The Iterated Prisoner's Dilemma

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Executive Summary

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Chapter 1

Introduction

Game theory is a set of analytical tools and solution concepts, which provide explanatory and predicting power in interactive decision situations, when the aims, goals and preferences of the participating players are potentially in conflict, Szabo and Fath (2007) [25]. The Prisoner's Dilemma(PD) is a well known example in Game Theory and in recent years has become the gold standard of understanding evolution of co-operative behavior [15]. Thus, it has been a topic of focus in various fields, such as biology, sociology, ecology and psychology.

In the example of the Prisoner's Dilemma(PD) two criminals have been arrested and interrogated, with no way of communicating, by the police. They are given only two choices, to either cooperate with each other or to defect. Now let us consider that the prisoners would be put back in their cells and would be asked the same question tomorrow. Furthermore, let this happen repeatably. This is referred to as the Iterated Prisoner's Dilemma(IPD) an example that has been a rich source of research material since the 1950s but has earned much interest in the 1980s due to the work done by the political scientist Robert Axelrod [2, 3, 4].

In 1980, Axelrod held the first ever IPD computer tournaments [2, 3], he invited academics from various fields to submit their strategies in computer code. The tournaments were of a round robin topology, the first competition included thirteen strategies, while the second one sixty-four. In both tournaments the strategy Tit for Tat was announced the winner and for many years it was consider to be the most successful strategy. Tit for Tat is a deterministic strategy that will always cooperate in the first round and afterwards it copies the opponents last move.

A large volume of literature emerged on the topic following this, including some criticism about these initial tournaments. Scientists questioned whether the conditions that the first tournament took place favored tit for tat. An argument was that the initial tournaments though they included a 1% chance of players misunderstanding their opponent's move in any round they did not examine noise. Noise is the probability that the player will submit the wrong move. David and Vivian Kraines [11] stated that TFT performed rather poorly when noise was introduced in the tournament. Another aspect, is the payoff matrix which according to Kretz [12] the precise choice of the payoff matrix is relevant to the results.

Furthermore, another aspect needed to be taken into account was the network topology underlying the tournament. In 1992 Nowak's and May's paper [19] spatial tournament are introduced. In which the players are placed on an two-dimensional spatial and allowed to play a game with only the immediate neighbors. Thus, squares that are adjoin. An example of this is shown in Figure ...

Their tournament considered the PD and the players could only defect or cooperate. They provided proof that cooperative behavior can emerge from a PD tournament in spatial topology. Many works on the IPD and spatial tournament were held due to their original paper. Such as [10, 20, 16, 19, 7, 18, 14]. These tournaments use either the PD or IPD and simple to complex strategies.

One can argue that the real life interactions are better represented by spatial tournament because in real life not all players interact with all opponents. Additionally, an interesting aspect of the spatial topology are the results compared to those of a round robin tournament. This dissertation will be focusing on reproducing a spatial tournament with some of the most successful strategies of various tournaments that have been held. For the spatial topology it will use various graphs compared to other works done only using lattices. Concluding how spatial topology, with any given graph, affects the effectiveness of these strategies.

1.1 The Prisoner's Dilemma

The PD was originally formulated in Merril [**M. Flood and Melvin Dresher**], who were working on the Flood-Desher Experiment at the RAND cooperation in 1950. Later in 1950, the mathematician Albert W. Tucker presented the first formal representation of the PD, titled A Two-Person Dilemma in a seminar at Stanford University [9].

A description of the PD, found in [13] is as follows: There are two players that simultaneously have to decide to whether Cooperate (C) or Defect (D) with each other, without exchanging information.

- If both players choose to cooperate they will both receive a reward (R)
- If a player defects and the other cooperates then the defector receives a temptation payoff (T) and the cooperator a sucker payoff (S)
- If both players defect they will both receive a penalty (P)

Figure 1.1 illustrates the payoffs matrix.

		Player II	
		Cooperate	Defect
Player I	Cooperate	***R*=3** / ***R*=3**	***T*=5** / ***S*=0**
	Defect	***S*=0** / ***T*=5**	***P*=1** / ***P*=1**

Figure 1.1: The payoff matrix for the Prisoners Dilemma

Taking into account the assumptions that both players are rational and that there is no way of communication between them. No matter what the other player does, defecting will be their dominant choice as it yields a higher payoff than cooperation. Thus a pure Nash Equilibrium exists when both players defect. Even though, both players would do better if they were to cooperate.Thus creating the dilemma.

Furthermore, for this to hold there are some extra assumptions for the relationship of the four outcomes. The T temptation to defect has to offer the highest payoff for a player and the worst he could get has to be the sucker S. Likewise, the reward for mutual cooperation should exceed that of mutual defection P. Thus the next condition is 1.1:

$$T > R > P > S \tag{1.1}$$

Moreover, it is assumed that the average of T and S is less than the reward for mutual cooperation 1.2 :

$$R > (T + S)/2 \tag{1.2}$$

Same conditions such as rationality, no communication and (1.1), (1.2), apply for the IPD. An IPD is nothing more than a PD were the players interact for an infinite number of times.

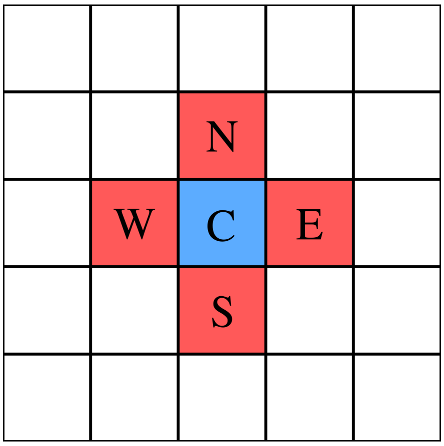
1.2 Problem Description

Axelrod's tournaments set a seed for generations of tournaments in the Prisoner's using computer modeling. Research has shown that by altering the environment of a tournament the effectiveness of some strategies can change radically. An aspect that has been investigated as to how the tournament results can bee affected was the topology. Nowak and May [19] introduced the spatial topology only to set yet another seed in the PD tournaments. Even so, spatial topology still has not been fully explored with only a small number of papers focusing on this specific topology. A goal of this dissertation is to understand the current state of the art in spatial prisoners dilemma tournaments.

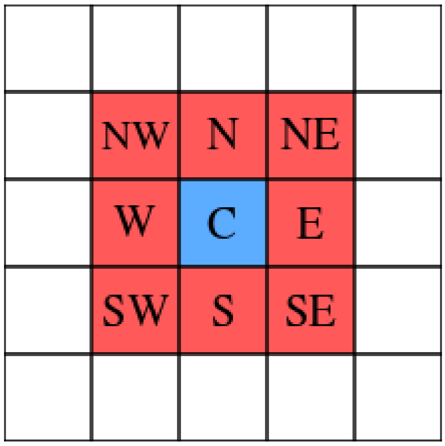
As described in [**Maschler**] the contemporary world is full of networks. That is one of the main reasons this dissertation will be focusing on such topology. Work done at this point consider spatial topology to be that of a square lattice were each edge represents a player which interacting/playing with only his neighbors in their von Neuman or Moore's neighborhood. A von Neuman neighborhood comprises the four cells orthogonally surrounding a central cell where the Moore's neighborhood eight cells. As shown in Figure 1.2.

Some further work address the spatial topology in a variate of graphs [17, 25]. This dissertation will also follow this approach. Szabo and Fth dealt with numerous graphs such as, lattices, small- world, scale free graphs and evolving networks. We will consider a spatial topology to be any given graph where the players are the nodes and only play other nodes that are linked to by an edge.

Another disadvantage of the aforementioned work is that it lacked in terms of best practice of reproducibility [2, 3, 22, 8, 24] . Due the work done by Axelrod library [26]. An open source python Package which allows for the easy reproducibility of experiments. As it allow to reproduce an IPD tournament and chose between



(a) von Neuman's neighborhood



(b) Moore's neighborhood

Figure 1.2: Possible neighborhoods. (a) A von Neuman's where each node has four neighbors. (b) Moore's where each node has eight neighbors.

131 strategies already given by the library. Code written for the purpose of this dissertation has been contributed to the library: see `https://github.com/Axelrod-Python`.

1.3 Structure of Dissertation

This dissertation is organized into 7 Chapters. Proceeding this introduction:

- In Chapter 2 we review previous literature dedicated to the PD/IPD and tournaments that have been conducted, different topologies and evolution.

Chapter 2

Literature Review

Following the initial work done by Axelrod, there are many other papers that have tried to tackle the PD and make their conclusions on cooperation in both a theoretical and real life setting. In this chapter a review of some of this work done in the IPD competitions, in spatial and evolutionary game theory will be carried out.

2.1 Tournaments

In order to identify the condition under which cooperation could emerge in the game of the Prisoner's Dilemma better, Robert Axelrod held a tournament in 1980. He invited a number of well-known game theorists to submit strategies for a computer tournament. Each strategy has to specify whether to cooperate or defect based on the history of previous moves made by both players. Strategies played again each other as well as a further Random strategy, that would randomly choose between C and D and with its own twin (same strategy). The tournament was a round robin with the payoff matrix 1.1. All entries knew the exact length (200 moves) of each game. To improve the reliability of the scores the entire round robin tournament was repeated five times. Fourteen strategies were submitted and by the end Tit for Tat was announced the winner. Surprisingly in the second tournament held where 64 strategies competed and all submitters had full knowledge of what have happened in the first tournament, Tit for Tat managed to get first place again[2].

As explained in [3], Tit for Tat, a simple strategy was able to beat sophisticated and more complex strategies thanks to three specific characteristics of the strategy:

- Niceness: A strategy is categorized as nice if it was not the first to defect, or at least, it will not do this until the last few moves.
- Forgiveness: The propensity to cooperate in the moves after the opponent defected.
- Clarity: After opponents identified that they were playing Tit for Tat choose to cooperate for the rest of the game.

The first tournaments were an innovation in combining computer modeling and Game Theory and in providing insights in the behavior emerging from simple dynamics. Moreover, Axelrod was the first to speak about niceness, forgiveness and gave an illustration that cooperation can be a victorious and advantageous strategy.

Another concept that had been developed was the Evolutionary Game Theory (EGT). EGT is an application of game theory to biological contexts, arising from the realization that frequency dependent fitness introduces a strategic aspect to evolution. In 1973, Maynard and Price introduced the concept of Evolutionary Stable Strategy (ESS), which is an extension of a Nash Equilibrium. If a population of the same strategies cannot be invaded by any alternative strategy that is initially rare then that strategy is an ESS. In his third tournament Axelrod [4] using the same set of strategies (63), the tournaments introduced a dynamical rule that mimics Darwinian selection. In this evolutionary computer tournament after a round robin game the score for each player was evaluated, and the strategies with high score would be adapted while the lowest ones ones would diminish. In most of these simulations, the success of Tit-for-Tat was confirmed because the population would end up with some mutually cooperating strategies prevailed by Tit-for-Tat.

There have been other tournaments, based off of Axelrods, exploring different environments and submitting new strategies. Boyd & Lorderbaum [15] state that no pure strategy is evolutionary stable because each can be invaded by the joint effect of two invading strategies when long term interaction occurs in th repeated game and future moves are discounted. In 1991 Bendor, Kramer and Stout [5] introduced noise to the IPD. Where noisy randomly flip the choice made by a strategy. The results of their tournament was that the strategies that were more generous, cooperated more than their opponents did, were more effective than Tit for Tat. Moreover, Kerts 2011 conducted a tournament where the payoff matrix was altered though satisfying the conditions 1.1, 1.2.

Furthermore, two more notable tournaments took place in 2005 and 2012. In the 2005 IPD competition a team from the University of Southampton participated using a group of strategies which won the top three propositions. These strategies were designed in such why that thought a predetermined sequence of five to ten moves would recognize each other. Once the two Southampton players recognized each other they would take up the roles of a ruler and a slave. The ruler would always cooperate where the slave would defect in order to maximize the payoff of the ruler. If the opponent was recognized to not being one of the team then the Southampton player would always choose to defect to minimize the score of the opponent [13]. Lastly, the Stewart- Plotkin [23] tournament which consisted of nineteen strategies, including a new set of strategies; the Zero- determinant(ZD) strategies. The ZD are strategies for the stochastic iterated prisoner's dilemma, discovered by Press and Dyson in 2012 [21]. The ZD apply a linear relationship between their own payoff and that of the opponent. Some review tournaments are listed on the Table 2.1:

Year	Reference	Number of Strategies	Type
1979	[2]	13	Standard
1979	[3]	64	Standard
1984	[4]	64	Evolutionary
1991	[5]	13	Noisy
2005	[8]	223	Varied
2012	[23]	13	Standard

Table 2.1: An overview of a selection of published tournaments.

In this section the work done for tournaments in the IPD has been cited and analyzed. Starting with the the work of Axelrod, the reasons the tournaments where organized and what where the fundings. Moreover, some research that has been generated the following years are stated. Below, a specialized case of these research will be studied. That case is that of the spatial structure tournaments.

2.2 Spatial Structure Tournaments

Further research was spawn in 1992 as to how the Prisoner's Dilemma could shade some insight into physics and biology. Where Nowak and May believed exist potential dynamics of spatially extended systems. Their tournament was a simple and purely deterministic spatial version of the PD in a two dimensional lattice. With players having no memory of the previous rounds and no strategical elaboration. Thus, the players could either always cooperate or defect. In each round each player interact with the immediate neighbor. [1] They used an evolution rule that after each round round the nodes with the lowest score in their neighborhood would copy the strategy of the player with the highest score. This was done to study which behavior, defection or cooperation , would last. The conclusion was that co-operational behavior is possible in the PD by using a spatial topology. Nowak produced more work on the topic on papers of his such as [20] (Nowak 1994). In his subsequent papers Nowak et al. (1994a,b) different spatial structures where studied. Including triangular and cubic lattices and a random grid. It turned out that cooperation can be maintained in spatial models even for some randomness.

On the other hand, in [14] players were allowed to have memory and therefore added complex strategies to the tournament such as Tit for Tat and Anti Tit for Tat. This was followed by the work of [7] which introduced even more complex strategies. Brauchli et all compares the spatial model with a randomly mixed model. A more complex strategy that they have tested was PAVLOV. A win-stay, lose-switch strategy. According to their fundings, there is more cooperative behavior in a spatial structure tournament and evolution is more less chaotic than in unstructured populations. Also as stated, generous variants of PAVLOV are found to be very successful strategies in playing the Iterated Prisoner's Dilemma.

Spatial topology has been defined by most scholars as a square lattice where the nodes - players only interact with their neighborhoods. Including connections between four or eight nearest neighbor sites, Neuman's or Moore's, according to Figure 1.2. A square lattice is a graph and one could argue that a round robin tournament itself is the complete graph on all players [6]. But in the above papers no authors defined the topology as a graph, apart from [18].

[1] In Nowak's and May's experiment, the result's hold for all three cases that the player interacts with 4, 6 and 8 neighbors.

In [18], an interesting approach was used. They presented a new spatial prisoner's dilemma game model in which the neighborhood size was increased onto two interdependent lattices. They implement the utility by integrating the payoff correlations between two lattices. A player would mimic a random player in his next move, base on a function that consider the utility of the player. It was characterized as a most realistic scenario.

Real life interactions are more likely to be like any given graph depending on the industry than a complete graph. Fatha et all [25], have considered a numerous of graphs, such as :

- Lattice, the interaction network is defined by the sites of a lattice. the distance between a pair does not exceed a given value. The most frequently used structure is the square lattice with von Neumann neighborhood and Moore neighborhood.
- Small word, a graph that is created from a square lattice by randomly rewiring a fraction of connections in a way that conserve the degree for each site.
- Scale-free graphs, a network that has a power-law degree distribution, regardless of any other structure.
- Evolving networks, networks that change as a function of time (this will not be considered in this dissertation).

The major theme of their review was how the graph structure of interactions could modify long term behavioral patterns emerging in evolutionary games. These graphs compose only a small fraction of graphs that exist. In this dissertation we will consider a list of graphs.

2.3 Axelrod Python Library

The Axelrod library [26] is an open source Python package that allows for reproducible game theoretic research into the Iterated Prisoner's Dilemma `https://github.com/Axelrod-Python`. For many of the tournaments aforementioned the original source code is almost never available and in no cases is the available code well-documented, easily modified or released with significant test suites. Due to that reproducing the results has not been an easy task.

However, Axelrod library manages to provide such a resource, with facilities for the design of new strategies and interactions between them, as well as conducting tournaments and ecological simulations for populations of strategies.

Strategies are implemented as classes which have a single method, `strategy()`. It only takes one argument, which is the opponent's previous moves and returns an action. These actions can be either to cooperate C or to defect D. At thins moment the Axelrod library consists of 131 strategies.Can be found in the Appendix. As an example we can see in Figure the source code for the famous strategy Tit for Tat.

```
class TitForTat(Player):
    """
    A player starts by cooperating and then mimics
    the previous action of the opponent.

    Note that the code for this strategy is written
    in a fairly verbose way. This is done so that it
    can serve as an example strategy for those who
    might be new to Python.
    """

    # These are various properties for the strategy
    name = 'Tit For Tat'
    classifier = {
        'memory_depth': 1,  # Four-Vector = (1.,0.,1.,0.)
        'stochastic': False,
        'makes_use_of': set(),
        'inspects_source': False,
        'manipulates_source': False,
        'manipulates_state': False
    }

    def strategy(self, opponent):
        """This is the actual strategy"""
        # First move
        if len(self.history) == 0:
            return C
        # React to the opponent's last move
        if opponent.history[-1] == D:
            return D
        return C
```

Additionally, tournament is a class responsible for coordinating the play of generated matches. It achieves that by calling a match generator class which returns all the single match parameters, such as turns, the game and the noise. Axelrod has the capability to write out the results into a csv file and also out put plots with the ranks of the strategies.

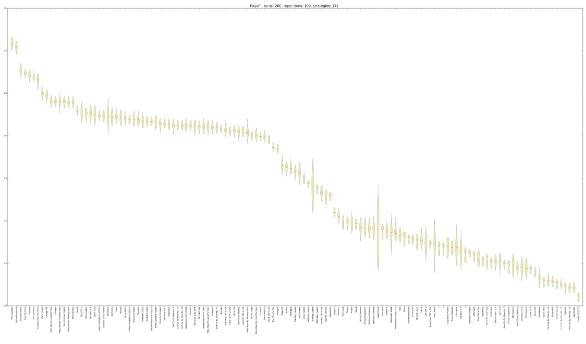
Furthermore, a basic tournament of 200 turns, 100 repetitions and the 131 strategies that exist in the library is being produced continuously. The current winner is called PSO gambler and it is a look up strategy. It uses a lookup table with probability numbers generated using a Particle Swarm Optimisation (PSO) algorithm, the sourse code can be found here : `http://axelrod.readthedocs.io/en/latest/_modules/axelrod/strategies/gambler.html?highlight=Gambler`) and a description of how this strategy was trained is given here: `https://gist.github.com/GDKO/60c3d0fd423598f3c4e4`. It uses a 64-key lookup table (keys are 3-tuples consisting of the opponent's starting actions, the opponent's recent actions, and our recent action) to decide whether to cooperate (C) or defect (D). The actions for each key were generated using an evolutionary algorithm.

To reproduce a basic tournament with the 131 strategies using Axelrod :

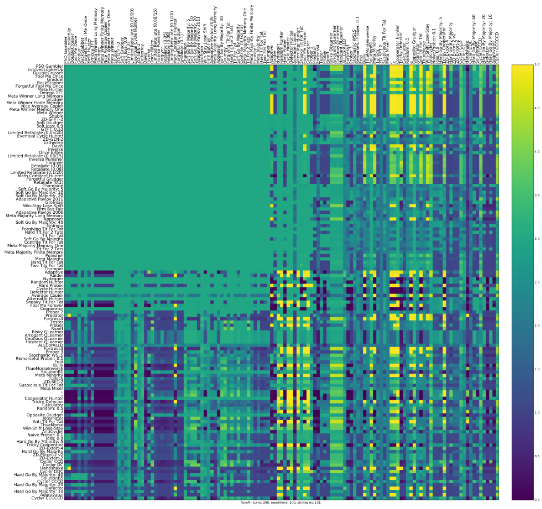
```
>>> import axelrod
>>> strategies = [s() for s in axelrod.ordinary_strategies]
>>> tournament = axelrod.Tournament(strategies)
>>> results = tournament.play()
>>> plot = axelrod.Plot(results)
>>> p = plot.payoff()
>>> p.show()
```

Listing 2.1: A simple set of commands to create a basic tournament. The output is shown in Figure 2.1.

Here are illustrated the results of the last tournament :



(a) Ranked violin plot



(b) Payoffs

Figure 2.1: Result Plots. (a) Ranked violin plot, the mean utility of each player. (b) Payoffs, the pair wise utilities of each player.

More details for the documentation of the library can be found here : `https://axelrod.readthedocs.io/en/latest/index.html`. Because is an open source library it makes it easy to contribute to it and make modifications needed for this dissertation.

Chapter 3

Implementation of spatial tournaments

3.1 Introduction

In this chapter we will discuss the source code committed to the Axelrod-Python library to implement spatial topology. In addition, some initial experiments and results will be discussed. These experiments will be performed using three chosen topologies and two different competitiros size.

3.1.1 Code Discussion

As analyzed in chapter 2, the Axelrod library uses a `Tournament()` class to run a round robin tournament. The `RoundRobinTournament()` itself calls upon another class the `Match Generator()` which is responsible to generate the matches. In a round robin cause(`RoundRobinMatches()`), it generates matches for each player against the rest. The parameters and the index of the pair are push to it by the `build single match()`. A generator that lives within the class. For a round robin tournament the structure of the code is illustrated in 3.1.

In order for us to implement a Spatial topology tournament we need to follow a similar approach. Firstly a new `Match Generator()` class was written. The `SpatialMatches()` is a class that generates spatially-structured matches. In these matches, players interact only with their neighbors rather than the entire population. According to [1] graphs can be represents in many different ways, one of which is by lists of edges. Due to a various number of python packages that

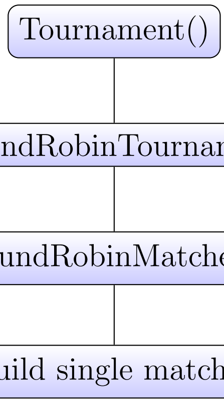


Figure 3.1: Code structure for a Round Robin tournament.

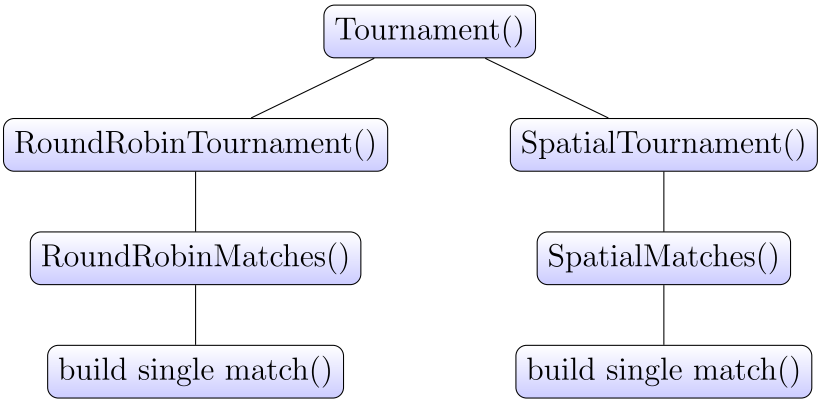


Figure 3.2: Code structure for when Round Robin and Spatial tournaments are implemented.

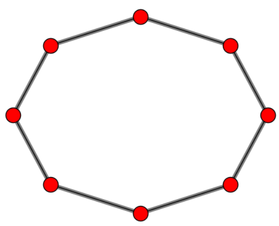
are used for graph manipulation, we want to generalize the representation of the edges. For the `SpatialMatches()` do understand which players are connected to each other, the edges are passed as a list argument. The `SpatialMatches()` will only create matches between the ending nodes of these edges. Finally the class `SpatialTournament()` runs the spatial tournament. The code structure now that the spatial tournaments have been added can be seen in Figure 3.2

In Axelrod library all the components are automatically tested using a combination of unit, property and integration tests(using `travis-ci.org`). Once a new feature is added to the library, corresponding test must also be written. The test are used to ensure compatibility and ensure that we get the expected results. The tests for the `SpatialTournament()` can be found here : `https://github.com/Axelrod-Python/Axelrod/blob/master/axelrod/tests/unit/test_tournament.py` under the class `TestSpatialTournament()`.

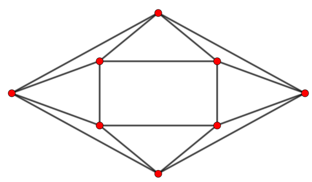
3.1.2 Experiment and the three topologies

In this chapter we will be running some simple examples of spatial topology and analyze the behavior of the strategies and the results. This will help us further down to tackle the problem of how topology can affect the outcome of an IPD competition and which strategies tend to perform well. Some of the most common networks, based o literature, are a square lattice with four degrees and a cycle.

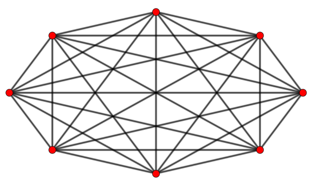
In a cycle or circular network consist of a single cycle. It has equal number of edges and nodes, and each node has degree 2. In [25] Szabo et all stated that "For spatial models the fixed interaction network is defined by the sites of a lattice and the edges between those pair whose distance does not exceed a given value." The most frequently used structure, and the one used in this experiment, is the square lattice with von Neumann neighborhood. Also the most common topology used, again based on literature, is that of a round robin. A round round topology is nothing else but a complete graph were every pair of distinct node is connected by a unique edge. Figure 3.3, shows an example of all the aforementioned topologies. Round robin will be mainly used for comparison reasons.



(a) Cycle network.



(b) Square lattice with degree 4 network.



(c) A complete or round robin network.

Figure 3.3: Network topologies.

Thus, we will be performing our experiment with three different topologies, that of a cycle, a lattice and a complete graph. For each topology, a fixed size of strategies out of the 132 of Axelrod-Python library are chosen randomly. These strategies create a neighborhood. The size of these neighborhoods will range. Let s be the size of the neighborhood, where s $\{s = 5 \text{and} s = 50\}$. Subsequently, the strategies are allocated on the graph, based on the topology, and they compete with their neighbors on a IPD tournament. For the circular and lattice topology, once the first game is complete, the strategies are randomly shuffled and allocated on the graph again. Another tournament is performed this time with different neighbors of a neighborhood interacting. This is repeated 10 times. The selection of strategies and creation of neighborhoods is repeated 100 times and each tournament of an IPD consists of 200 turns and 10 repetitions.

By setting an axelrod-seed [1], the 100 neighborhoods for the topologies are the same, of course their allocation to the graphs differs because of the random shuffle.

3.1.3 Initial Analysis

For each of the experiment we have forced to keep in track for a set of parameters, such as the list of players in a neighborhood, the score, normalized score and average score for each strategy etc. From a quick look at these data sets we can obtain the following informations.

For the spatial tournaments, for both neighborhood sizes (5 and 50) we achieved 1000 tournaments. Containing 100 different neighborhoods, where each the exists for 10 tournaments.

Moreover, for the cycle experiment we can see in Table [], that degree is fixed at 2 and the payoffs are fixed to $R = 3, P = 1, S = 0, T = 5$ for both $s = 5$ and $s = 50$. For $s = 5$, the mean average score is 2.45, with a minimum value of 0.0175 and a maximum value of 4.95. The mean average score of the neighbors score is 980.95 with a standard deviation of 219.87. Moreover the minimum value is set at 141.55 and the maximum at 1756.50. The mean average score does not seem to differ for $s = 5$, which is at 2.39. Though, in the experiment with a size of fifty a strategy achieved a score of 0. Additionally the average score of the neighbors

[1] A function used by the library, which sets both seeds for Numpy and the standard library.In general, seeds allow us reproduce the same players and tournaments.

ranges from 19.30 to 1884.5. In both cases of size the clustering coefficient is zero and the connectivity due to fixed degree is 2.

Cycle	s=5 and s=50			s=5			s=50		
	(R,P,S,T)	degree	connectivity	average score	average neighbors score	clustering	average score	average neighbors score	clustering
mean	(3,1,0,5)	2.0	2.0	2.456690	980.952580	0.00	2.394577	957.238664	0.00
std	(0,0,0,0)	0.0	0.0	0.748772	219.875803	0.00	0.777189	231.321350	0.00
min	(3,1,0,5)	2.0	2.0	0.017500	141.550000	0.00	0.000000	19.300000	0.00
max	(3,1,0,5)	2.0	2.0	4.950000	1756.500000	0.00	5.000000	1884.500000	0.00

Table 3.1: Summary table for topology circle.

For the lattice experiment a table that summarize the data set is shown in Table 3.2. For both size's values the payoffs are the same $(R = 3, P = 1, S = 0, T = 5)$ and the degree is fixed at 4. For $s = 50$ the mean average score varies between 0.018 and 4.97. The average score of the neighbors varies from 832.67 and 2895.42. Much higher than both the cycle experiments achieve. Logical based on the fact that the number of neighbors is now doubled. For $s = 5$ the mean score is 0.57 and the mean average neighbor score 2.45. The clustering coefficient is 1 and for $s = 50$ is 0.5. This shows that in the lattice example the strategies tend to create groups.

Lattice	s=5 and s=50			s=5			s=50		
	(R,P,S,T)	degree	connectivity	average score	average neighbors score	clustering	average score	average neighbors score	clustering
mean	(3,1,0,5)	4.0	4.0	2.450782	1958.569220	1.0	2.393000	1912.748200	0.5
std	(0,0,0,0)	0.0	0.0	0.576812	287.638191	0.0	0.590971	268.375436	0.00
min	(3,1,0,5)	4.0	4.0	0.527500	1059.775000	1.0	0.018750	832.675000	0.5
max	(3,1,0,5)	4.0	4.0	4.245000	2518.700000	1.0	4.973750	2895.425000	0.5

Table 3.2: Summary table for topology lattice.

Finally, for the round robin tournaments 100 tournaments were performed for both neighborhood sizes. Parameters such as neighborhood size and neighbor's score were not computed for the round robin topology. Because all players interact with each other we would not get any additional information. In Table 3.3, we can see the average score the strategies achieved in this topology for both sizes. In a neighborhood of a size 50, the mean average score is 2.39 with an standard deviation of 0.335. For $s = 5$ we can notice that the values are almost equal to those of the lattice topology with $s = 5$.

Round Robin	s=5	s=50
	average score	average score
mean	2.447105	2.393220
std	0.576014	0.335552
min	0.527500	1.523673
max	4.245000	3.339592

Table 3.3: Summary table for round robin topology.

In this section we gone through the structure of the source code for implementing the Spatial Tournament, by adding to the Axelrod-Python library. Furthermore, now that the code is usable various experiments were conducted with different topologies and number of players participating in each tournament. An overview of the results was done but now in the following sections we will go though some more intense analysis.

3.2 Analyzing the effect of the topologies

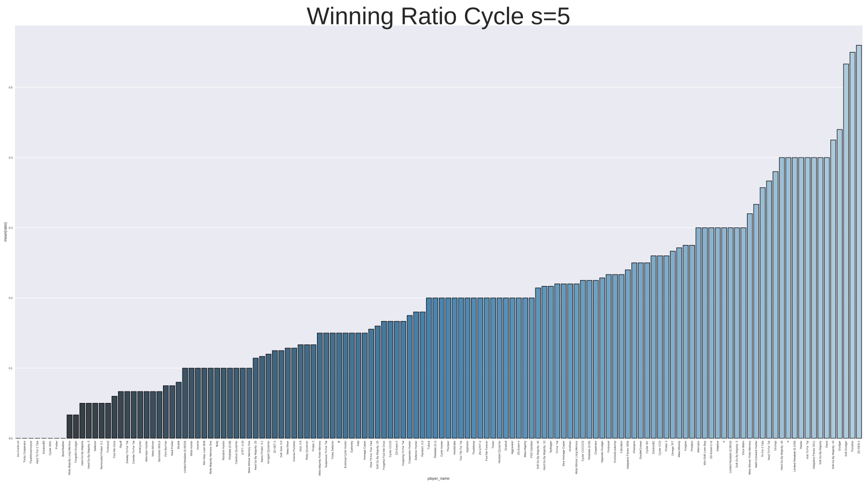
For all experiments all 132 strategies of Axelrod-Python library have participated at least in a one tournament. Here we will go through the strategies and their performance in each experiment. Not all strategies participated in an equal number of tournaments. Forcing the strategies to perform in a uniform number is not an option as we want to keep the random effect for validation of the results. On could argue that by participating in a larger number of tournaments, the probability of winning increases as well. Thus, instead of number of winning tournaments our analysis will be using the ratio of wins. By wining ratio we mean the number of tournaments a strategy first place divided by the number of tournaments the strategy competed in. Additionally, the normalized average score each strategy achieved will also be studied. Mainly to study the variation, helping us to make conclusions on how the neighborhood could affect the strategy. Lastly, having all different factors such as degree, neighbors, clustering, scores etc building a regression model could point out effects on the results.

3.2.1 Winning Ratio

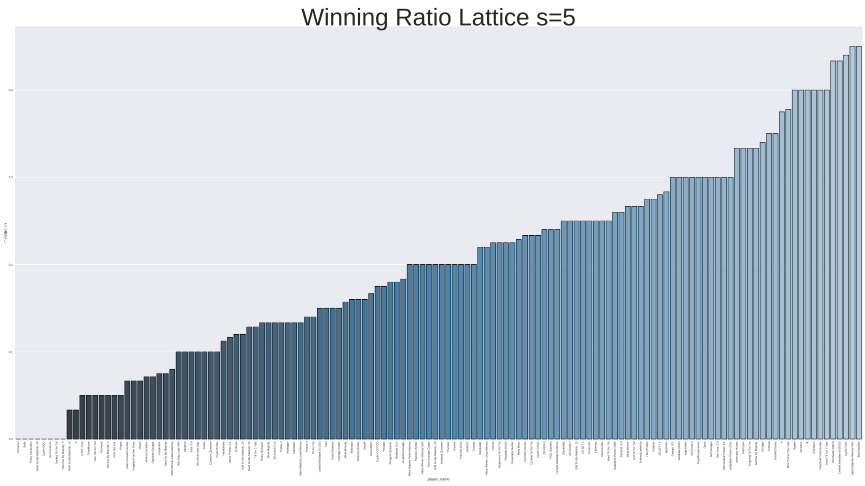
The winning rations for all three topologies for a neighborhood size equal to 5 are shown in Figure 3.4. In the experiment where the strategies compete on a cycle topology 124 strategies out of the 132 have a winning ration greater than zero. The strategies which did not are ALLCorALLD, Tricky Cooperator, ThueMorseInverse, Hard Tit For 2 Tats, SolutionB5, Cycler DDC, Prober and BackStabber. In the lattice topology as well, 8 strategies do not have a non- zero winning ratio. Fortress4, Bully, Tricky Cooperator, Hard Go By Majority: 40, 5 and 10. Also Cycler DDC and ALLCorALLD have a similar performance in this topology as well. In the round robin tournament various strategies ranked last, among them Fortress4 and the whole family of Hard Go By Majority.

On the other hand the strategies with the highest winning ration for each topology are as follow :

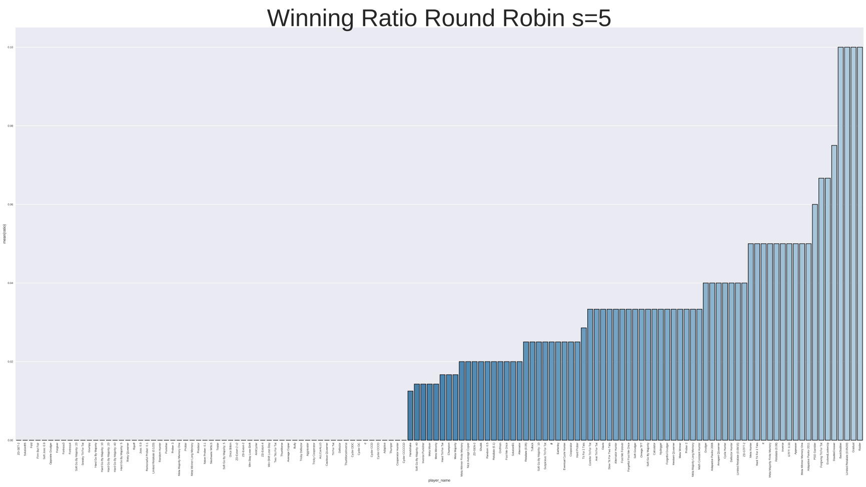
- Cycle topology with a winning ratio of 0.56 ZD-GEN-2, followed by Punisher with 0.55
- Lattice topology with a ratio of 0.45 BackStabber and Meta Majority Memory one
- Round Robin topology with a winning ratio of 1 : Raider, Gradual, Limited Retaliate(0.05/20) and BackStabber



(a) Winning ration cycle s=5.



(b) Winning ration lattice s=5.



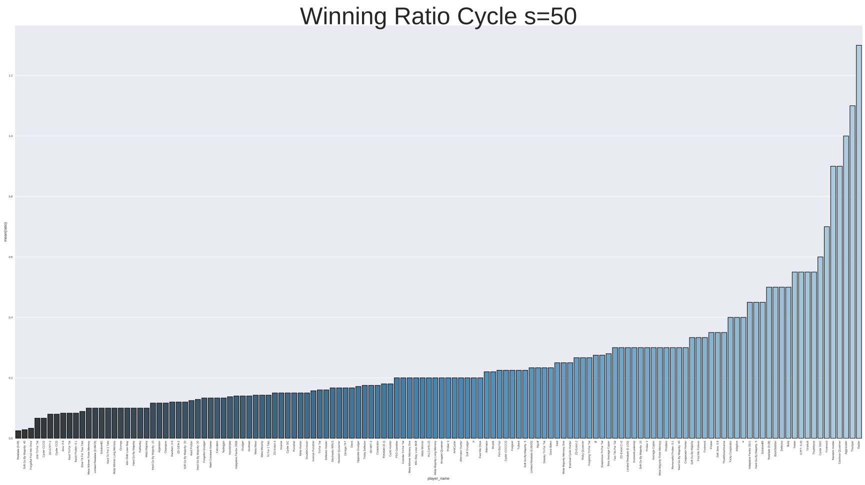
(c) Winning ration round robin s=5.

Figure 3.4: Winning ratio for all three topologies s=5.

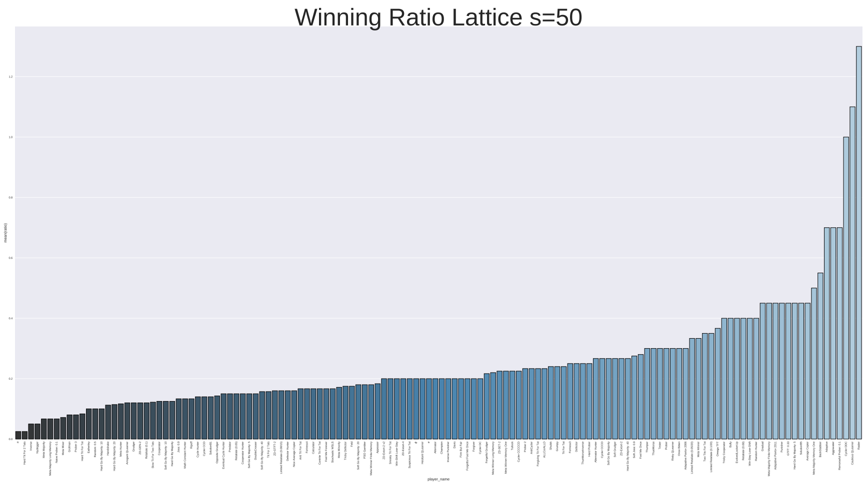
Same approach for a neighborhood size of 50. In these experiments non strategies have a zero ratio. Figure 3.5, illustrates the ratios for each topology in ascending order. The strategies with the highest winning ratio for each respective topologies are as follow :

- Cycle topology with a winning ratio of 1.3 Raider
- Lattice topology with a winning ratio of 1.3 Raider
- Round Robin topology with a winning ration of 0.833 EvolvedLookerUp

Unexpectedly, strategy Raider seems to have dominated the first place in both the cycle and the lattice topology. It achieved a winning ration of 1.3, 0.5 more than the strategy ranked second. Another surprise in the results is that of the round robin topology. Where almost all strategies have a winning ratio of zero. Only 14 strategy managed a ratio higher than hero. The strategy with the highest on of all was EvolvedLookerUp.



(a) Winning ration cycle s=50.



(b) Winning ration lattice s=50.

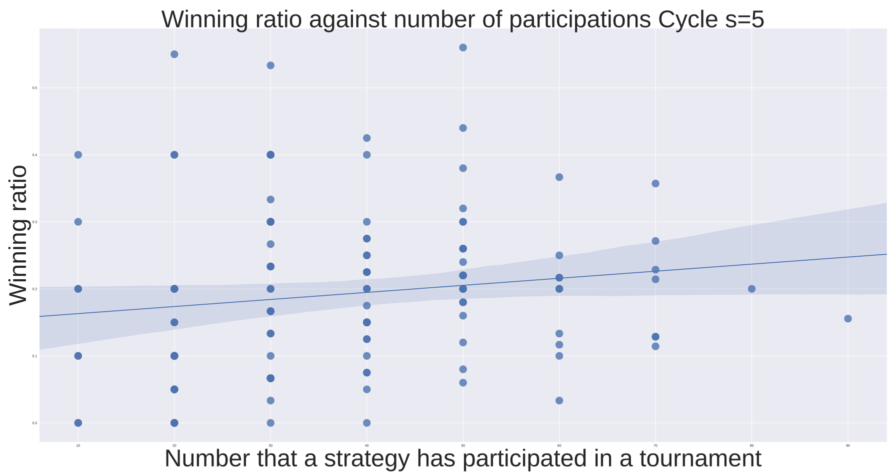
(c) Winning ration round robin s=50.

Figure 3.5: Winning ratio for all three topologies s=50.

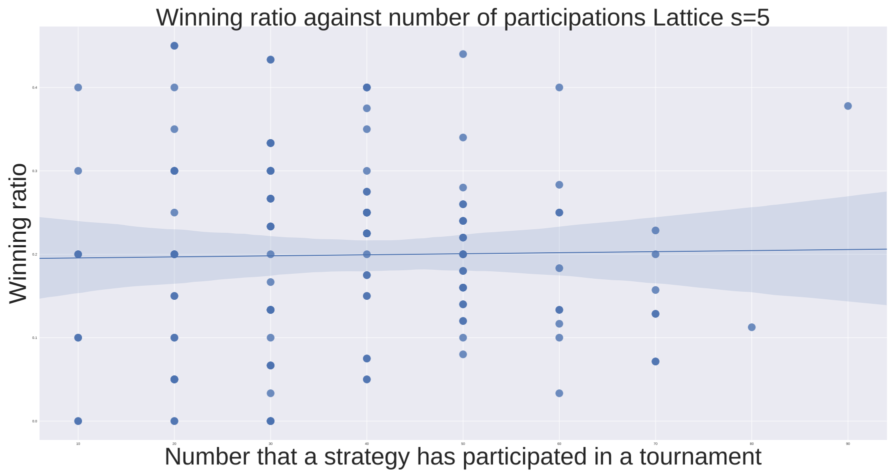
The strategies with the highest ratios seem to vary depending on the topology and the size. Though for $s = 5$ it did not seemed to be an similarity between the highest rank strategies for $s = 50$ is dominated by the strategy Raider. To further investigate this behavior, the winning ratio was plotted against the numbers of participations the strategy had. Figure 3.6 illustrates the plots for $s = 5$ and Figure 3.7 for $s = 50$.

For $s = 5$ we observe that for the topologies lattice and round robin there is no correlation between the participations and the winning ratio. Thus, the number of tournaments that a strategy has competed in does not affect its the winning percentage. In a cycle topology, there tends to be a positive correlation. Thus, we can guess that the strategies that were ranked first in these topology had high participating rates as well. Cycle seems to have different results and that could be because of the degree number.On the contrary, for $s = 5$ topologies cycle and lattice show that the number of participations have a negative effect on your winning ratio. Thus, playing less in these experiments could help you achieve a better outcome. This results were unexpected as well. For the round robin topology, that so little strategies achieved a winning ratio of non zero, participating seems to now have any effect.

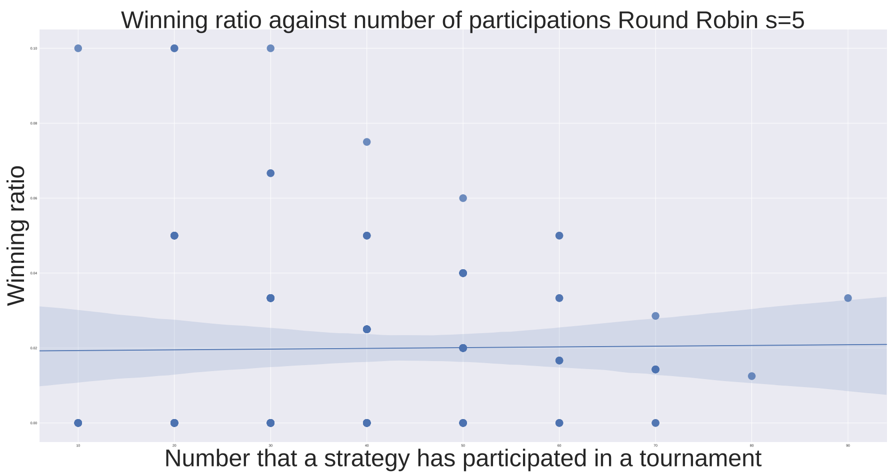
In conclusion, the only strategy that performed well in more than one experiment was the strategy Raider. Raider's winning ratio varies from 1 to 1.3 depending on the topology. The only experiment it did not perform as well as the others was in that of cycle topology size 5, which is the only topology that has a positive correlation between winning ratio and participations. And Raider only competed 10 times which could be the reason behind it's lack of performance.



(a) Winning ration against number of participations cycle s=5.

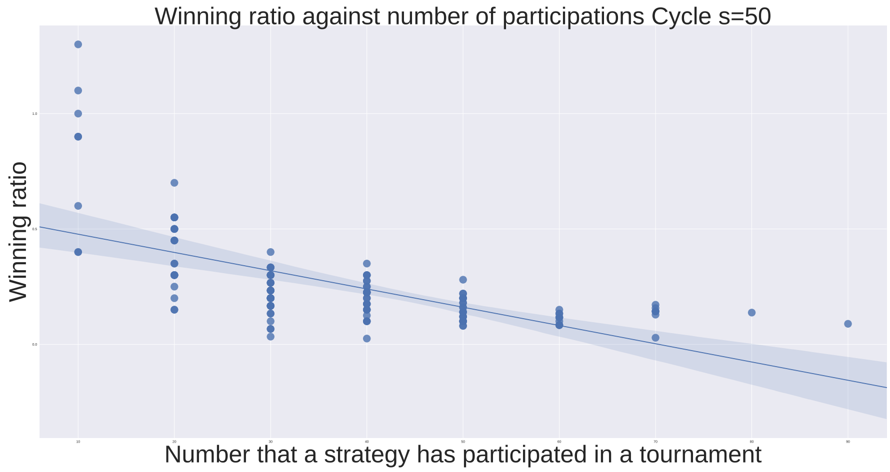


(b) Winning ration against number of participations lattice s=5.

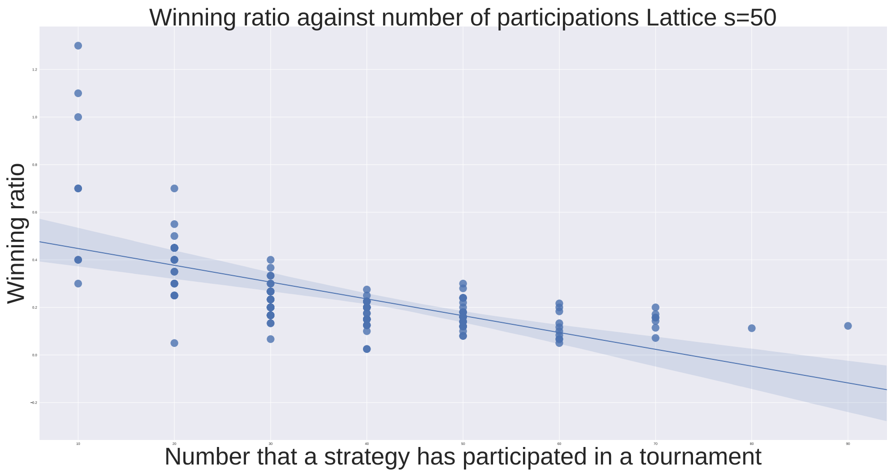


(c) Winning ration against number of participations round robin s=5.

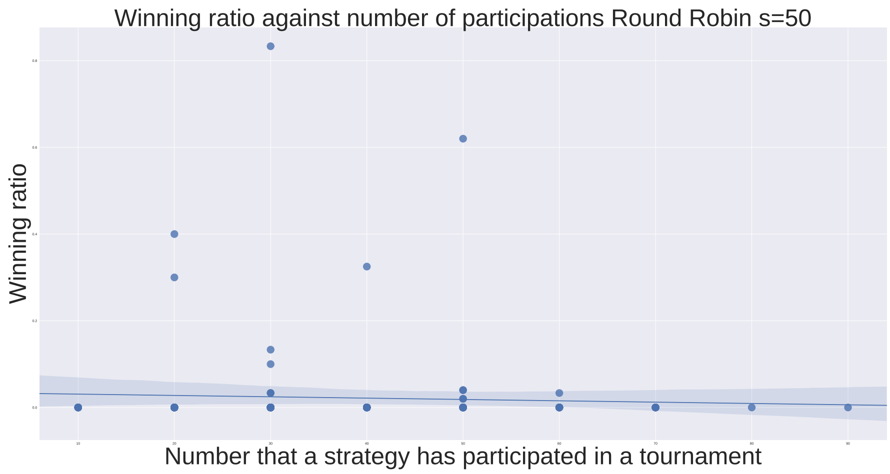
Figure 3.6: Winning ratio against number of participating in a tournament for all three topologies s=5.



(a) Winning ration against number of participations cycle s=50.



(b) Winning ration against number of participations lattice s=50.

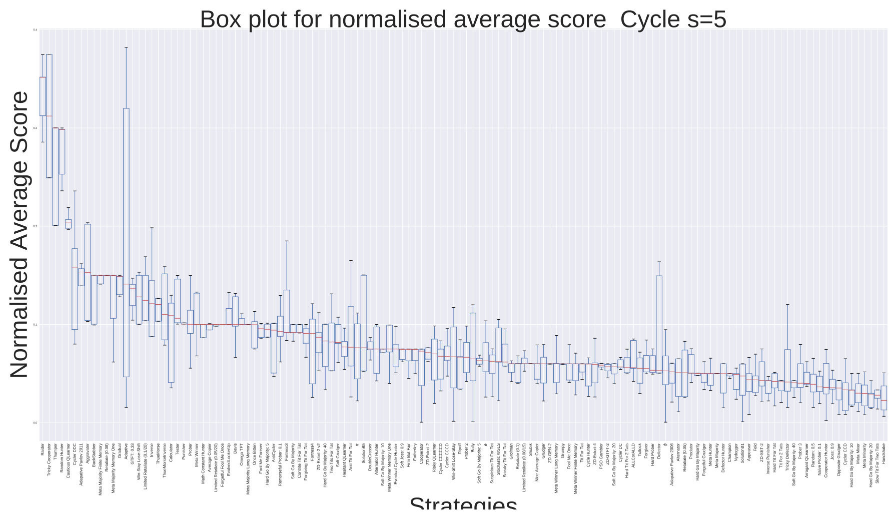


(c) Winning ration against number of participations round robin s=50.

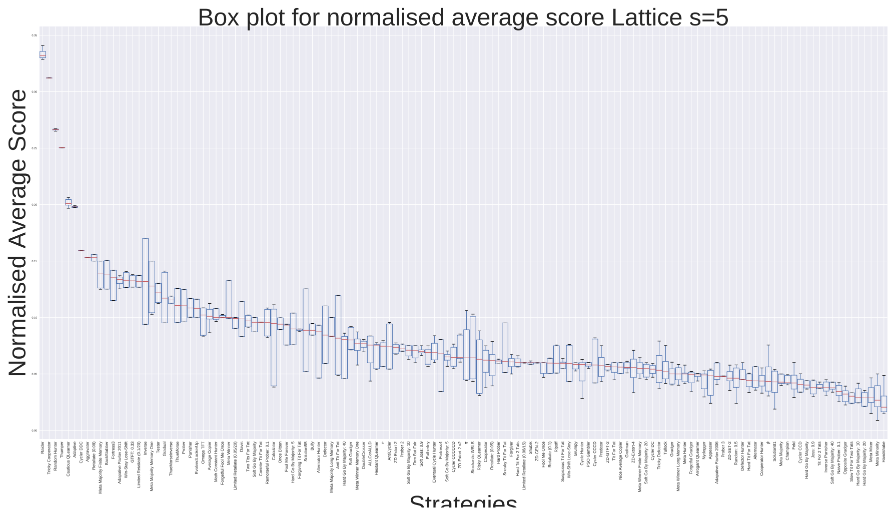
Figure 3.7: Winning ratio against number of participating in a tournament for all three topologies s=50.

3.2.2 Average Scores

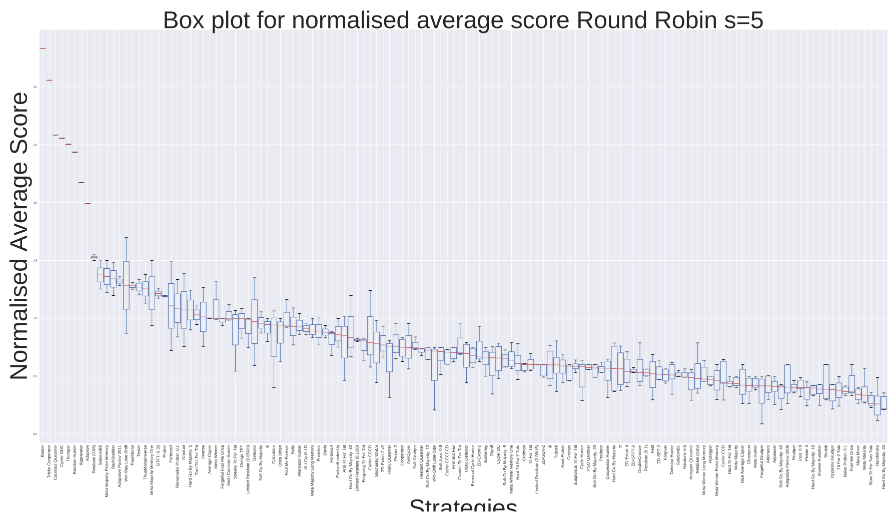
The normalized average score is calculated by diving the average score of each strategy with their participating rates. Then the average score is plotted against the strategies. As shown in both Figure 3.8 and Figure 3.9, there is variation everywhere. Thus, all strategy had different levels of average score for each experiment. Meaning the performance in your experiments was not based so much on the strategy itself. Other reasons could be, participating rates, neighborhood or it could even be completely randomized. Furthermore, the average score is plotted against participations to identify any hidden effects.



(a) Normalized average score cycle s=5.

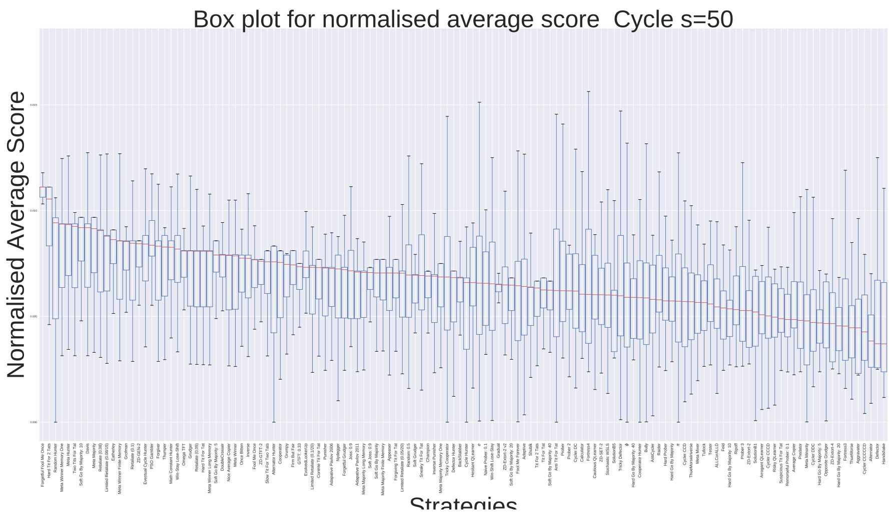


(b) Normalized average score cycle s=5.

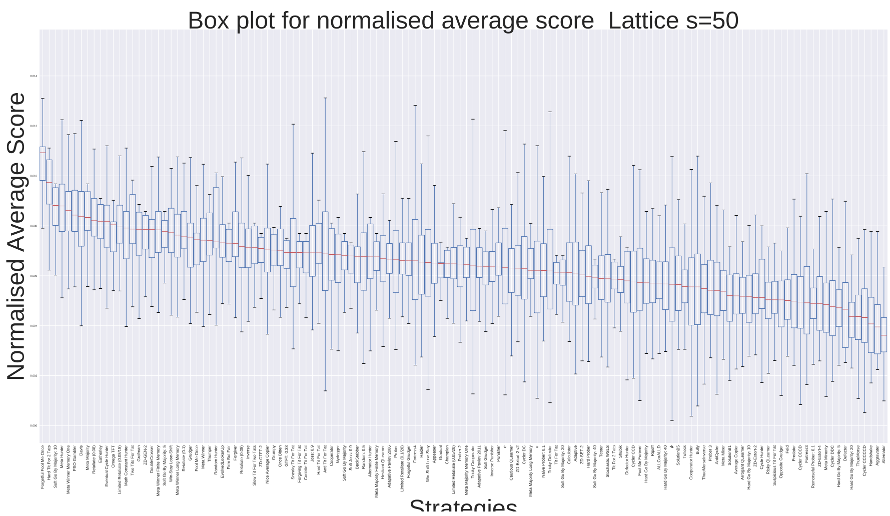


(c) Normalized average score cycle s=5.

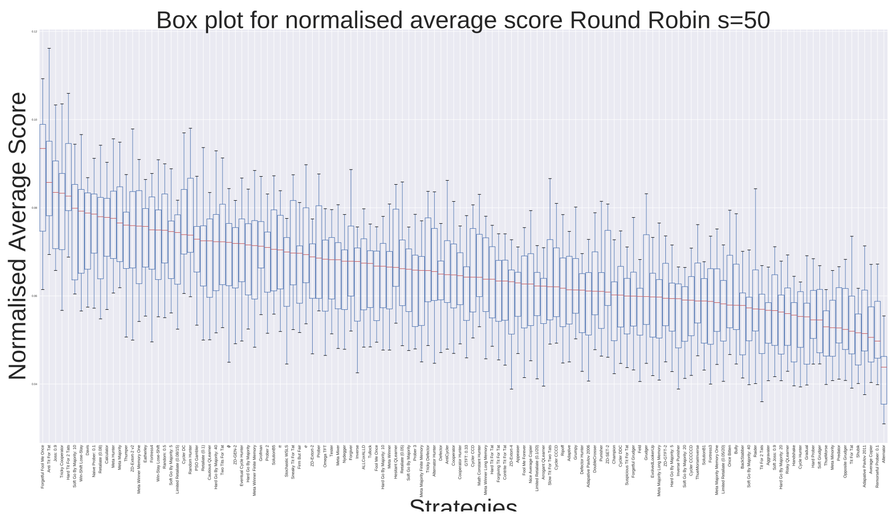
Figure 3.8: Normalized average score for the three topologies s=5.



(a) Normalized average score cycle s=50.



(b) Normalized average score cycle s=50.



(c) Normalized average score cycle s=50.

Figure 3.9: Normalized average score for the three topologies s=50.

3.2.3 Regression

Finally, a common methodology when investing factors as predictors is building a regression model. We are building a model wanting to identify any factor that can explain the average score of a strategy. The model is the following :

$$\begin{aligned} normalized\ average\ score = degree + average\ neighborhood\ score + \\ clustering + number\ of\ participations \end{aligned} \tag{3.1}$$

The round robin topology is not included in this subsection analysis. As explained above, parameters as average neighborhood score was not monitored. We believe that in a round robin topology factors like do not have significant effects. Also round robin topology was mainly used for comparison reasons. The model was used to each of the experiments for lattice and cycle topologies. The results of models are shown below, Table 3.4 :

Size	Topology	Intercept		degree		average neighborhood score		connectivity		participations		R-square
		coef	p	coef	p	coef	p	coef	p	coef	p	
s=5	Cycle	0.028	0.00	0.0559	0.00	-3.763e-06	0.043	0.0	NA	-0.0016	0.00	0.457
	Lattice	0.0064	0.00	0.0256	0.00	1.079e-05	0.00	0.0064	0.00	-0.0016	0.00	0.549
s=50	Cycle	0.0025	0.00	0.0051	0.00	-2.168e-07	0.00	0	NA	-1.602e-05	0.00	0.120
	Lattice	0.0006	0.00	0.0024	0.00	1.033e-06	0.00	0.0003	0.00	-1.601e-05	0.00	0.216

Table 3.4: Regression results for model 3.1

In the output we can see that Degree, average neighborhood score and participations are significant predictors for all the experiments with a p value less than an 0.00. Lower than the common alpha level of 0.05 For the cycle topology, average neighborhood score and participations have a negative coefficients. For example a decrease in participations by one would increase the average score by 0.0016. Degree on the other hand has a positive coefficient and connectivity has no effect at all. Furthermore the model for $s = 5$ has a R-square value of 0.457, thus it only explains 0.4 variation of the data which is quite insignificant. For $s = 50$ it is even lower at only 0.12.

Finally, for the lattice topology only participations have a negative coefficients. Thus the only factor with a reverse influence on average score in the lattice topology. Connectivity is an significant predictor as well with a coefficient 0.0064 and 0.0003 respectively. Though the R-square value is still significant small, with a value 0.547 and 0.216 respectively. Even if there are predictors with a significant p value, the overall performance of the model does not give us enough evidence. Thus no validate conclusion can be made for the model.

3.3 Summary

In this section we will make a summary of all the previous analysis that was made in 3.2. Furthermore, compare the results produced by all six experiments. The strategies which managed to get a high winning ratio and identify any further relationship between the performance of a strategy and the topology.

From the analysis that was performed in 3.2.1, the wining ratio for each strategy for all 6 experiments indicated the following :

- For the cycle topology of $s = 5$ ZD-GEN-2 had the highest winning ratio and for the lattice BackStabber and Meta Majority Memory one
- For round robin s=50, most of the strategies finished the experiment with winning ratio of zero.
- Raider seem to have successful performance in both lattice and cycle $s = 50$ and in round robin $s = 5$. robin

An attempt to find any significant reason as to why these strategies outperform the rest returned the findings below :

- Participating rate has a negative effect on the winning ratio for $s = 50$ in lattice and cycle topology and a positive in cycle for $s = 5$
- There is high variation is the average score of each strategy for all experiments
- Regression model for the normalized average score did not return any significant results.

In conclusion, the network topologies used in these examples seem to have be simple and no real results were able to emerge. There were quite a few surprises. Raider outperforming so many other strategies, the negative effect of participation and winning ration. Moving forwards new experiments a list of recommended additions would be :

- More complicated and random topologies

- Different neighborhood sizes. Probably randomize the size as well
- To be able to compare the strategies that ranked high we will keep track of their cooperation rate. Thus we will be able to classify them on how cooperative they are.

Chapter 4

Title of Chapter 3

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References

[1] D. Archdeacon. "Topological graph theory-a survey". In: *Cong. Num* (1996), pp. 1–67. URL: `http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.28.1728$%5Cbackslash$npapers://92fd5174-9e31-4983-9b1c-5a79350234b8/Paper/p12584` (cit. on p. 14).

[2] R. Axelrod. "Effective Choice in the Prisoner's Dilemma". In: *Journal of Conflict Resolution* 24.1 (1980), pp. 3–25 (cit. on pp. 1, 4, 6, 8).

[3] R. Axelrod. "More Effective Choice in the Prisoner's Dilemma". In: *Journal of Conflict Resolution* 24.3 (1980), pp. 379–403. ISSN: 0022-0027. DOI: `10.1177/002200278002400301` (cit. on pp. 1, 4, 6, 8).

[4] R. Axelrod and W. D. Hamilton. *The Evolution of Cooperation The Evolution of Cooperation.* Vol. 211. 4489. 1981, pp. 1390–1396. ISBN: 0036807518110. DOI: `10.1086/383541`. arXiv: `t8jd4qr3m [13960]` (cit. on pp. 1, 7, 8).

[5] J. Bendor, R. M. Kramer, and S. Stout. "When in Doubt ... Cooperation in a Noisy Prisoner ' s Dilemma". In: *ournal of Con ict Resolution* 35.4 (1991), pp. 691–719. URL: `http://jcr.sagepub.com/content/35/4/691` (cit. on pp. 7, 8).

[6] B. Bollobas. "Graph Theory: An Introductory Course (Graduate Texts in Mathematics)". In: (1990). URL: `http://www.amazon.com/Graph-Theory-Introductory-Graduate-Mathematics/dp/0387903992%3FSubscriptionId%3D0JYN1NVW651KCA56C102%26tag%3Dtechkie-20%26linkCode%3Dxm2%26camp%3D2025%26creative%3D165953%26creativeASIN%3D0387903992` (cit. on p. 9).

[7] K. Brauchli, T. Killingback, and M. Doebeli. "Evolution of cooperation in spatially structured populations". In: *Journal of theoretical biology* 200.4 (1999), pp. 405–17. ISSN: 1095-8541. DOI: `10.1006/jtbi.1999.1000`. URL: `http://www.ncbi.nlm.nih.gov/pubmed/10525399` (cit. on pp. 2, 9).

[8] S. Y. Chong et al. "Chapter 1 The Iterated Prisoner ' s Dilemma : 20 Years On". In: (2004), pp. 1–21 (cit. on pp. 4, 8).

[9] S. I. Gass and A. a. Assad. "an Annotated Timeline of Operations Research". In: (2005), p. 125. DOI: `1402081138`. URL: `http://ebooks.kluweronline.com` (cit. on p. 3).

[10] J. Gruji et al. "A comparative analysis of spatial Prisoner's Dilemma experiments: Conditional cooperation and payoff irrelevance." In: *Scientific reports* 4 (2014), p. 4615. ISSN: 2045-2322. DOI: `10.1038/srep04615`. URL: `http://www.pubmedcentral.nih.gov/articlerender.fcgi?artid=3983604%7B%5C&%7Dtool=pmcentrez%7B%5C&%7Drendertype=abstract` (cit. on p. 2).

[11] D. Kraines and V. Kraines. "Pavlov and the Prisoner's Dilemma". In: *Theory and Decision* 26 (1989), pp. 47–79 (cit. on p. 2).

[12] T. Kretz. "A Round-Robin Tournament of the Iterated Prisoner's Dilemma with Complete Memory-Size-Three Strategies". In: *Transport* (2011), pp. 1–29. arXiv: `1101.0340` (cit. on p. 2).

[13] J. Li, P. Hingston, and G. Kendall. "Engineering design of strategies for winning iterated prisoner's dilemma competitions". In: *Ieee Transactions on Computational Intelligence and Ai in Games* 3.4 (2011), pp. 348–360. ISSN: 1943-068X. DOI: `10.1109/TCIAIG.2011.2166268`. URL: `http://www.scopus.com/inward/record.url?eid=2-s2.0-83655198233%7B%5C&%7DpartnerID=40%7B%5C&%7Dmd5=86fc01bcb87e488fac376d4ffb08ebc7$%5Cbackslash$nhttp://ieeexplore.ieee.org/ielx5/4804728/6099654/06004823.pdf?tp=%7B%5C&%7Darnumber=6004823%7B%5C&%7Disnumber=6099654` (cit. on pp. 3, 8).

[14] K. Lindgren and M. Nordahl. "Evolutionary dynamics of spatial games". In: *Physica D: Nonlinear Phenomena* 75.1-3 (1994), pp. 292–309. ISSN: 01672789. DOI: `10.1016/0167-2789(94)90289-5` (cit. on pp. 2, 9).

[15] J. P. Lorberbaum. "No strategy is evolutionarily stable in the repeated Prisoner's Dilemma game". In: *Journal of Theoretical Biology* 168.2 (1994), pp. 117–130 (cit. on pp. 1, 7).

[16] E. Maciver. "Spatial Prisoner's Dilemma". PhD thesis. 2014. ISBN: 9780857009289. DOI: `10.1177/0022002793037003008.JSTOR` (cit. on p. 2).

[17] E. Maciver. "Spatial Prisoner's Dilemma". PhD thesis. 2014. ISBN: 9780857009289. DOI: `10.1177/0022002793037003008.JSTOR` (cit. on p. 4).

[18] X. K. Meng et al. "Spatial prisoner's dilemma games with increasing neighborhood size and individual diversity on two interdependent lattices". In: *Physics Letters, Section A: General, Atomic and Solid State Physics* 379.8 (2015), pp. 767–773. ISSN: 03759601. DOI: `10.1016/j.physleta.2014.`

12.051. URL: http://dx.doi.org/10.1016/j.physleta.2014.12.051 (cit. on pp. 2, 9, 10).

[19] M. a. Nowak and R. M. May. *Evolutionary games and spatial chaos*. 1992. DOI: 10.1038/359826a0. arXiv: arXiv:1011.1669v3 (cit. on pp. 2, 4).

[20] M. a. Nowak and R. M. May. "The spatial dilemmas of evolution". In: *International Journal of Bifurcation and Chaos* 03.01 (Feb. 1993), pp. 35–78. ISSN: 0218-1274. DOI: 10.1142/S0218127493000040. URL: http://www.worldscientific.com/doi/abs/10.1142/S0218127493000040 (cit. on pp. 2, 9).

[21] W. H. Press and F. J. Dyson. "Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent." In: *Proceedings of the National Academy of Sciences of the United States of America* 109.26 (2012), pp. 10409–13. ISSN: 1091-6490. DOI: 10.1073/pnas.1206569109. URL: http://www.pnas.org/content/109/26/10409.abstract (cit. on p. 8).

[22] D. W. Stephens, C. M. McLinn, and J. R. Stevens. "Discounting and reciprocity in an Iterated Prisoner's Dilemma." In: *Science (New York, N.Y.)* 298.2000 (2002), pp. 2216–2218. ISSN: 00368075. DOI: 10.1126/science.1078498 (cit. on p. 4).

[23] a. J. Stewart and J. B. Plotkin. "Extortion and cooperation in the Prisoner's Dilemma". In: *Proceedings of the National Academy of Sciences* 109.26 (2012), pp. 10134–10135. ISSN: 0027-8424. DOI: 10.1073/pnas.1208087109 (cit. on p. 8).

[24] A. J. Stewart and J. B. Plotkin. "From extortion to generosity, evolution in the Iterated Prisoner's Dilemma". In: *Proceedings of the National Academy of Sciences* 110.38 (2013), pp. 15348–53. ISSN: 1091-6490. DOI: 10.1073/pnas.1306246110/-/DCSupplemental.www.pnas.org/cgi/doi/10.1073/pnas.1306246110. arXiv: arXiv:1304.7205v2 (cit. on p. 4).

[25] G. Szab and G. Fth. "Evolutionary games on graphs". In: *Physics Reports* 446.4-6 (2007), pp. 97–216. ISSN: 03701573. DOI: 10.1016/j.physrep.2007.04.004. arXiv: 0607344 [cond-mat] (cit. on pp. 1, 4, 10, 16).

[26] The Axelrod project developers. *Axelrod: ¡RELEASE TITLE¿*. Apr. 2016. DOI: <DOIINFORMATION>. URL: http://dx.doi.org/10.5281/zenodo.%3CDOI%20NUMBER%3E (cit. on pp. 4, 10).

Appendix A

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Appendix B

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