	19.30	0.00
max	(3,1,0,5)	2.0	2.0	4.95	1756.50	0.00	5.00	1884.50	0.00

Table 3.1: Summary table for topology circle.

Much higher than both the cyclic experiments achieved. This is understood to be based on the fact that the number of neighbors is now doubled. For size 5 the mean score is 0.57 and the mean average neighbor score 2.45. The clustering coefficient is 1 and for size 50 is 0.5. This shows that in the lattice example the strategies tend to create groups.

Lattice	tournament size 5 and 50			tournament size 5			tournament size 50		
	(R,P,S,T)	degree	connectivity	average score	average neighbors score	clustering	average score	average neighbors score	clustering
mean	(3,1,0,5)	4.0	4.0	2.45	1958.56	1.0	2.39	1912.74	0.5
std	(0,0,0,0)	0.0	0.0	0.57	287.63	0.0	0.59	268.37	0.00
min	(3,1,0,5)	4.0	4.0	0.52	1059.77	1.0	0.01	832.67	0.5
max	(3,1,0,5)	4.0	4.0	4.24	2518.70	1.0	4.97	2895.42	0.5

Table 3.2: Summary table for the lattice topology.

Finally, for the round robin tournaments, 100 tournaments were performed for both sizes. Parameters such as neighborhood size and neighbor's score were not computed for the round robin topology. This is because all players interact with each other thus there were not any additional information to be gained. In Table 3.3, the average score the strategies achieved in this topology for both sizes is shown. In a tournament of size 50, the mean average score is 2.39 with a standard deviation of 0.335.

Round Robin	tournament size 5	tournament size 50
	average score	average score
mean	2.447105	2.393220
std	0.576014	0.335552
min	0.527500	1.523673
max	4.245000	3.339592

Table 3.3: Summary table for round robin topology.

In this section the structure of the source code for implementing the Spatial Tournament, by adding to the Axelrod-Python library was analyzed. Furthermore, now that the code is usable various experiments were conducted with different topologies and number of players participating in each tournament. An overview of the data sets produced was done but now in the following sections some more analysis on the results will be performed.

3.2 Analyzing the effect of the topologies

In the data all 132 strategies of Axelrod-Python library have participated at least in one tournament. Not all strategies participated in an equal number of tournaments. Forcing the strategies to perform in a uniform number is not an option because the random effect is needed for validation of the results. But one could argue that by participating in a larger number of tournaments, the probability of winning increases as well. Thus, for a measure of performance, instead of using the number of tournaments won, the analysis will be using the ratio of wins in subsection 3.2.1. The winning ratio is defined as the number of tournaments a strategy was ranked first divided by the number of tournaments the strategy competed in. Additionally, the normalized average score each strategy achieved will also be studied in subsection 3.2.2. Mainly to study the variation, helping with conclusions on the performance of a strategy. Lastly in subsection 3.2.3, having all different factors such as degree, neighbors, clustering, scores etc building a regression model could point out effects on the results.

3.2.1 Winning Ratio

In the experiment where the strategies compete on a cyclic 124 strategies out of the 132 have a winning ration greater than zero. In other words 8 strategies won no tournaments. These strategies are shown in Table 3.4 :

Strategies list	Description
ALLCorALLD	Simply repeats its last move, and so mimics ALLC or ALLD after round one.
Tricky Cooperator	Almost always cooperates, but will try to trick the opponent by defecting.
ThueMorseInverse	Defects or cooperates according to the Thue-Morse sequence (Inverse of ThueMorse).
SolutionB5	
Hard Tit For 2 Tats	A variant of Tit For Two Tats that uses a longer history for retaliation.
BackStabber	Forgives the first 3 defections but on the fourth,will defect forever. Defects on the last 2 rounds unconditionally.
Prober	Plays D, D, C, C initially. Defects forever if opponent cooperated in moves 2 and 3. Otherwise plays TFT.
Cycler DDC	A player that repeats the sequence DDC indefinitely.

Table 3.4: Strategies with winning ratio 0 in the cyclic experiment with tournament size 5.

In the lattice topology as well, 8 strategies do not have a non-zero winning ratio. Among them Tricky Cooperator, Cycle DDC and AllCorAllD again. The rest strategies and a simple explanation of them can be found in Table 3.5.

In the round robin tournament 61 strategies ranked a winning ratio of zero. Among them Cycler CCD, Tricky Cooperator, Fortress4, Bully, and the whole

Strategies list	Description
Bully	A player that behaves opposite to Tit For Tat, including first move.
Fortress4	A finite state machine player
Hard Go By Majority: 5	A player examines the history of the opponent: if the opponent has more defections than cooperations then the player defects. Here the player has a memory of 5.
Hard Go By Majority: 40	A Hard Go By Majority player, with memory 40.
Sneaky Tit For Tat	Tries defecting once and repents if punished.

Table 3.5: Strategies with winning ratio 0 in the periodic lattice experiment with tournament size 5.

family of Hard Go By Majority.

On the other hand the strategies with the highest winning ratio for each topology are as follows:

- Cyclic topology with a winning ratio of 0.56 ZD-GEN-2, followed by Punisher with 0.55 and Soft Grudger with 0.53
- Lattice topology with a ratio of 0.45 BackStabber and Meta Majority Memory one. Followed by Cycler CCD with 0.44, Limited Retaliate(0.05/20) and Stochastic WSLS with 0.43 winning ratio
- Round Robin topology with a winning ratio of 1 : Raider, Gradual, Limited Retaliate(0.05/20) and BackStabber.

In every topology the highest ranking strategy is different. Only BackStabber seems to be repeated, even so in the cyclic topology it had a winning ratio of zero. BackStabber is one of highest ranking strategies in the overall round robin tournament performed by the Axelrod-Python library. ALLCorALLD, Cycler CCD, Tricky Cooperator and Bully did badly in all three topologies.

When considering tournaments of size 50 no strategies have a zero ratio (in any topology). The number of tournaments participated in tournament size 5 ranges from 10 to 90, where in the tournament size 50 from 270 to 570. The higher the number of tournaments participated the higher probabilities of winning at least one. This could explain the existence of only non zero winning ratios.

The strategies with the highest winning ratio for each respective topologies are as follow :

- Cyclic topology with a winning ratio of 0.038 Soft Go By Majority:10. Followed by Nice Average Copier with 0.037 and e with 0.036. Raider has the fourth higher winning ratio on 0.034

- Lattice topology with a winning ratio of 0.037 Adapative Pavlov 2006. Followed by Fool Me Once with 0.036 and Raider with 0.035. Inverse Punisher had a winning ratio of 0.033, the fourth highest of all
- Round Robin topology with a winning ration of 0.86 PSO Gambler, followed by EvolvedLookerUp with 0.60

In the cyclic and periodic lattice topology the highest winning ration achieved was 0.038. The number is lower than any other experiment due the range of tournaments participated. In the round robin experiment the maximum number of participations was 57, explaining the big difference in the winning ration between these experiments. PSO Gambler that achieved the highest ratio is also the current winner of the Axelrod-Python tournament.

Overall, for the results of all six experiments no similarities stick out. A more appropriate visualization of the top ranking strategies for both sizes are shown in Figures 3.5 3.6 3.7 and Figures 3.8 3.9 3.10. This will improve the understanding on the correlation of the top ranking strategies. For experiments of tournament size 5, 3 pairs for the experiments are illustrated. Overall, the strategies with an average good ranking in all experiments were Punisher, Raider and BackStabber. A similarly approach for experiments of size 50. The overall successful strategies are Nice Average Copier, Raider and Fool Me Once.

Raider is a repeatedly successful strategies for more than one experiment. It is a finite state machine strategy found in [2]. For now Raider strategy seems to be a well performed strategy for any given random situation. Further investigation will be conducted.

Illustration of the ratios for each experiment in asceding order can be found in Appendix A, Figure A.2 and Figure A.1.

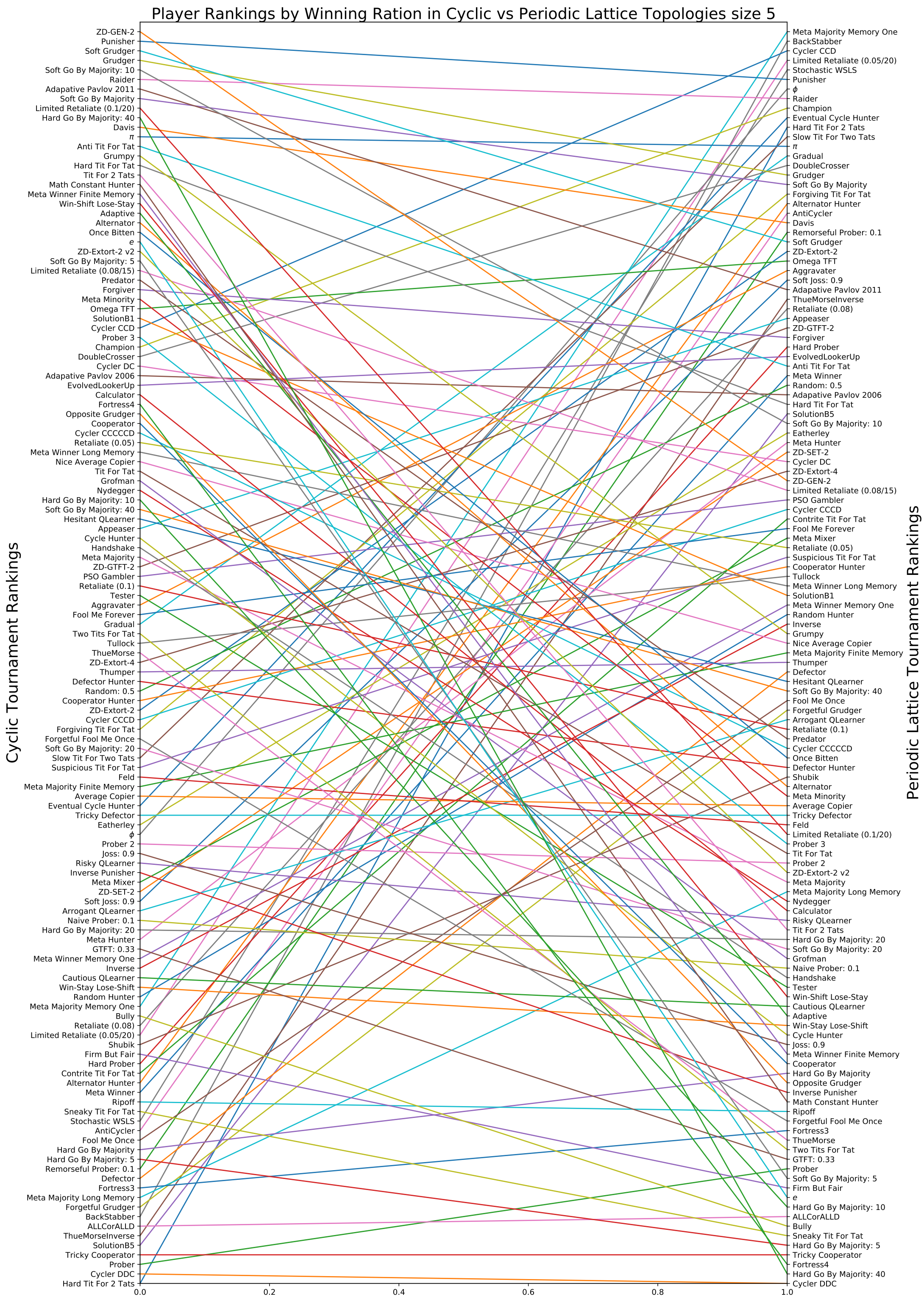


Figure 3.5: Cyclic vs Periodic Lattice Topologies Size 5.

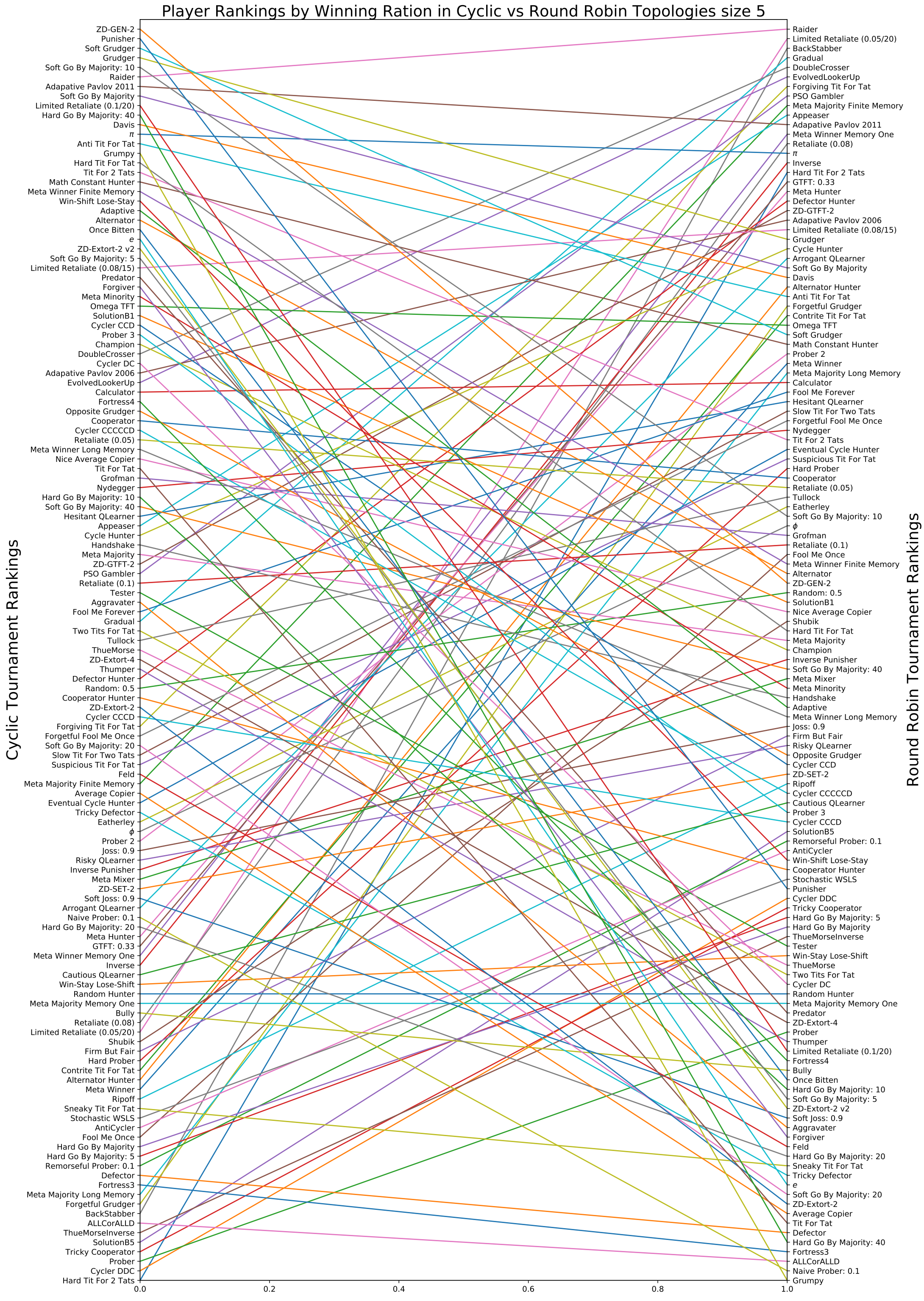


Figure 3.6: Cyclic vs Round Robin Topologies Size 5.

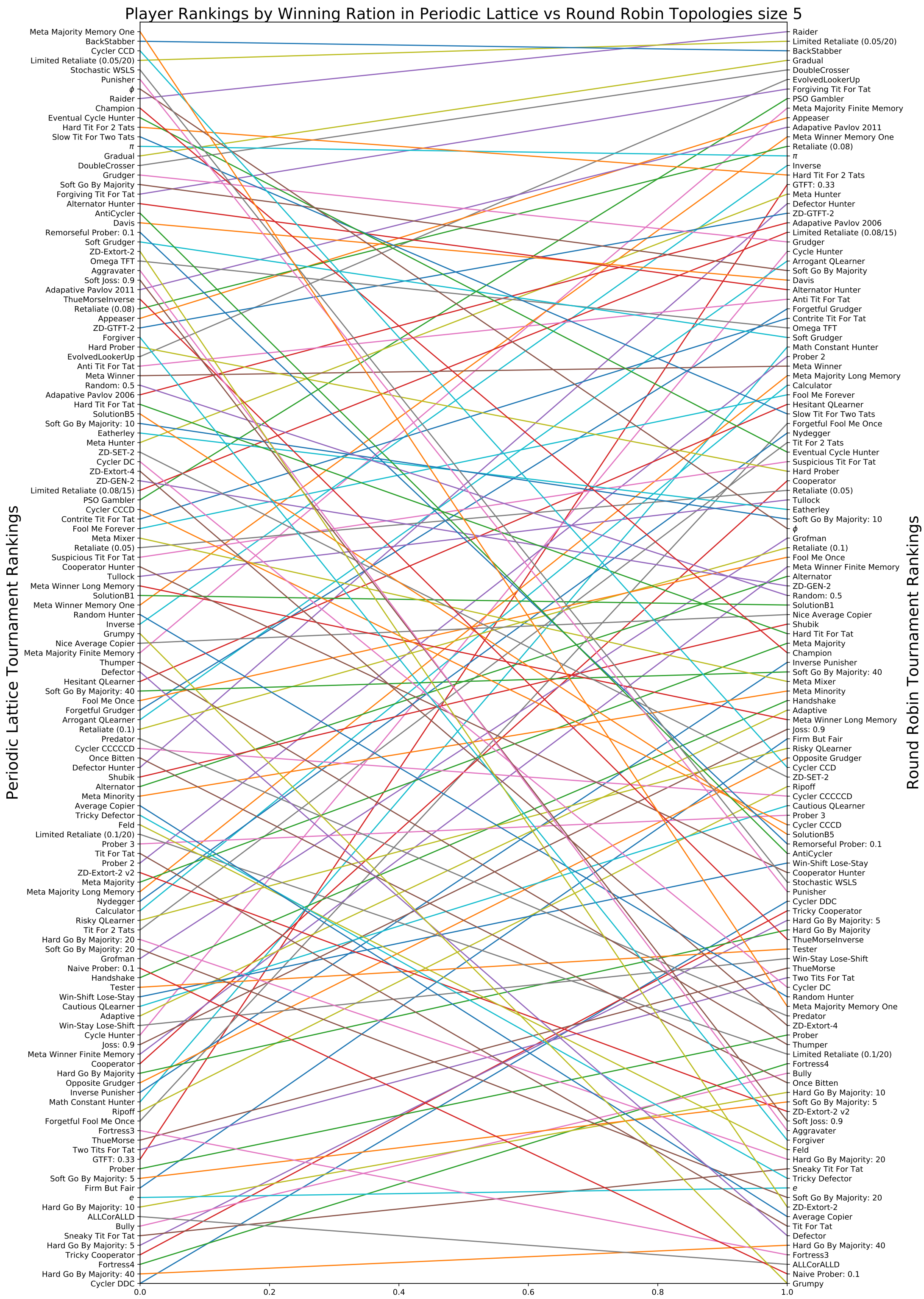


Figure 3.7: Periodic Lattice vs Round Robin Topologies Size 5.

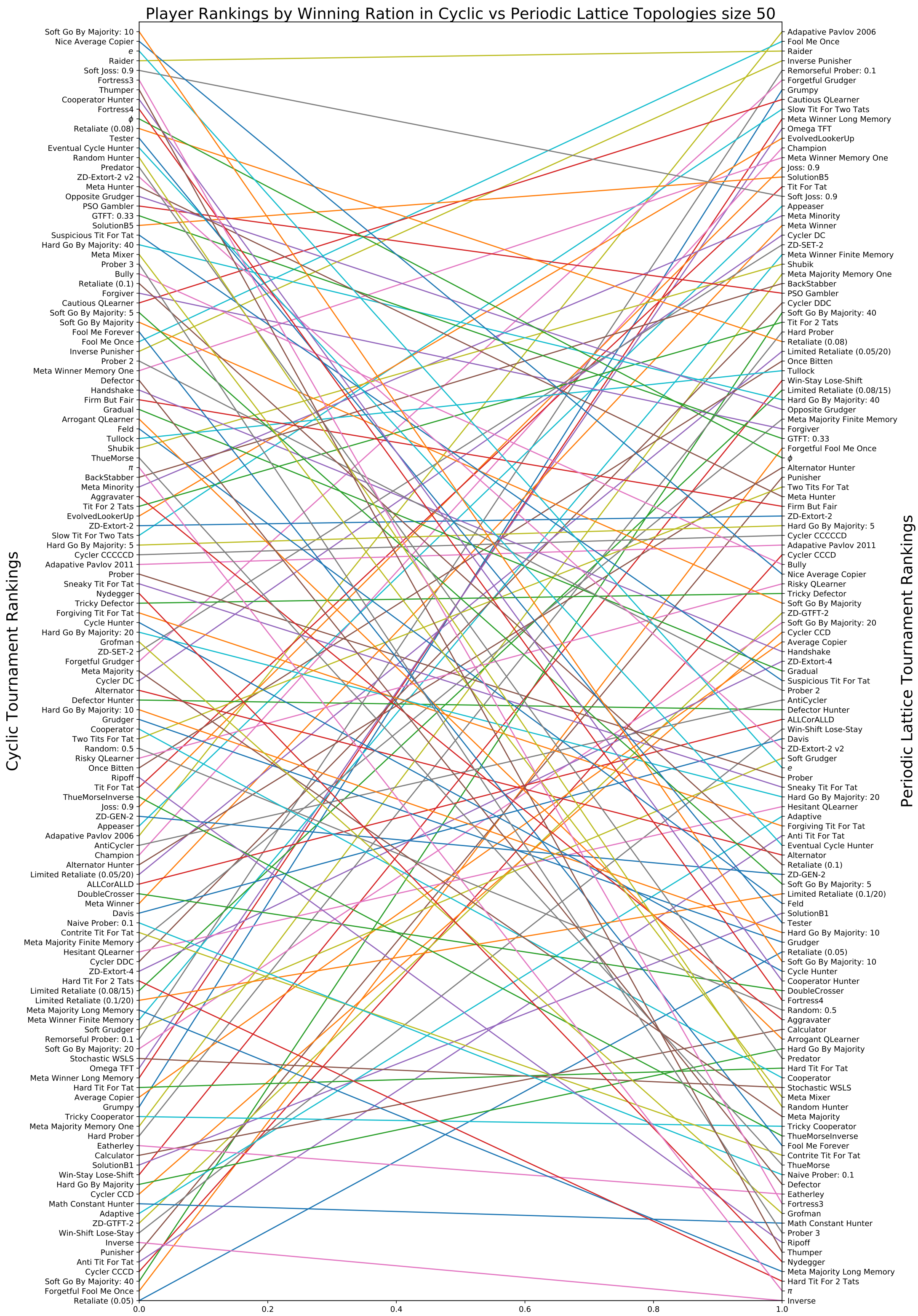


Figure 3.8: Cyclic vs Periodic Lattice Topologies Size 50.

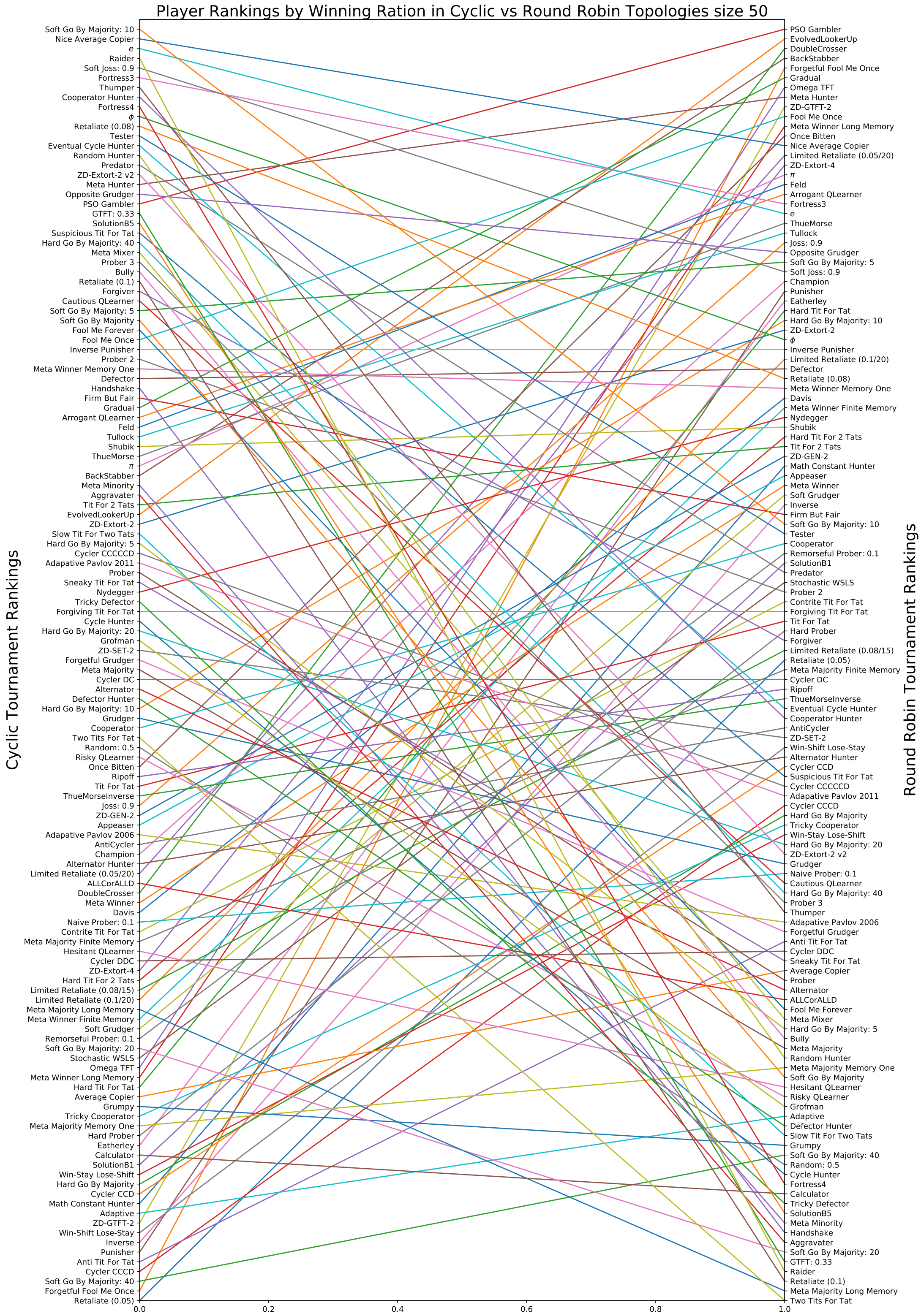


Figure 3.9: Cyclic vs Round Robin Topologies Size 50.

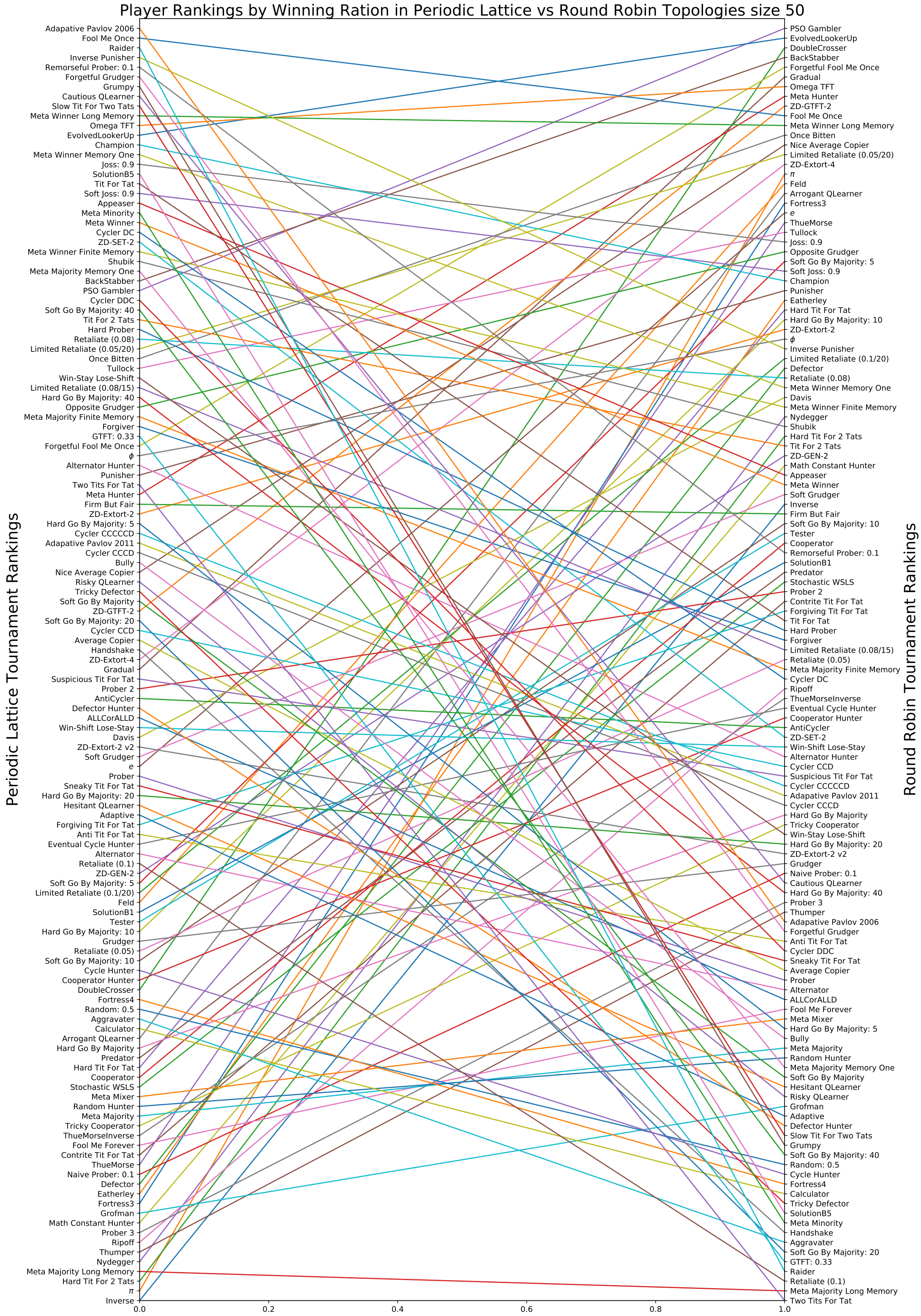


Figure 3.10: Periodic Lattice vs Round Robin Topologies Size 50.

The strategies can be classified based on the number of tournament participations. For tournament size 5, 9 classes have been created for the cyclic and lattice, [10, 20, 30, 40, 50, 60, 70, 80, 90]. For tournament size 50, [270, 280, 290, 310, 320, 330, 340, 350, 360, 370, 380, 390, 400, 410, 420, 430, 440, 450, 460, 470, 570]. Round robin topology experiments were classified as [1, 2, 3, 4, 5, 6, 7, 8, 9] and [27, 28, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 57], respectively. The winning rations were plotted against participation number in Figures 3.11 and 3.12. An ANOVA analysis was used to analyze the importance of the differences in the winning ratios among the participating groups.

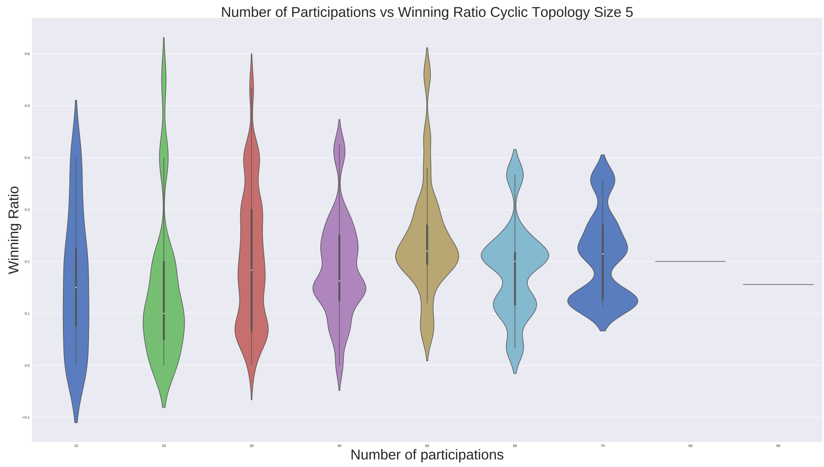
In Figure 3.11, for all topologies, a significant variation does seems to be pointed out. Though the Anova results 3.6 indicate that for the topologies cyclic and periodic lattice the difference of rations between the participations group is significant (p values are lower that 0.05). Equivalent, in 3.12 for tournament size 50. For topologies cyclic and periodic lattices there is variation. Compared to that of a round robin topology, where there seems to be no variation at all. In the Anova results from Table 3.7, p value of round robin and cyclic topology are greater that 0.05. For a lattice topology is below 0.05. and, therefore, there is a statistically significant difference. Thus, participating number affects the winning ratios for only two out of the six experiments. The specific groups that differed are not clear. Post-hoc tests could be performed but for now is not needed.

Topology	Anova results tournament size 5		
	Sum of Squares	F ratio	p value
Cyclic	0.41	33.83	6.379416e-09
Periodic Lattice	0.04	4.24	0.039
Round Robin	0.008	0.18	0.66

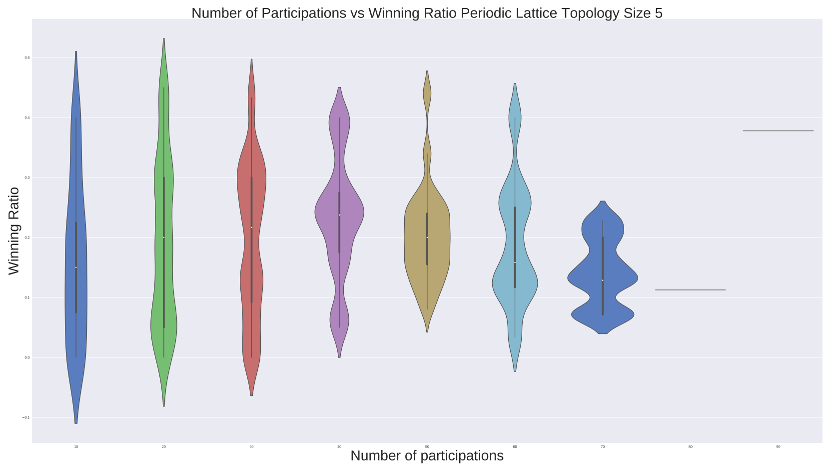
Table 3.6: Anova results tournament size 5

Topology	Anova results tournament size 5		
	Sum of Squares	F ratio	p value
Cyclic	0.000056	1.12	0.28
Periodic Lattice	0.045	1110.45	7.777650e-241
Round Robin	0.00098	0.10	0.74

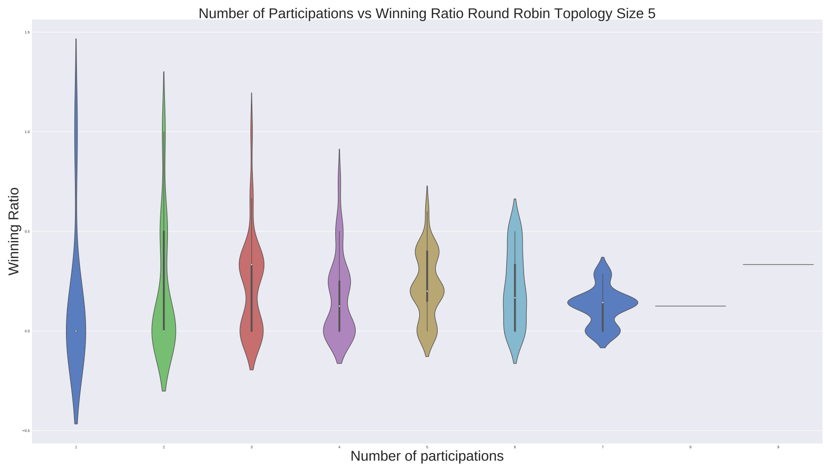
Table 3.7: Anova results tournament size 50



(a) Winning Ration vs Number of Participations Cyclic Tournament Size 5.

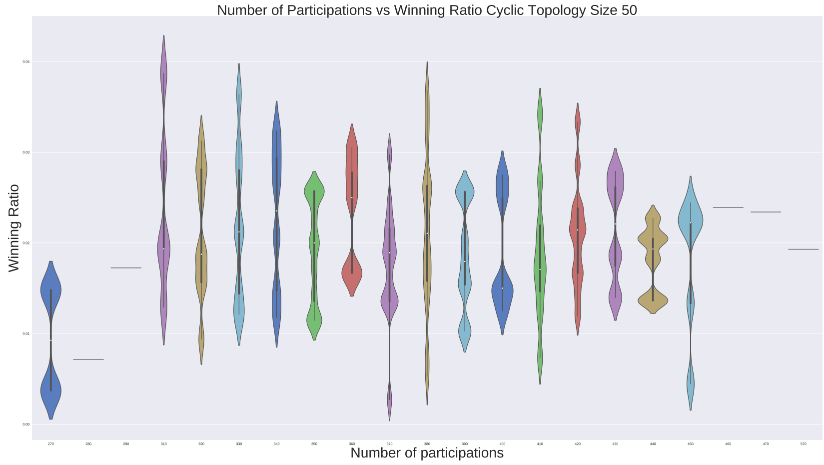


(b) Winning Ration vs Number of Participations Periodic Lattice Tournament Size 5.

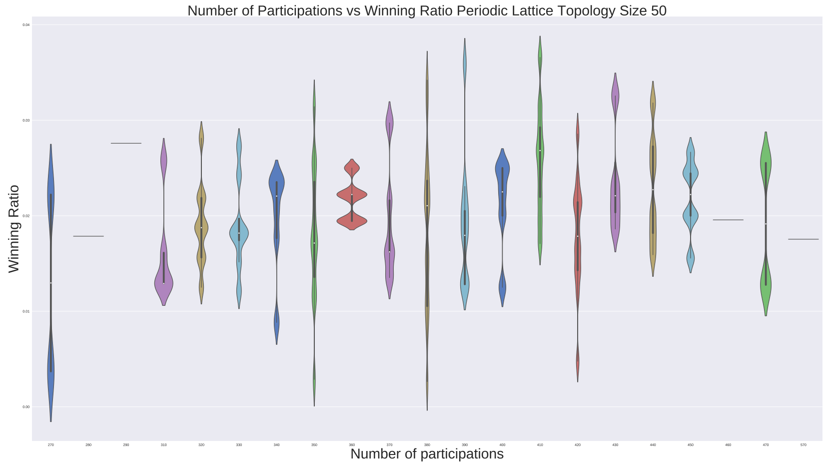


(c) Winning Ration vs Number of Participations Round Robin Tournament Size 5.

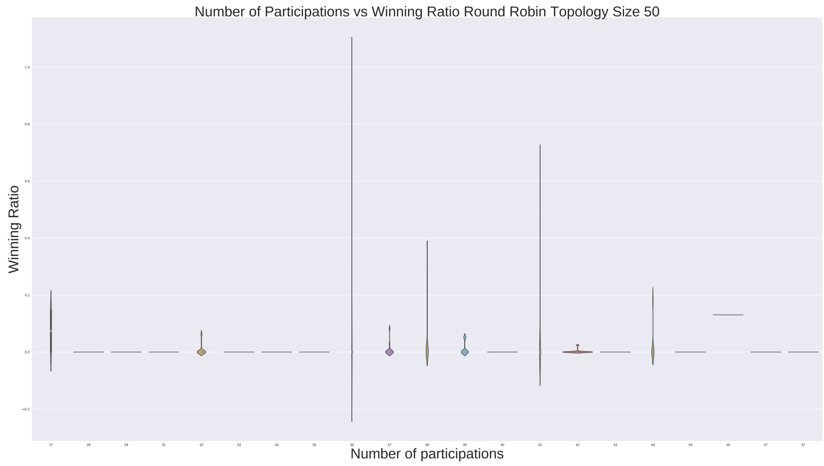
Figure 3.11: Winning Ration vs Number of Participations Tournament Size 5.



(a) Winning Ration vs Number of Participations Cyclic Tournament Size 50.



(b) Winning Ration vs Number of Participations Periodic Lattice Tournament Size 50.



(c) Winning Ration vs Number of Participations Round Robin Tournament Size 50.

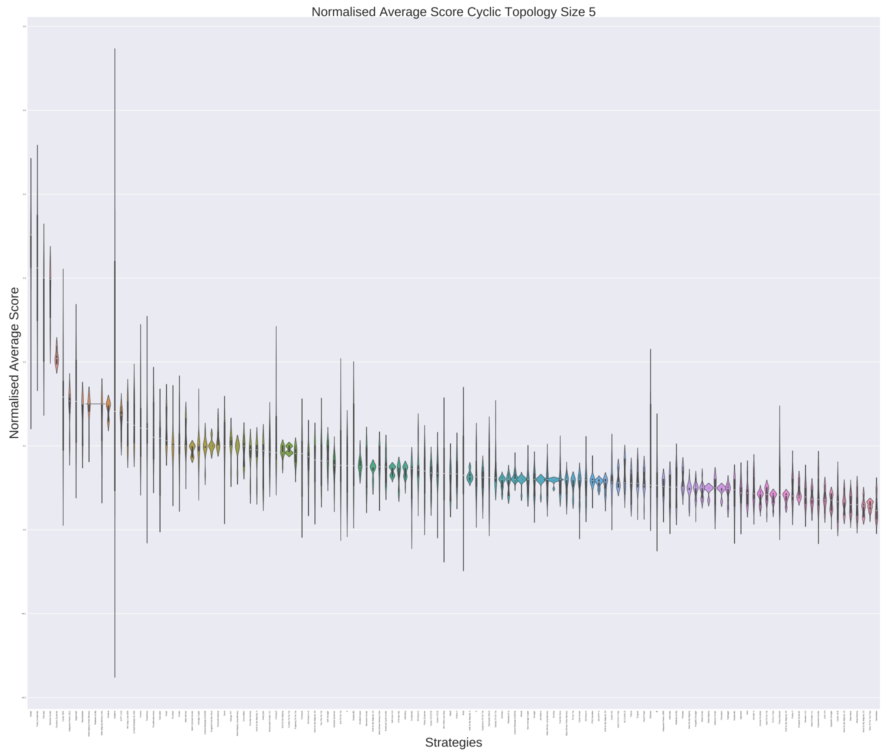
Figure 3.12: Winning Ration vs Number of Participations Tournament Size 50.

In this subsection the winning ratio has been covered. The winning ratio is defined as the number of ranking first in a tournament divided by the number you participated in one. The strategies were ranked based on their winning ratio in each of the six experiments. Line plots that illustrate the devolution in the strategies rankings, were conducted. Indicating that Raider could be an overall well performed strategy. Reasons as to why are being investigated. For example the correlation of the winning ratio and number of participation has been examined. Though they were not many significant results, connecting winning ratio and participation. In the next subsection, the Normalized Average Score achieved by the strategies will be investigated.

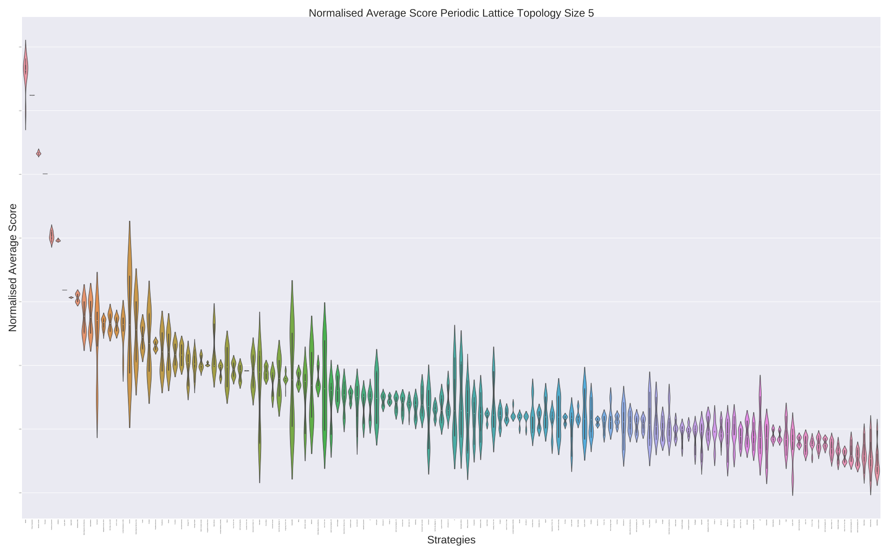
3.2.2 Normalized Average Scores

The normalized average score is calculated by dividing the average score per turn per opponent of each strategy with their participation counts. Then the average score is plotted against the strategies. As shown in both Figures 3.13 and 3.14.

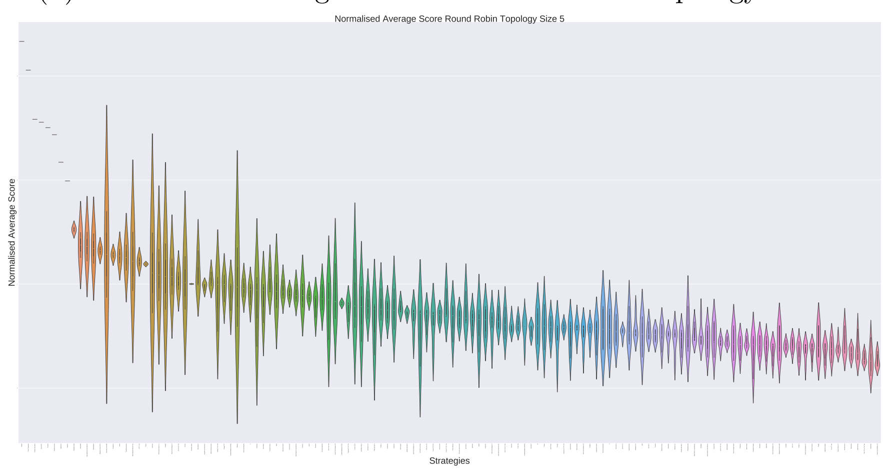
For all six experiments the normalized average score seems to have a lot of variation. Which indicates each strategy performed differently at given points of the same experiment. These could be because of the opponent or the whole neighborhood.



(a) Normalized Average Score Cyclic Topology Size 5.

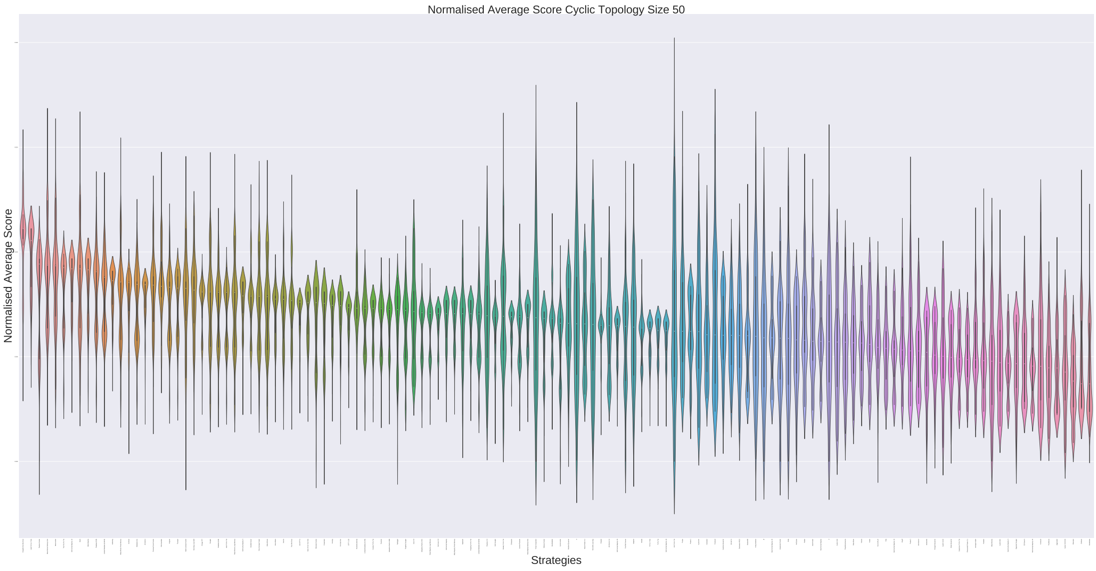


(b) Normalized Average Score Periodic Lattice Topology Size 5.

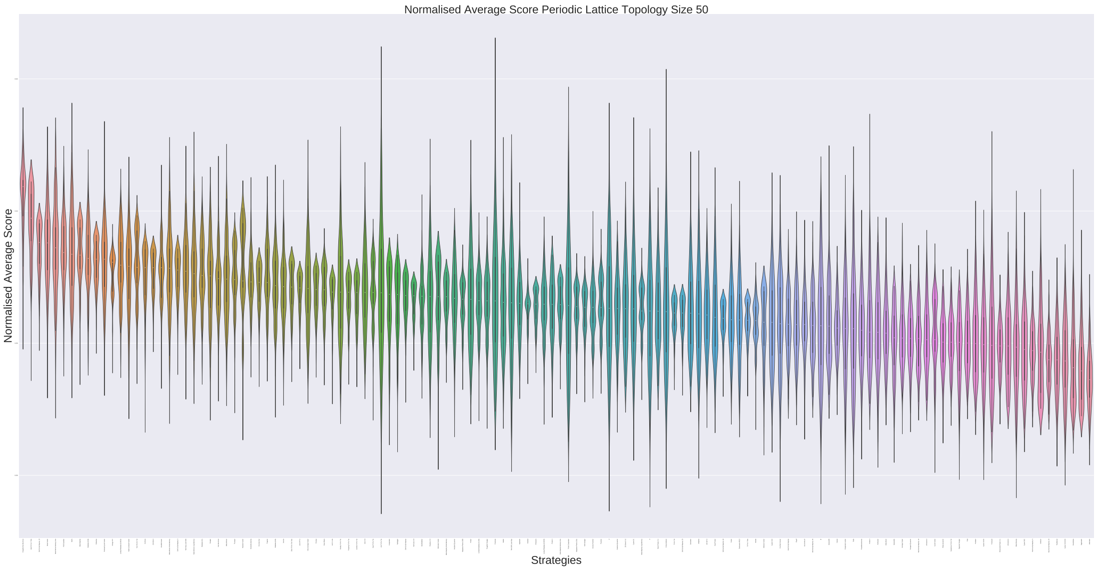


(c) Normalized Average Score Round Robin Topology Size 5.

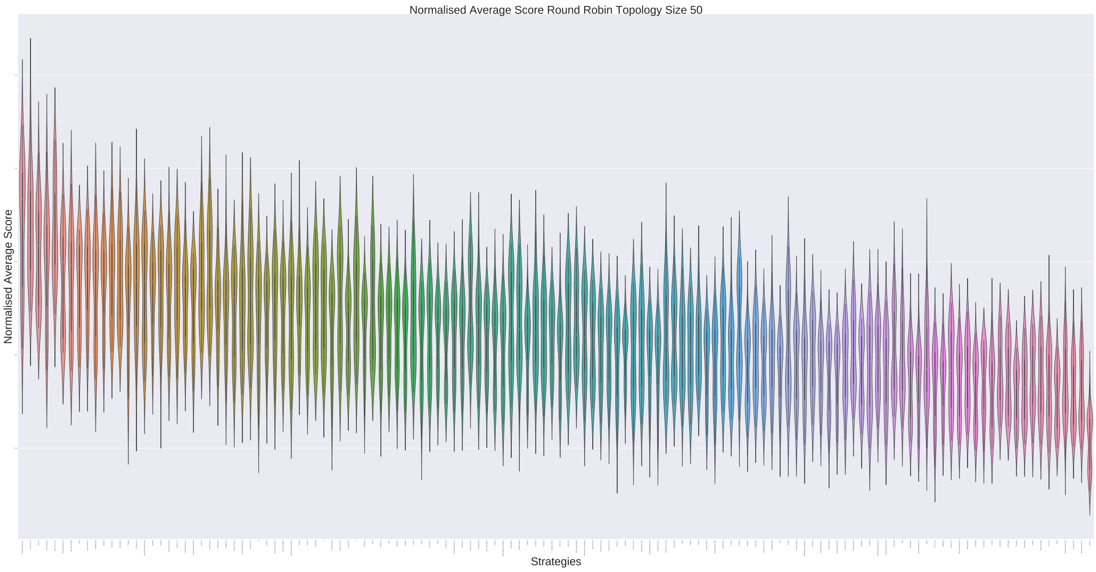
Figure 3.13: Normalized Average Score for all Three Topologies Size 5.



(a) Normalized average score cycle s=50.



(b) Normalized average score cycle s=50.



(c) Normalized Average Score Round Robin Topology Size 50.

Figure 3.14: Normalized Average Score for all Three Topologies Size 50.

Not many conclusions can be made out of this. The high variation could indicate that the results are completely random. Unfortunately, this could mean that for any random situation there does not seem to be a strategy that could achieve a particularly high score although as described in Section 3.2.1 it is possible to ensure a good performance. In the following subsection, a linear regression model is build for more conclusions.

3.2.3 Regression Analysis

Finally, a common methodology when investing factors as predictors is building a regression model. In this subsection two regression models are used to identify any factor that can explain the winning ratio and average score of a strategy. The round robin topology is not included in this subsection analysis as this investigation aims to understand the effect of network topology.

The first model was build with the winning ratio as the depended variable, shown here:

$$\begin{aligned} \text{winning.ratio}_t = \alpha &+ \beta_1 \text{degree}_t && (3.1)\\ &+ \beta_2 \text{average.neighborhood.score}_t && (3.2)\\ &+ \beta_3 \text{clustering}_t && (3.3)\\ &+ \beta_4 \text{number.of.participations}_t + \epsilon && (3.4) \end{aligned}$$

Size	Topology	Intercept		degree		average neighborhood score		clustering		participations		R-square
		coef	p	coef	p	coef	p	coef	p	coef	p	
size 5	Cyclic	0.0344	0.00	0.0688	0.00	3.239e-06	0.651	0.0	NA	0.0006	0.00	0.007
	Lattice	0.0108	0.00	0.0431	0.00	7.447e-06	0.159	0.0108	0.00	-0.0002	0.036	0.001
size 50	Cyclic	0.0043	0.00	0.0087	0.00	-1.386e-06	0.00	0	NA	-8.156e-07	0.216	0.002
	Lattice	0.0008	0.00	0.0031	0.00	-4.549e-07	0.00	0.0004	0.00	2.005e-05	0.00	0.022

Table 3.8: Regression Results for Winning Ratio Model.

From the results it is shown that only degree seems to be a stable significant predictor for all three experiments. With a p value less that 0.001. Average neighborhood score affects only the size 50 experiments. In both experiments, of size 50, it has a negative correlation with the winning ratio. Thus, the better the neighbors score the less score will a strategy achieve.

Periodic lattice experiments are affected by clustering. Both p values are less than 0.05 but are affected with an insignificant amount, less that 0.1. Participations seems to have a negative effect on the lattice size 5 experiment but a positive one for the cyclic size 5 and lattice size 50.

Overall, all R-square values are really low. With the highest R- value of a model being 0.022 for lattice size 50. Thus, the performance of the model can be characterized as insignificant,

The second model using the normalized average score is the following.

$$\begin{aligned} \text{average.normalised.score}_t = \alpha + \beta_1 \text{degree}_t && (3.5)\\ + \beta_2 \text{average.neighborhood.score}_t && (3.6)\\ + \beta_3 \text{clustering}_t && (3.7)\\ + \beta_4 \text{number.of.participations}_t + \epsilon && (3.8) \end{aligned}$$

The model was used to each of the experiments for lattice and cycle topologies individually. The results of models are shown below, Table 3.9 :

Size	Topology	Intercept		degree		average neighborhood score		connectivity		participations		R-square
		coef	p	coef	p	coef	p	coef	p	coef	p	
size 5	Cyclic	0.028	0.00	0.0559	0.00	-3.763e-06	0.043	0.0	NA	-0.0016	0.00	0.457
	Lattice	0.0064	0.00	0.0256	0.00	1.079e-05	0.00	0.0064	0.00	-0.0016	0.00	0.549
size 50	Cyclic	0.0025	0.00	0.0051	0.00	-2.168e-07	0.00	0	NA	-1.602e-05	0.00	0.120
	Lattice	0.0006	0.00	0.0024	0.00	1.033e-06	0.00	0.0003	0.00	-1.601e-05	0.00	0.216

Table 3.9: Regression Results for Average Score Model.

In the output we can see that Degree, average neighborhood score and participations are significant predictors for all the experiments with a p value less than an 0.0001. For the cyclic topology, average neighborhood score and participations have negative coefficients. For example a decrease in participations by one would increase the average score by 0.0016. Degree on the other hand has a positive coefficient and connectivity has no effect at all. Furthermore the model for size 5 has a R-square value of 0.457, thus it only explains 0.4 variation of the data which is quite low. For size 50 it is even lower at only 0.12.

Finally, for the lattice topology only participations have a negative coefficients. Thus the only factor with a reverse influence on average score in the lattice topology. Connectivity is a significant predictor as well with a coefficient 0.0064 and 0.0003 respectively. Though the R-square value is still small, with a value 0.547 and 0.216 respectively. Even if there are predictors with a significant p value, the overall performance of the model is moderate.

For both models and all set of 6 experiments some predictors can e characterized as significant. More than one factors have a p value less than a 0.05, the aplha that has been set. Even so, the coefficients and the overall R square values are quite low. Thus, it can not be said that any of this model can predict the winning ratio neither the average score of a strategy.

3.3 Summary

In this section we will make a summary of all the previous analysis that was made in ?? and list further research that could be performed.

From the analysis that was performed in 3.2.1, the wining ratio for each strategy for all 6 experiments indicated the following :

- For the size 5 experiments, the well performed strategies were Punisher, Raider and BackStabber
- For the size 50 experiments, the well performed strategies were Nice Average Copier, Raider and Fool Me Once
- BackStabber, which is one of the highest ranked strategies in the Axelrod-Python tournament. Did not win a single tournament in the cyclic 5 experiment
- PSO Gambler, the current winner of the Axelrod-Python tournament had the highest winning ratio only in the Round Robin 50 experiment
- For a Round Robin size 50, most of the strategies finished the experiment with winning ratio of zero.
- Raider seem to have successful performance in general

An attempt to find any significant reason as to why these strategies outperform the rest gave the findings below:

- Winning ratio has a significant difference based on the participations number only for the cyclic and lattice topologies of size 5

- There is high variation is the average score of each strategy for all experiments
- Both regression models for the winning ratio and the normalized average score did not return any significant results.

In conclusion, the strategies winning ratio differs in every topology and tournament size. Between the two sizes experiments only Raider seems a repeating strategy in the high ranked winning rations. Even so, valid arguments as to how to successful perform to any given experiment returned zero fundings.

Comparing the results to any other work done in the literature is not possible. Because this kind experiments have not be conducted before. The Axelrod- Python library tournament could be used a measure of comparison. Still two of the best overall strategies of that tournament have only made a brief appearance.

Further actions to the experiments and can be taken for this point on. Producing a more complete data set for each experiment and making a more deep analysis. One could argue that all above experiments were conducted in simple topologies and small number of repetitions. Thus, the next step is to use more complex networks for topologies and conduct a larger number of tournaments.

Chapter 4

Complex Networks Experiments

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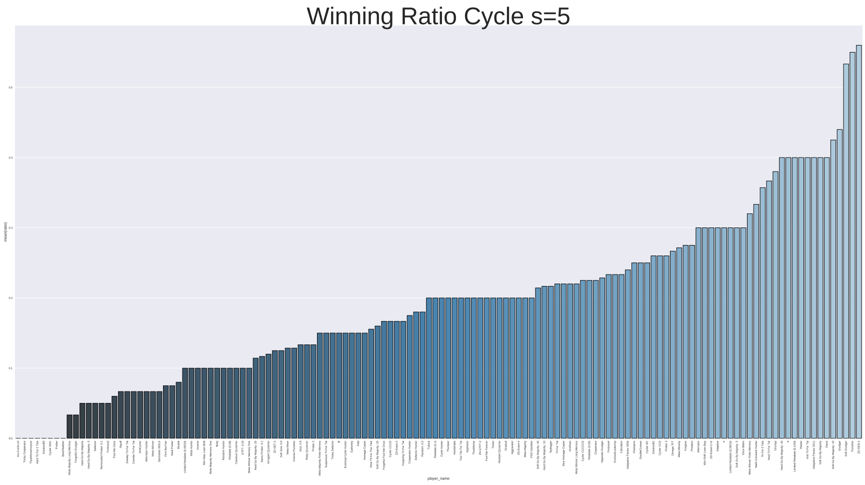
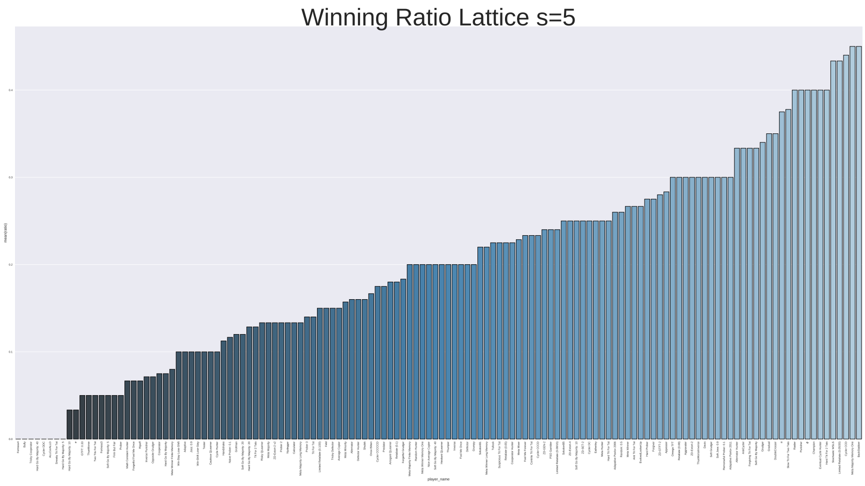
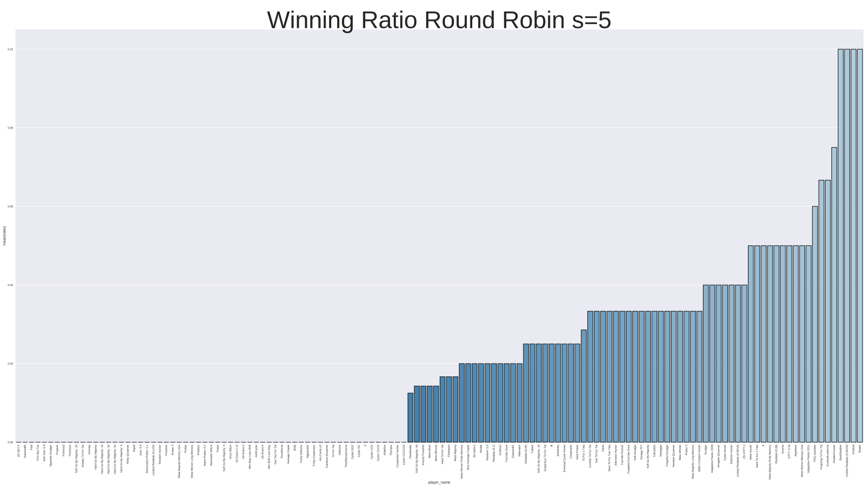


(a) Winning ration cycle s=5.



(b) Winning ration lattice s=5.

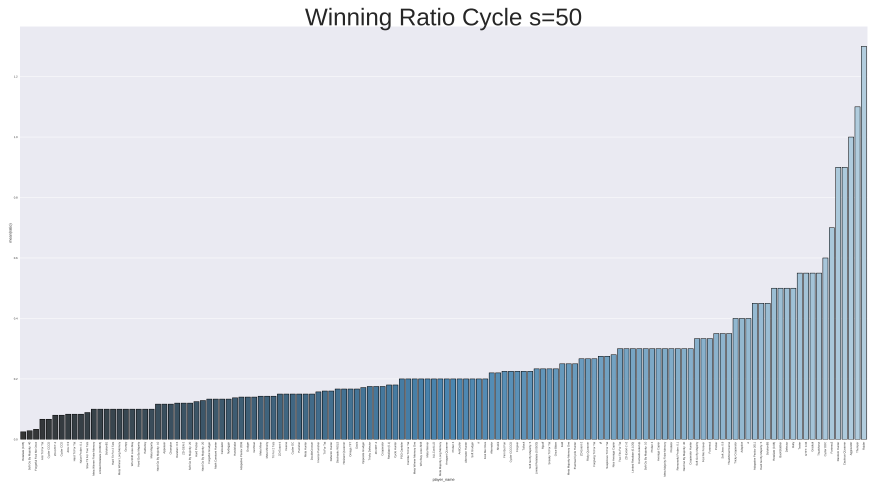


(c) Winning ration round robin s=5.

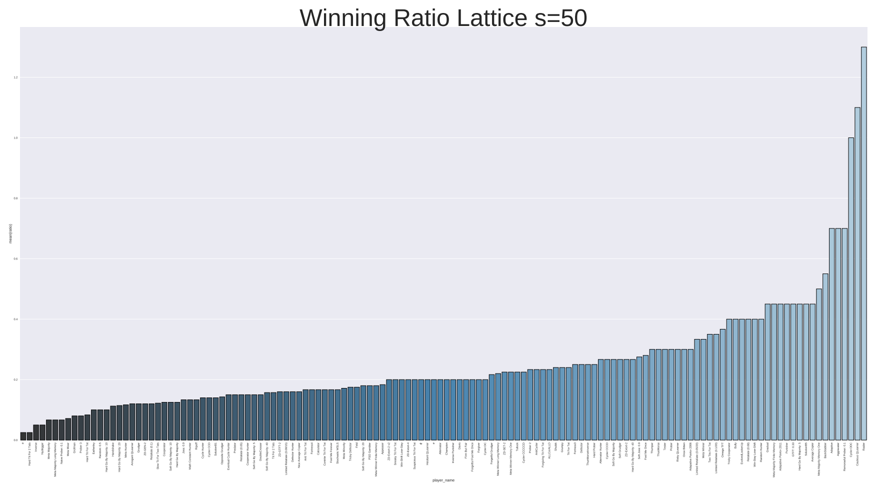
Figure A.1: Winning ratio for all three topologies s=5.

Appendix A

First Appendix Title



(a) Winning ration cyclic s=50.



(b) Winning ration lattice s=50.

Appendix B

Second Appendix Title

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