The Iterated Prisoner's Dilemma

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A dissertation submitted in partial fulfilment of the requirements for MSc (in Operational Research and Applied Statistics) by taught programme.

Executive Summary

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Acknowledgements

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Chapter 1

Complex Networks Experiments

1.1 Introduction

In this chapter, experiments with more complex networks have been performed. An introduction to these networks and the reasons behind their selection has been covered in the begging of the chapter. The experiments structure, for each topology, have also been analyzed and illustrated with pseudo code. Finally, an analysis on the data produced for these experiments, has been performed.

The issues raised in ??, have been address in the current chapter, altering various approaches. For example, a measure of similarity between the strategies has been used. The measure used, is based on the average cooperating ratio of the strategies. Furthermore, median ranking of each strategy has been looked into, for it is believed to be a more appropriate measure of performance, compared to winning ratio. Regression models for the median rank have been build and Kruskal Wallis analysis has been held. Furthermore, round robin in treated as a spatial tournament of a complete topology.

1.1.1 Networks

A complex network is a graph with non-trivial topological features; features that can not occur in networks such as the ones studied in ??. Such features include, a heavy tail in the degree distribution, a high clustering coefficient, and hierarchical structure. Complex networks are commonly used to study computer network, technological networks and social networks.

They will also be used in the experiment conducted in this dissertation. Small world and random graphs, and more specifically, the following networks are studied :

- Watts-Strogatz, a model for generating graphs with small-world properties
- ErdsRnyi, a model for generating random graphs
- Complete Graph, a graph with the properties of a round robin topology

Each of the experiment has a different set of parameters, thus the size of data produced by each experiment is different. The structure of each experiment is analyzed on the following subsection.

1.1.2 Experiments Structure

Small-world

Watts Strogatz, is the small world graph generator model used in this experiment. The Watts Strogatz model, was proposed by D. Watss and S. Strogatz, (whose name was used), in their work in 1998 [2]. Their works was aimed to reproduce a network/graph, which was somewhere between a regular and a random network. To interpolate between regular and random networks, they considered a random rewiring procedure. Starting from a ring lattice with n vertices and k edges per vertex, each edge is being rewired, at random, with a probability of p.

Networkx, provides a function for the given model. The documentation for the Watts Strogatz function, can be found here `https://networkx.github.io/documentation/development/reference/generated/networkx.generators.random_graphs.watts_strogatz_graph.html#networkx.generators.random_graphs.watts_strogatz_graph` , and the given arguments are n, k and p. Moreover, the tournament size is not going to be deterministic, it will range from 2 to 50. Similarly for the initial size, it will range from 2 to tournament size minus one. The chosen players are also shuffled and re placed on the graph. A pseudo code illustrating the structure of the experiment is shown in 1:

Algorithm 1 Small world Experiment

procedure SMALL WORLD
loop:
  for *tournament.size* ← *2 to 50* **do**
    for *initial.neighborhood.size* ← *1 to tournament.size-1* **do**
      for i ← *0 to 10* **do**
        player ← *random.strategies.*
        for p ← *0 to 10* **do**
          G ← *create.watts.strogatz.graph.*
          edges ← *G.egdes*
          results ← *play.tournament. loop.*

Random

For recreating a random network experiment, ErdösRényi model was used. The model is named after P. Erdös and A. Rényi, who introduced the model in 1959 [1]. It generates a random graph of n vertices and chooses each of the possible edges with probability p.

Once again, networkx provide a function to easily generate such graph `https://networkx.github.io/documentation/development/reference/generated/networkx.generators.random_graphs.binomial_graph.html#networkx.generators.random_graphs.binomial_graph`. The arguments taken but the function are n and p as the probability for edge creation. Similarly, the tournament size is not going to be deterministic. The tournament size will range from 2 to 50. The chosen players are also shuffled and re placed on the graph. A pseudo code illustrating the structure of the experiment is shown in 2:

Algorithm 2 Random Experiment

procedure RANDOM
loop:
  for *tournament.size* ← *2 to 50* **do**
    for i ← *0 to 100* **do**
      player ← *random.strategies.*
      for p ← *0 to 10* **do**
        G ← *create.erdos.renyi.graph.*
        edges ← *G.egdes*
        results ← *play.tournament. loop.*

Complete

In the previous chapter, ??, the round robin experiment was conducted by the `RoundRobinTournament` class, in this experiment the `SpatialTournament` class is used. The following networx function is used(`https://networkx.github.io/documentation/development/reference/generated/networkx.generators.classic.complete_graph.html#networkx.generators.classic.complete_graph`). , to generate a complete graph. The following pseudo-code explain the structure of the experiment 3.

Algorithm 3 Complex Experiments Rules

```
procedure COMPLETE
loop:
    for tournament.size ← 2 to 50 do
        for i ← 0 to 100 do
            player ← random.strategies.
            G ← create.complete.graph.
            edges ← G.egdes
            results ← play.tournament. loop.
```

1.2 Preliminary Analysis

In this section the analysis for the complex networks experiments is described. Starting with an analysis on the networks for all three topologies, following a preliminary analysis on the results of the experiments. The next subsection is focused on the classification of the strategies based on the cooperation rate.

1.2.1 Network Analysis

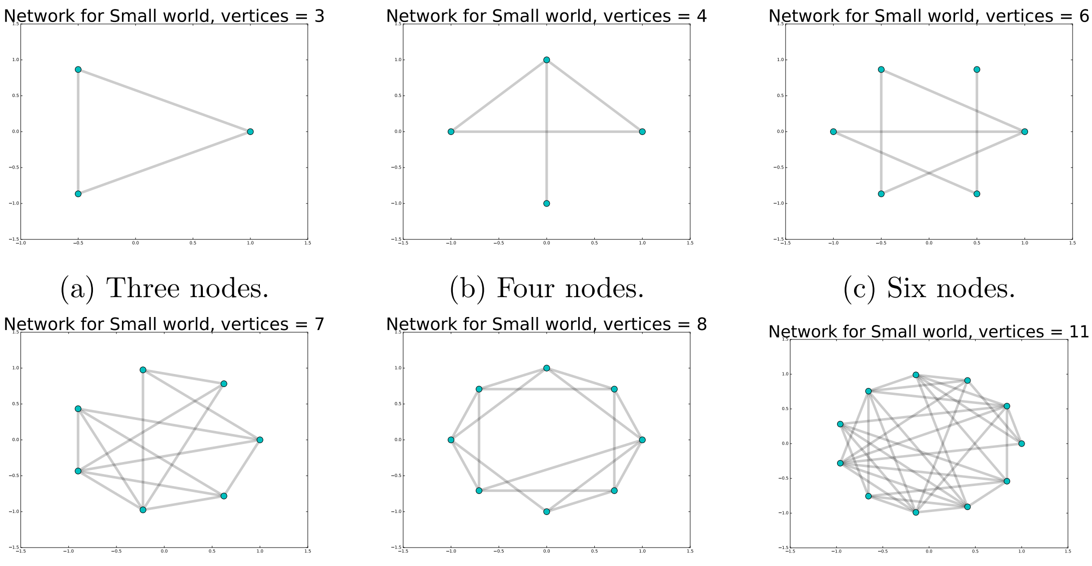
The networks of each experiment are studied. As introduced in ??, the connectivity and clustering concepts are summarized and reviewed. Due to the size of experiments and the time in hands, not all experiments managed to maximize their tournament sizes, as they have been described in 1.1.2.

More specifically, for the small world experiment, a total of 59051 graphs have been created. The maximum number of nodes achieved is 13. Moreover, the median degree, has been 4.75. Thus indicating that in most tournaments, the average number of neighbors per strategy was 4. The maximum median degree is equal to 12. Further, the mean connectivity of the Watts Strogatz network is 3.40. Minimum value is 0 and maximum 9. Lastly, clustering coefficient ranges from 0 to 1, with a mean of 0.48.

Small world Networks Summary Table				
	connectivity	clustering	degree	nodes
mean	3.40	0.48	4.75	10.00
std	2.37	0.33	2.65	-
min	0.00	0.00	1.00	3.00
max	9.00	1.00	12.00	13.00

Table 1.1: Small world Summary Table

Illustrating 59051 graphs for understanding the networks of the experiment, is an impossible task. For this reason, the summary table 1.1 has been used. A mean of 0.48, for the clustering coefficients, indicates that strategies have a neutral trend towards creating cliques (complete graphs). Eight networks have been randomly selected and displayer here 1.1. Mainly to get an ideal of the look of the networks in this experiment.



(a) Three nodes. (b) Four nodes. (c) Six nodes.

(d) Seven nodes. (e) Eight nodes. (f) Eleven nodes.

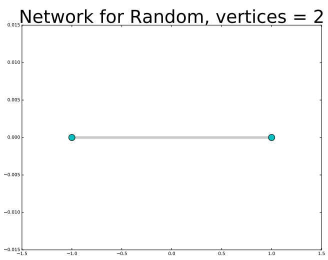
Figure 1.1: Various Watss Strogatz networks.

Likewise for the Erdös Rényi network. The experiment did not manage to achieve the maximum tournament size that had been set. Instead, 21924 have been created and the maximum nodes number is equal to 29. As shown in table 1.2, the mean number of nodes of the experiments has been 20 and the minimum 2. The minimum number of nodes differs to these of the small world experiment, where tournaments of two have not been able to be played. Probably, because a node was disconnected. Binomial seems to have handled that. This can been shown also by the connectivity, where the mean connectivity, of a random network, is approximately 8.26. Compared to the small world that has been fairly lower, approximately 3.

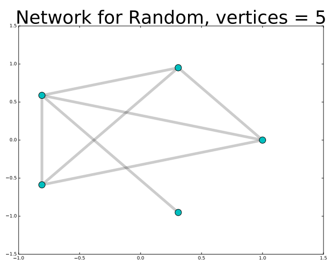
Random Networks Summary Table				
	connectivity	clustering	degree	nodes
mean	8.26	0.60	11.22	19.80
std	6.46	0.30	6.50	-
min	0.00	0.00	1.00	2.00
max	28.00	1.00	28.00	29.00

Table 1.2: Random Summary Table

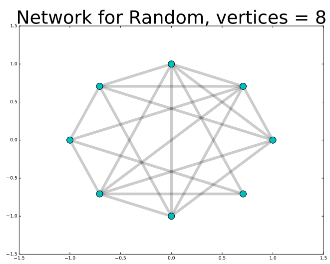
Another differs between the two network experiments, have occurred in the clustering coefficient. The clustering coefficient, of the random networks has a mean value of 0.60. Thus, the strategies tend to form groups, more in this experiment. The mean node number is equal to 20 with a mean degree 11. Thus, in most of the tournaments that occurred, the strategy had to compete with 11 of it's neighbors. Once again, six random have been picked and are illustrated in 1.2.



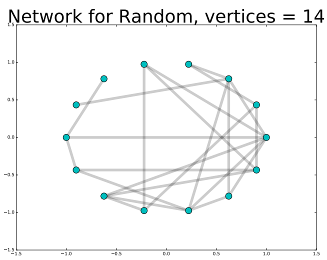
(a) Two nodes.



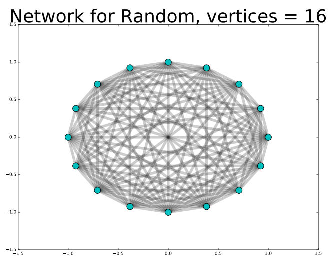
(b) Five nodes.



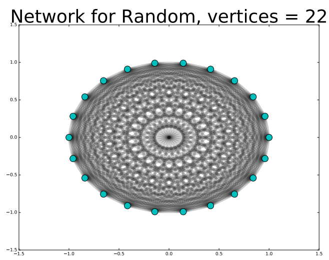
(c) Eight nodes.



(d) Fourteen nodes.



(e) Sixteen nodes.



(f) Twenty-two nodes.

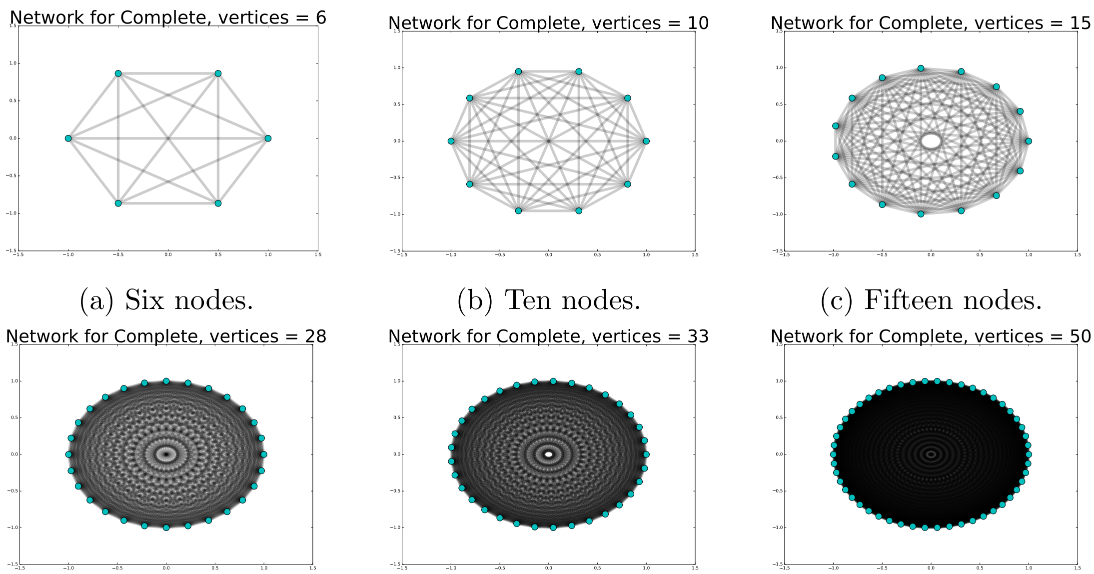
Figure 1.2: Various Erdös Rényi networks.

Lastly, the complete networks experiment, was the only experiment where the tournament size ranges from 2 to 50. A total of 500 networks, with complete topology, have been constructed and played through. Table 1.3 summarizes, a few of the networks measures. For the complete networks, the measures could have been predicted, due to the 'all connected rule'. The connectivity coefficient it is equal to the degree. They vary from 1 to 49. One degree value corresponds to the first network, with a minimum number of nodes 2, and 49 to the last networks, with a maximum number of 50 nodes.

Complete Networks Summary Table				
	connectivity	clustering	degree	nodes
mean	32.70	0.99	32.70	33.70
std	11.87	0.03	11.87	-
min	1.00	0.00	1.00	2.00
max	49.00	1.00	49.00	50.00

Table 1.3: Complete Summary Table

The mean clustering coefficients is set at 0.99 Indicating strong clusters throughout the experiment, an anticipated result. Furthermore, eight random graphs are displayed in 1.3.



(a) Six nodes. (b) Ten nodes. (c) Fifteen nodes.

(d) Twenty-eight nodes. (e) Thirty-three nodes. (f) FIfty nodes.

Figure 1.3: Various Complete networks.

In this subsection, an initial summary of the overall graphs conducted, in each of the experiment has been held. The number of tournaments, as well as the tournament size differ. That is not a negative fact, what has been tried to achieved in the complex networks experiments, is more complex and random networks. In the following subsection, an initial analysis on the data produced by these experiments in held.

1.2.2 Data Analysis

This subsection summarizes the data that have been extracted for the experiments. Individual tables have been managed for each topology. The payoff matrix, for the game of the prisoners dilemmas, follows the payoffs as explained in **??**. Thus, punishment payoff is equal to 1 temptation to 5, reward is equal to 3 and the sucker's payoff is set to 0.

For the small word, Watts Strogratx, experiment the description of the average score, average neighborhood score, cooperating ratio and neighborhood size are shown in 1.4. The average score and average neighborhood score, are fairly equal, both ranging form 0 to 5 with an average of 2.4. This could indicate that all strategies performed similarly. The mean cooperation rating, is equal to 0.64. Thus, strategies behave mostly cooperative. Mean neighborhood size is 4, agrees with 1.2.1. Maximum neighbors is 9, so even if the maximum players has been 11, non strategy compete against 10 strategies.

Small world Data Summary Table				
	average score	average neighborhood	cooperating ratio	neighborhood size
mean	2.40	2.40	0.64	4.74
std	0.63	0.40	0.30	2.65
min	0.00	0.00	0.00	1.00
max	5.00	5.00	1.00	12.00

Table 1.4: Small world Data Summary Table

For the random, Erdös Rényi, Table 1.5, is an initial summary, for basic measure of the experiment. Similarly, the average score and average neighborhood score, are fairly equal. This time, the average neighborhood score, is slightly lower, 2.38, and the maximum average score achieved by a strategy is 4.99. Instead of the maximum of a 5. The mean cooperating ration, is higher than 0.50 again. Thus, strategies tend to cooperate and the neighborhood size, verifies the results of 1.2.1.

Finally, for the complete experiment, Table 1.6. The average scores, are lower for this experiment. Both mean average scores are equal to 2.38. But the maximum

Random Data Summary Table				
	average score	average neighborhood	cooperating ratio	neighborhood size
mean	2.38	2.38	0.63	11.00
std	0.48	0.24	0.25	6.57
min	0.00	0.00	0.00	1.00
max	5.00	4.99	1.00	28.00

Table 1.5: Random Data Summary Table

average scores, strategies and neighbors, are below 4. On the other hand, the minimum score are non zero. The behavior of the scores can be explained by the topology. Complete experiment, is the experiment with highest number of games played. Considering that for each tournament $n - 1$ games take place. Thus, scoring zero is less possible, because the games you play are increased by far, and the overall score falls, because of the number of the opponents. Cooperating ratio, is similarly once again and the neighborhood size verify what was discussed in 1.2.1.

Complete Data Summary Table				
	average score	average neighborhood	cooperating ratio	neighborhood size
mean	2.38	2.38	0.63	32.70
std	0.13	0.35	0.23	11.90
min	0.54	0.55	0.00	1.00
max	3.58	3.89	1.00	49.00

Table 1.6: Complete Data Summary Table

In this subsection, a very brief summary was done to the data. In the section to follow, more important topics will be raised and an analysis of the best ranking strategies will take place.

1.2.3 Classification

As stated in [], the Axelrod-Python library (version 1.16.00), consists of 132. Reading though the documentation for each strategy, enough to be able to understand and distinguish, any similarities between their player, would be an unbearable task. Even so, another concept provided by the Axelrod- Python library is the cooperating ratio. The cooperating ratio, is the ratio of times a strategy cooperated, divided by the number of turns in a single tournament.

Initially the data of all experiments have been combined, and afterwards have been grouped by each strategy. The average cooperating ratio for each strategy is then computed. Furthermore, by using the k-means algorithm, the boundaries for any given number of categories can be found. For the cooperation, five individual categories have been chosen. The lowest category representing the non - cooperative strategies, the middle category, the medium cooperatives and the highest the cooperative ones. The two remaining categories, represent the moderate and weak cooperation strategies respectively.

The results, for performing the algorithm, in each experiment can seen in Tables, 1.7 and a more detailed table, with the respective category of each strategy can be found in the Appendix A.1.

Overall six strategies are in the low category. Their average cooperating ration ranges between 0.00 and 0.20. In the weak category, the ratio is between 0.23 and 0.40, and is consisted by 17 strategies. Twenty-eight strategies are in the mid category, with upper bound of ratio 0.57 and lower bound a 0.42. The moderate category, is for strategies which ratio is between 0.58 and 0.75. Lastly, the high category, with 48 strategies and a mean ratio of 0.87.

Classification Table				
Categories	count	cooperating ratio		
		min	mean	max
low	6	0.00	0.10	0.20
weak	17	0.23	0.32	0.40
mid	28	0.42	0.50	0.57
moderate	33	0.58	0.65	0.75
high	48	0.76	0.87	1.00

Table 1.7: Classification Table

The classification of the strategies will be a useful tool in the following subsection. For now the categories have been introduced and more details, for each strategy, can be found in the Appendix. This section have been for the preliminarily analysis. Where an overview of the networks and data have been conducted. In the section that follows, the analysis on performance is undertaken.

1.3 Performance Analysis

In this section, three measure will be scrutinized, to assess performance of the strategies. These measures are the winning ratio, the normalized average score and the median rank. Various regression models have been constructed, for the median rank and lastly, the findings are summarized.

As aforementioned, our experiments did not manage to achieve maximum capability. Additionally, not all strategies participated in each of the experiments. Even so, the goal of this chapter remains the observation of the strategies, for any given topology. Thus, from here onwards the data that have been used, are a combination of the data sets produced by each experiment. The analysis is focused solidarily, on the overall performance of each strategy and the identification of any network attributes, affecting the performance.

1.3.1 Winning Ratio

In this subsection, a ranking of the strategies has been carried out, based on the winning ratio. Shown in Figure 1.4, is a point plot illustrating the order of the strategies based on this rank. The highest ranked strategies, are Fool Me Forever, Grumpy, Ripoff, Cycler DDC and, with wining ratio of 0.23 Calculator. Even though, Fool Me Forever, Grumpy, are overall cooperative strategies Ripoff, Cycler DDC and Calcuter have a cooperative ratio around 0.35. It can be assumed that these strategies have been favored by their environment.

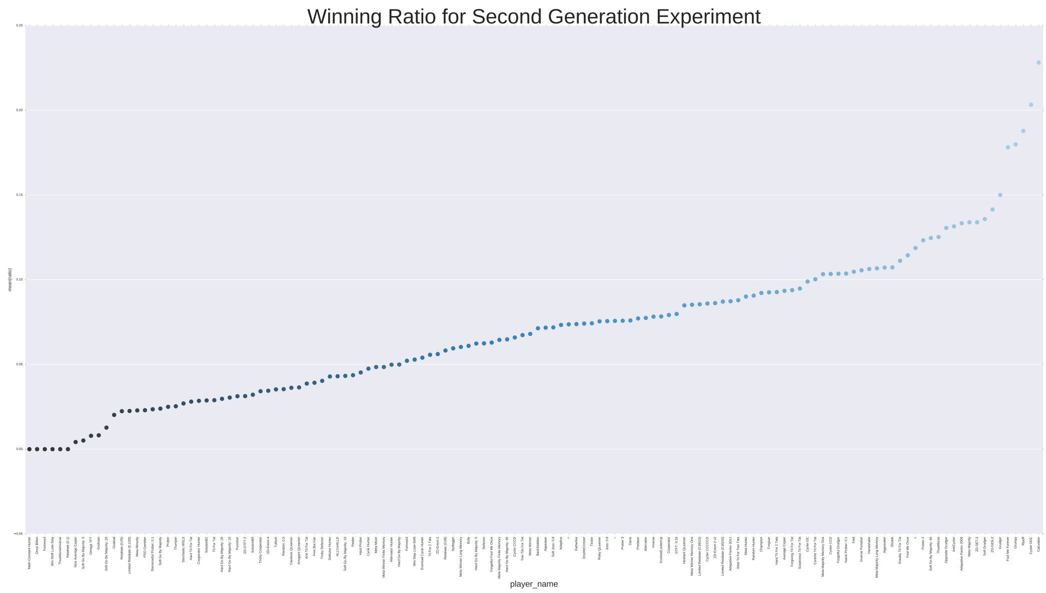


Figure 1.4: Winning Ratio Complex Networks Experiments

Even so, by winning ratio it assumed that a well performed strategy, is a strategy that have won. This raises questions as to, is the performance only based by that? Is there any reassurance that this strategy did not stand 'lucky' in the small number of tournaments that it had participated, because it was favored by the circumstances and environment, and did not do poorly in the rest? Thus, no more attention will be given to winning ratio, as it does not express the overall well performance that is tried to be observed.

The following subsection, is focused on the normalized average score.

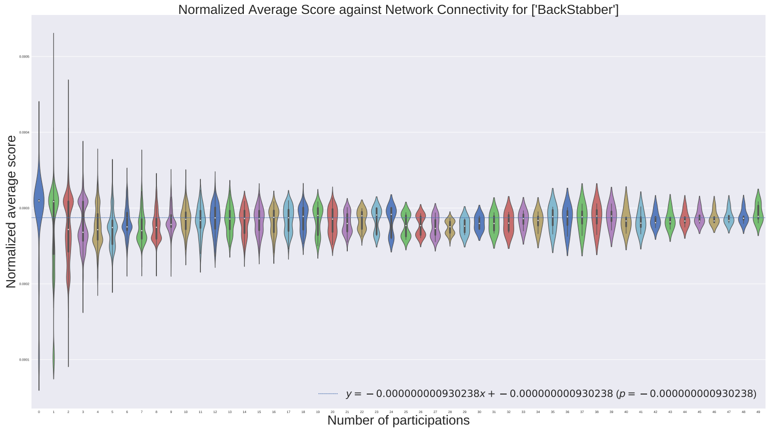
1.3.2 Normalized Average Score

In this subsection, an individual data frame for each strategy, with each of their participations, is created.Thus, 132 frames. Therefore, by reading thought these frames the normalized average score is studied. To understand how the network topologies affect the scoring of a strategy an Anova analysis has been executed. To identify the effects of the topologies, the existence of a statistically significant differences of the normalized score, between the clustering and connectivity groups is being tested.

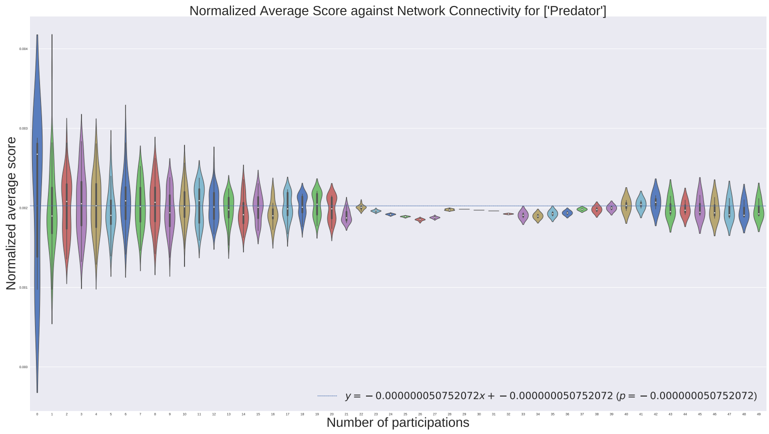
For the particular statistical analysis on this subsection, the scipy python library is being used once again. Firstly, the normality of the average score is being tested. By looping through the frames and using the scipy command `scipy.stats.mstats.normaltest`, the tests are carried out quite simply. The results indicate that for each frame, all 132, the p value is lower than 0.05. Thus the average normalized score is not normally distributed. For this reason, the Kruskal Wallis test, the corresponding non parametric test, has been used.

For testing the difference between the connectivity groups, of each frame, the following results have been returned. Most of the strategies, have a significant difference in their scores. They returned a p value less than 0.05. Thus, their scores have been affected by the connectivity of the topology the strategy played in. Even so, 45 strategies did not. These 45 strategies include, Cycler DDC, Raider, Inverse Punisher, Hard Go By Majority and more. Randomly chosen strategies, out of 132, have been chosen and the variation in their average score is illustrated in the violin plots, in Figure :

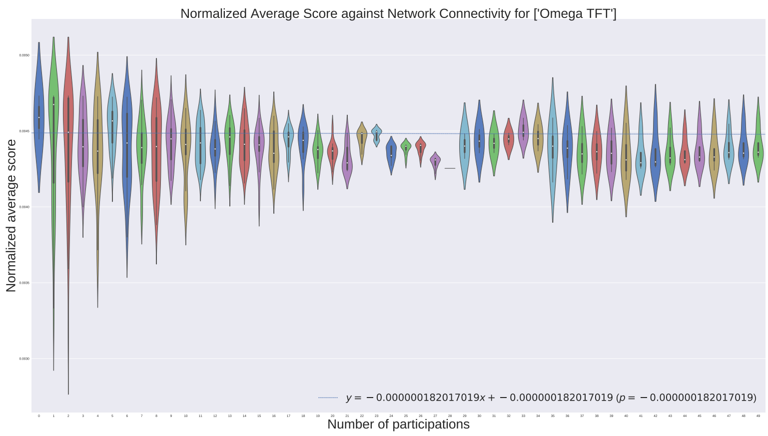
The fact that numerous, Kruskal Wallis tests return that there was no significant difference between groups could indicate, that some strategies where not affected by the topology, thus could have performed well in any given topology. To further investigate this, a more appropriate measure of performance is introduced in



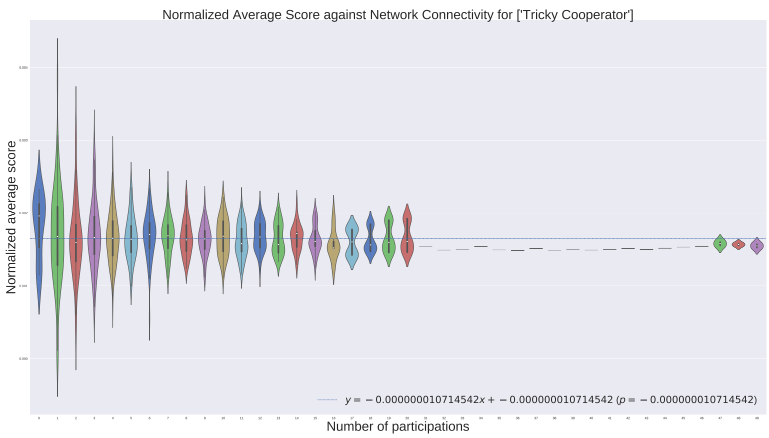
(a) Normalized Average Score BackStabber



(b) Normalized Average Score Predator



(c) Normalized Average Score Omega TFT



(d) Normalized Average Score Tricky Cooperator the following subsection. Where more Kruskal wallis tests and other statistical analysis has been held.

1.3.3 Normalized Median Rank

This part of the chapter, focuses on a new measure of performance. The normalized median rank of the strategies. To compute the normalized median rank, the rank for each strategy, for each participation is, divided by the corresponding size of tournament. The median rank is believe to be a more accurate measure, compared to the winning ratio. It will reveal strategies, that did not necessarily achieved the maximum wins, but had an overall satisfying and smoothly performance throughout the experiments.

By once again, concreting the data set by each strategy and computing the median of the normalized ranks of each strategy, a point plot can have been created. As illustrated in Figure 1.6. [1] The strategies with the worst ranking are Meta Majority Finite Memory, Meta Minority and Limited Retaliate (0.1/20). On the other hand, the highest ranked strategies are the following:

- Third with a median ranking equal to 0.27, Nydegger
- Second with a median ranking equal to 0.25, Cautious QLearner
- First with a median ranking equal to 0.23, Gradual

Nydegger, is a strategy which begins by playing tit for tat for the first three moves. Except if it was the only one to cooperate on the first move and the only one to defect on the second move, it defects on the third move. After the third move, its choice is determined from the 3 preceding outcomes. More detail information about the 3 preceding outcomes can be found in Appendix[].

Cautious QLearner, is a strategy that learns the best strategies through the q-learning algorithm. Lastly Gradual, is a player that punishes defections with a growing number of defections but after punishing enters a calming state and cooperates no matter what the opponent does for two rounds. All three strategies are categorized as high cooperators and Gradual is a well performed strategy in the actual Axelrod tournament as well.

[1] A reminder,the lowest the rank the higher a strategy scored. 0 is the highest rank a strategy can achieve in a tournament.

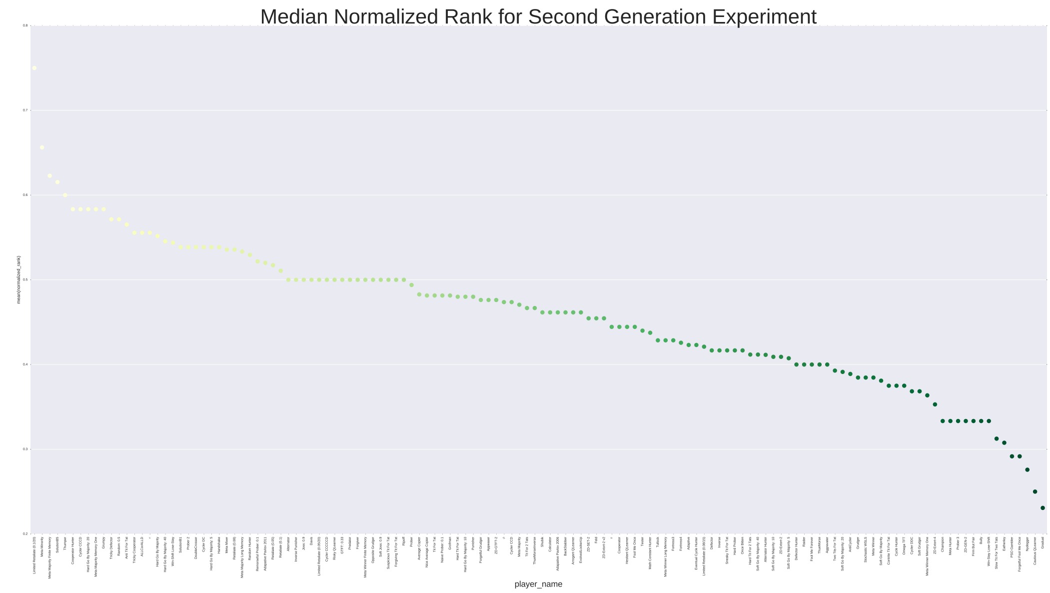


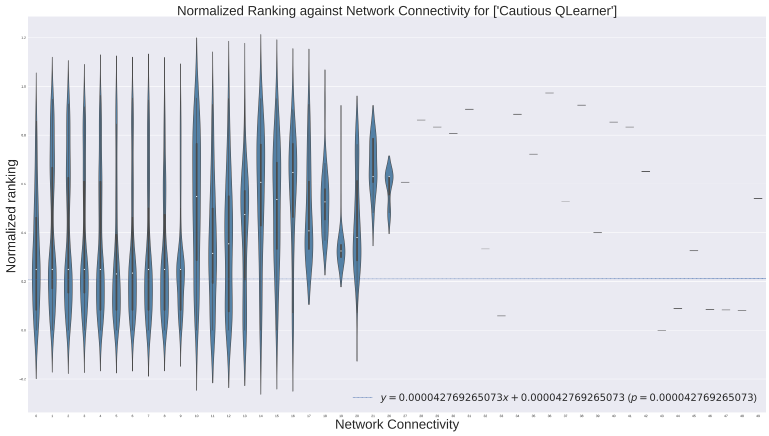
Figure 1.6: Normalized Median Rank Complex Networks Experiments

For further research on these three strategies, a Kruskal Wallis test has been performed for their normalized rankings. Whether there is a significant difference on the rankings between groups of, networks connectivity, clustering and cooperating ratio has been examined. A non parametric test is used, because ranks are not a normally distributed measure.

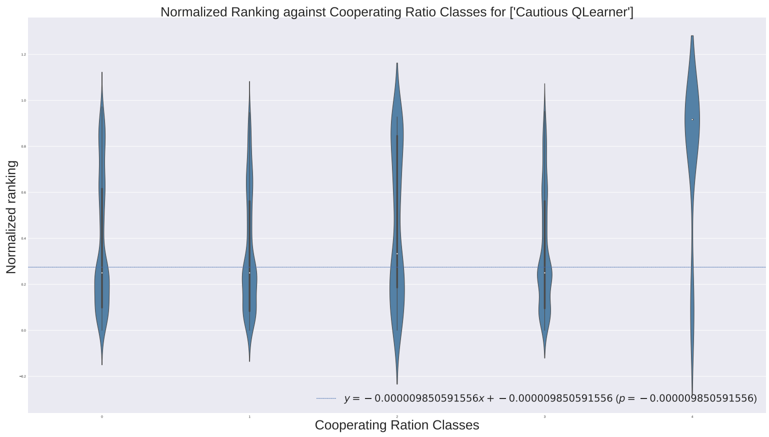
For the clustering and cooperating ratio, a k-means algorithm has been used, to create groups, 5 and 10 respectively. A table with the classification groups can be found in the Appendix A.2.

The finding of the tests returned the following results. All p values, for all three strategies, and all three categories are lower that 0.05. Thus, there is significant difference between the groups. The ranking of each three highest ranks strategies has been ranging, in different topological environments. Violin plots for Cautious are shown in Figure 1.7, for in Nydegger in Figure 1.8 and finally, in Figure 1.9 Gradual.

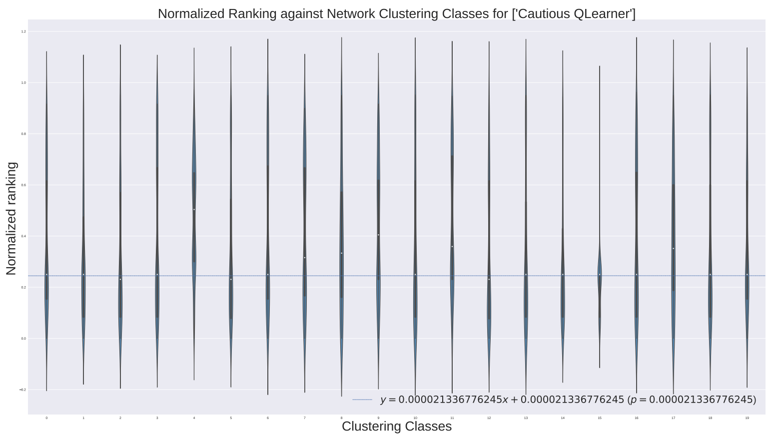
In this subsection, the best performed strategies, Nydegger, Cautious QLearner, Gradual, have been defined based on the median ranking. Furthermore, the difference in these strategies ranks have been studied, based on groups of network clustering, connectivity and cooperating ration. The results indicated, statistically significant difference. The following subsection focal point is building a regression model.



(a) Normalized Ranking against Network Connectivity

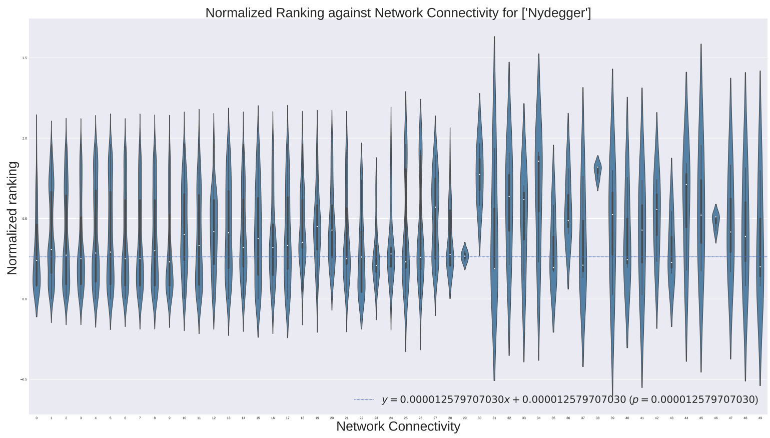


(b) Normalized Ranking against Cooperating Ratio Classes

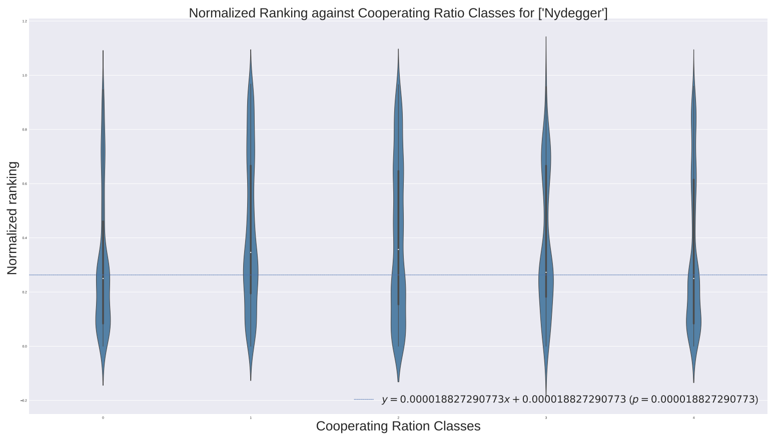


(c) Normalized Ranking against Network Clustering Classes

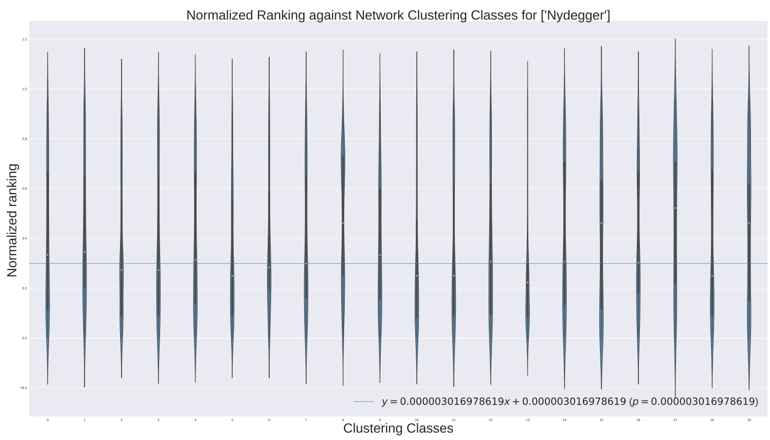
Figure 1.7: Normalized Ranking Violin Plots for Cautious QLearner



(a) Normalized Ranking against Network Connectivity

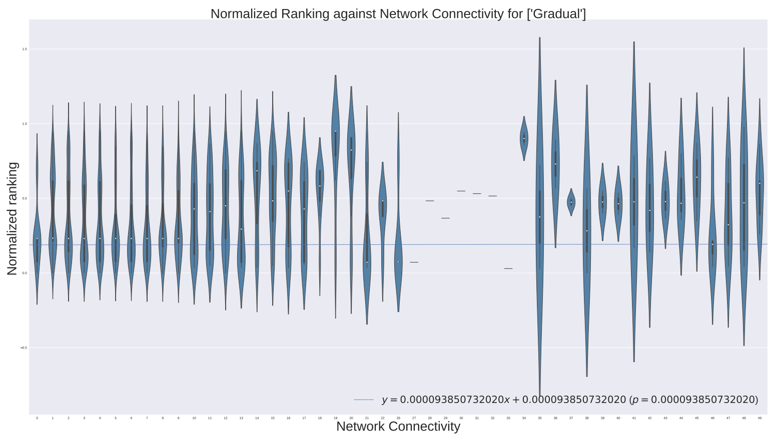


(b) Normalized Ranking against Cooperating Ratio Classes

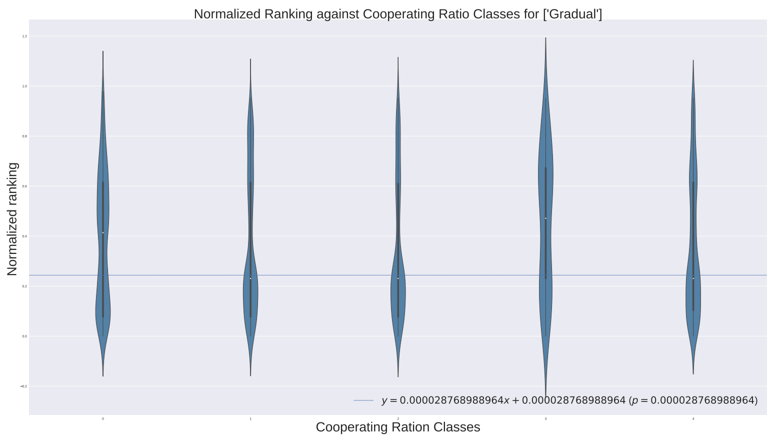


(c) Normalized Ranking against Network Clustering Classes

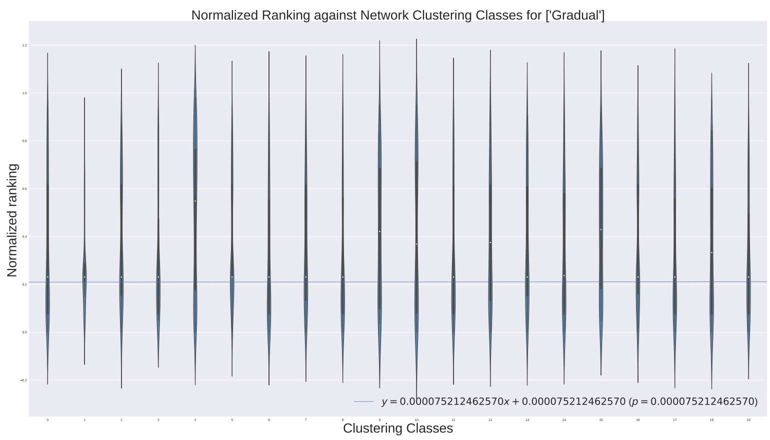
Figure 1.8: Normalized Ranking Violin Plots for Nydegger



(a) Normalized Ranking against Network Connectivity



(b) Normalized Ranking against Cooperating Ratio Classes



(c) Normalized Ranking against Network Clustering Classes

Figure 1.9: Normalized Ranking Violin Plots for Gradual

1.3.4 Regression

This subsection concentrates, into build a regression model, for predicting the normalized ranking of s strategy. This has been done, by identifying any correlation between the topology measures and the participations of the strategy. Initially, an analysis was performed for the entire data set, using the following model.

$$
\begin{aligned}
\text{normalised.ranking}_t = \alpha &+ \beta_1 \text{average.neighborhood.score}_t && (1.1)\\
&+ \beta_2 \text{normalized.average.score}_t && (1.2)\\
&+ \beta_3 \text{cooperating.ratio}_t && (1.3)\\
&+ \beta_4 \text{connectivity}_t && (1.4)\\
&+ \beta_5 \text{clustering}_t && (1.5)\\
&+ \beta_6 \text{neighborhood.size}_t && (1.6)\\
&= \beta_7 \text{tournament.size}_t && (1.7)\\
&+ \beta_8 \text{number.of.participations}_t + \epsilon && (1.8)
\end{aligned}
$$

The result of the model have been the following 1.8. All the predictors, expect clustering and neighborhood size, are significant. Due their p value being less than 0.05. Apart from normalized average score, connectivity and cooperating ratio, all variables have a positive correlation with the normalized ranking. Because the objective function, is to minimize the ranking, normalized average score, connectivity and cooperating ratio are the only predictors affecting the performance of a strategy positively. The model itself has a R squared value of 0.014.

Regression Results			
	coefficient	p value	R-squared
Intercept	0.3712	0.000	0.006
average.neighborhood.score	0.0206	0.000	-
normalized.average.score	-0.6150	0.001	-
clustering	0.0033	0.033	-
connectivity	-0.0011	0.000	-
cooperating.ratio	-0.0403	0.000	-
tournament.size	0.0033	0.000	-
frequency	1.35e-06	0.000	-
neighborhood.size	0.0005	0.017	-

Table 1.8: Regression Results

explaingn

Furthermore, a model for each of the top ranking strategies has been build. Using the same model, the individual results are shown in Table 1.9. For Nydegger and Gradual the R square values of the models are low, compared to Cautious QLeaner. Cuatious QLeaner is the only strategy were more than two predictors are significant. The findings of these approach are the following,

- Nydegger's, performance was significantly affected by clustering of the network and the tournament size
- Cautious QLearner's, performance was significantly affected by the tournament size , the average neighborhood score and the normalized average score
- Gradual's, performance can not be anticipated by the model

Regression Results									
	Nydegger			Cautious QLearner			Gradual		
	coefficient	p value	R-squared	coefficient	p value	R-squared	coefficient	p value	R-squared
Intercept	2.729e-09	0.269	0.042	1.508e-08	0.114	0.129	8.626e-08	0.058	0.020
average.neighborhood.score	0.0475	0.043	-	0.1716	0.000	-	-0.0301	0.478	-
normalized.average.score	114.0857	0.163	-	-653.0862	0.000	-	83.8451	0.339	-
connectivity	-0.0019	0.316	-	0.0043	0.110	-	0.0006	0.838	-
clustering	-0.0749	0.004	-	-0.0594	0.059	-	-0.0108	0.785	-
cooperating.ratio	0.0456	0.331	-	0.0673	0.497	-	-0.1310	0.082	-
tournament.size	0.0100	0.000	-	0.0176	0.000	-	0.0037	0.080	-
frequency	4.884e-06	0.758	-	3.942e-05	0.196	-	0.0002	0.054	-
neighborhood.size	0.0004	0.844	-	-0.0066	0.033	-	0.0005	0.894	-

Table 1.9: Regression results for Nydegger, Cautious QLearner and Gradual

In this subsection, the performance of the strategies has been tried to be predicted by a regression model. The results, for running this regression model to the hole data set returned that cooperating ratio, connectivity and the normalized average score were significant predictors. Furthermore, the analysis of the model to each top ranked strategy individual returned, that their performance can not be accurately predicted by the possible predictors given. In the next section, a summary of the findings of this chapter have been stated.

1.4 Summary

In this chapter, experiments with more complex networks topologies have been performed. Networks such as small world, random and complete have been chosen, and more specifically models, to recreate each of the networks. Due to the large cpu power and time needed for the experiments, only the complete experiment, have reached maximum capabilities. Even so, a data set, created by combining the results retrieved from each experiment, has been used to carry out the analysis.

Before combining the results, an initial analysis on the networks and their results has been carried out. For the research on the combined data, a brief mention on the winning ratio and the normalized average score have been made. In this chapter, a new measure has been used, the normalized median rank, and the three top performed strategies of the complex networks experiment have been named. Nydegger, Cautious QLearner and Gradual.

An analysis on the normalized ranking was then conducted. The findings are the following. Normalized ranking, for the top strategies have been significantly different between groups of connectivity, clustering and cooperating. Also cooperating behavior, has been showed to be affecting the performance. All three strategies have been characterized as highly cooperators by the analysis in 1.2.3. Regression showed that, predicting the performance of this strategies will not be impossible. Even so, the overall model did return a few finding compared to **??**.

In conclusion, further research with more data and different random topology is needed. Thought connectivity and cooperating ratio seems to have played a role, there is not enough validation in the results. The topology does in did affect the strategies, but a strategy that could outperform any topology has now been found yet. Whether a strategy like this exist or not in the Axlerod Python Library is not sure. Thus, in the following chapter an strategy will be trained for this reason.

References

[1] P. Erds and a. Rnyi. “On random graphs”. In: *Publicationes Mathematicae* 6 (1959), pp. 290–297. ISSN: 00029947. DOI: 10.2307/1999405. arXiv: 1205.2923 (cit. on p. 3).

[2] D. J. Watts and S. H. Strogatz. “Watts Strogatz Collective dynamics small world networks.1998”. In: 393.June (1998), pp. 440–442. ISSN: 0028-0836. DOI: 10.1038/30918. arXiv: 0803.0939v1 (cit. on p. 2).

Appendix A

First Appendix Title

A.1 Cooperating Ratio Classification

Here the table with the list of strategies and their respective cooperating category, as explained in 1.2.3, are illustrated.

Categories based on cooperating ratio			
	strategy	cooperating ratio	category
0	ϕ	0.622042	moderate
1	π	0.786273	low
2	e	0.769091	low
3	ALLCorALLD	0.597729	moderate
4	Adapative Pavlov 2006	0.591843	moderate
5	Adapative Pavlov 2011	0.794202	low
6	Adaptive	0.603062	moderate
7	Aggravater	0.092448	mid
8	Alternator	0.500000	weak
9	Alternator Hunter	0.999616	low
10	Anti Tit For Tat	0.593823	moderate
11	AntiCycler	0.910000	low
12	Appeaser	0.614020	moderate
13	Arrogant QLearner	0.821084	low
14	Average Copier	0.350241	high
15	BackStabber	0.690611	moderate
16	Bully	0.510216	weak

17	Calculator	0.230031	high
18	Cautious QLearner	0.834256	low
19	Champion	0.878732	low
20	Contrite Tit For Tat	0.848997	low
21	Cooperator	1.000000	low
22	Cooperator Hunter	0.959154	low
23	Cycle Hunter	0.987100	low
24	Cycler CCCCCD	0.835000	low
25	Cycler CCCD	0.750000	moderate
26	Cycler CCD	0.670000	moderate
27	Cycler DC	0.500000	weak
28	Cycler DDC	0.330000	high
29	Davis	0.402531	high
30	Defector	0.000000	mid
31	Defector Hunter	0.993341	low
32	DoubleCrosser	0.625702	moderate
33	Eatherley	0.941412	low
34	Eventual Cycle Hunter	0.775378	low
35	EvolvedLookerUp	0.700659	moderate
36	Feld	0.293793	high
37	Firm But Fair	0.853547	low
38	Fool Me Forever	0.904576	low
39	Fool Me Once	0.620283	moderate
40	Forgetful Fool Me Once	0.820023	low
41	Forgetful Grudger	0.624388	moderate
42	Forgiver	0.435188	weak
43	Forgiving Tit For Tat	0.866466	low
44	Fortress3	0.253173	high
45	Fortress4	0.201929	mid
46	GTFT: 0.33	0.902416	low
47	Gradual	0.839741	low
48	Grofman	0.925631	low
49	Grudger	0.541712	weak
50	Grumpy	0.803507	low
51	Handshake	0.071420	mid

52	Hard Go By Majority	0.434785	weak
53	Hard Go By Majority: 10	0.316090	high
54	Hard Go By Majority: 20	0.302703	high
55	Hard Go By Majority: 40	0.461587	weak
56	Hard Go By Majority: 5	0.419470	weak
57	Hard Prober	0.558885	weak
58	Hard Tit FormidTats	0.856890	low
59	Hard Tit For Tat	0.674626	moderate
60	Hesitant QLearner	0.807483	low
61	Inverse	0.681649	moderate
62	Inverse Punisher	0.579137	moderate
63	Joss: 0.9	0.477184	weak
64	Limited Retaliate (0.05/20)	0.599711	moderate
65	Limited Retaliate (0.08/15)	0.675263	moderate
66	Limited Retaliate (0.1/20)	0.547303	weak
67	Math Constant Hunter	0.636059	moderate
68	Meta Hunter	0.571483	weak
69	Meta Majority	0.806281	low
70	Meta Majority Finite Memory	0.782167	low
71	Meta Majority Long Memory	0.607384	moderate
72	Meta Majority Memory One	0.844402	low
73	Meta Minority	0.600192	moderate
74	Meta Mixer	0.465104	weak
75	Meta Winner	0.454301	weak
76	Meta Winner Finite Memory	0.631975	moderate
77	Meta Winner Long Memory	0.369008	high
78	Meta Winner Memory One	0.513786	weak
79	Naive Prober: 0.1	0.384372	high
80	Nice Average Copier	0.494468	weak
81	Nydegger	0.896419	low
82	Omega TFT	0.819010	low
83	Once Bitten	0.776105	low
84	Opposite Grudger	0.887569	low
85	PSO Gambler	0.697883	moderate
86	Predator	0.287886	high

87	Prober	0.334014	high
88	Probermid	0.658765	moderate
89	Probermoderate	0.101165	mid
90	Punisher	0.674391	moderate
91	Raider	0.389103	high
92	Random Hunter	0.961781	low
93	Random: 0.5	0.500214	weak
94	Remorseful Prober: 0.1	0.519303	weak
95	Retaliate (0.05)	0.628877	moderate
96	Retaliate (0.08)	0.747511	moderate
97	Retaliate (0.1)	0.688798	moderate
98	Ripoff	0.444200	weak
99	Risky QLearner	0.886197	low
100	Shubik	0.534640	weak
101	Slow Tit For Two Tats	0.899662	low
102	Sneaky Tit For Tat	0.669040	moderate
103	Soft Go By Majority	0.869060	low
104	Soft Go By Majority: 10	0.838410	low
105	Soft Go By Majority: 20	0.852889	low
106	Soft Go By Majority: 40	0.820777	low
107	Soft Go By Majority: 5	0.838984	low
108	Soft Grudger	0.757100	moderate
109	Soft Joss: 0.9	0.855790	low
110	SolutionB1	0.924119	low
111	SolutionB5	0.322532	high
112	Stochastic WSLS	0.562218	weak
113	Suspicious Tit For Tat	0.442323	weak
114	Tester	0.569160	weak
115	ThueMorse	0.500000	weak
116	ThueMorseInverse	0.500000	weak
117	Thumper	0.682443	moderate
118	Tit FormidTats	0.944449	low
119	Tit For Tat	0.762837	low
120	Tricky Cooperator	0.789430	low
121	Tricky Defector	0.286726	high

122	Tullock	0.504201	weak
123	Two Tits For Tat	0.586879	moderate
124	Win-Shift Lose-Stay	0.558327	weak
125	Win-Stay Lose-Shift	0.697654	moderate
126	ZD-Extort-2	0.299234	high
127	ZD-Extort-2 v2	0.360133	high
128	ZD-Extort-4	0.154946	mid
129	ZD-GEN-2	0.899807	low
130	ZD-GTFT-2	0.862014	low
131	ZD-SET-2	0.452609	weak

A.2 Classification Tables for Nydegger, QLearner and Gradual

In this section, tables for the classes of clustering and cooperating ratio, for the top three ranking strategies of the complex networks experiments, are illustrated.

Appendix B

Second Appendix Title

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