The Iterated Prisoner's Dilemma

Nikoleta Evdokia Glynatsi

September 2016

School of Mathematics,

Cardiff University

A dissertation submitted in partial fulfilment of the requirements for MSc (in Operational Research and Applied Statistics) by taught programme.

Executive Summary

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo.

Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetuer at, consectetuer sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui.

Morbi luctus, wisi viverra faucibus pretium, nibh est placerat odio, nec commodo wisi enim eget quam. Quisque libero justo, consectetuer a, feugiat vitae, porttitor eu, libero. Suspendisse sed mauris vitae elit sollicitudin malesuada. Maecenas ultricies eros sit amet ante. Ut venenatis velit. Maecenas sed mi eget dui varius euismod. Phasellus aliquet volutpat odio. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Pellentesque sit amet pede ac sem eleifend consectetuer. Nullam elementum, urna vel imperdiet sodales, elit ipsum pharetra ligula, ac pretium ante justo a nulla. Curabitur tristique arcu eu metus. Vestibulum lectus. Proin mauris. Proin eu nunc eu urna hendrerit faucibus. Aliquam auctor, pede consequat laoreet varius, eros tellus scelerisque quam, pellentesque hendrerit ipsum dolor sed augue. Nulla nec lacus.

Acknowledgements

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Contents

List of Figures

List of Tables

Summary

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Chapter 1

Introduction

Game theory is a set of analytical tools and solution concepts, which provide explanatory and predicting power in interactive decision situations, when the aims, goals and preferences of the participating players are potentially in conflict, Szabo and Fath (2007) [25]. The Prisoner's Dilemma(PD) is a well known example in Game Theory and in recent years has become the gold standard of understanding evolution of co-operative behavior [15]. Thus, it has been a topic of focus in various fields, such as biology, sociology, ecology and psychology.

In the example of the Prisoner's Dilemma(PD) two criminals have been arrested and interrogated, with no way of communicating, by the police. They are given only two choices, to either cooperate with each other or to defect. Now let us consider that the prisoners would be put back in their cells and would be asked the same question tomorrow. Furthermore, let this happen repeatably. This is referred to as the Iterated Prisoner's Dilemma(IPD) an example that has been a rich source of research material since the 1950s but has earned much interest in the 1980s due to the work done by the political scientist Robert Axelrod [2, 3, 4].

In 1980, Axelrod held the first ever IPD computer tournaments [2, 3], he invited academics from various fields to submit their strategies in computer code. The tournaments were of a round robin topology, the first competition included thirteen strategies, while the second one sixty-four. In both tournaments the strategy Tit for Tat was announced the winner and for many years it was consider to be the most successful strategy. Tit for Tat is a deterministic strategy that will always cooperate in the first round and afterwards it copies the opponents last move.

A large volume of literature emerged on the topic following this, including some criticism about these initial tournaments. Scientists questioned whether the conditions that the first tournament took place favored tit for tat. An argument was that the initial tournaments though they included a 1% chance of players misunderstanding their opponent's move in any round they did not examine noise. Noise is the probability that the player will submit the wrong move. David and Vivian Kraines [11] stated that TFT performed rather poorly when noise was introduced in the tournament. Another aspect, is the payoff matrix which according to Kretz [12] the precise choice of the payoff matrix is relevant to the results.

Furthermore, another aspect needed to be taken into account was the network topology underlying the tournament. In 1992 Nowak's and May's paper [19] spatial tournament are introduced. In which the players are placed on an two-dimensional spatial and allowed to play a game with only the immediate neighbors. Thus, squares that are adjoin. An example of this is shown in Figure ...

Their tournament considered the PD and the players could only defect or co-operate. They provided proof that cooperative behavior can emerge from a PD tournament in spatial topology. Many works on the IPD and spatial tournament were held due to their original paper. Such as [10, 20, 16, 19, 7, 18, 14]. These tournaments use either the PD or IPD and simple to complex strategies.

One can argue that the real life interactions are better represented by spatial tournament because in real life not all players interact with all opponents. Additionally, an interesting aspect of the spatial topology are the results compared to those of a round robin tournament. This dissertation will be focusing on reproducing a spatial tournament with some of the most successful strategies of various tournaments that have been held. For the spatial topology it will use various graphs compared to other works done only using lattices. Concluding how spatial topology, with any given graph, affects the effectiveness of these strategies.

1.1 The Prisoner's Dilemma

The PD was originally formulated in Merril [**M. Flood and Melvin Dresher**], who were working on the Flood-Desher Experiment at the RAND cooperation in 1950. Later in 1950, the mathematician Albert W. Tucker presented the first formal representation of the PD, titled A Two-Person Dilemma in a seminar at Stanford University [9].

A description of the PD, found in [13] is as follows: There are two players that simultaneously have to decide to whether Cooperate (C) or Defect (D) with each other, without exchanging information.

- If both players choose to cooperate they will both receive a reward (R)
- If a player defects and the other cooperates then the defector receives a temptation payoff (T) and the cooperator a sucker payoff (S)
- If both players defect they will both receive a penalty (P)

Figure 1.1 illustrates the payoffs matrix.

		Player II	
		Cooperate	Defect
Player I	Cooperate	***R*=3** / ***R*=3**	***T*=5** / ***S*=0**
	Defect	***S*=0** / ***T*=5**	***P*=1** / ***P*=1**

Figure 1.1: The payoff matrix for the Prisoners Dilemma

Taking into account the assumptions that both players are rational and that there is no way of communication between them. No matter what the other player does, defecting will be their dominant choice as it yields a higher payoff than cooperation. Thus a pure Nash Equilibrium exists when both players defect. Even though, both players would do better if they were to cooperate.Thus creating the dilemma.

Furthermore, for this to hold there are some extra assumptions for the relationship of the four outcomes. The T temptation to defect has to offer the highest payoff for a player and the worst he could get has to be the sucker S. Likewise, the reward for mutual cooperation should exceed that of mutual defection P. Thus the next condition is 1.1:

$$T > R > P > S \tag{1.1}$$

Moreover, it is assumed that the average of T and S is less than the reward for mutual cooperation 1.2 :

$$R > (T + S)/2 \tag{1.2}$$

Same conditions such as rationality, no communication and (1.1), (1.2), apply for the IPD. An IPD is nothing more than a PD were the players interact for an infinite number of times.

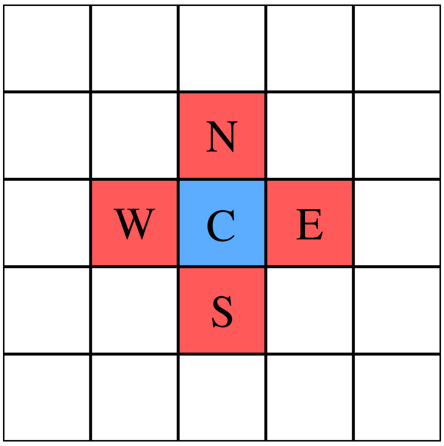
1.2 Problem Description

Axelrod's tournaments set a seed for generations of tournaments in the Prisoner's using computer modeling. Research has shown that by altering the environment of a tournament the effectiveness of some strategies can change radically. An aspect that has been investigated as to how the tournament results can bee affected was the topology. Nowak and May [19] introduced the spatial topology only to set yet another seed in the PD tournaments. Even so, spatial topology still has not been fully explored with only a small number of papers focusing on this specific topology. A goal of this dissertation is to understand the current state of the art in spatial prisoners dilemma tournaments.

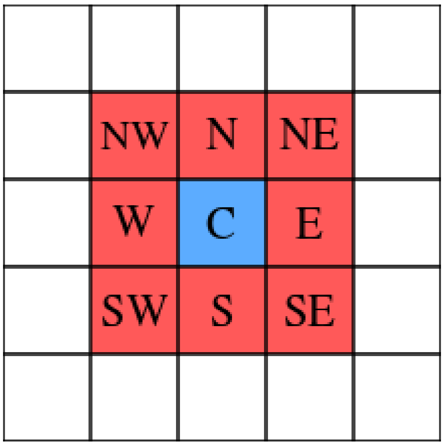
As described in [**Maschler**] the contemporary world is full of networks. That is one of the main reasons this dissertation will be focusing on such topology. Work done at this point consider spatial topology to be that of a square lattice were each edge represents a player which interacting/playing with only his neighbors in their von Neuman or Moore's neighborhood. A von Neuman neighborhood comprises the four cells orthogonally surrounding a central cell where the Moore's neighborhood eight cells. As shown in Figure 1.2.

Some further work address the spatial topology in a variate of graphs [17, 25]. This dissertation will also follow this approach. Szabo and Fth dealt with numerous graphs such as, lattices, small- world, scale free graphs and evolving networks. We will consider a spatial topology to be any given graph where the players are the nodes and only play other nodes that are linked to by an edge.

Another disadvantage of the aforementioned work is that it lacked in terms of best practice of reproducibility [2, 3, 22, 8, 24] . Due the work done by Axelrod library [26]. An open source python Package which allows for the easy reproducibility of experiments. As it allow to reproduce an IPD tournament and chose between



(a) von Neuman's neighborhood



(b) Moore's neighborhood

Figure 1.2: Possible neighborhoods. (a) A von Neuman's where each node has four neighbors. (b) Moore's where each node has eight neighbors.

131 strategies already given by the library. Code written for the purpose of this dissertation has been contributed to the library: see `https://github.com/Axelrod-Python`.

1.3 Structure of Dissertation

This dissertation is organized into 7 Chapters. Proceeding this introduction:

- In Chapter 2 we review previous literature dedicated to the PD/IPD and tournaments that have been conducted, different topologies and evolution.

Chapter 2

Literature Review

Following the initial work done by Axelrod, there are many other papers that have tried to tackle the PD and make their conclusions on cooperation in both a theoretical and real life setting. In this chapter a review of some of this work done in the IPD competitions, in spatial and evolutionary game theory will be carried out.

2.1 Tournaments

In order to identify the condition under which cooperation could emerge in the game of the Prisoner's Dilemma better, Robert Axelrod held a tournament in 1980. He invited a number of well-known game theorists to submit strategies for a computer tournament. Each strategy has to specify whether to cooperate or defect based on the history of previous moves made by both players. Strategies played again each other as well as a further Random strategy, that would randomly choose between C and D and with its own twin (same strategy). The tournament was a round robin with the payoff matrix 1.1. All entries knew the exact length (200 moves) of each game. To improve the reliability of the scores the entire round robin tournament was repeated five times. Fourteen strategies were submitted and by the end Tit for Tat was announced the winner. Surprisingly in the second tournament held where 64 strategies competed and all submitters had full knowledge of what have happened in the first tournament, Tit for Tat managed to get first place again[2].

As explained in [3], Tit for Tat, a simple strategy was able to beat sophisticated and more complex strategies thanks to three specific characteristics of the strategy:

- Niceness: A strategy is categorized as nice if it was not the first to defect, or at least, it will not do this until the last few moves.
- Forgiveness: The propensity to cooperate in the moves after the opponent defected.
- Clarity: After opponents identified that they were playing Tit for Tat choose to cooperate for the rest of the game.

The first tournaments were an innovation in combining computer modeling and Game Theory and in providing insights in the behavior emerging from simple dynamics. Moreover, Axelrod was the first to speak about niceness, forgiveness and gave an illustration that cooperation can be a victorious and advantageous strategy.

Another concept that had been developed was the Evolutionary Game Theory (EGT). EGT is an application of game theory to biological contexts, arising from the realization that frequency dependent fitness introduces a strategic aspect to evolution. In 1973, Maynard and Price introduced the concept of Evolutionary Stable Strategy (ESS), which is an extension of a Nash Equilibrium. If a population of the same strategies cannot be invaded by any alternative strategy that is initially rare then that strategy is an ESS. In his third tournament Axelrod [4] using the same set of strategies (63), the tournaments introduced a dynamical rule that mimics Darwinian selection. In this evolutionary computer tournament after a round robin game the score for each player was evaluated, and the strategies with high score would be adapted while the lowest ones ones would diminish. In most of these simulations, the success of Tit-for-Tat was confirmed because the population would end up with some mutually cooperating strategies prevailed by Tit-for-Tat.

There have been other tournaments, based off of Axelrods, exploring different environments and submitting new strategies. Boyd & Lorderbaum [15] state that no pure strategy is evolutionary stable because each can be invaded by the joint effect of two invading strategies when long term interaction occurs in th repeated game and future moves are discounted. In 1991 Bendor, Kramer and Stout [5] introduced noise to the IPD. Where noisy randomly flip the choice made by a strategy. The results of their tournament was that the strategies that were more generous, cooperated more than their opponents did, were more effective than Tit for Tat. Moreover, Kerts 2011 conducted a tournament where the payoff matrix was altered though satisfying the conditions 1.1, 1.2.

Furthermore, two more notable tournaments took place in 2005 and 2012. In the 2005 IPD competition a team from the University of Southampton participated using a group of strategies which won the top three propositions. These strategies were designed in such why that thought a predetermined sequence of five to ten moves would recognize each other. Once the two Southampton players recognized each other they would take up the roles of a ruler and a slave. The ruler would always cooperate where the slave would defect in order to maximize the payoff of the ruler. If the opponent was recognized to not being one of the team then the Southampton player would always choose to defect to minimize the score of the opponent [13]. Lastly, the Stewart- Plotkin [23] tournament which consisted of nineteen strategies, including a new set of strategies; the Zero- determinant(ZD) strategies. The ZD are strategies for the stochastic iterated prisoner's dilemma, discovered by Press and Dyson in 2012 [21]. The ZD apply a linear relationship between their own payoff and that of the opponent. Some review tournaments are listed on the Table 2.1:

Year	Reference	Number of Strategies	Type
1979	[2]	13	Standard
1979	[3]	64	Standard
1984	[4]	64	Evolutionary
1991	[5]	13	Noisy
2005	[8]	223	Varied
2012	[23]	13	Standard

Table 2.1: An overview of a selection of published tournaments.

In this section the work done for tournaments in the IPD has been cited and analyzed. Starting with the the work of Axelrod, the reasons the tournaments where organized and what where the fundings. Moreover, some research that has been generated the following years are stated. Below, a specialized case of these research will be studied. That case is that of the spatial structure tournaments.

2.2 Spatial Structure Tournaments

Further research was spawn in 1992 as to how the Prisoner's Dilemma could shade some insight into physics and biology. Where Nowak and May believed exist potential dynamics of spatially extended systems. Their tournament was a simple and purely deterministic spatial version of the PD in a two dimensional lattice. With players having no memory of the previous rounds and no strategical elaboration. Thus, the players could either always cooperate or defect. In each round each player interact with the immediate neighbor. [1] They used an evolution rule that after each round round the nodes with the lowest score in their neighborhood would copy the strategy of the player with the highest score. This was done to study which behavior, defection or cooperation , would last. The conclusion was that co-operational behavior is possible in the PD by using a spatial topology. Nowak produced more work on the topic on papers of his such as [20] (Nowak 1994). In his subsequent papers Nowak et al. (1994a,b) different spatial structures where studied. Including triangular and cubic lattices and a random grid. It turned out that cooperation can be maintained in spatial models even for some randomness.

On the other hand, in [14] players were allowed to have memory and therefore added complex strategies to the tournament such as Tit for Tat and Anti Tit for Tat. This was followed by the work of [7] which introduced even more complex strategies. Brauchli et all compares the spatial model with a randomly mixed model. A more complex strategy that they have tested was PAVLOV. A win-stay, lose-switch strategy. According to their fundings, there is more cooperative behavior in a spatial structure tournament and evolution is more less chaotic than in unstructured populations. Also as stated, generous variants of PAVLOV are found to be very successful strategies in playing the Iterated Prisoner's Dilemma.

Spatial topology has been defined by most scholars as a square lattice where the nodes - players only interact with their neighborhoods. Including connections between four or eight nearest neighbor sites, Neuman's or Moore's, according to Figure 1.2. A square lattice is a graph and one could argue that a round robin tournament itself is the complete graph on all players [6]. But in the above papers no authors defined the topology as a graph, apart from [18].

[1] In Nowak's and May's experiment, the result's hold for all three cases that the player interacts with 4, 6 and 8 neighbors.

In [18], an interesting approach was used. They presented a new spatial prisoner's dilemma game model in which the neighborhood size was increased onto two interdependent lattices. They implement the utility by integrating the payoff correlations between two lattices. A player would mimic a random player in his next move, base on a function that consider the utility of the player. It was characterized as a most realistic scenario.

Real life interactions are more likely to be like any given graph depending on the industry than a complete graph. Fatha et all [25], have considered a numerous of graphs, such as :

- Lattice, the interaction network is defined by the sites of a lattice. the distance between a pair does not exceed a given value. The most frequently used structure is the square lattice with von Neumann neighborhood and Moore neighborhood.
- Small word, a graph that is created from a square lattice by randomly rewiring a fraction of connections in a way that conserve the degree for each site.
- Scale-free graphs, a network that has a power-law degree distribution, regardless of any other structure.
- Evolving networks, networks that change as a function of time (this will not be considered in this dissertation).

The major theme of their review was how the graph structure of interactions could modify long term behavioral patterns emerging in evolutionary games. These graphs compose only a small fraction of graphs that exist. In this dissertation we will consider a list of graphs.

2.3 Axelrod Python Library

The Axelrod library [26] is an open source Python package that allows for reproducible game theoretic research into the Iterated Prisoner's Dilemma `https://github.com/Axelrod-Python`. For many of the tournaments aforementioned the original source code is almost never available and in no cases is the available code well-documented, easily modified or released with significant test suites. Due to that reproducing the results has not been an easy task.

However, Axelrod library manages to provide such a resource, with facilities for the design of new strategies and interactions between them, as well as conducting tournaments and ecological simulations for populations of strategies.

Strategies are implemented as classes which have a single method, `strategy()`. It only takes one argument, which is the opponent's previous moves and returns an action. These actions can be either to cooperate C or to defect D. At thins moment the Axelrod library consists of 131 strategies.Can be found in the Appendix. As an example we can see in Figure the source code for the famous strategy Tit for Tat.

```
class TitForTat(Player):
    """
    A player starts by cooperating and then mimics
    the previous action of the opponent.

    Note that the code for this strategy is written
    in a fairly verbose way. This is done so that it
    can serve as an example strategy for those who
    might be new to Python.
    """

    # These are various properties for the strategy
    name = 'Tit For Tat'
    classifier = {
        'memory_depth': 1,  # Four-Vector = (1.,0.,1.,0.)
        'stochastic': False,
        'makes_use_of': set(),
        'inspects_source': False,
        'manipulates_source': False,
        'manipulates_state': False
    }

    def strategy(self, opponent):
        """This is the actual strategy"""
        # First move
        if len(self.history) == 0:
            return C
        # React to the opponent's last move
        if opponent.history[-1] == D:
            return D
        return C
```

Additionally, tournament is a class responsible for coordinating the play of generated matches. It achieves that by calling a match generator class which returns all the single match parameters, such as turns, the game and the noise. Axelrod has the capability to write out the results into a csv file and also out put plots with the ranks of the strategies.

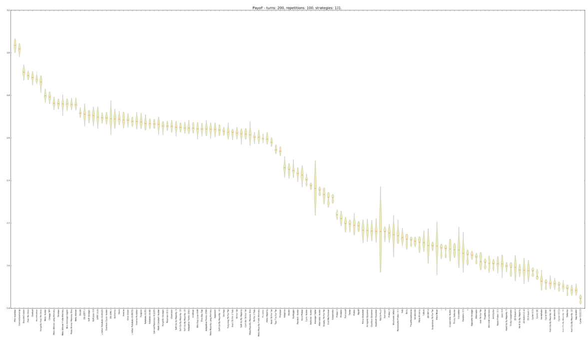
Furthermore, a basic tournament of 200 turns, 100 repetitions and the 131 strategies that exist in the library is being produced continuously. The current winner is called PSO gambler and it is a look up strategy. It uses a lookup table with probability numbers generated using a Particle Swarm Optimisation (PSO) algorithm, the sourse code can be found here : `http://axelrod.readthedocs.io/en/latest/_modules/axelrod/strategies/gambler.html?highlight=Gambler`) and a description of how this strategy was trained is given here: `https://gist.github.com/GDKO/60c3d0fd423598f3c4e4`. It uses a 64-key lookup table (keys are 3-tuples consisting of the opponent's starting actions, the opponent's recent actions, and our recent action) to decide whether to cooperate (C) or defect (D). The actions for each key were generated using an evolutionary algorithm.

To reproduce a basic tournament with the 131 strategies using Axelrod :

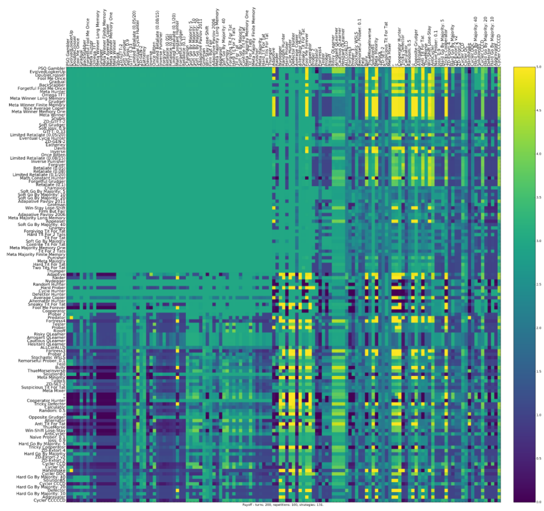
```
>>> import axelrod
>>> strategies = [s() for s in axelrod.ordinary_strategies]
>>> tournament = axelrod.Tournament(strategies)
>>> results = tournament.play()
>>> plot = axelrod.Plot(results)
>>> p = plot.payoff()
>>> p.show()
```

Listing 2.1: A simple set of commands to create a basic tournament. The output is shown in Figure 2.1.

Here are illustrated the results of the last tournament :



(a) Ranked violin plot



(b) Payoffs

Figure 2.1: Result Plots. (a) Ranked violin plot, the mean utility of each player. (b) Payoffs, the pair wise utilities of each player.

More details for the documentation of the library can be found here : `https://axelrod.readthedocs.io/en/latest/index.html`. Because is an open source library it makes it easy to contribute to it and make modifications needed for this dissertation.

Chapter 3

Implementation of spatial tournaments

In this chapter we will discuss the source code committed to the axelrod library to implement spatial topology. In addition, some initial experiments and results will be discussed.

3.1 Code Discussion

As analyzed in chapter 2, the Axelrod library uses a `Tournament()` class to run a round robin tournament. The `RoundRobinTournament()` itself calls upon another class the `Match Generator()` which is responsible to generate the matches. In a round robin cause(`RoundRobinMatches()`), it generates matches for each player against the rest. The parameters and the index of the pair are push to it by the `build single match()`. A generator that lives within the class. For a round robin tournament the structure of the code is illustrated in 3.1.

In order for us to implement a Spatial topology tournament we need to follow a similar approach. Firstly a new `Match Generator()` class was written. The `SpatialMatches()` is a class that generates spatially-structured matches. In these matches, players interact only with their neighbors rather than the entire population. According to [1] graphs can be represents in many different ways, one of which is by lists of edges. Due to a various number of python packages that are used for graph manipulation, we want to generalize the representation of the edges. For the `SpatialMatches()` do understand which players are connected to each other, the edges are passed as a list argument. The `SpatialMatches()` will

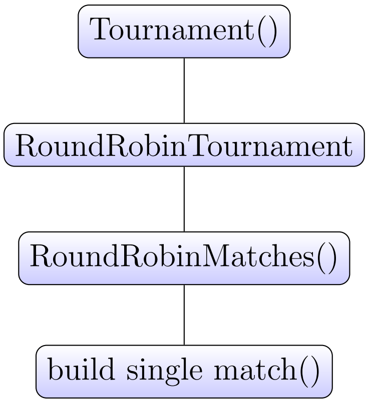


Figure 3.1: Code structure for a Round Robin tournament.

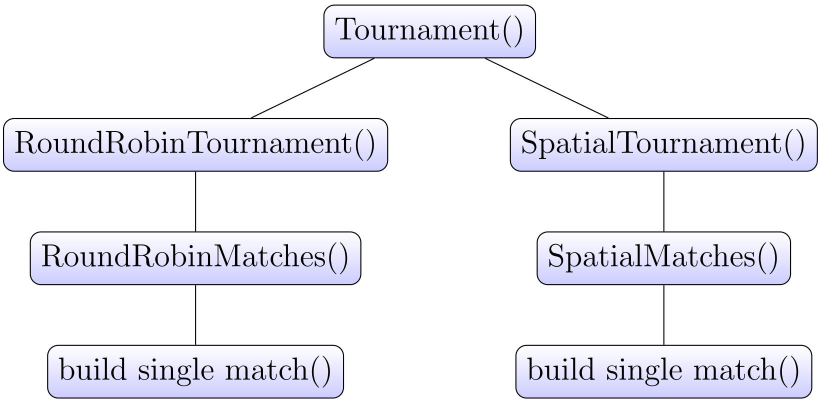


Figure 3.2: Code structure for when Round Robin and Spatial tournaments are implemented.

only create matches between the ending nodes of these edges. Finally the class `SpatialTournament()` runs the spatial tournament. The code structure now that the spatial tournaments have been added can be seen in Figure 3.2

In Axelrod library all the components are automatically tested using a combination of unit, property and integration tests(using `travis-ci.org`). Once a new feature is added to the library, corresponding test must also be written. The test are used to ensure compatibility and ensure that we get the expected results. The tests for the `SpatialTournament()` can be found here : `https://github.com/Axelrod-Python/Axelrod/blob/master/axelrod/tests/unit/test_tournament.py` under the class `TestSpatialTournament()`.

In this section we gone through the structure of the source code for implementing the Spatial Tournament, by adding to the Axelrod-Python library. Furthermore, now that the code is usable in the following section we will go though some initial analysis.

3.2 First Experiment

In this chapter we will be running some simple examples of spatial topology and analyze the behavior of the strategies and the results. This will help us further down to tackle the problem of how topology can affect the outcome of an IPD competition and which strategies tend to perform well. Some of the most common networks, based o literature, are a 2-D square lattice with four degrees and a circle. Figure 3.3, shows an example of these topologies.

(a) 2-D square lattice network. (b) Circle network.

Figure 3.3: Network topologies.

Thus, we will be performing our experiment with two different topologies, that of a lattice and a circle. For each topology, 50 strategies out of the 132 of Axelrod-Python library are chosen randomly. These 50 strategies create a neighborhood. They are allocated on the graph, based on the topology, and they compete with their neighbors on a IPD tournament. Once the first game is complete, the strategies are randomly shuffled and allocated on the graph again. They play another game of the IPD but this time with different neighbors of they neighborhood. This is repeated 10 times and the random selection of 50 players and creation of neighborhood is repeated 100 times. Thus we achieve, 1000 tournaments with 100 different neighbors ,where each neighborhood exists for 10 tournaments. Furthermore, each game of the IPD consists of 200 turns and 10 repetitions. By setting an axelrod-seed [1], the 100 neighborhoods for the topologies are the same, of course their allocation to the graphs differs because of the random shuffle.

[1] A function used by the library, which sets both seeds for Numpy and the standard library.In general, seeds allow us reproduce the same players and tournaments.

3.2.1 Circle Topology

In the tournament with circle lattice while fifty players participated in each of the hundred tournaments all 132 strategies of the Axelrod-Python library were used at least once. Moreover, all strategies won, ranked 0, at least one tournament. In Figure 3.4 we can see the number of times each strategy has won a tournament in ascending order :

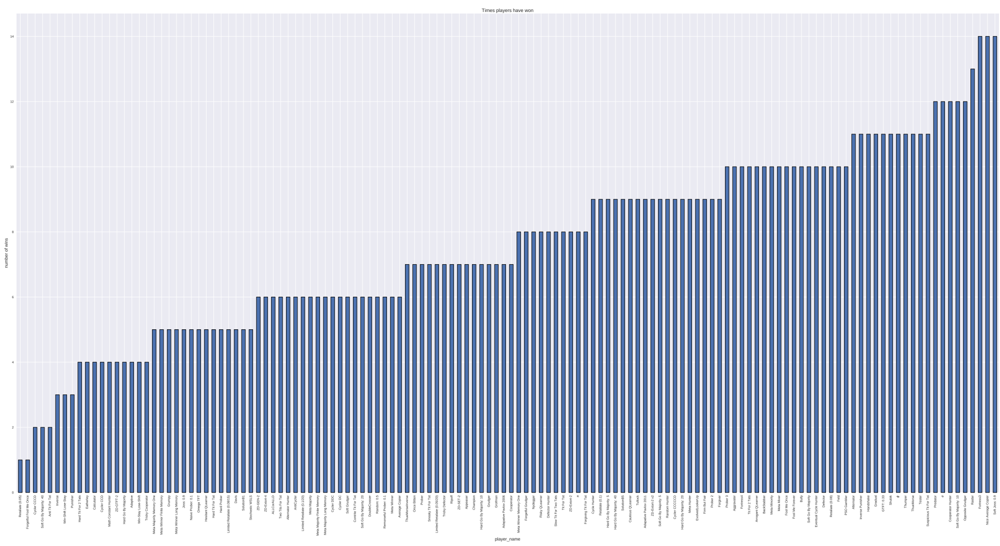


Figure 3.4: Number of winning tournaments by strategy in a circle topology.

As shown, the three strategies which achieved the most wins in the 1000 tournaments are : Soft Joss: 0.9, Fortress3, Nice Average Copier. Soft Joss : 0.9, is an altered version of Tit-For-Tat. The strategy emulates Tit-For-Tat but defects with a probability of 0.9 when the opponent defects. Nice average copier will cooperate with a probability p if the opponent's cooperation ratio is p. Starts by cooperating. All three strategies achieved 14 wins. In a simple round robin tournament all these have significant different cooperating rating. Nice Average Copier, is a cooperating strategy where Soft Joss: 0.9 has an average cooperating rating and Fortress3 a low one. We want to investigate any key factor for these strategies achievement of first place even so in the original tournament the score quite low.

The number that each strategy participated in the 1000 tournaments is not clear. Thus, we can plot the number of participations of each strategy as show in Figure 3.5. It is shown that the range of participations varies from 270 to 570 played tournaments and that Altenator played the most. Furthermore, a more clear representation of the participations and strategies is given by table in the Appendix. There is clear that the most common played strategies were Altenator, ThueMorse, Shubik and Gradual. None of the first ranked strategies.

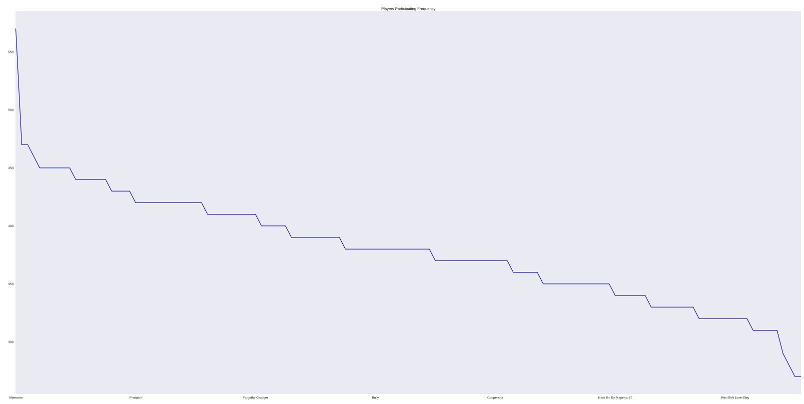
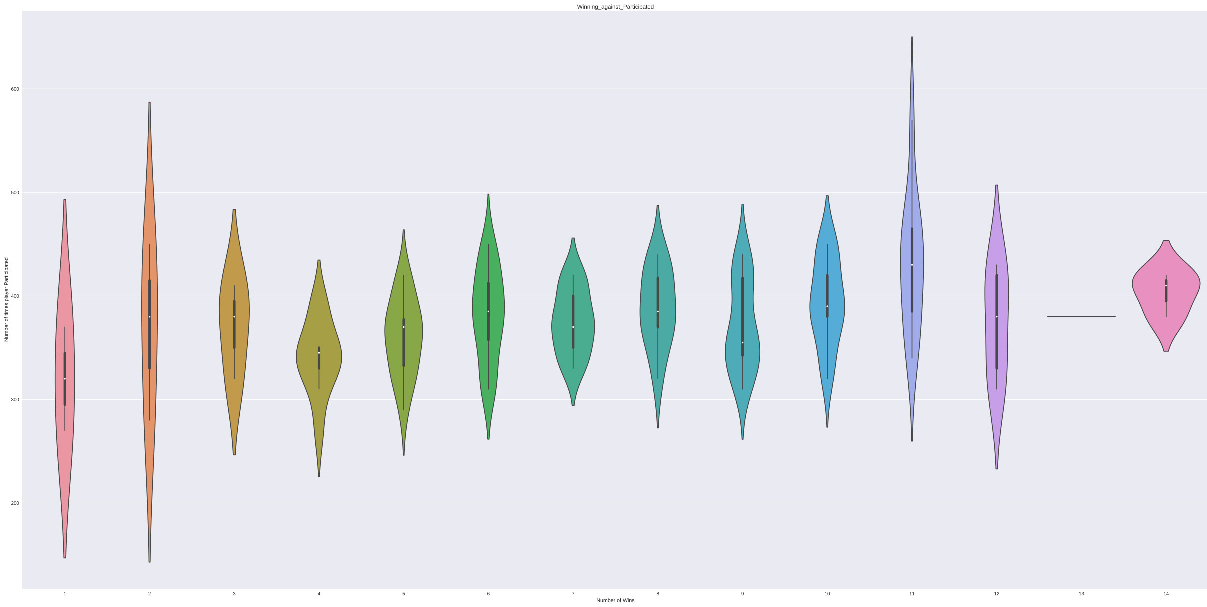
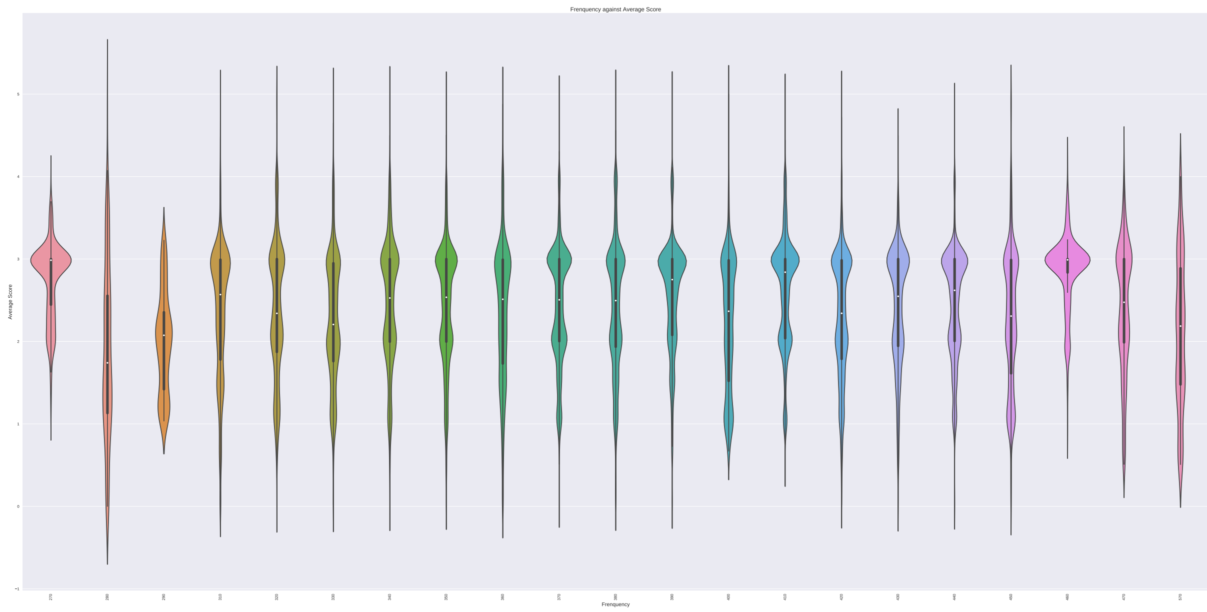


Figure 3.5: Number of participations in tournaments of each strategy.

Moreover, we can plot the number of participation against the number of times the specific strategy has won to identify any correlation between these facts Figure 3.6. Though the strategies with the highest participation ratio seem to have won tournaments, it is only a insignificant amount. strategies with the highest number of wins see to have place an average number of tournaments. By plotting the Frequency of the strategies against average score per turn, we anticipated to have a positive correlation. As the more tournaments a strategy competes in there is a higher probability that the strategy will be declared the winner. Even so, from Figure 3.6 we can assume that they are is no significant correlation between these two facts.



(a) Number of participations against the number of winning tournaments.



(b) Frequency of strategies against winning tournaments.

Figure 3.6: (a) Number of participations against the number of winning tournaments. (b) Frequency of strategies against winning tournaments.

Another factor we can investigate is the score of the neighborhood. Each strategy belongs in a neighborhood of 50 strategies during the experiment. Plotting the average score per turn against the neighbors overall average score. In Figure 3.7, see an see as expected that each when the higher the average score of the neighborhood the lowest the individuals score per turn. Thus, a strategy in a neighborhood with winners is performing poorly.

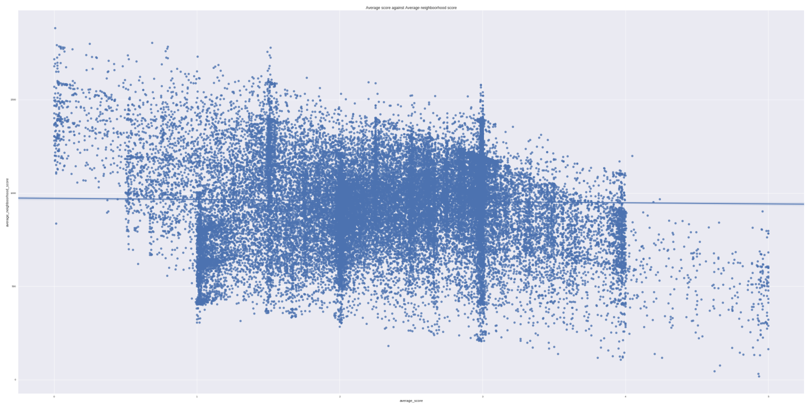


Figure 3.7: Scatter plot. Average score per turn and average neighborhood score. Circle topology.

Finally, a common methodology is building a regression model. We are building a model wanting to identify any factor that can explain the average score of a strategy. The model is the following :

$$average\_score = degree + average\_neighboorhood\_score + connectivity + frenquency \tag{3.1}$$

and the output produced using python's statistic's tools is shown in Figure 3.8. In the output we can see that Degree, average neighborhood score and connectivity are significant predictors with a p value less than an 0.00. However, the p-value of frequency (0.446) is greater than the common alpha level of 0.05, which indicates that frequency is not significant. We could foresee this from the undertaken analysis. The coefficients of degree and connectivity point out the positivity relationship of these variables and the average score. Average scores, again something we could foresee, has a negative effect on the average score. Additionally, an R square value of zero indicated that the model explains none of the variability of the response data around its mean.

OLS Regression Results

Dep. Variable:	average_score	**R-squared:**	0.000
Model:	OLS	**Adj. R-squared:**	0.000
Method:	Least Squares	**F-statistic:**	10.17
Date:	Mon, 01 Aug 2016	**Prob (F-statistic):**	3.83e-05
Time:	01:13:32	**Log-Likelihood:**	-58333.
No. Observations:	50000	**AIC:**	1.167e+05
Df Residuals:	49997	**BIC:**	1.167e+05
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[95.0% Conf. Int.]
Intercept	0.2756	0.004	77.817	0.000	0.269 0.282
degree	0.5511	0.007	77.817	0.000	0.537 0.565
average_neighboorhood_score	-6.698e-05	1.5e-05	-4.458	0.000	-9.64e-05 -3.75e-05
connectivity	0.5511	0.007	77.817	0.000	0.537 0.565
freq	-5.543e-05	7.27e-05	-0.762	0.446	-0.000 8.72e-05

Omnibus:	864.471	**Durbin-Watson:**	1.968
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	909.368
Skew:	-0.330	**Prob(JB):**	3.41e-198
Kurtosis:	2.956	**Cond. No.**	8.99e+20

Figure 3.8: Regression for circle topology results.

3.2.2 Lattice Topology

A similar approach as described above was taken for an experiment where the tournaments had a lattice topology. The most frequented winner this time was Adapative Pavlov 2006 (APavlov) with 15 wins 3.9. Runner ups with 14 wins were Remorseful Prober: 0.1, Fool Me Once and Inverse Punisher. APavlov is a strategy that attempts to classify its opponent as one of five strategies: Cooperative, ALLD, STFT, PavlovD, or Random. Then responds in a manner intended to achieve mutual cooperation or to defect against uncooperative opponents. None of this strategies ranked first in the circle topology. Furthermore, all 132 strategies participated at least on in this experiment and all achieved first place at least once.

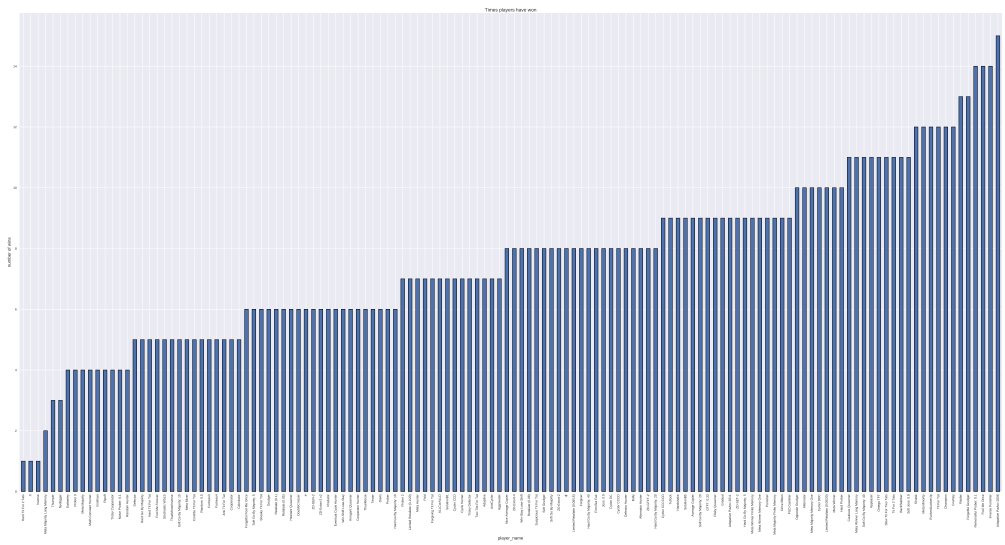
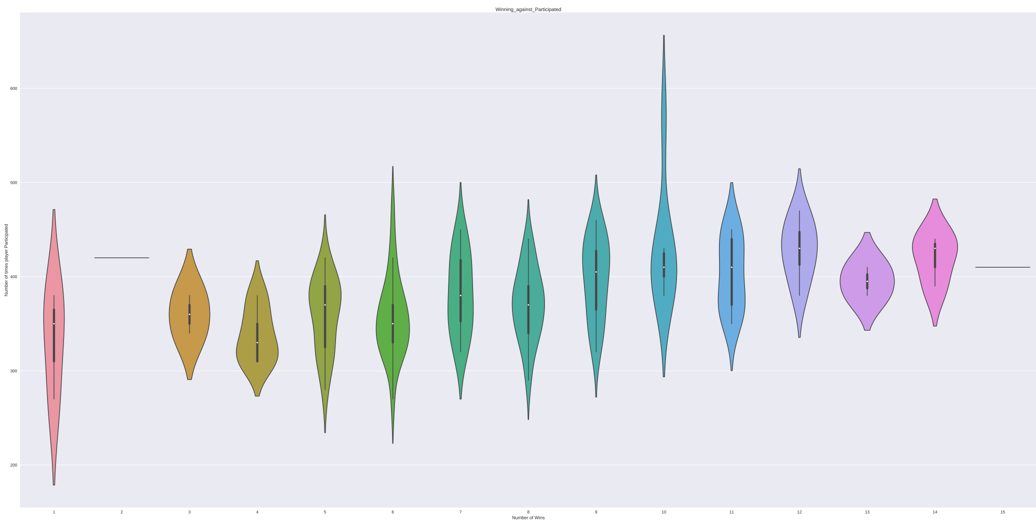
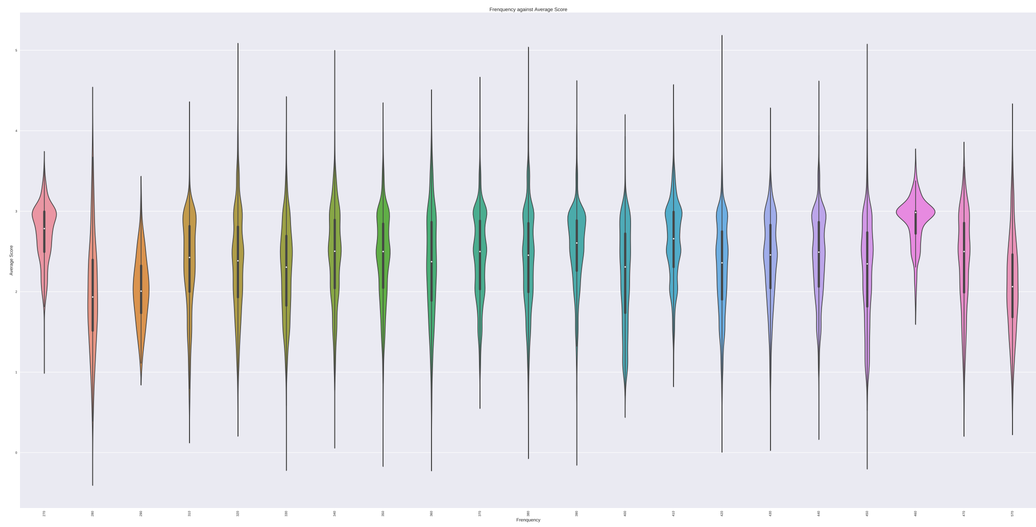


Figure 3.9: Number of winning tournaments by strategy in a lattice topology.

An analysis to identify and factor for these results was performed. The number of participations of each strategy, the ranks and average score achieved by them were analyzed one again. Because the neighborhood are the same for both topologies and what changes is only the degree, the plot for number of participations should remain the same as shown in Figure 3.5. Furthermore, the number of wins against number of participations are plot again as well as the frequency against the winning tournaments. In the Participated plot, a positive linear relation could exists but in the frequency plot we can see no significant variation to the mean.



(a) Number of participations against the number of winning tournaments.



(b) Frequency of strategies against winning tournaments.

Figure 3.10: (a) Number of participations against the number of winning tournaments. (b) Frequency of strategies against winning tournaments. In a lattice topology.

Moreover, a scatter plot to identify relationship with the average neighborhood score is produced as well. Compared to our previous founding this plot shows a positive correlation between the two factors 3.11. Meaning, that you are more likely to win if you are in a winning neighborhood. This could be an affect of the topology. More nodes interact with each other in a lattice and can create cluster more easy than in circle topology.

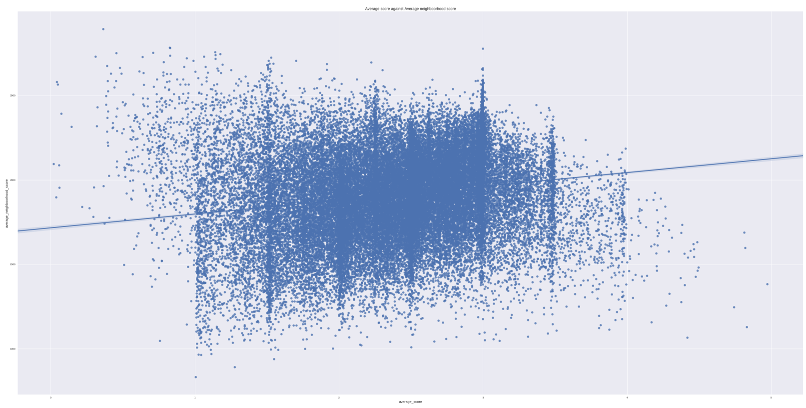


Figure 3.11: Scatter plot. Average score per turn and average neighborhood score. Lattice topology

To test our hypothesis we will again create a regression model, same as in 3.1. The results are shown here 3.12. Once more, degree, average neighborhood score and connectivity are significant predictors with a p-value less than 0.00. Frequency, in the lattice topology does not seem to have any effect either. What worth to point out is that now all the significant predictors have a positive effect on the average score. Finally, the R -square value indicates that only explains 3 % of the variability of the data around its mean.

OLS Regression Results

Dep. Variable:	average_score	**R-squared:**	0.032
Model:	OLS	**Adj. R-squared:**	0.032
Method:	Least Squares	**F-statistic:**	839.1
Date:	Mon, 01 Aug 2016	**Prob (F-statistic):**	0.00
Time:	12:15:54	**Log-Likelihood:**	-43822.
No. Observations:	50000	**AIC:**	8.765e+04
Df Residuals:	49997	**BIC:**	8.768e+04
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[95.0% Conf. Int.]**
Intercept	0.0502	0.001	58.871	0.000	0.049 0.052
degree	0.2009	0.003	58.871	0.000	0.194 0.208
average_neighboorhood_score	0.0004	9.69e-06	40.943	0.000	0.000 0.000
connectivity	0.2009	0.003	58.871	0.000	0.194 0.208
freq	-5.972e-05	5.44e-05	-1.097	0.272	-0.000 4.69e-05

Omnibus:	1300.814	**Durbin-Watson:**	2.113
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	1512.615
Skew:	-0.356	**Prob(JB):**	0.00
Kurtosis:	3.467	**Cond. No.**	1.05e+21

Figure 3.12: Regression for circle topology results.

3.2.3 Round Robin Tournament

Once the experiment with the both topology was performed we wanted to compare our results further and make an conclusion on the winning strategies. In order to achieve that it was decided to run an experiment with the same 100 neighborhoods. In time they strategies would compete on a round robin topology. Again the number of turns was set to 200 but the repetitions to 100. All 132 strategies compete at least once but only 72 of them achieved first place. In Figure the results of this experiment are shown.

to be continuous ...

3.2.4 Comparing Results

In this section we will make a summary of all the previous analysis that was made in ??. Furthermore, we will to compare the winning strategies of each topology and identify any further relationship between the performance of a strategy and the topology. From the analysis that was performed in 3.2.1, the winner came out to be 3 strategies that performed the same, Soft Joss: 0.9, Fortress3, Nice Average Copier. These strategies have not a similar characteristics like cooperating rating. An attempt to find any significant reason as to why these strategies outperform the rest return zero findings. Factors such as the connectivity of the graph, the average score of the neighbors, the frequency that each strategy participated and the degree of the graph had zero or non strongly significant effects. In the circle topology the neighborhood score has a negative effect on the performance of the strategy and the degree and connectivity of the graph a positive one.

Furthermore, the same approach was taken for 3.2.2. The winning strategy of this experiment was announced to be Adapative Pavlov 2006, which outperformed all the other strategies. Though the results as to why APavlov did not differ much from the circle experiment. The were non strongly significant factors identified from the analysis and the regression model. A difference that was observed is that in the lattice example the average score of your neighbors seem to have a positive effect on you. This is due to the structure of the topology. The last experiment that was held was in 3.2.3, where the neighborhoods competed in an all out round robin tournament.

An illustration of the performance in all three topology for each strategy can been seen in Figure.

In conclusion, the network topologies used in these examples seem to have be simple and no real results were able to emerge. Moving forwards experiments with more complicated and random topologies will be performed with all the 132 strategies of the Axelrod-Python library.

Chapter 4

Title of Chapter 3

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas. Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetuer.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetuer at, consectetuer sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui.

Morbi luctus, wisi viverra faucibus pretium, nibh est placerat odio, nec commodo wisi enim eget quam. Quisque libero justo, consectetuer a, feugiat vitae, porttitor eu, libero. Suspendisse sed mauris vitae elit sollicitudin malesuada. Maecenas ultricies eros sit amet ante. Ut venenatis velit. Maecenas sed mi eget dui varius euismod. Phasellus aliquet volutpat odio. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Pellentesque sit amet pede ac sem eleifend consectetuer. Nullam elementum, urna vel imperdiet sodales, elit ipsum pharetra ligula, ac pretium ante justo a nulla. Curabitur tristique arcu eu metus. Vestibulum lectus. Proin mauris. Proin eu nunc eu urna hendrerit faucibus. Aliquam auctor, pede consequat laoreet varius, eros tellus scelerisque quam, pellentesque hendrerit ipsum dolor sed augue. Nulla nec lacus.

Suspendisse vitae elit. Aliquam arcu neque, ornare in, ullamcorper quis, commodo eu, libero. Fusce sagittis erat at erat tristique mollis. Maecenas sapien libero, molestie et, lobortis in, sodales eget, dui. Morbi ultrices rutrum lorem. Nam elementum ullamcorper leo. Morbi dui. Aliquam sagittis. Nunc placerat. Pellentesque tristique sodales est. Maecenas imperdiet lacinia velit. Cras non urna. Morbi eros pede, suscipit ac, varius vel, egestas non, eros. Praesent malesuada, diam id pretium elementum, eros sem dictum tortor, vel consectetuer odio sem sed wisi.

References

[1] D. Archdeacon. "Topological graph theory-a survey". In: *Cong. Num* (1996), pp. 1–67. URL: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.28.1728$%5Cbackslash$npapers://92fd5174-9e31-4983-9b1c-5a79350234b8/Paper/p12584 (cit. on p. 14).

[2] R. Axelrod. "Effective Choice in the Prisoner's Dilemma". In: *Journal of Conflict Resolution* 24.1 (1980), pp. 3–25 (cit. on pp. 1, 4, 6, 8).

[3] R. Axelrod. "More Effective Choice in the Prisoner's Dilemma". In: *Journal of Conflict Resolution* 24.3 (1980), pp. 379–403. ISSN: 0022-0027. DOI: 10.1177/002200278002400301 (cit. on pp. 1, 4, 6, 8).

[4] R. Axelrod and W. D. Hamilton. *The Evolution of Cooperation The Evolution of Cooperation*. Vol. 211. 4489. 1981, pp. 1390–1396. ISBN: 0036807518110. DOI: 10.1086/383541. arXiv: t8jd4qr3m [13960] (cit. on pp. 1, 7, 8).

[5] J. Bendor, R. M. Kramer, and S. Stout. "When in Doubt ... Cooperation in a Noisy Prisoner ' s Dilemma". In: *ournal of Con ict Resolution* 35.4 (1991), pp. 691–719. URL: http://jcr.sagepub.com/content/35/4/691 (cit. on pp. 7, 8).

[6] B. Bollobas. "Graph Theory: An Introductory Course (Graduate Texts in Mathematics)". In: (1990). URL: http://www.amazon.com/Graph-Theory-Introductory-Graduate-Mathematics/dp/0387903992%3FSubscriptionId%3D0JYN1NVW651KCA56C102%26tag%3Dtechkie-20%26linkCode%3Dxm2%26camp%3D2025%26creative%3D165953%26creativeASIN%3D0387903992 (cit. on p. 9).

[7] K. Brauchli, T. Killingback, and M. Doebeli. "Evolution of cooperation in spatially structured populations". In: *Journal of theoretical biology* 200.4 (1999), pp. 405–17. ISSN: 1095-8541. DOI: 10.1006/jtbi.1999.1000. URL: http://www.ncbi.nlm.nih.gov/pubmed/10525399 (cit. on pp. 2, 9).

[8] S. Y. Chong et al. "Chapter 1 The Iterated Prisoner ' s Dilemma : 20 Years On". In: (2004), pp. 1–21 (cit. on pp. 4, 8).

[9] S. I. Gass and A. a. Assad. "an Annotated Timeline of Operations Research". In: (2005), p. 125. DOI: 1402081138. URL: http://ebooks.kluweronline.com (cit. on p. 3).

[10] J. Gruji et al. "A comparative analysis of spatial Prisoner's Dilemma experiments: Conditional cooperation and payoff irrelevance." In: *Scientific reports* 4 (2014), p. 4615. ISSN: 2045-2322. DOI: 10.1038/srep04615. URL: http://www.pubmedcentral.nih.gov/articlerender.fcgi?artid=3983604%7B%5C&%7Dtool=pmcentrez%7B%5C&%7Drendertype=abstract (cit. on p. 2).

[11] D. Kraines and V. Kraines. "Pavlov and the Prisoner's Dilemma". In: *Theory and Decision* 26 (1989), pp. 47–79 (cit. on p. 2).

[12] T. Kretz. "A Round-Robin Tournament of the Iterated Prisoner's Dilemma with Complete Memory-Size-Three Strategies". In: *Transport* (2011), pp. 1–29. arXiv: 1101.0340 (cit. on p. 2).

[13] J. Li, P. Hingston, and G. Kendall. "Engineering design of strategies for winning iterated prisoner's dilemma competitions". In: *Ieee Transactions on Computational Intelligence and Ai in Games* 3.4 (2011), pp. 348–360. ISSN: 1943-068X. DOI: 10.1109/TCIAIG.2011.2166268. URL: http://www.scopus.com/inward/record.url?eid=2-s2.0-83655198233%7B%5C&%7DpartnerID=40%7B%5C&%7Dmd5=86fc01bcb87e488fac376d4ffb08ebc7$%5Cbackslash$nhttp://ieeexplore.ieee.org/ielx5/4804728/6099654/06004823.pdf?tp=%7B%5C&%7Darnumber=6004823%7B%5C&%7Disnumber=6099654 (cit. on pp. 3, 8).

[14] K. Lindgren and M. Nordahl. "Evolutionary dynamics of spatial games". In: *Physica D: Nonlinear Phenomena* 75.1-3 (1994), pp. 292–309. ISSN: 01672789. DOI: 10.1016/0167-2789(94)90289-5 (cit. on pp. 2, 9).

[15] J. P. Lorberbaum. "No strategy is evolutionarily stable in the repeated Prisoner's Dilemma game". In: *Journal of Theoretical Biology* 168.2 (1994), pp. 117–130 (cit. on pp. 1, 7).

[16] E. Maciver. "Spatial Prisoner's Dilemma". PhD thesis. 2014. ISBN: 9780857009289. DOI: 10.1177/0022002793037003008.JSTOR (cit. on p. 2).

[17] E. Maciver. "Spatial Prisoner's Dilemma". PhD thesis. 2014. ISBN: 9780857009289. DOI: 10.1177/0022002793037003008.JSTOR (cit. on p. 4).

[18] X. K. Meng et al. "Spatial prisoner's dilemma games with increasing neighborhood size and individual diversity on two interdependent lattices". In: *Physics Letters, Section A: General, Atomic and Solid State Physics* 379.8 (2015), pp. 767–773. ISSN: 03759601. DOI: 10.1016/j.physleta.2014.

12.051. URL: http://dx.doi.org/10.1016/j.physleta.2014.12.051 (cit. on pp. 2, 9, 10).

[19] M. a. Nowak and R. M. May. *Evolutionary games and spatial chaos*. 1992. DOI: 10.1038/359826a0. arXiv: arXiv:1011.1669v3 (cit. on pp. 2, 4).

[20] M. a. Nowak and R. M. May. "The spatial dilemmas of evolution". In: *International Journal of Bifurcation and Chaos* 03.01 (Feb. 1993), pp. 35–78. ISSN: 0218-1274. DOI: 10.1142/S0218127493000040. URL: http://www.worldscientific.com/doi/abs/10.1142/S0218127493000040 (cit. on pp. 2, 9).

[21] W. H. Press and F. J. Dyson. "Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent." In: *Proceedings of the National Academy of Sciences of the United States of America* 109.26 (2012), pp. 10409–13. ISSN: 1091-6490. DOI: 10.1073/pnas.1206569109. URL: http://www.pnas.org/content/109/26/10409.abstract (cit. on p. 8).

[22] D. W. Stephens, C. M. McLinn, and J. R. Stevens. "Discounting and reciprocity in an Iterated Prisoner's Dilemma." In: *Science (New York, N.Y.)* 298.2000 (2002), pp. 2216–2218. ISSN: 00368075. DOI: 10.1126/science.1078498 (cit. on p. 4).

[23] a. J. Stewart and J. B. Plotkin. "Extortion and cooperation in the Prisoner's Dilemma". In: *Proceedings of the National Academy of Sciences* 109.26 (2012), pp. 10134–10135. ISSN: 0027-8424. DOI: 10.1073/pnas.1208087109 (cit. on p. 8).

[24] A. J. Stewart and J. B. Plotkin. "From extortion to generosity, evolution in the Iterated Prisoner's Dilemma". In: *Proceedings of the National Academy of Sciences* 110.38 (2013), pp. 15348–53. ISSN: 1091-6490. DOI: 10.1073/pnas.1306246110/-/DCSupplemental.www.pnas.org/cgi/doi/10.1073/pnas.1306246110. arXiv: arXiv:1304.7205v2 (cit. on p. 4).

[25] G. Szab and G. Fth. "Evolutionary games on graphs". In: *Physics Reports* 446.4-6 (2007), pp. 97–216. ISSN: 03701573. DOI: 10.1016/j.physrep.2007.04.004. arXiv: 0607344 [cond-mat] (cit. on pp. 1, 4, 10).

[26] The Axelrod project developers. *Axelrod: ¡RELEASE TITLE¿*. Apr. 2016. DOI: <DOIINFORMATION>. URL: http://dx.doi.org/10.5281/zenodo.%3CDOI%20NUMBER%3E (cit. on pp. 4, 10).

Appendix A

First Appendix Title

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Appendix B

Second Appendix Title

Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.