Learning in games

International Symposium on Dynamic Games and Applications

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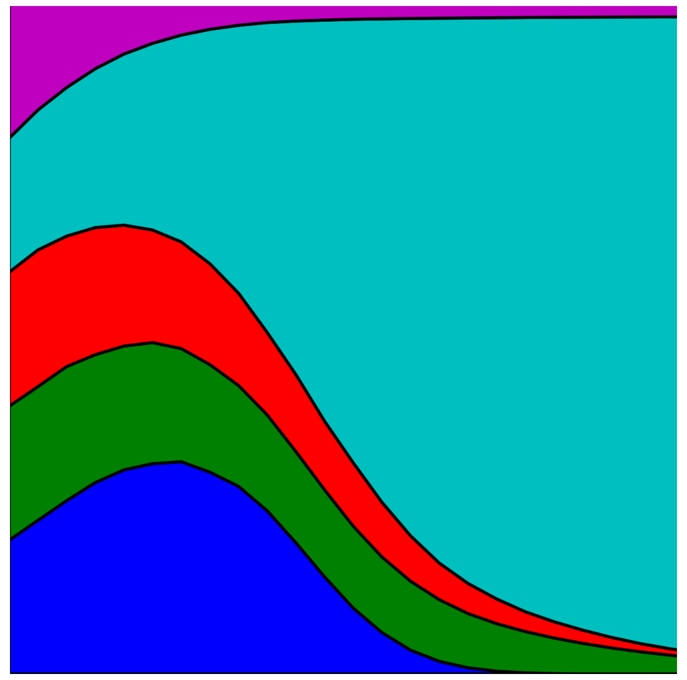
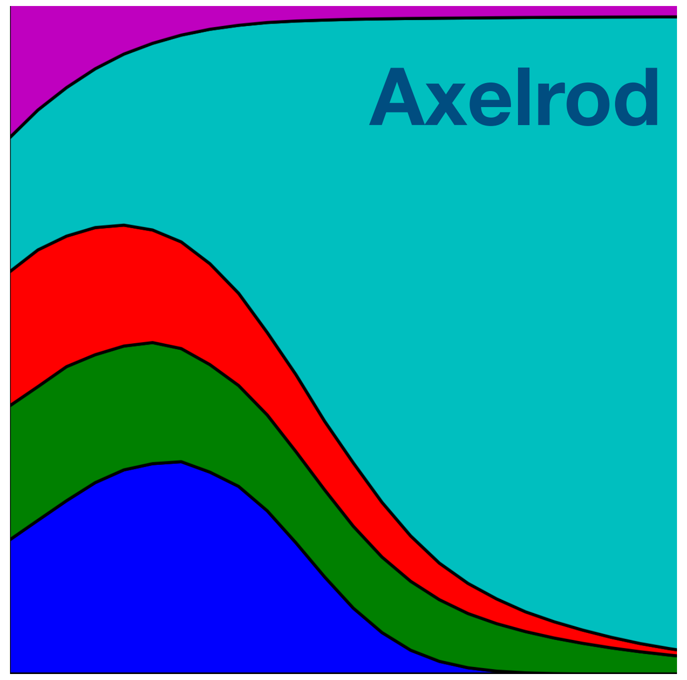
Introduction



Introduction



Introduction

Axelrod

Software
Sustainability
Institute

Social Behavior

Understand Cooperation

- Advance theoretical models
- Identify factors that can help sustain cooperation
- Train and understand autonomous learning agents

A.

```
N ← population size;
resident ← starting resident;
while t < maximum number of steps do
    mutant ← random strategy;
    fixation probability ← ρ_M;
    if ρ_M > random: i → [0, 1] then
        resident ← mutant;
    end
end
```

A.

$N \leftarrow$ population size;
resident $\leftarrow$ starting resident;
while $t <$ *maximum number of steps* **do**
  mutant $\leftarrow$ random strategy;
  fixation probability $\leftarrow \rho_{\mathrm{M}}$;
  if $\rho_M >$ *random:* $i \rightarrow [0, 1]$ **then**
    resident $\leftarrow$ mutant;
  end
end

B.

```
>>> import nashpy as nash
```

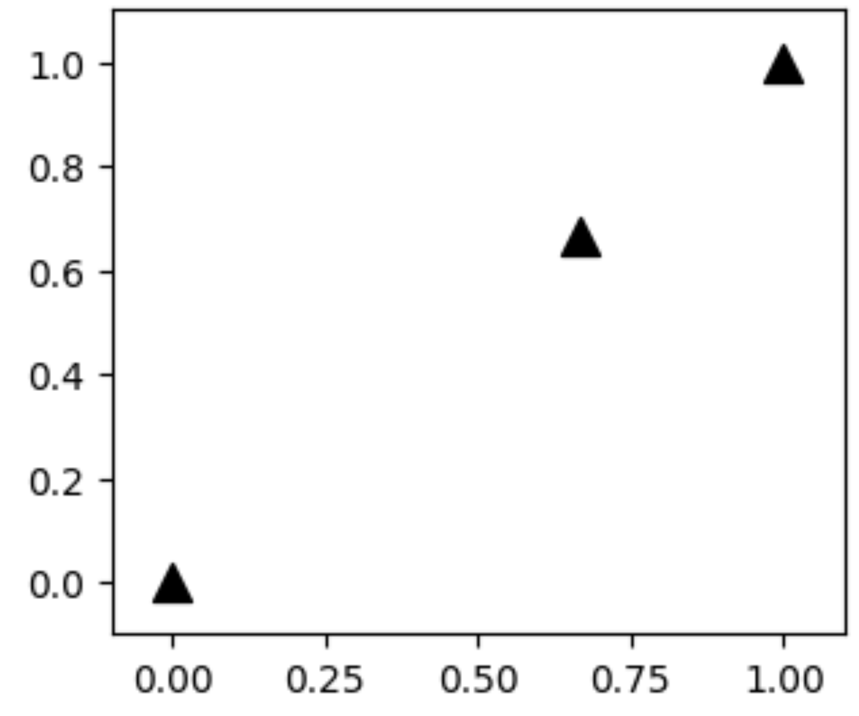
A.

$N \leftarrow$ population size;
resident $\leftarrow$ starting resident;
while $t <$ *maximum number of steps* **do**
    mutant $\leftarrow$ random strategy;
    fixation probability $\leftarrow \rho_{\mathrm{M}}$;
    if $\rho_M >$ *random:* $i \rightarrow [0, 1]$ **then**
        resident $\leftarrow$ mutant;
    end
end

B.

```
>>> import nashpy as nash
```

C.



1. **Introduction to Games**

   2×2 *normal form games*

2. **Learning Dynamics**

   Replicator dynamics,
   Pairwise comparison process,
   Introspection dynamics

3. **Repeated Games**

   Repeated prisoner's dilemma

4. **Revisit Learning Dynamics**

   Applying them to repeated games

5. **Conclusions**

Disclaimer

1. Introduction to Games

2×2 normal form games

Normal form games

Normal form games

$$\begin{array}{ccc} & C & D \\ \begin{array}{c} C \\ D \end{array} & \multicolumn{2}{c}{\begin{pmatrix} (r, l) & (s, m) \\ (t, n) & (p, k) \end{pmatrix}} \end{array}$$

Normal form games

$$\begin{array}{ccc} & C & D \\ \begin{array}{c} C \\ D \end{array} & \multicolumn{2}{c}{\begin{pmatrix} (r,l) & (s,m) \\ (t,n) & (p,k) \end{pmatrix}} \end{array}$$

$$r = l, s = n, t = m, p = k$$

Normal form games

$$
\begin{array}{c c}
 & \begin{array}{cc} C & D \end{array} \\
\begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (r, r) & (s, t) \\ (t, s) & (p, p) \end{pmatrix}
\end{array}
$$

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$$

$$r > t, p > s$$

Normal form games

$$\begin{array}{ccc} & C & D \\ \begin{array}{c} C \\ D \end{array} & \multicolumn{2}{c}{\begin{pmatrix} (\boxed{r}, r) & (s, t) \\ (t, s) & (\boxed{p}, p) \end{pmatrix}} \end{array}$$

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$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (\boxed{r}, \boxed{r}) & (s, t) \\ (t, s) & (\boxed{p}, \boxed{p}) \end{pmatrix} \end{array}$$

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Class of game	Payoffs	Nash equilibria	Example

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (r, r) & (s, t) \\ (t, s) & (p, p) \end{pmatrix} \end{array}$$

Class of game	Payoffs	Nash equilibria	Example
Coordination	$r > t, p > s$.		

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (r,r) & (s,t) \\ (t,s) & (p,p) \end{pmatrix} \end{array}$$

Class of game	Payoffs	Nash equilibria	Example
Coordination	$r > t, p > s$.	Two pure strategy $(C,C),(D,D)$ and one mixed strategy NE.	

$$
\begin{array}{ccc}
 & C & D \\
\begin{matrix} C \\ D \end{matrix} & \multicolumn{2}{c}{\begin{pmatrix} (r,r) & (s,t) \\ (t,s) & (p,p) \end{pmatrix}}
\end{array}
$$

Class of game	Payoffs	Nash equilibria	Example
Coordination	$r > t, p > s$.	Two pure strategy $(C,C), (D,D)$ and one mixed strategy NE.	Stag-hunt game.

$$\begin{array}{ccc} & C & D \\ \begin{array}{c} C \\ D \end{array} & \multicolumn{2}{c}{\begin{pmatrix} (r,r) & (s,t) \\ (t,s) & (p,p) \end{pmatrix}} \end{array}$$

Class of game	Payoffs	Nash equilibria	Example
Coordination	$r > t, p > s$.	Two pure strategy $(C,C),(D,D)$ and one mixed strategy NE.	Stag-hunt game.
Anti-coordination	$r < t, s > p$.		

$$\begin{array}{c|cc} & C & D \\ \hline C & (r,r) & (s,t) \\ D & (t,s) & (p,p) \end{array}$$

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$$

Class of game	Payoffs	Nash equilibria	Example
Coordination	$r > t, p > s$.	Two pure strategy $(C, C), (D, D)$ and one mixed strategy NE.	Stag-hunt game.
Anti-coordination	$r < t, s > p$.	Two pure strategy $(C, D), (D, C)$ and one mixed strategy NE.	Chicken game.

$$\begin{array}{ccc} & C & D \\ \begin{array}{c} C \\ D \end{array} & \multicolumn{2}{c}{\begin{pmatrix} (r,r) & (s,t) \\ (t,s) & (p,p) \end{pmatrix}} \end{array}$$

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Anti-coordination	$r < t, s > p$.	Two pure strategy $(C,D),(D,C)$ and one mixed strategy NE.	Chicken game.
Dominance-solvable	$r > t, s > p$; $r < t, s < p$		

$$
\begin{array}{cc}
 & \begin{array}{cc} C & D \end{array} \\
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$$

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Dominance-solvable	$r > t, s > p$; $r < t, s < p$	Unique pure strategy NE.	

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Class of game	Payoffs	Nash equilibria	Example
Coordination	$r > t, p > s$.	Two pure strategy $(C, C), (D, D)$ and one mixed strategy NE.	Stag-hunt game.
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Dominance-solvable	$r > t, s > p$; $r < t, s < p$	Unique pure strategy NE.	Prisoner's dilemma.

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Dominance-solvable	$r > t, s > p$; $r < t, s < p$	Unique pure strategy NE.	Prisoner's dilemma.	

Normal form games

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (4,4) & (1,3) \\ (3,1) & (3,3) \end{pmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (2,2) & (1,3) \\ (3,1) & (0,0) \end{pmatrix} \end{array}$$

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (3,3) & (0,5) \\ (5,0) & (1,1) \end{pmatrix} \end{array}$$

Normal form games

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (4,4) & (1,3) \\ (3,1) & (3,3) \end{pmatrix} \end{array}$$

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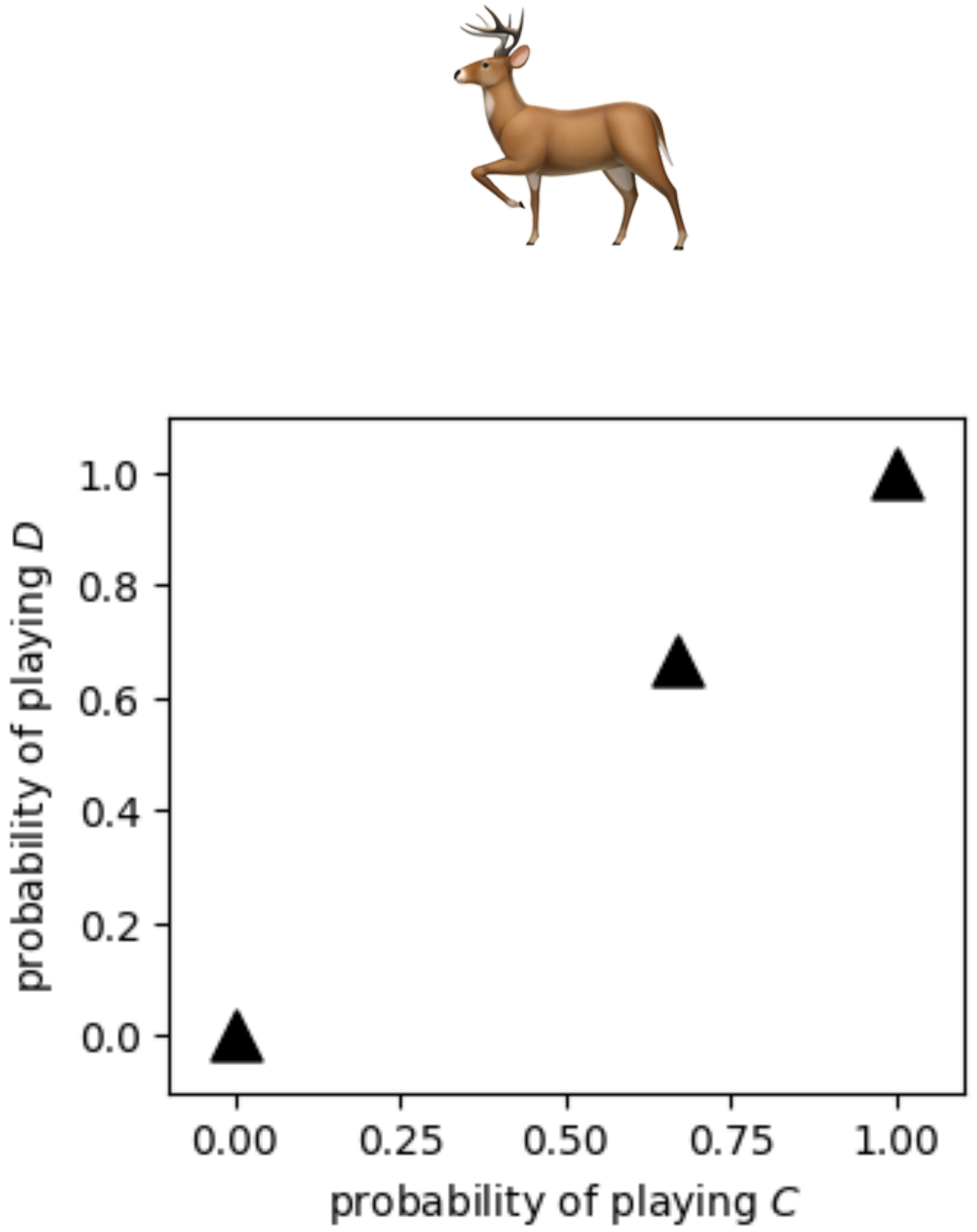
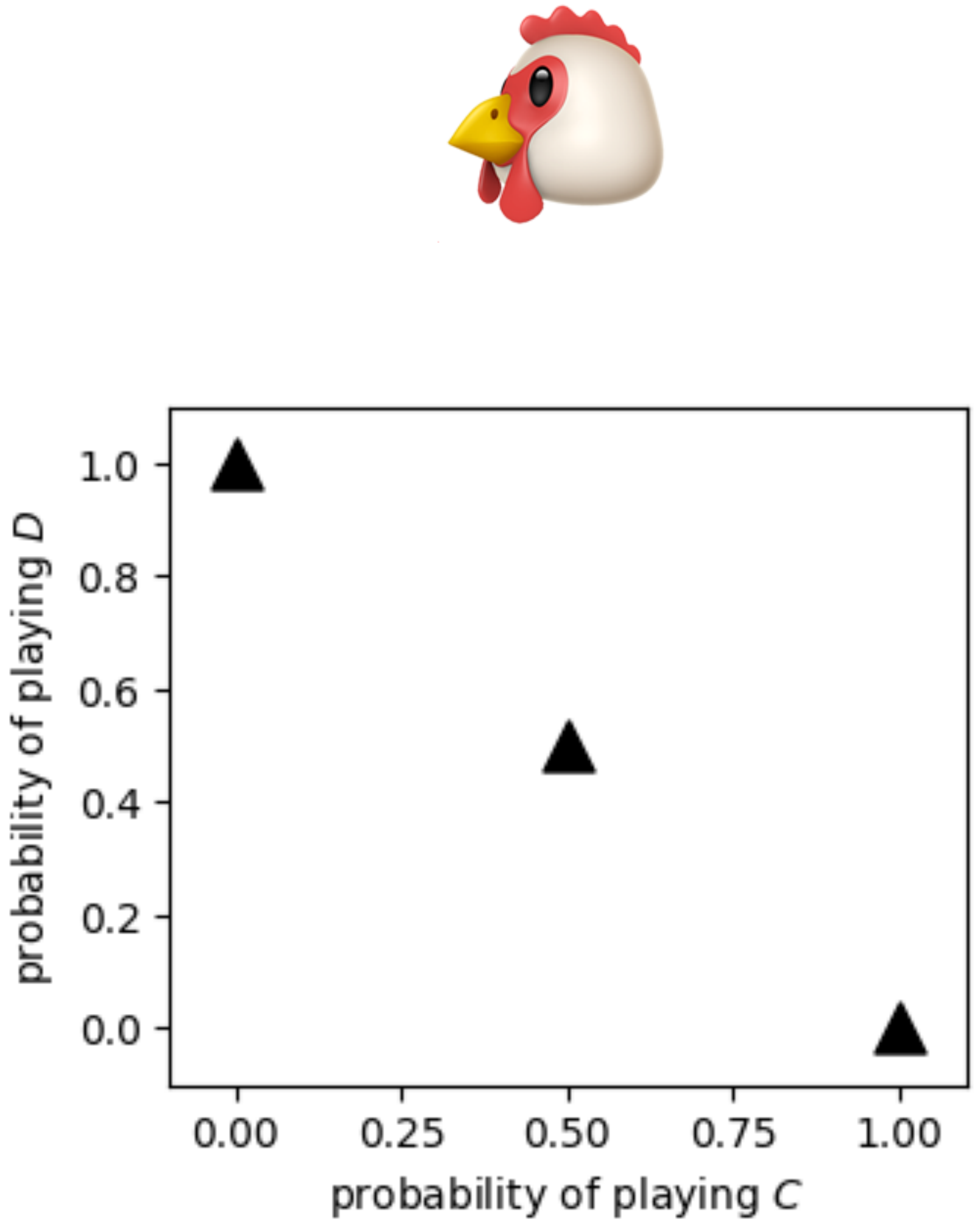
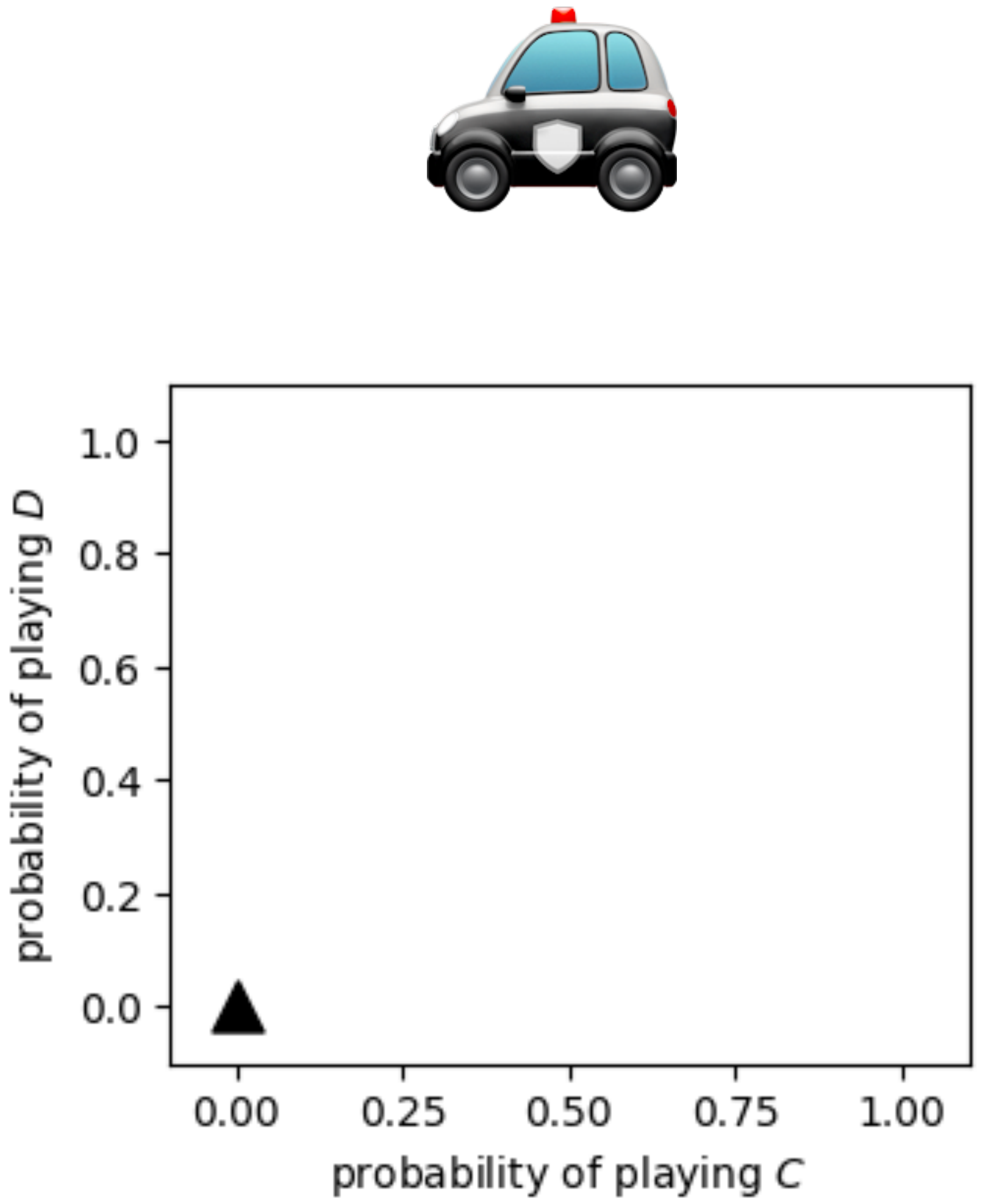
Normal form games

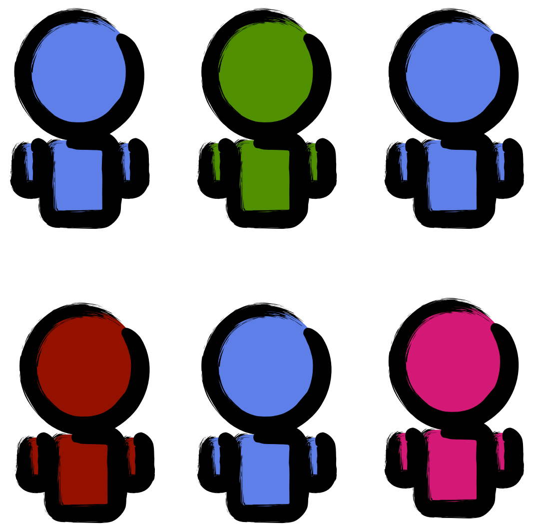




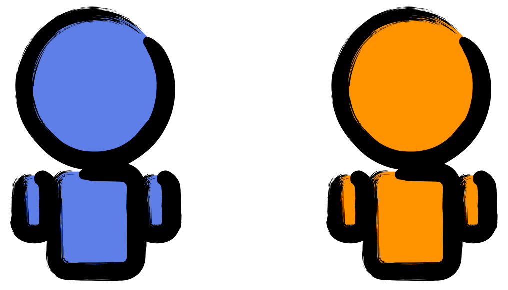


2. Learning Dynamics

Evolutionary game theory



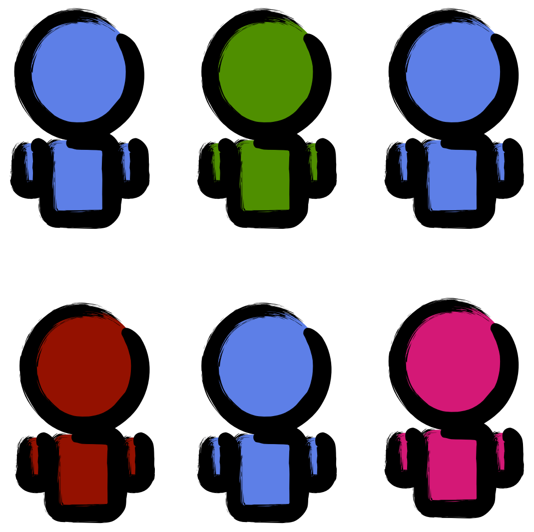
Reinforcement learning



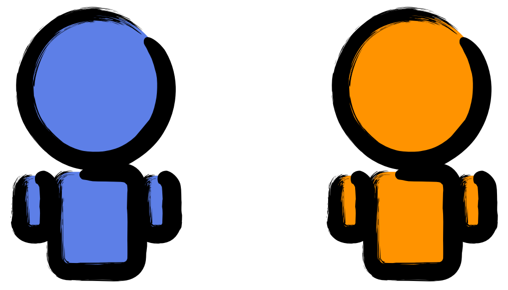
2. Learning Dynamics

Replicator dynamics,
Pairwise comparison process,
Introspection dynamics

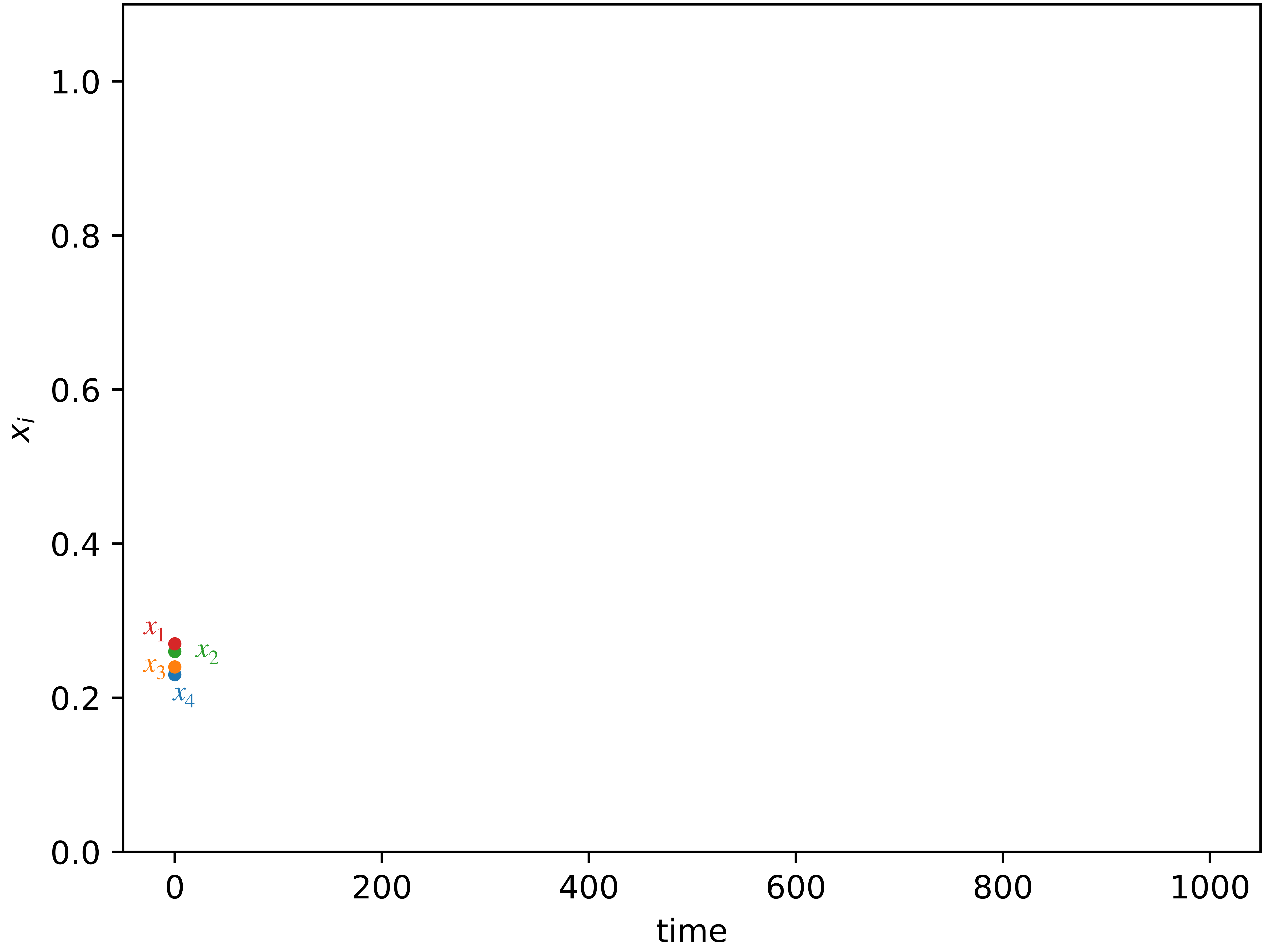
Evolutionary game theory



Reinforcement learning

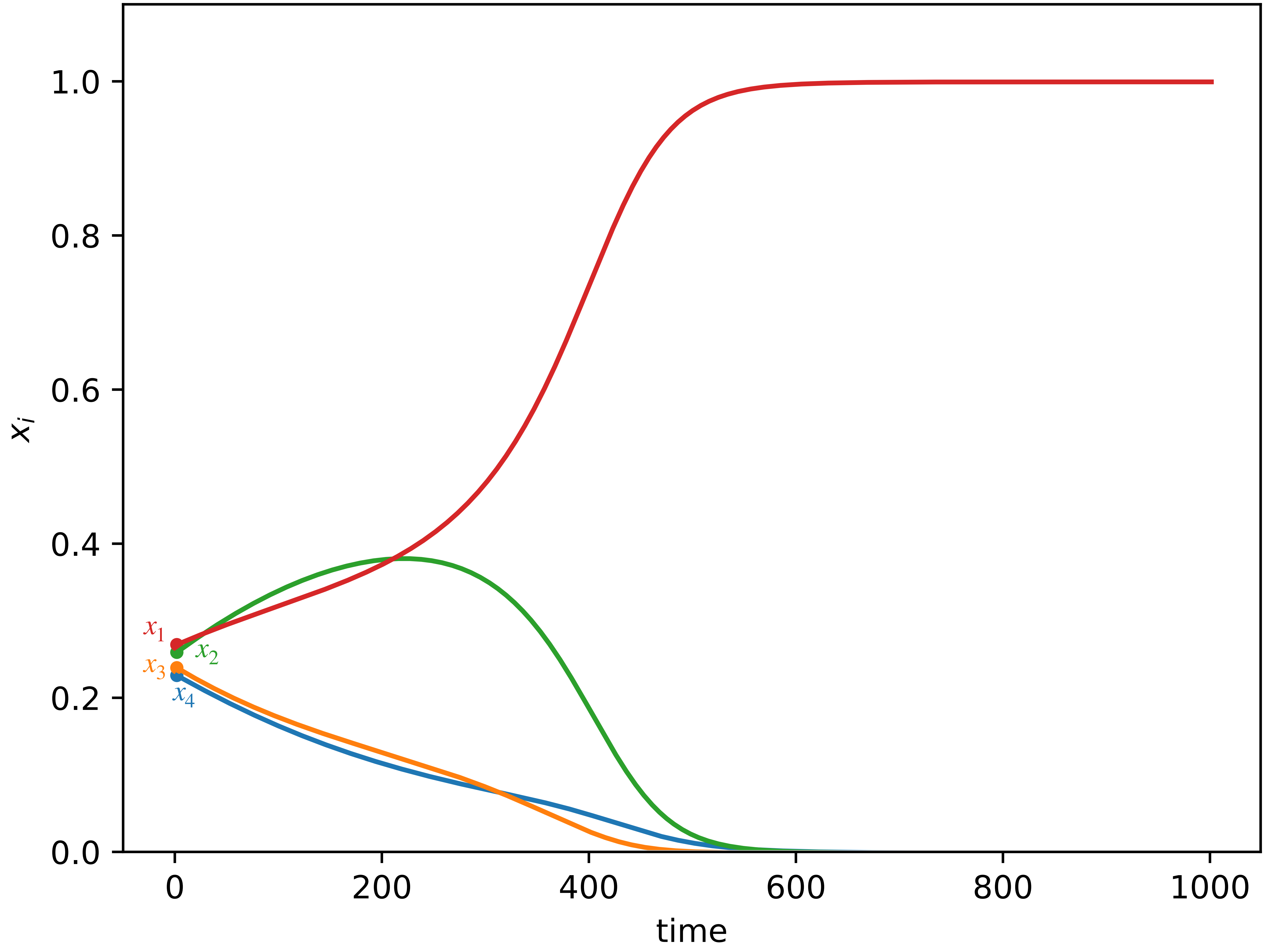


Replicator dynamics



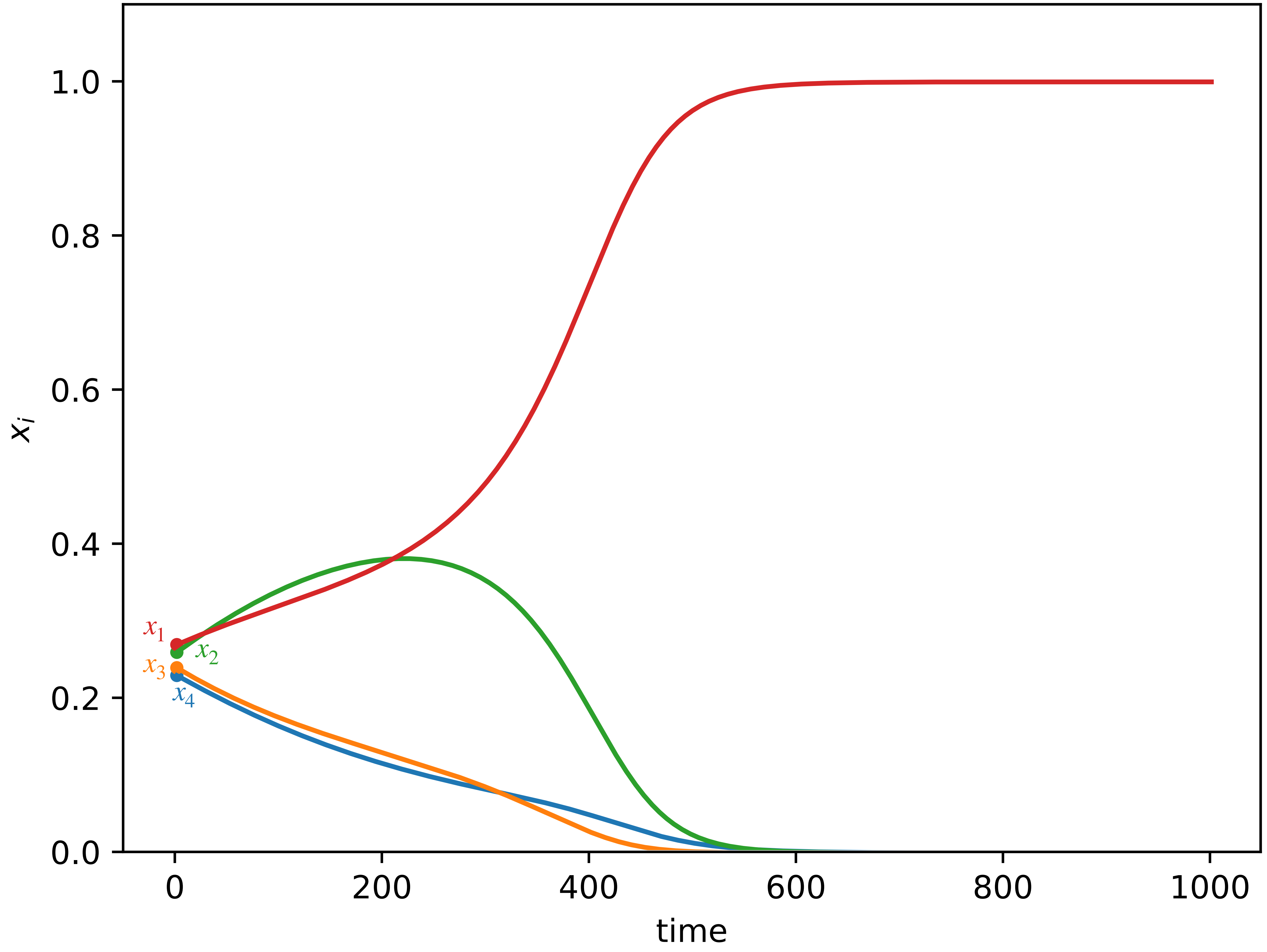
$$\dot{x}_i = x_i(\pi_i - \bar{\pi})$$

Replicator dynamics



$$\dot{x}_i = x_i(\pi_i - \bar{\pi})$$

Replicator dynamics



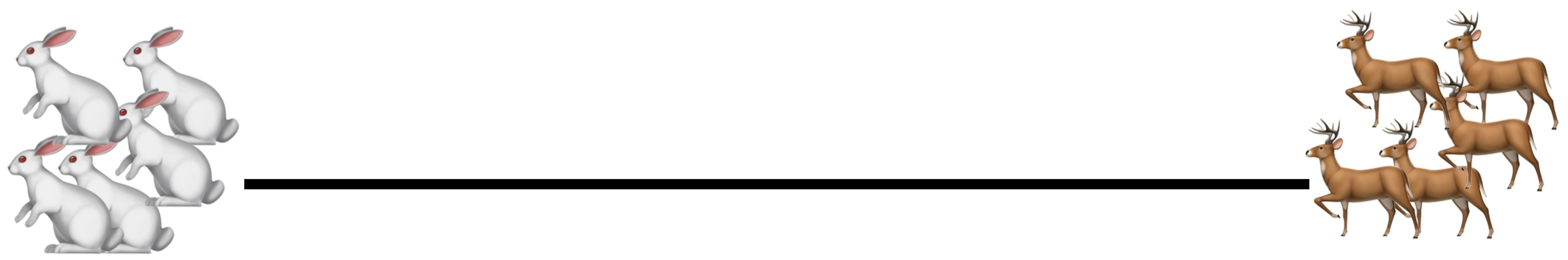
$$\dot{x}_i = x_i(\pi_i - \bar{\pi})$$

Replicator dynamics

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (4,4) & (1,3) \\ (3,1) & (3,3) \end{pmatrix} \end{array}$$

Replicator dynamics

$$
\begin{array}{ccc}
 & C & D \\
\begin{array}{c} C \\ D \end{array} & \multicolumn{2}{c}{\begin{pmatrix} (4,4) & (1,3) \\ (3,1) & (3,3) \end{pmatrix}}
\end{array}
$$



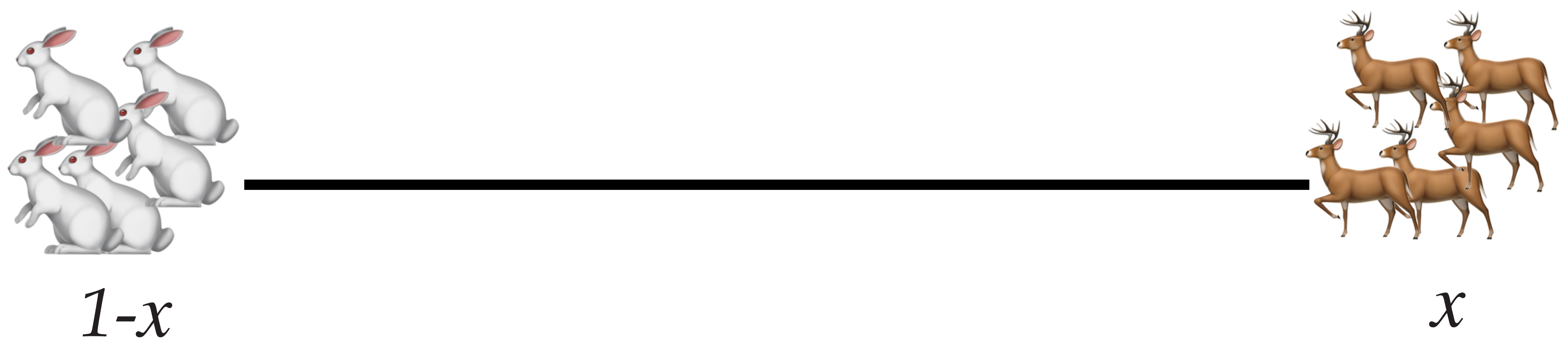
$1\text{-}x$ $\qquad$ x

Replicator dynamics

$$\begin{array}{ccc} & C & D \\ \begin{array}{c} C \\ D \end{array} & \multicolumn{2}{c}{\begin{pmatrix} (4,4) & (1,3) \\ (3,1) & (3,3) \end{pmatrix}} \end{array}$$

$$\pi_{\text{🦌}} = 4x + (1 - x)$$

$$\pi_{\text{🐇}} = 3x + 3(1 - x)$$



Replicator dynamics

$$\begin{array}{cc} & \begin{array}{cc} C & \quad D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (4,4) & (1,3) \\ (3,1) & (3,3) \end{pmatrix} \end{array}$$

$$\pi_{\text{🦌}} = 4x + (1 - x)$$

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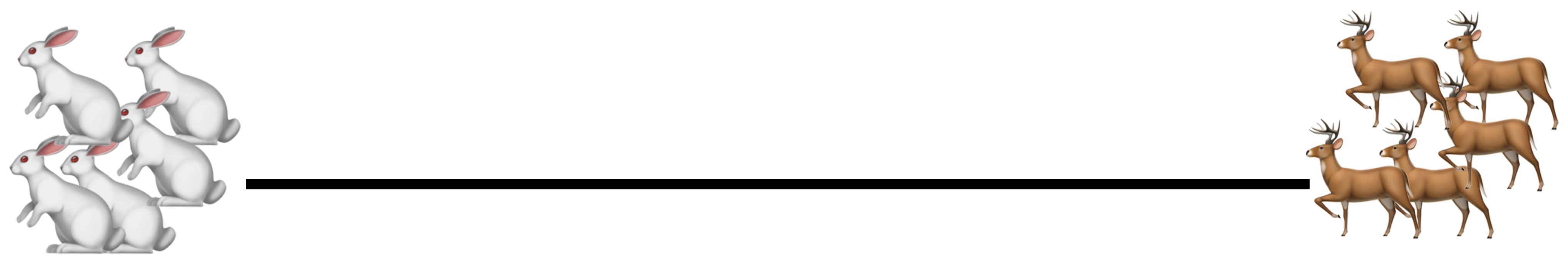
$$\dot{x} = x(1 - x)[3x - 2]$$



1-x *x*

Replicator dynamics

$$
\begin{array}{cc}
 & \begin{array}{cc} C & D \end{array} \\
\begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (4,4) & (1,3) \\ (3,1) & (3,3) \end{pmatrix}
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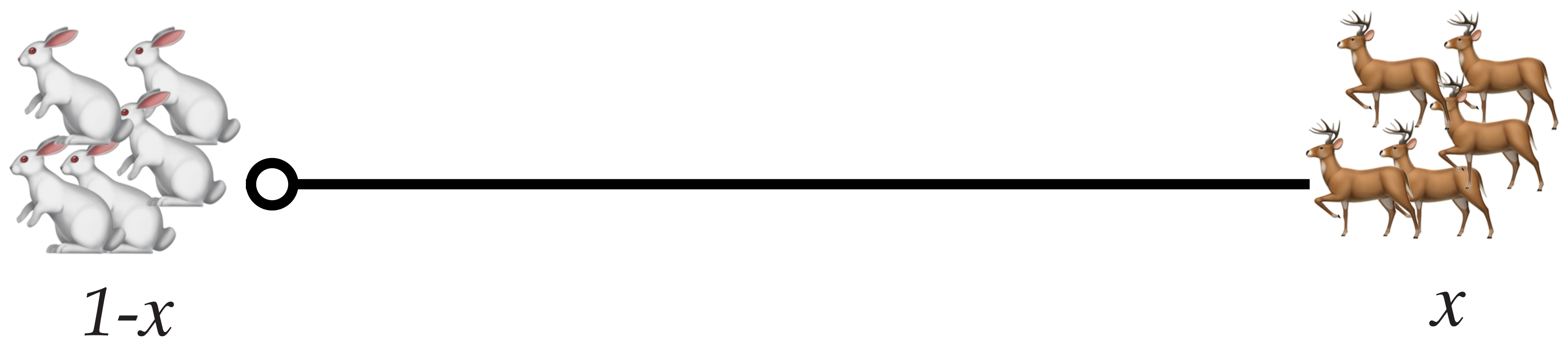

Replicator dynamics

$$
\begin{array}{c c}
 & \begin{array}{cc} C & D \end{array} \\
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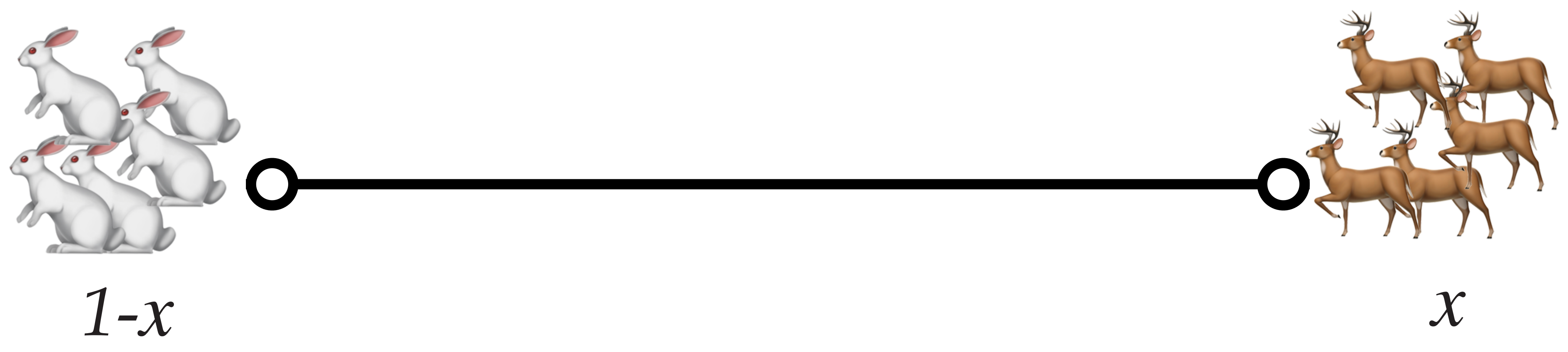

Replicator dynamics

$$
\begin{array}{cc}
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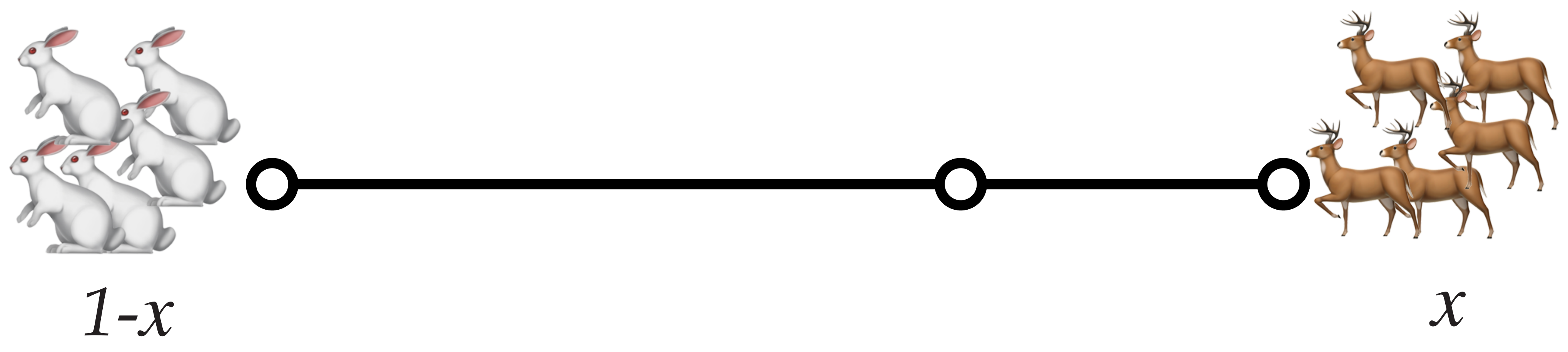

Replicator dynamics

$$
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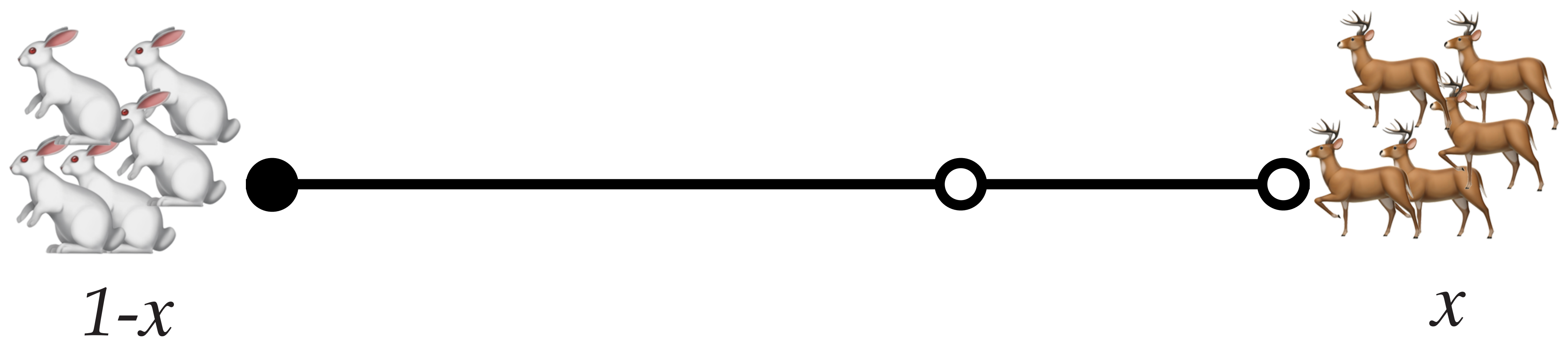

Replicator dynamics

$$\begin{array}{ccc} & C & D \\ C & \multirow{2}{*}{$\begin{pmatrix} (4,4) & (1,3) \\ (3,1) & (3,3) \end{pmatrix}$} & \\ D & & \end{array}$$

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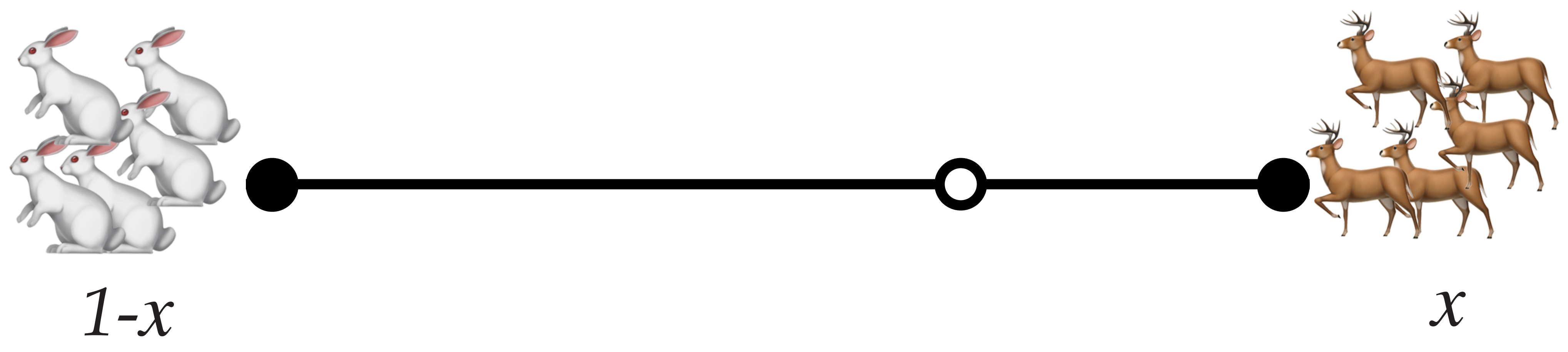

Replicator dynamics

$$
\begin{array}{c c}
 & \begin{array}{cc} \quad C & \quad\quad D \end{array} \\
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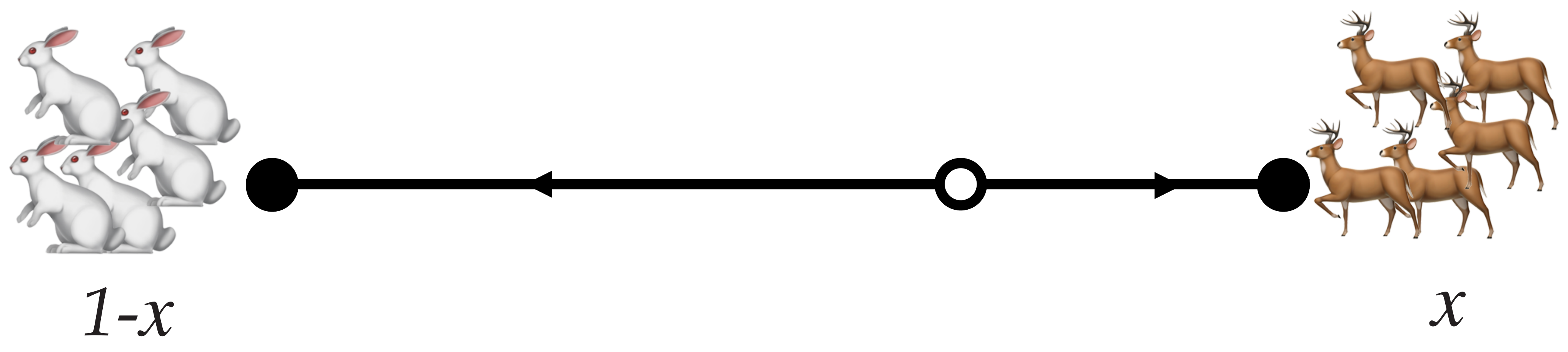

Replicator dynamics

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\begin{array}{ccc}
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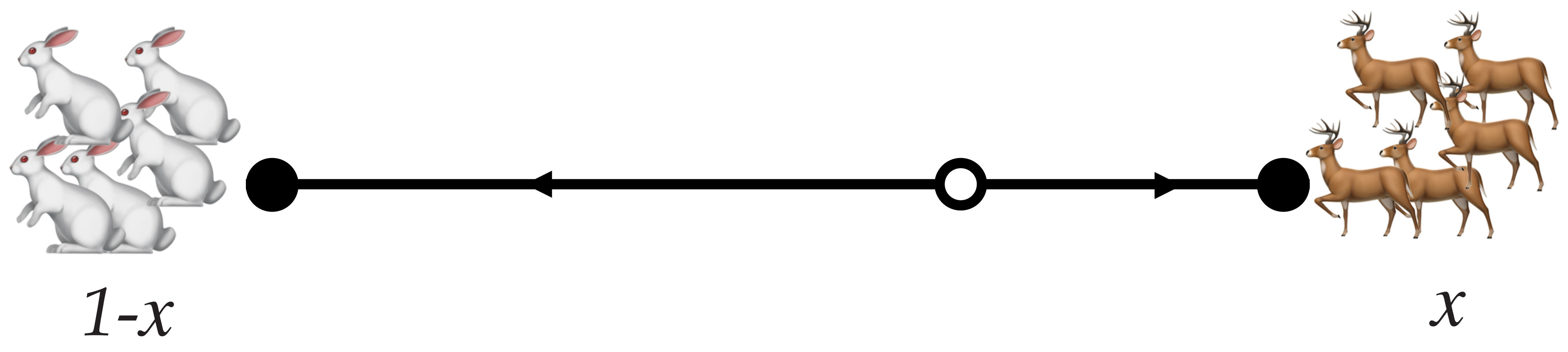
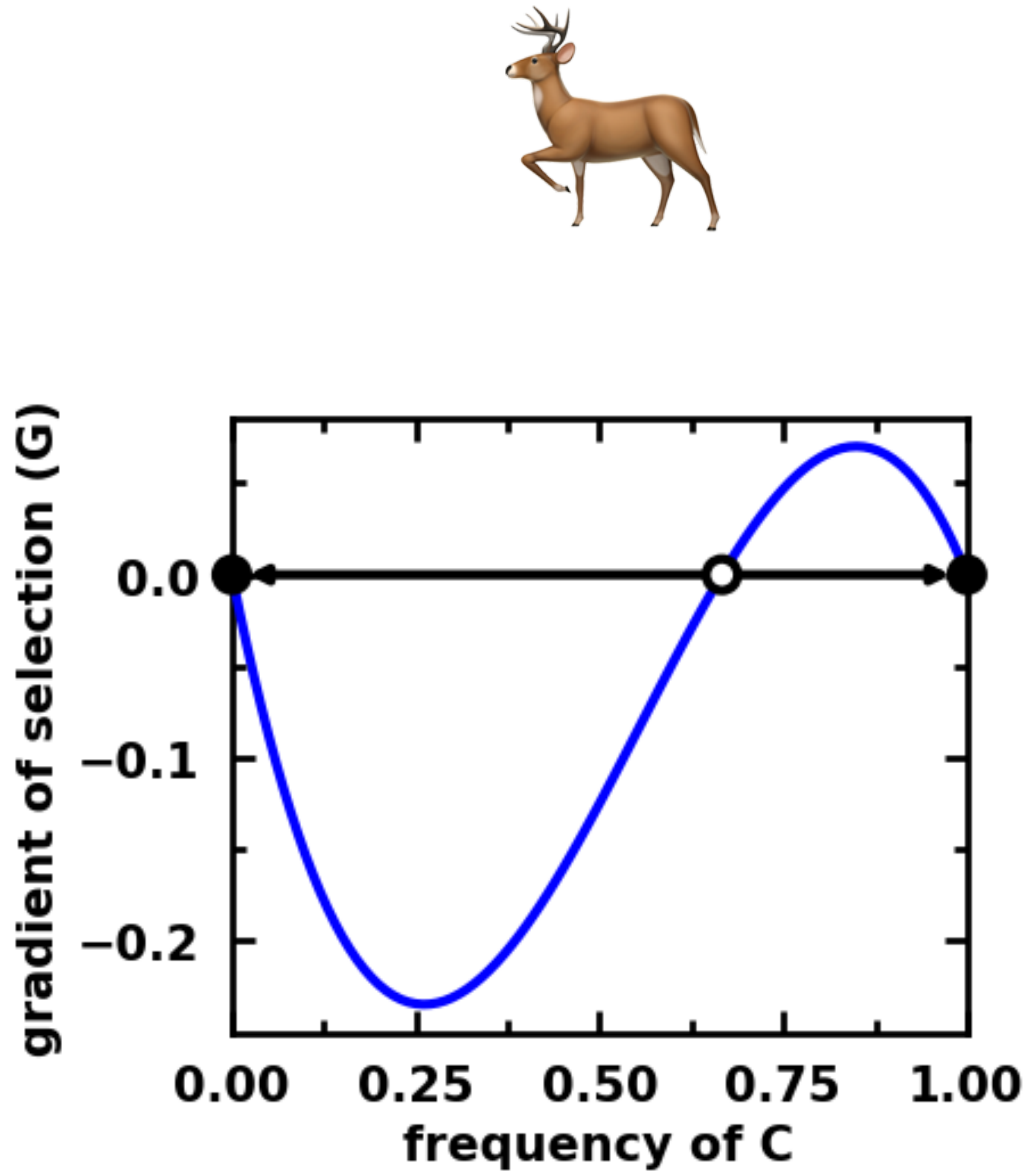
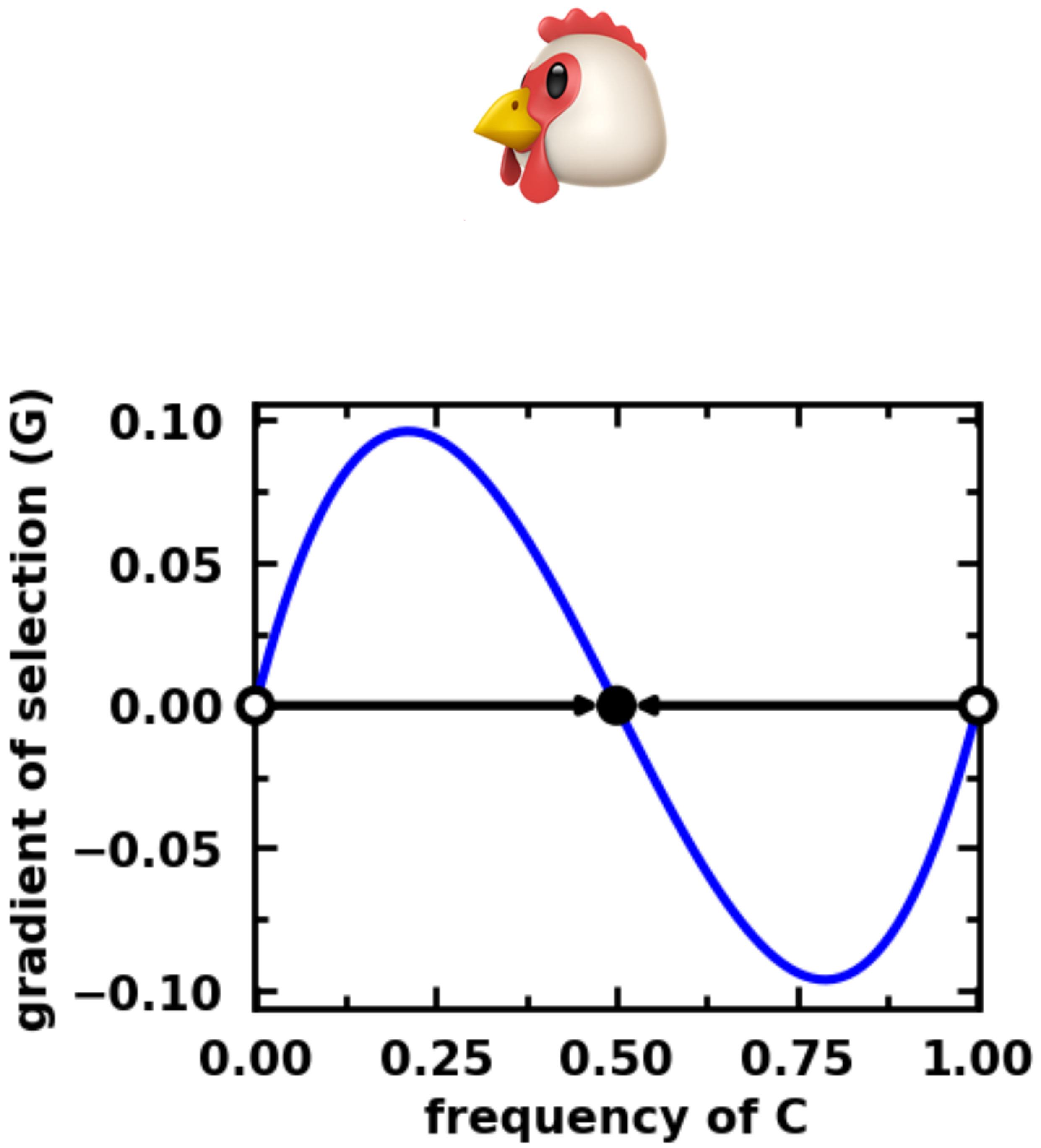
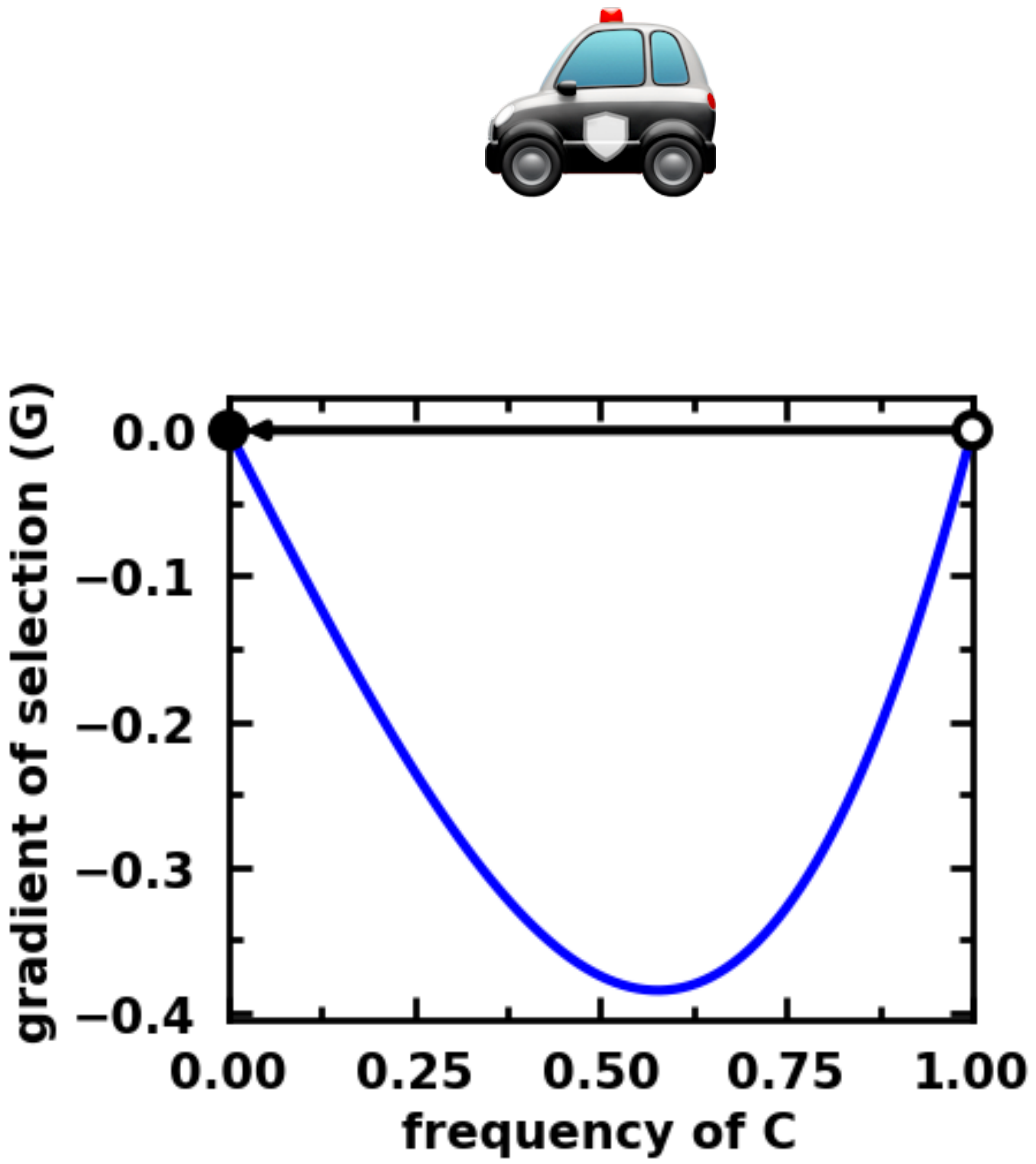


Replicator dynamics

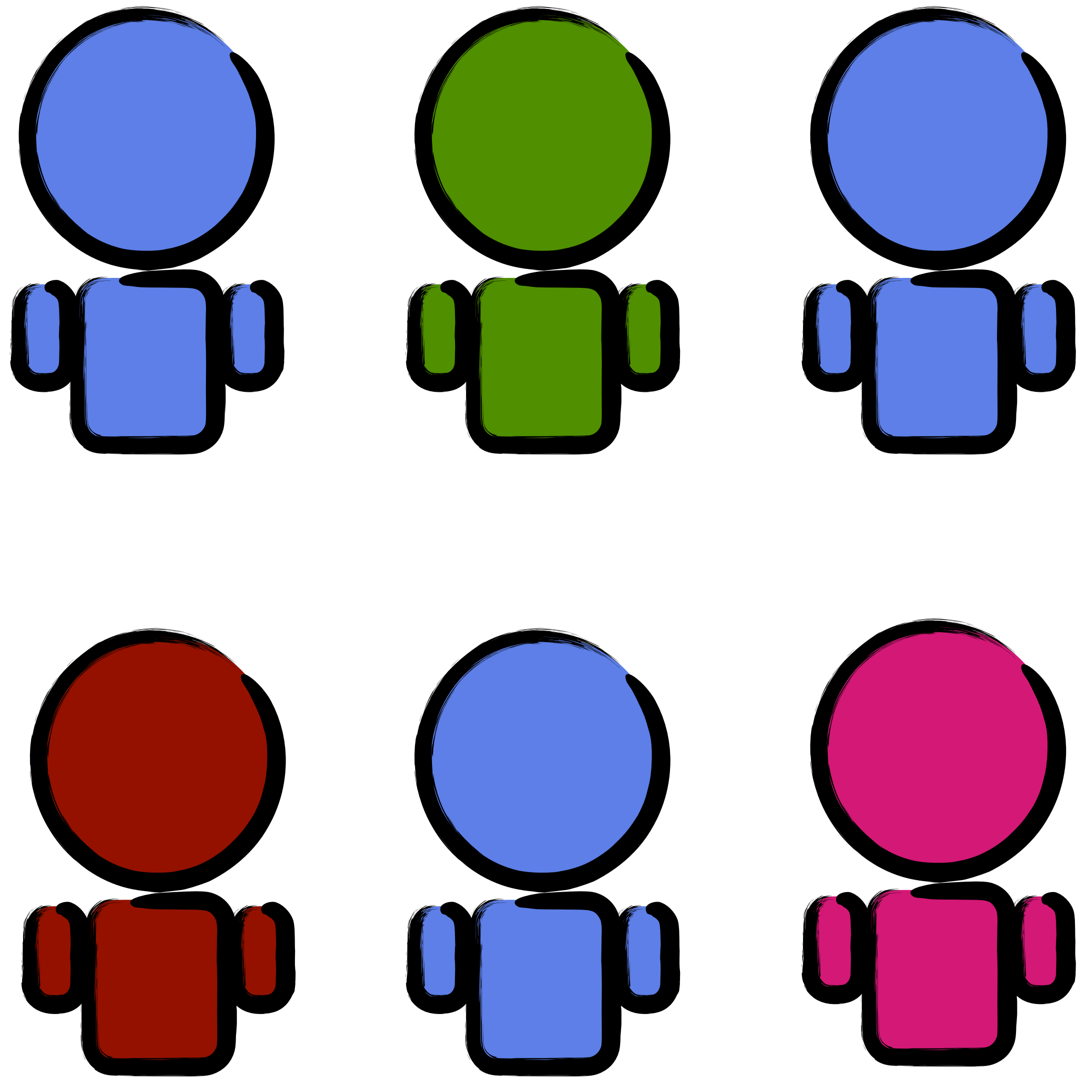
$\dot{x}$



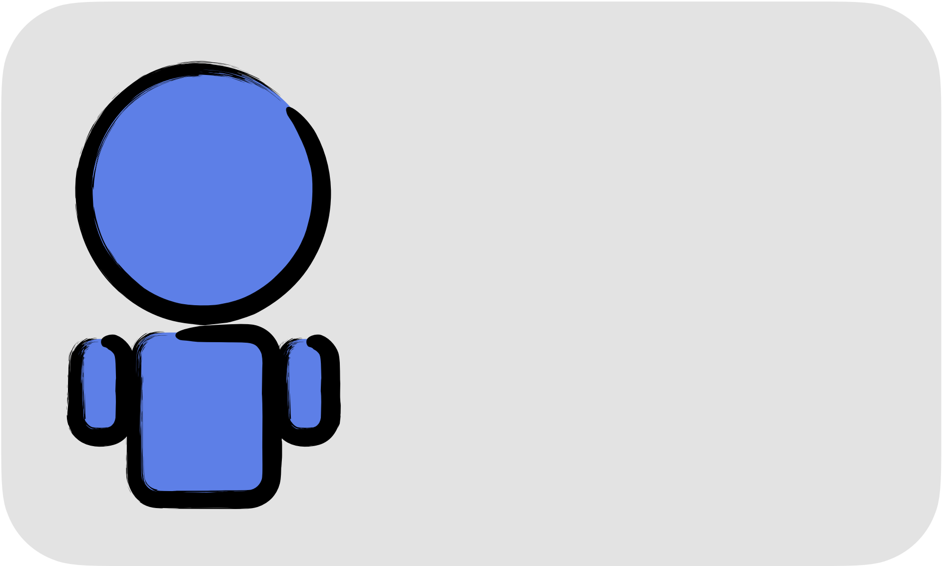




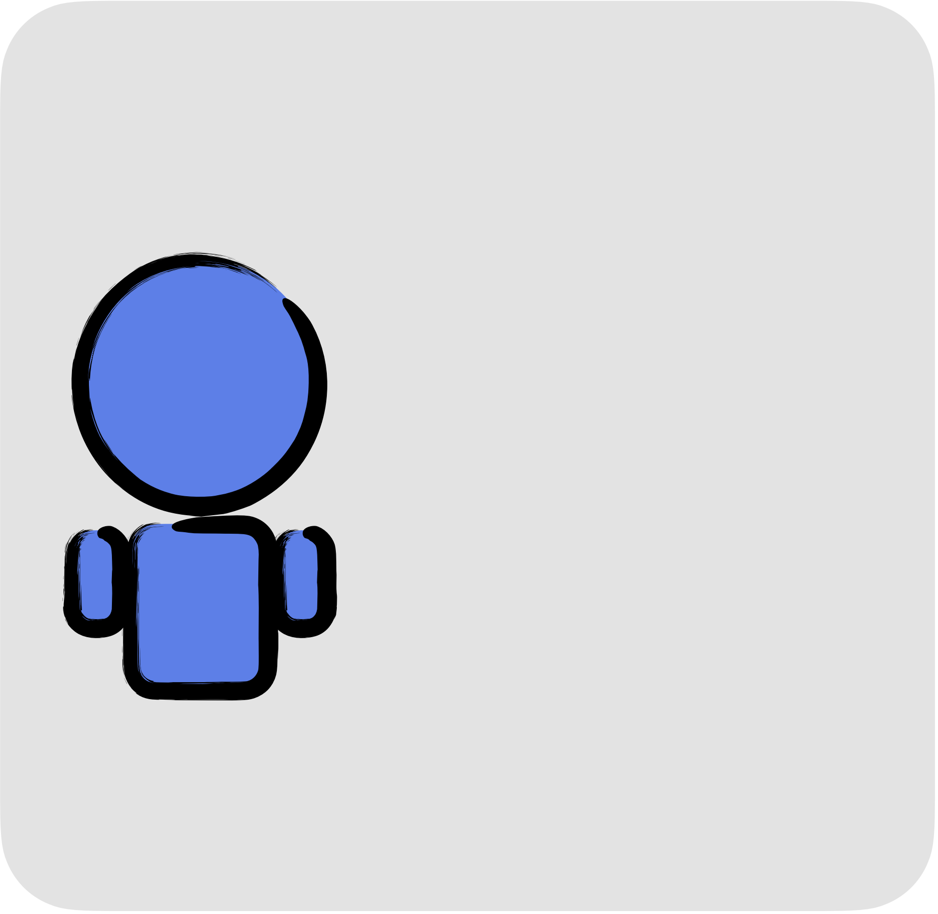
Pairwise comparison process



N: **population size**

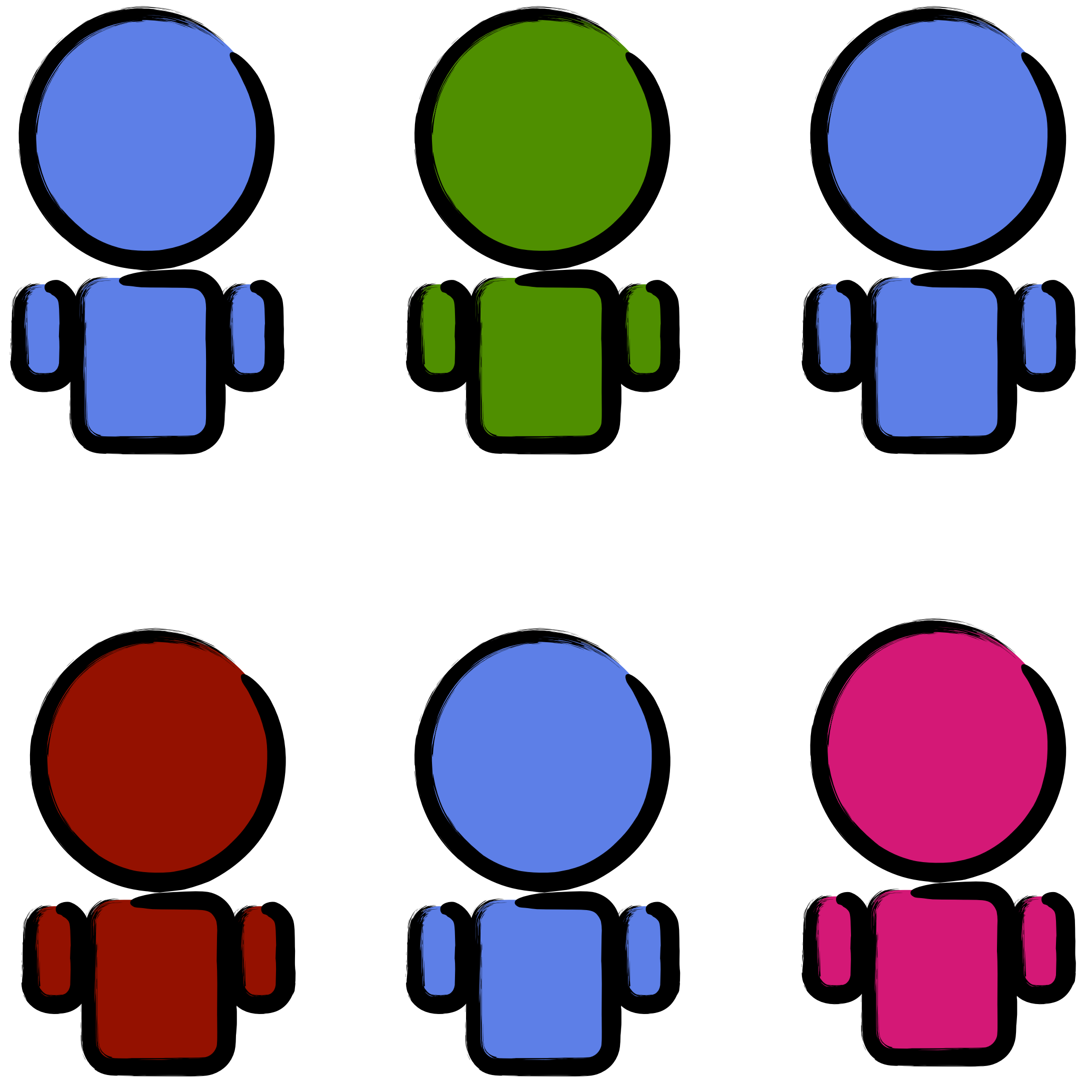


μ: **imitaton**

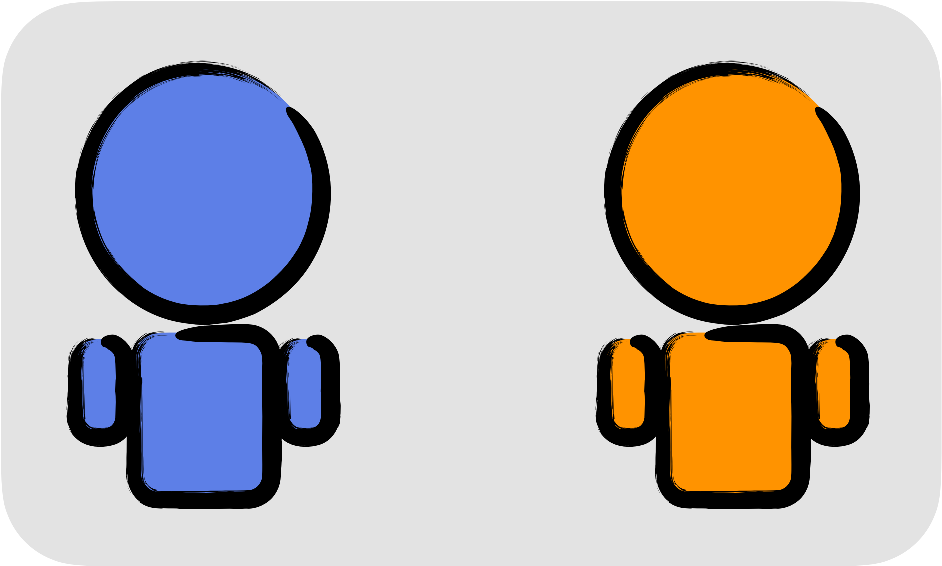


$1 - \mu$: **imitaton**

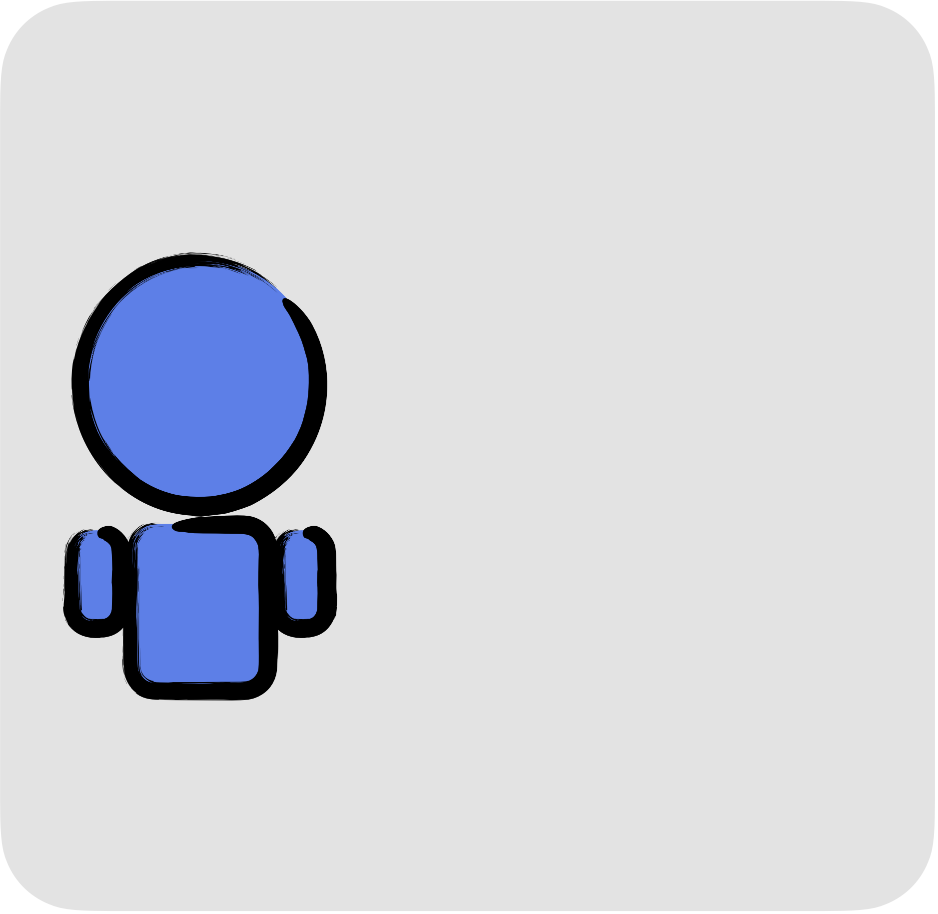
Pairwise comparison process



N: **population size**

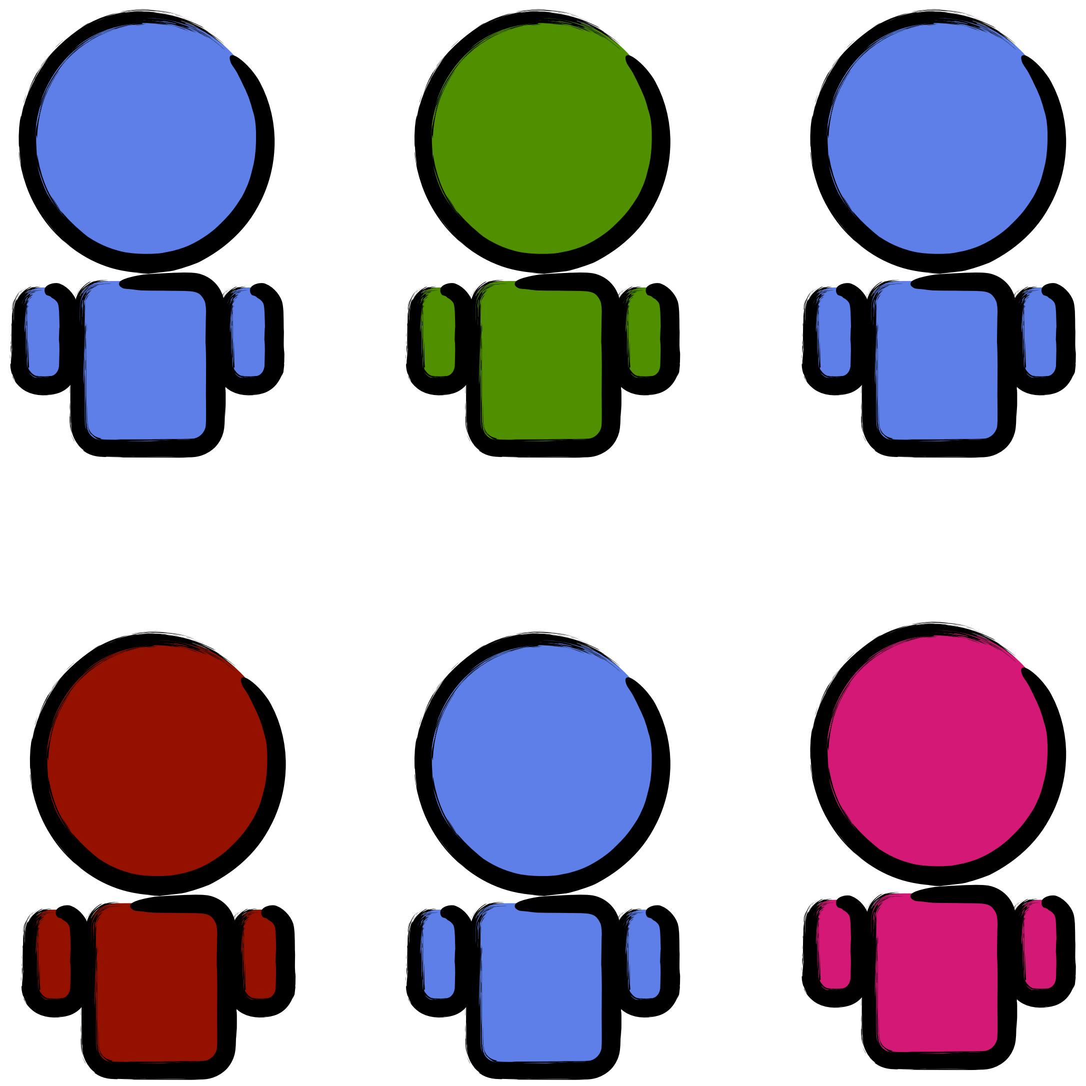


μ: **imitaton**

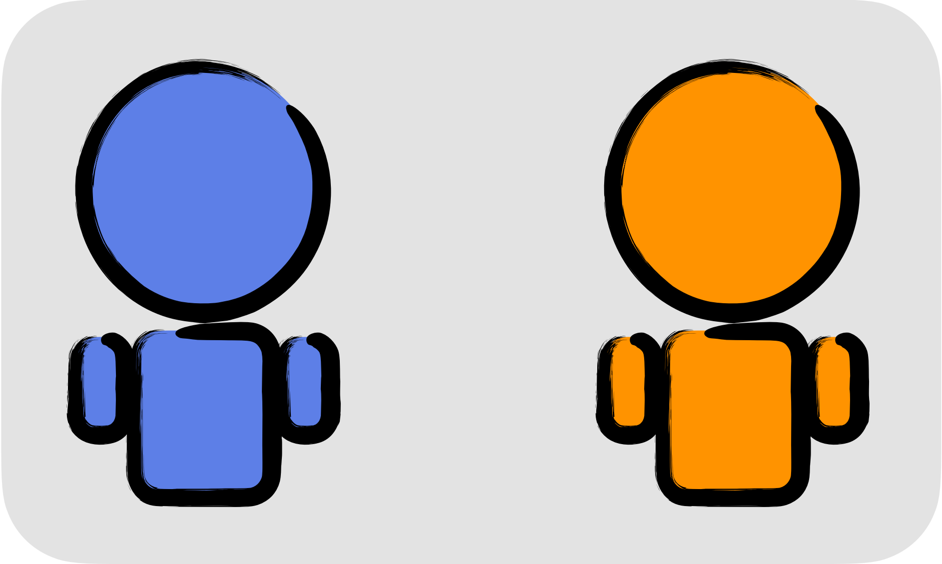


$1 - \mu$: **imitaton**

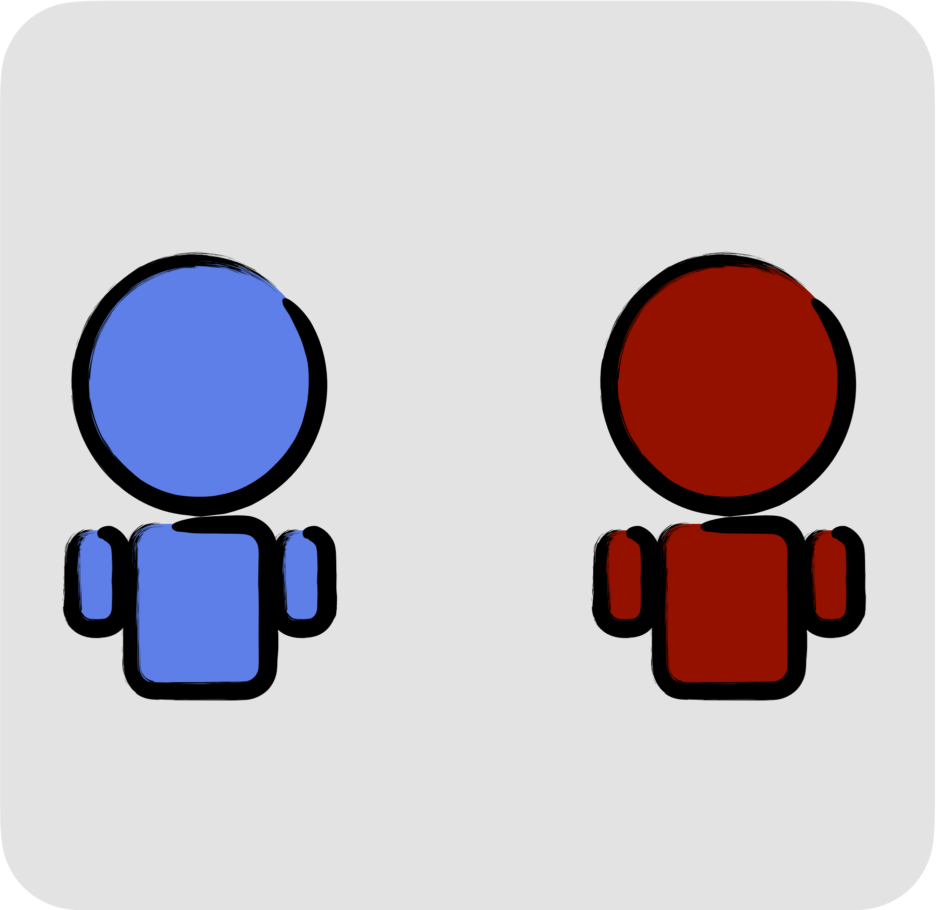
Pairwise comparison process



N: **population size**

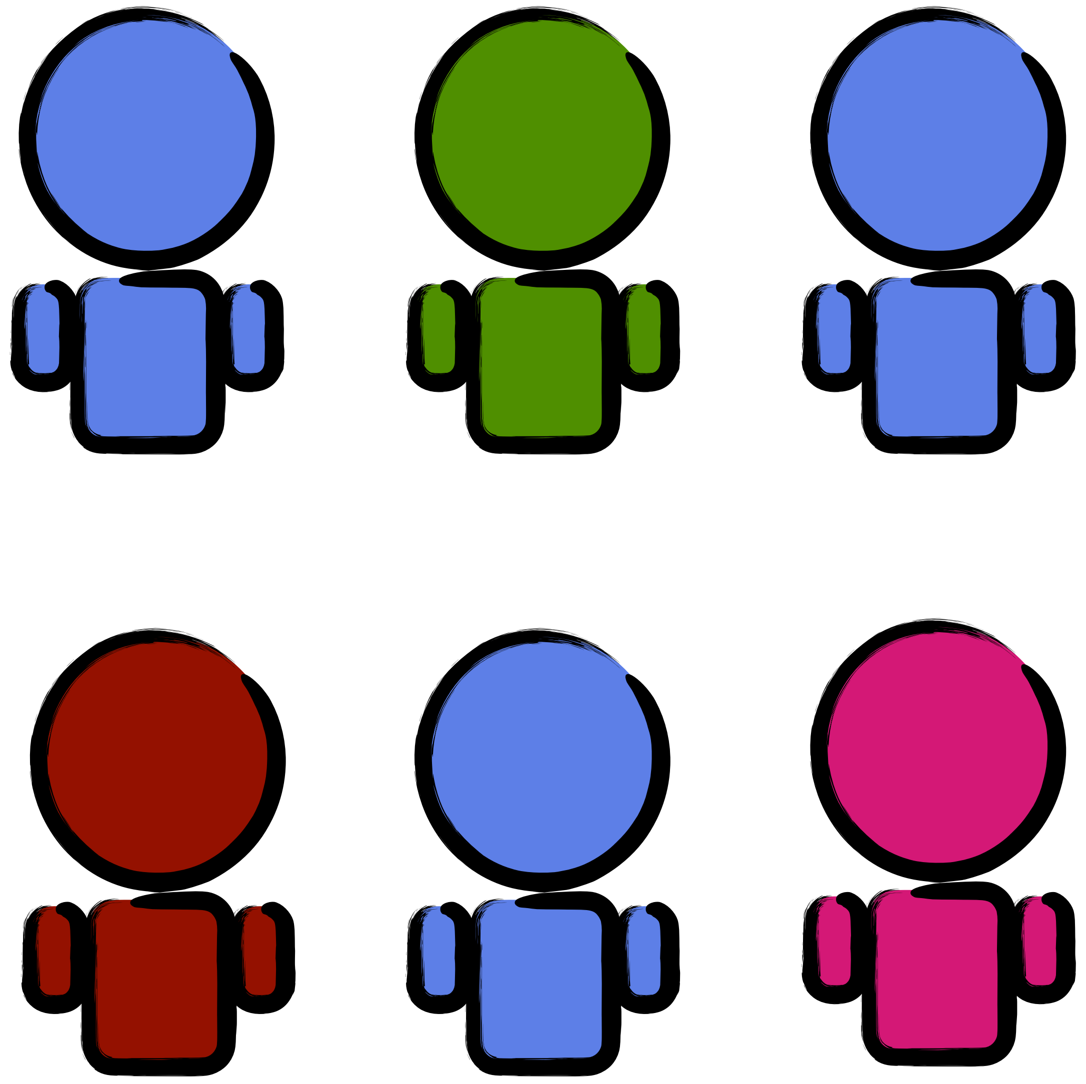


μ: **imitaton**

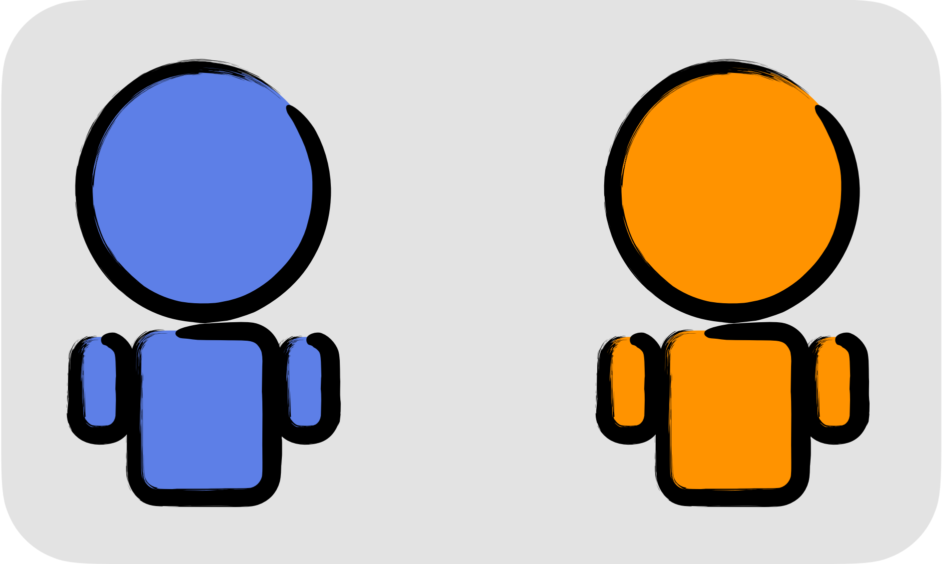


$1-\mu$: **imitaton**

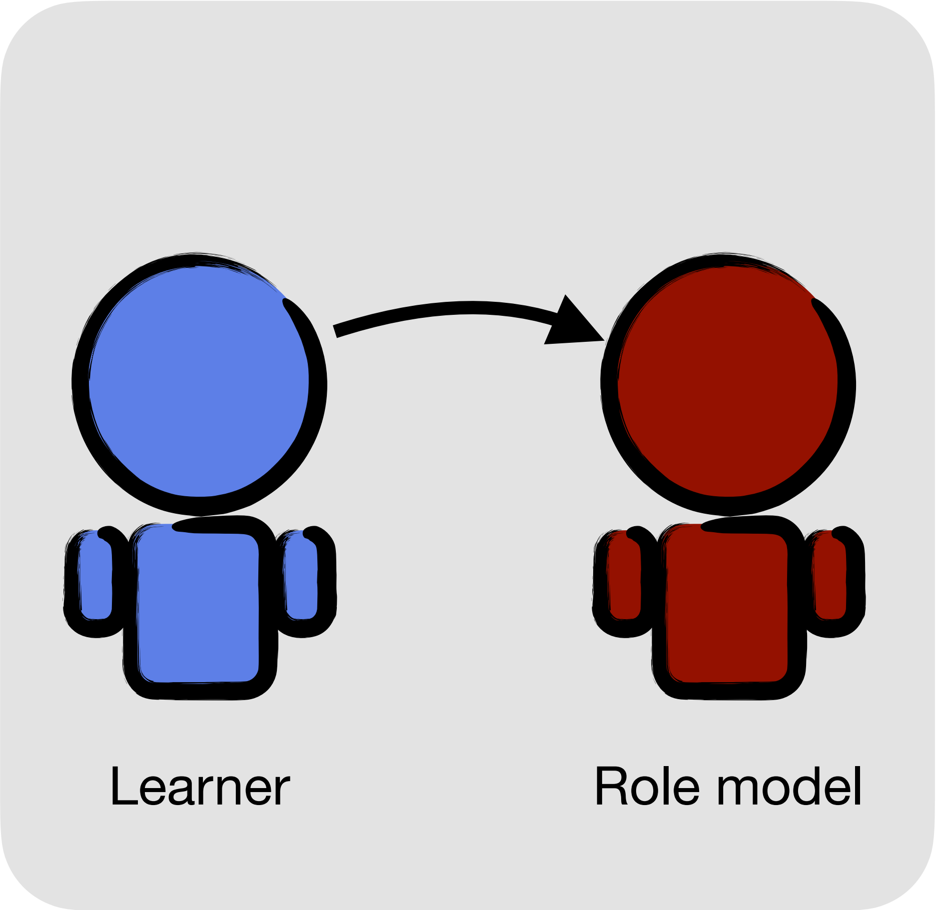
Pairwise comparison process



N: **population size**

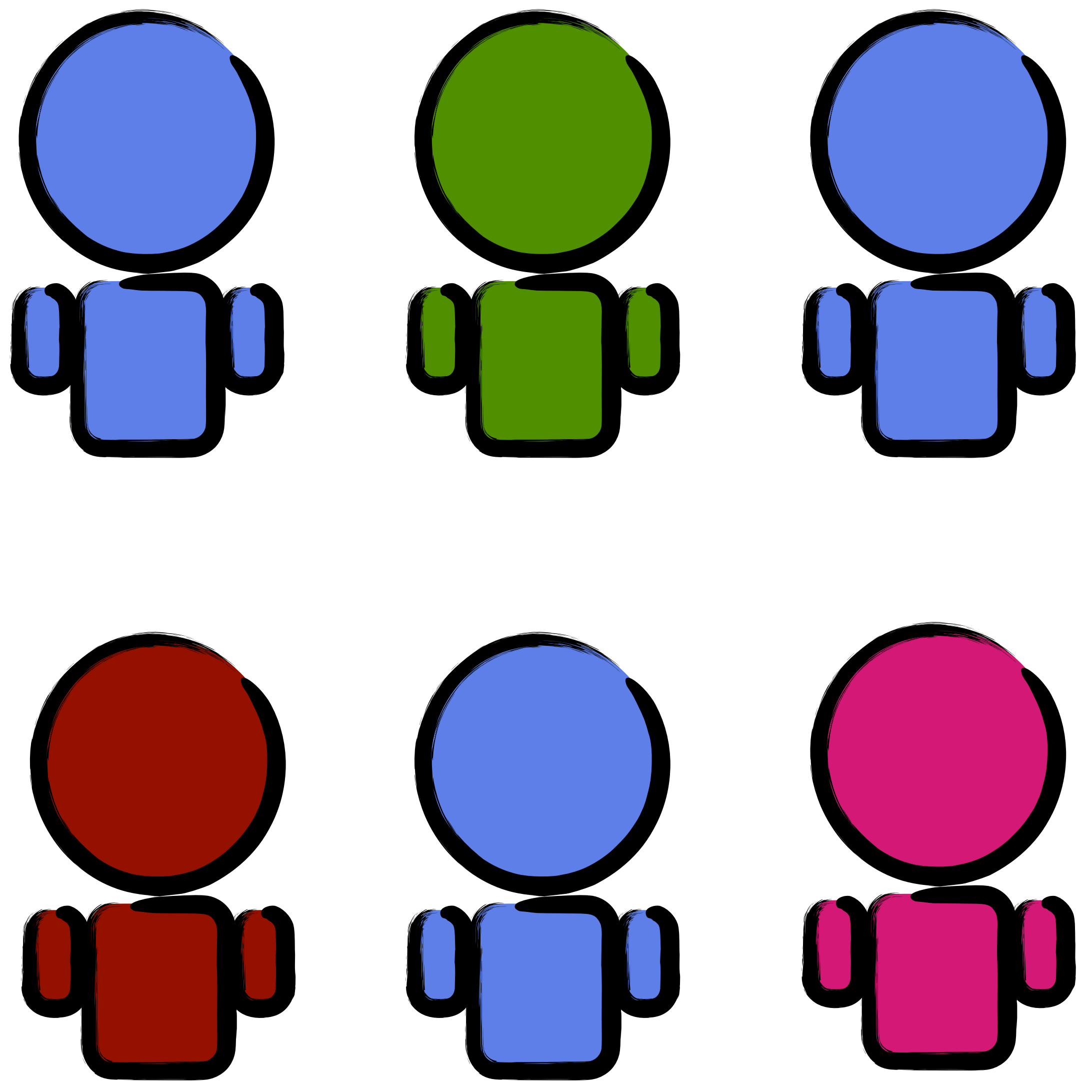


μ: **imitaton**

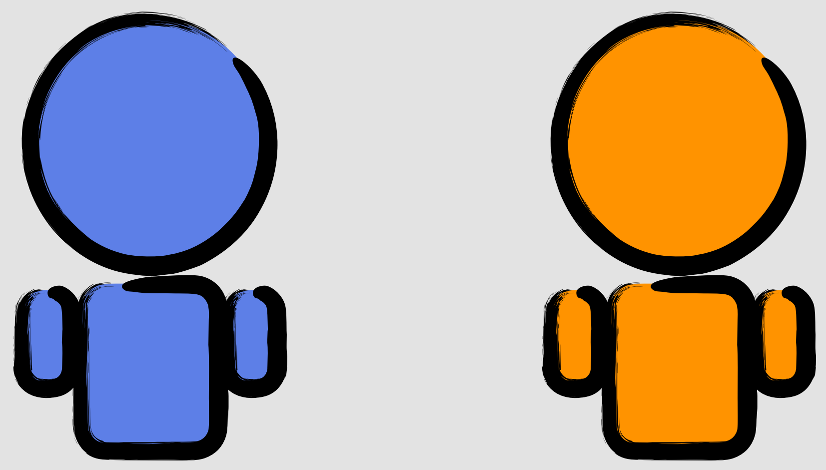


$1-\mu$: **imitaton**

Pairwise comparison process



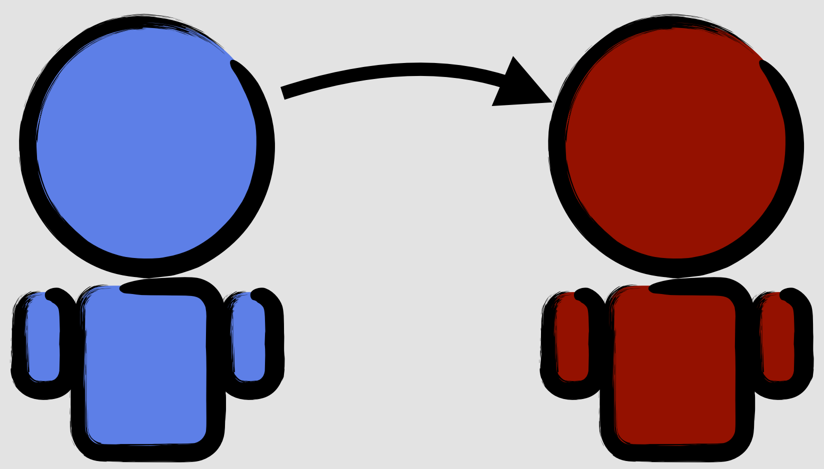
N: **population size**



μ: **imitaton**

$$\frac{1}{1+e^{-\beta(\pi_{red}-\pi_{blue})}}$$

β: **strength of selection**



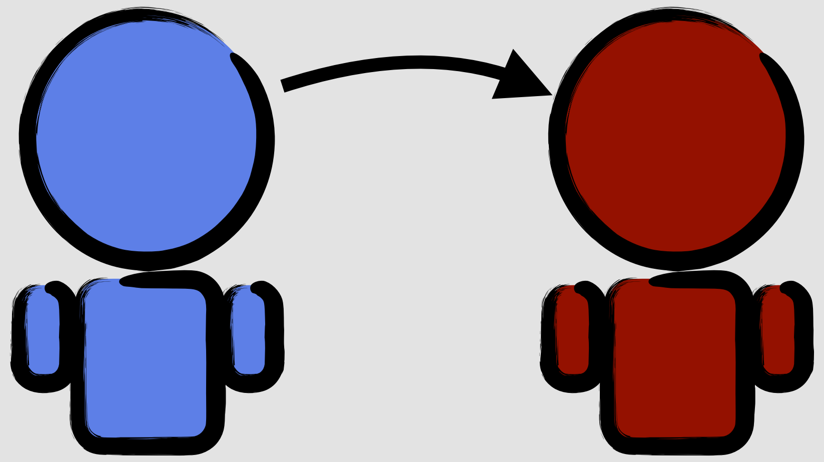
Learner

Role model

$1-\mu$: **imitaton**

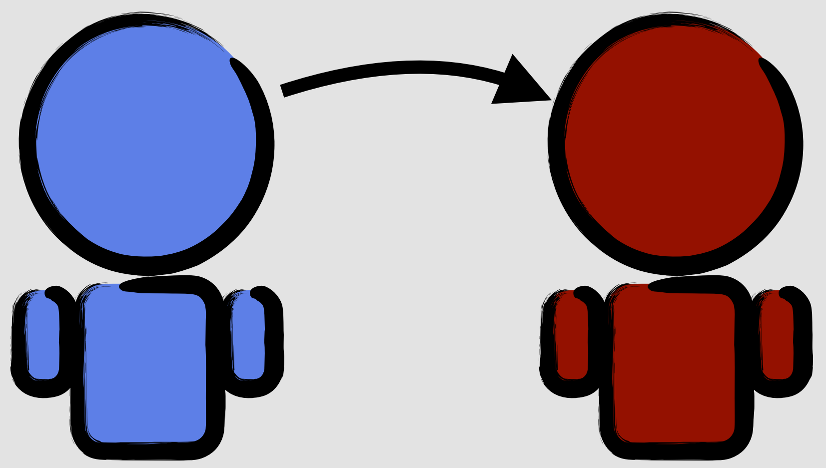
Pairwise comparison process

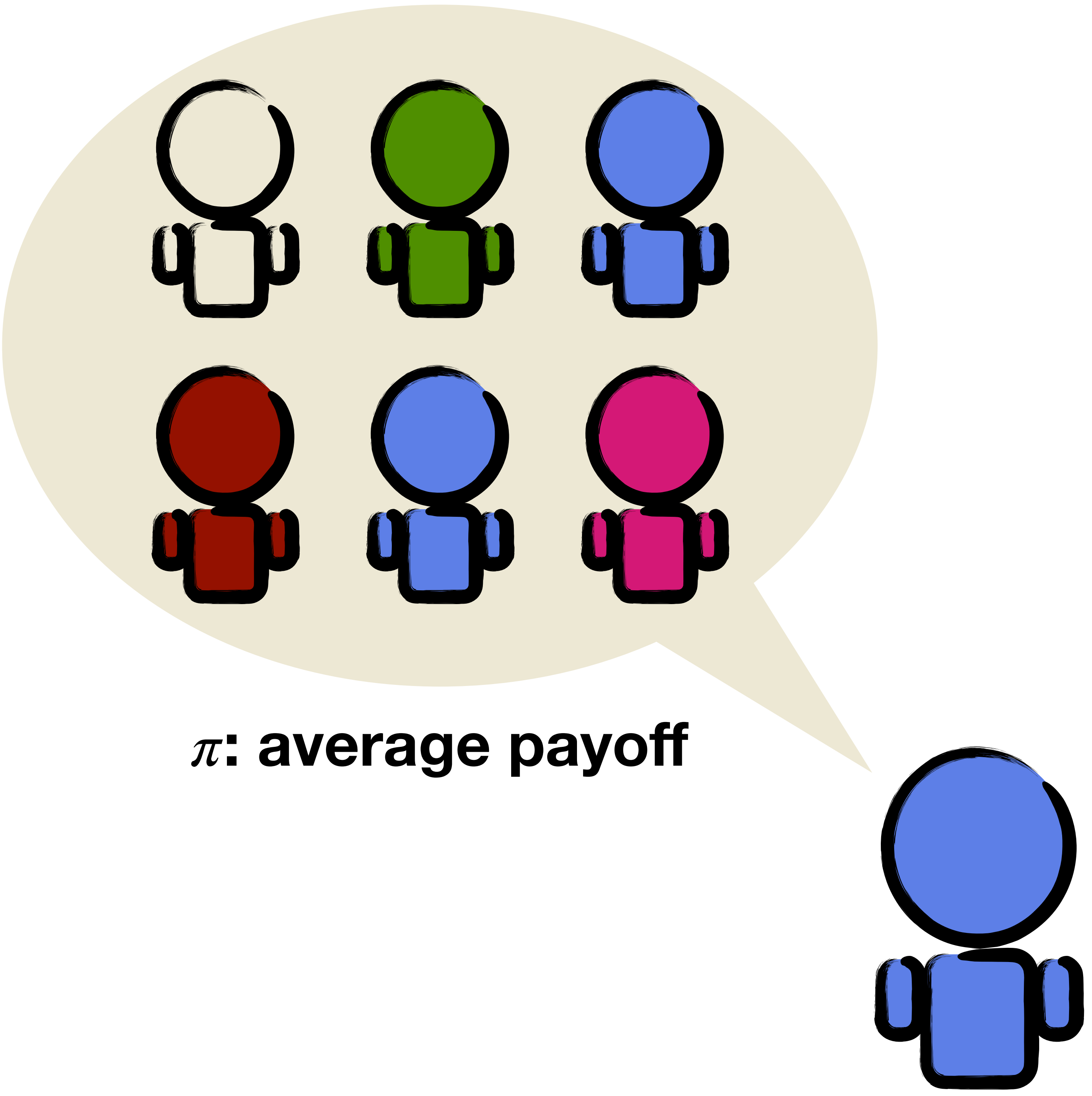
$$\frac{1}{1+e^{-\beta(\pi_{red}-\pi_{blue})}}$$



Pairwise comparison process

$$\frac{1}{1+e^{-\beta(\pi_{red}-\pi_{blue})}}$$

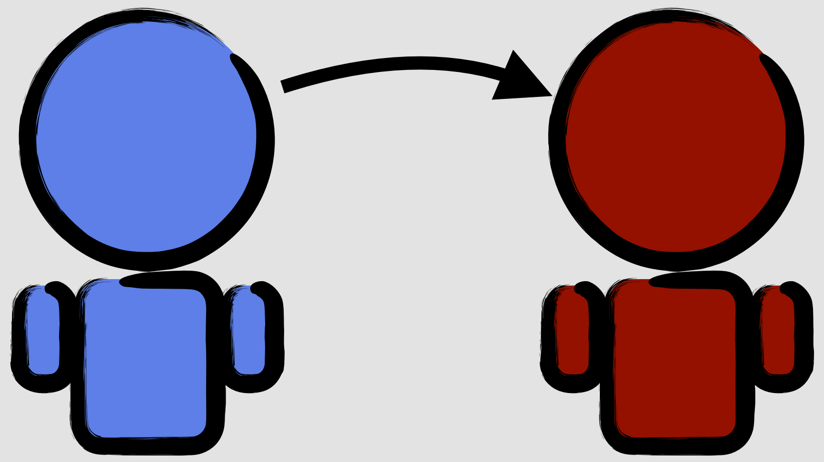


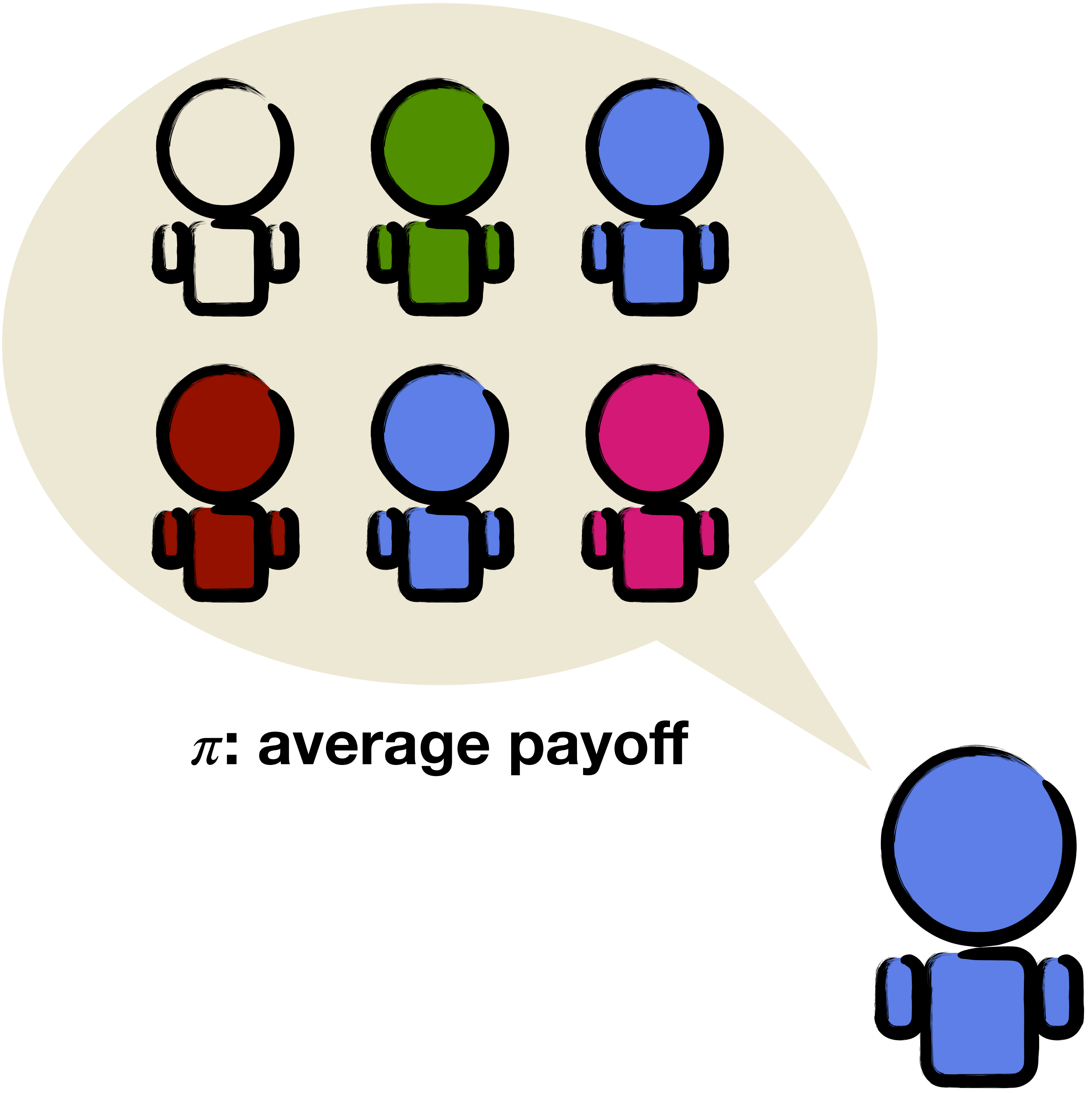


π: **average payoff**

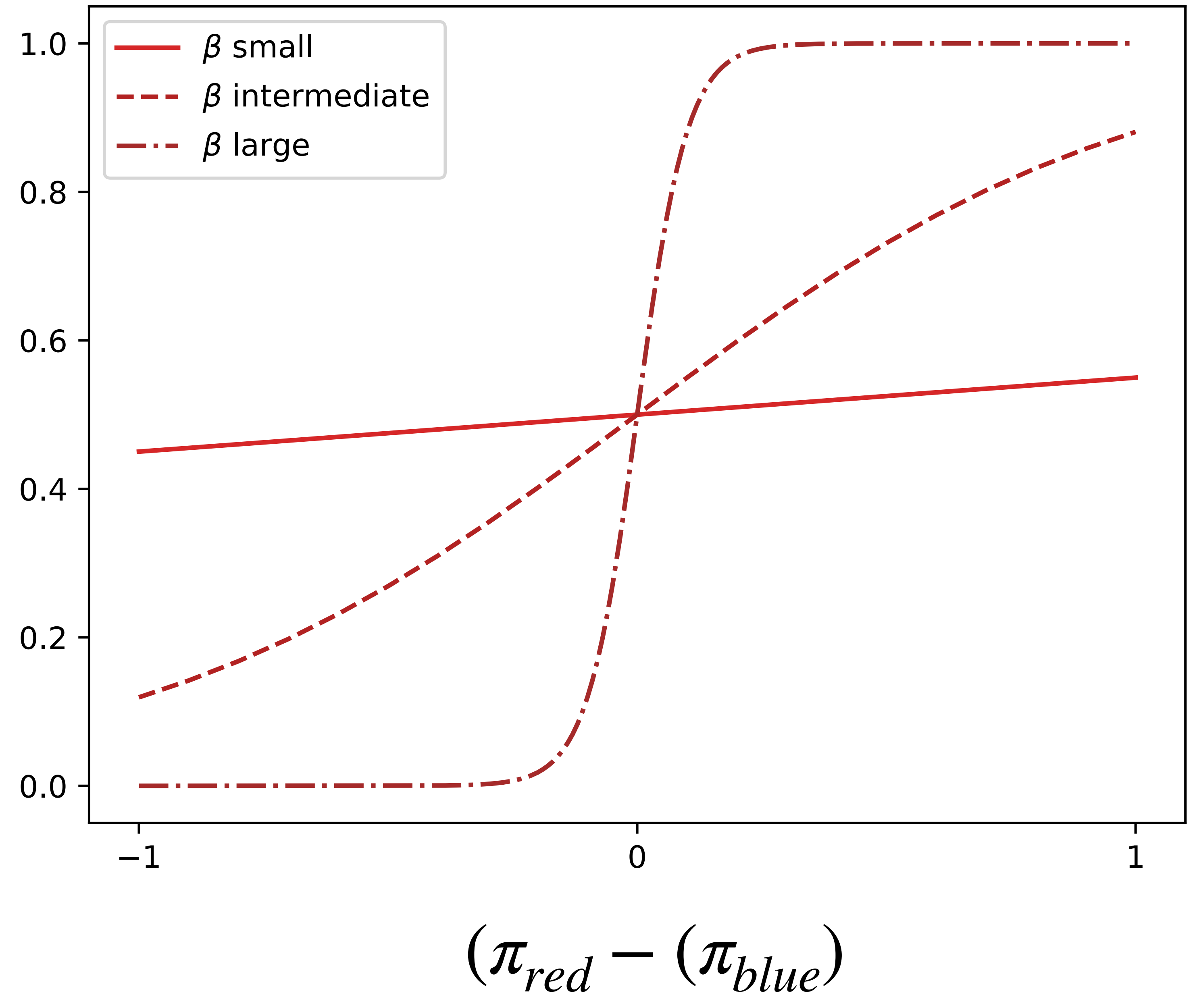
Pairwise comparison process

$$\frac{1}{1+e^{-\beta(\pi_{red}-\pi_{blue})}}$$



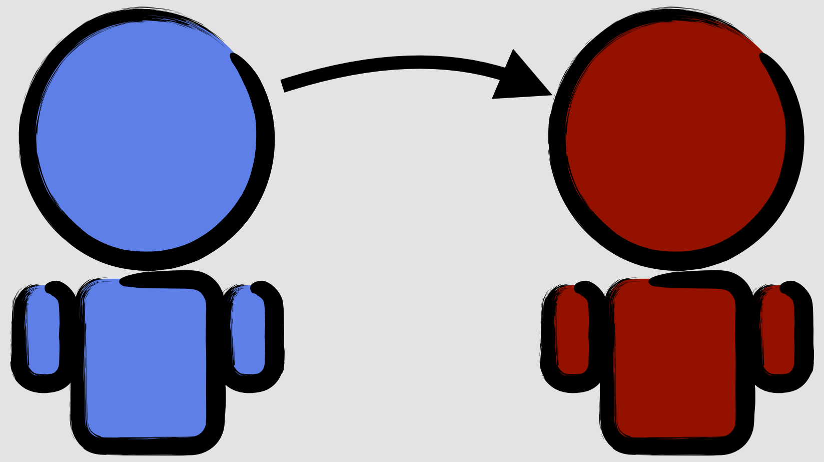


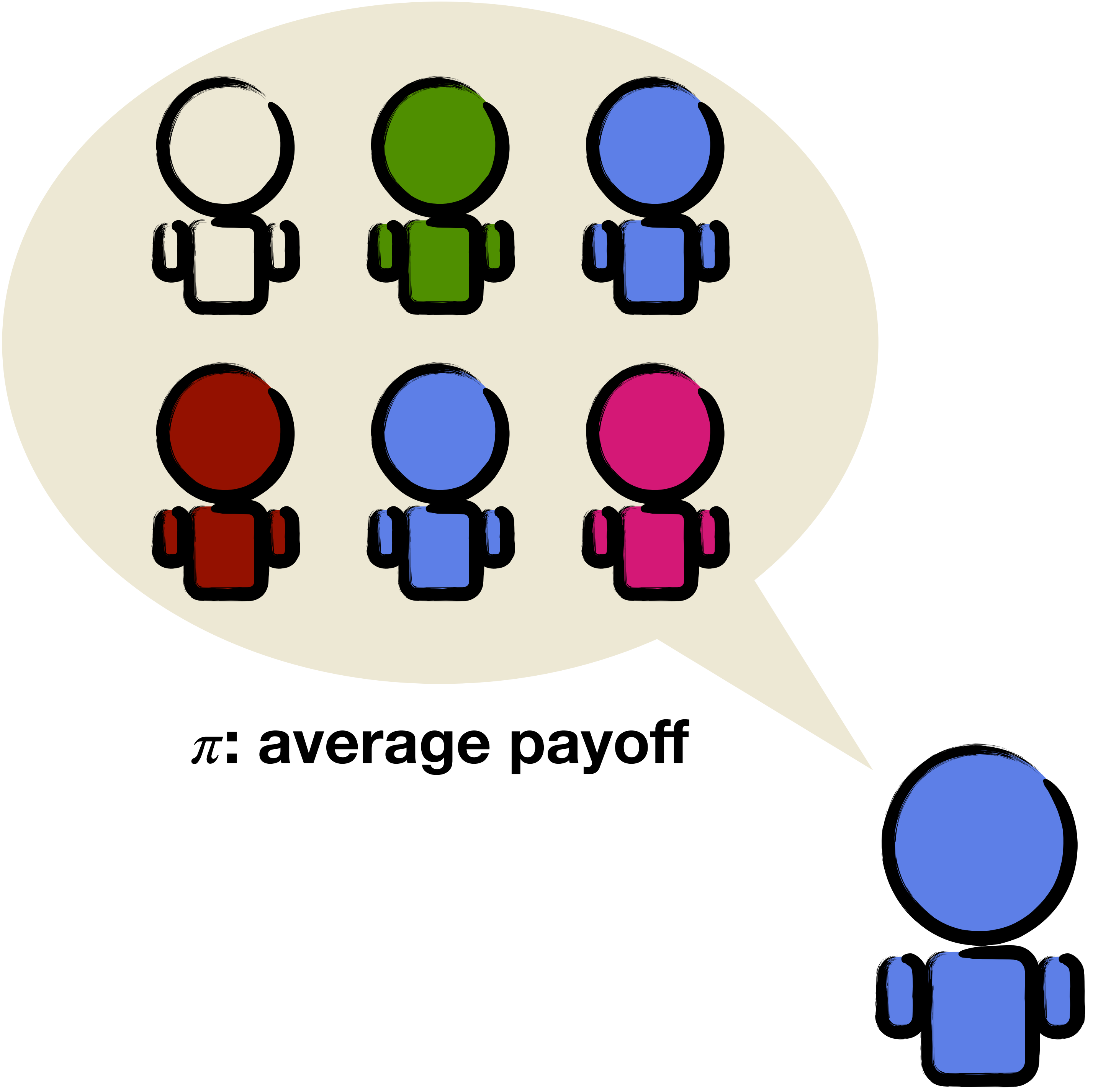
β: strength of selection



Pairwise comparison process

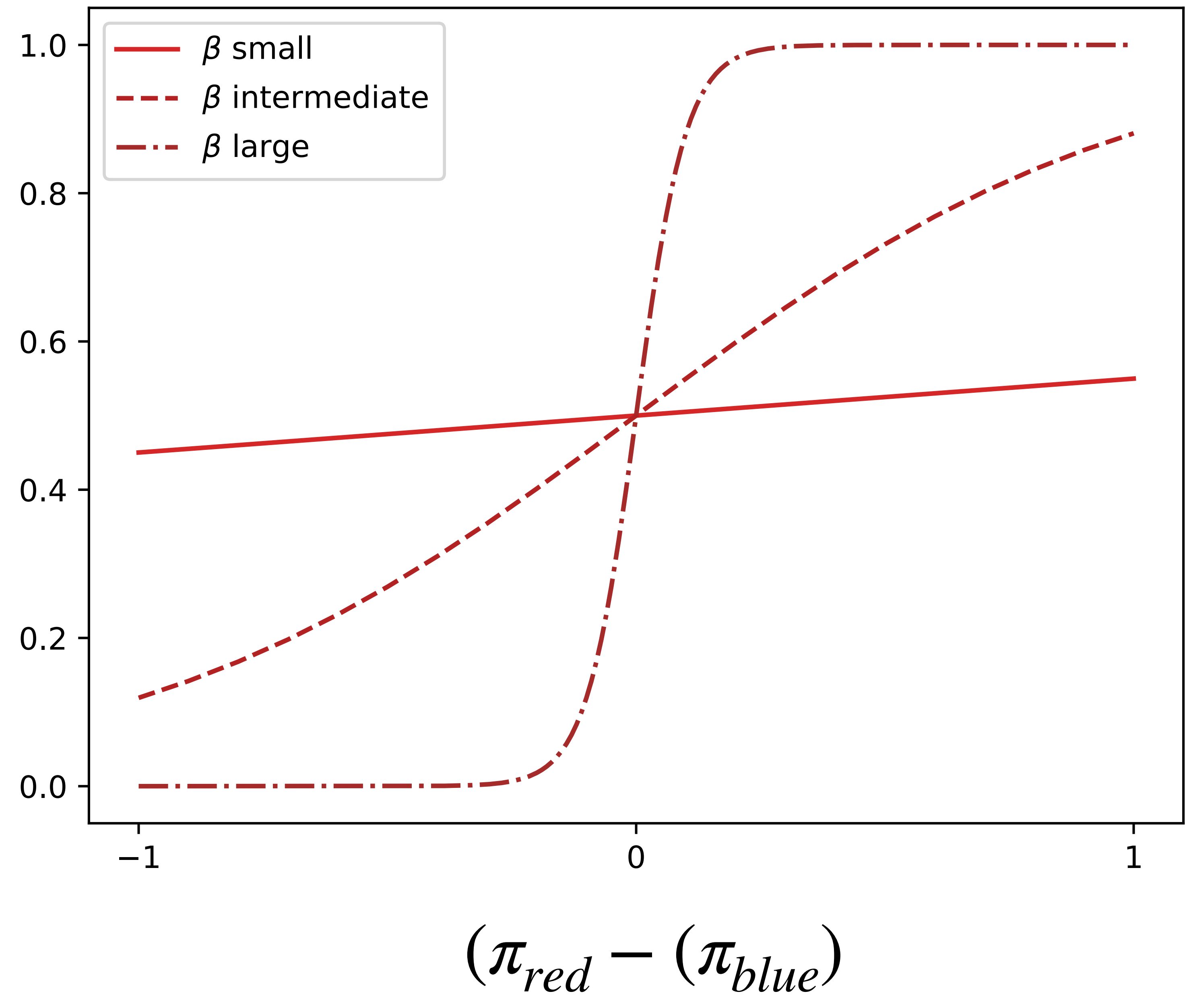
$$\frac{1}{1+e^{-\beta(\pi_{red}-\pi_{blue})}}$$





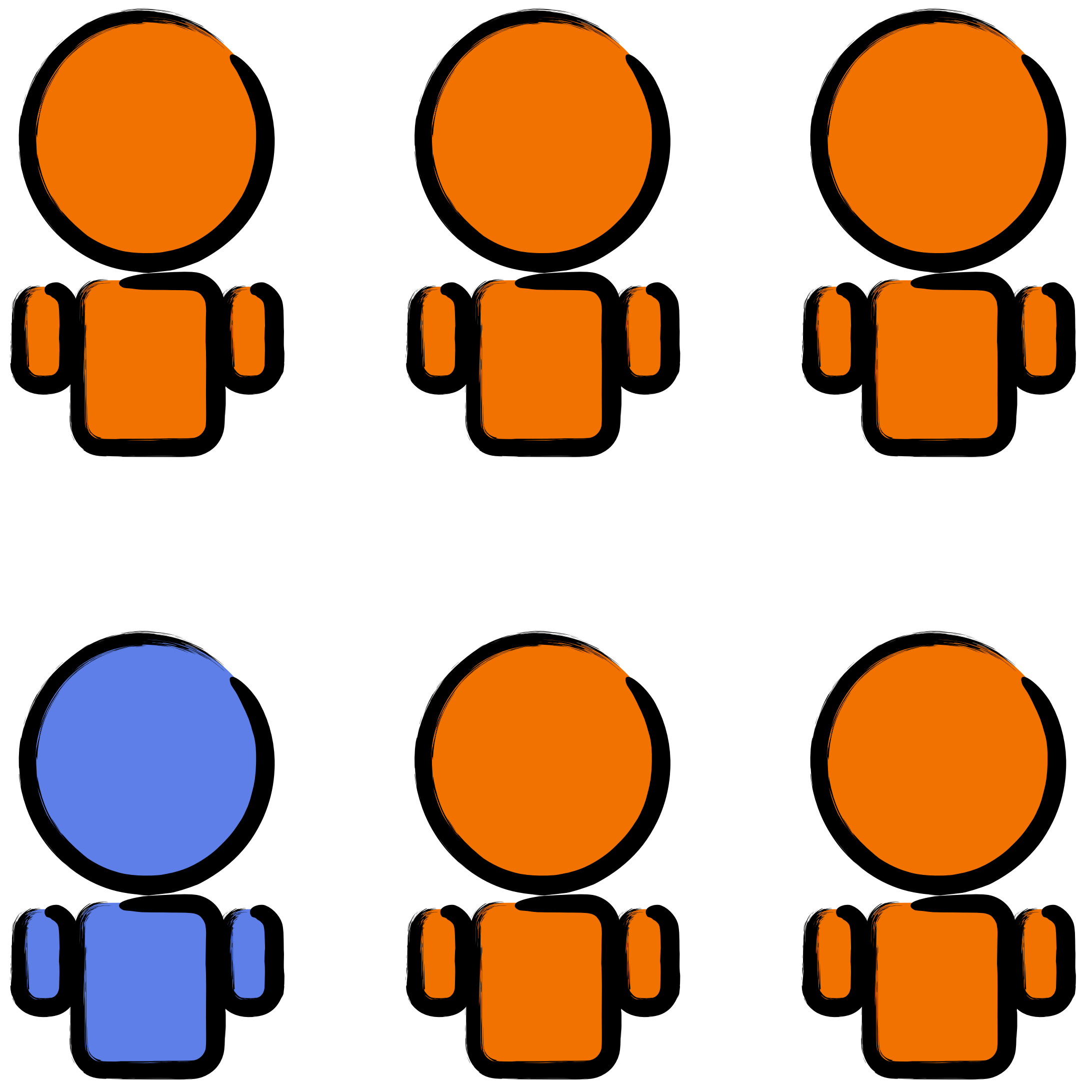
π: **average payoff**

β: strength of selection



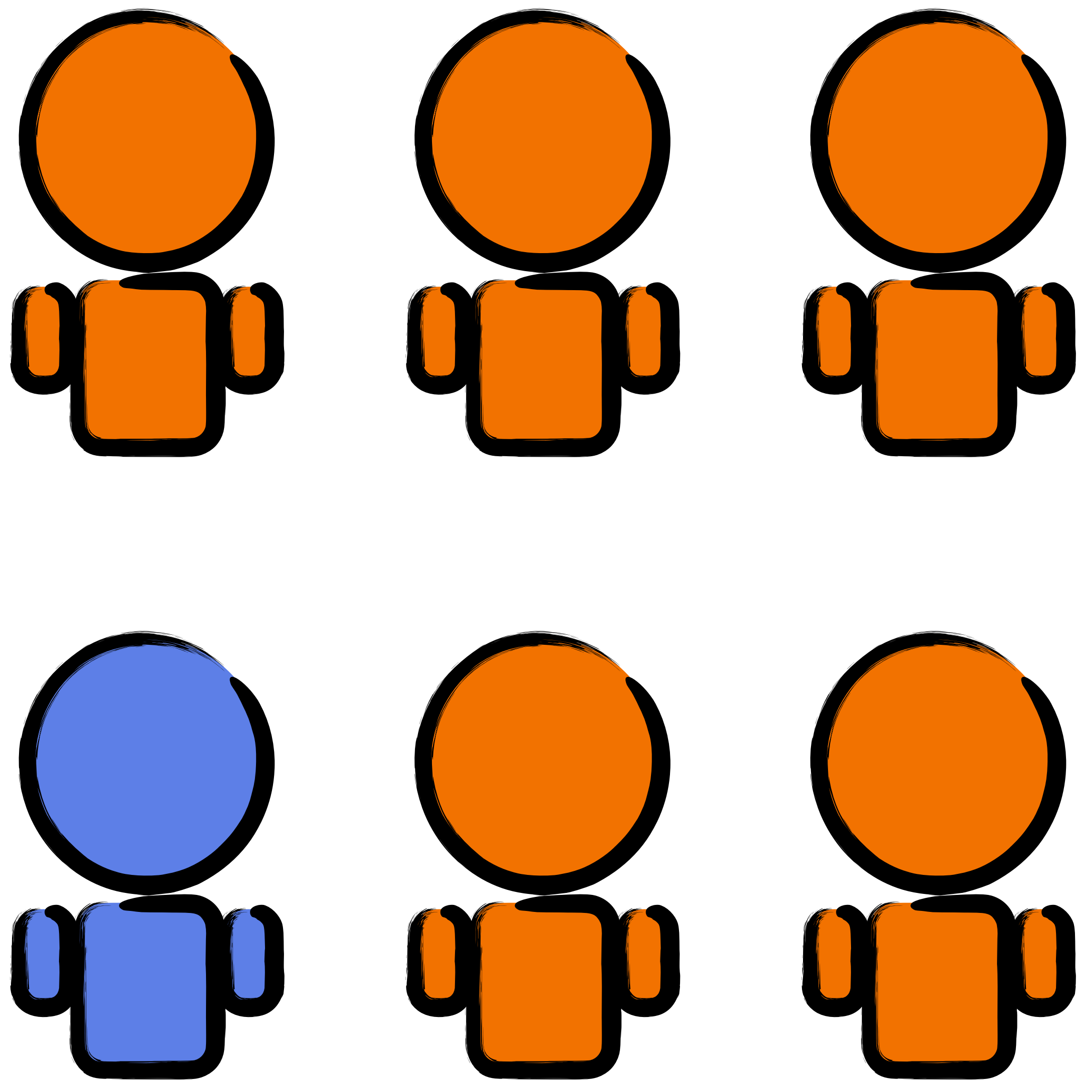
Pairwise comparison process

Limit of weak selection



Pairwise comparison process

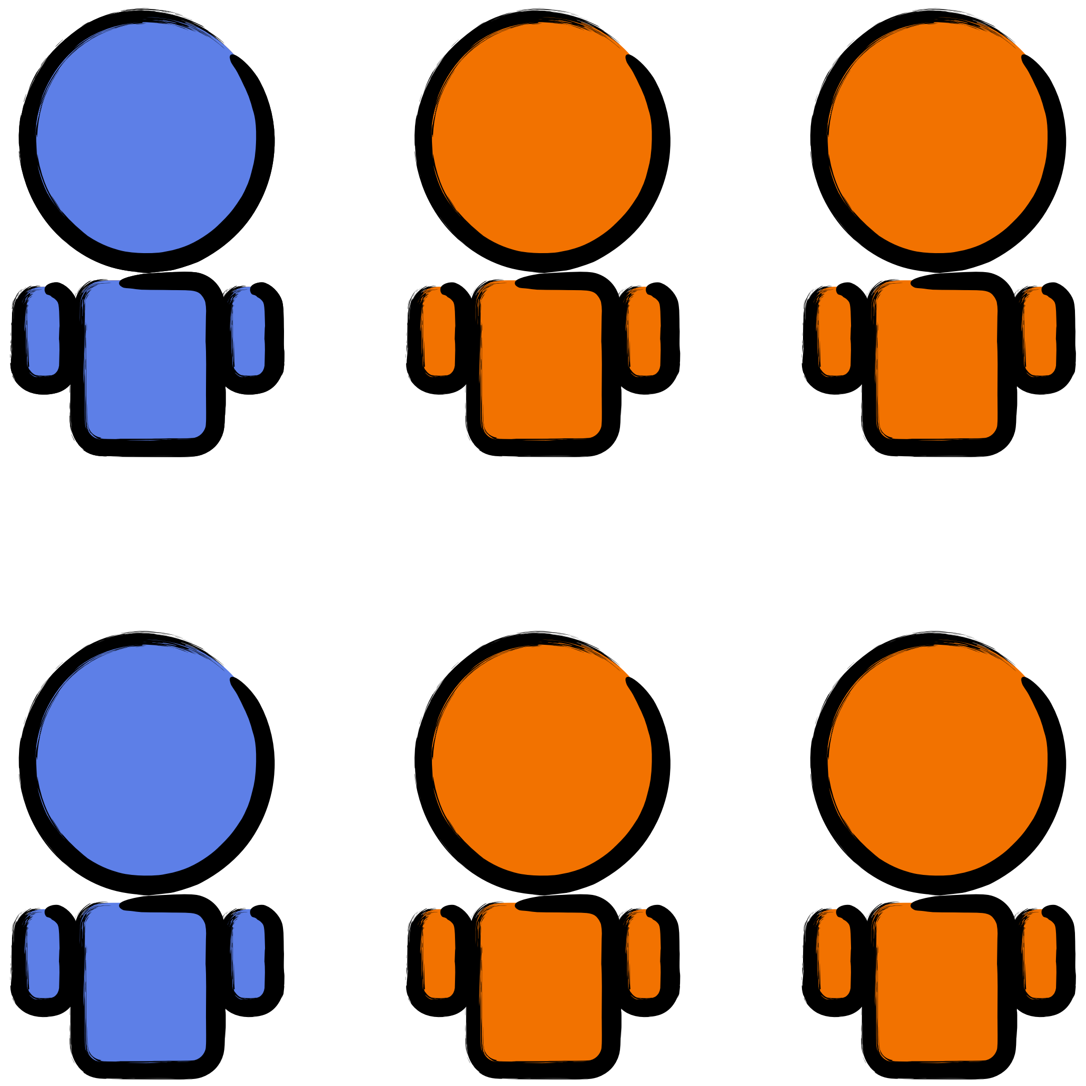
Limit of weak selection



What's the probability that Blue fixes in the population?

Pairwise comparison process

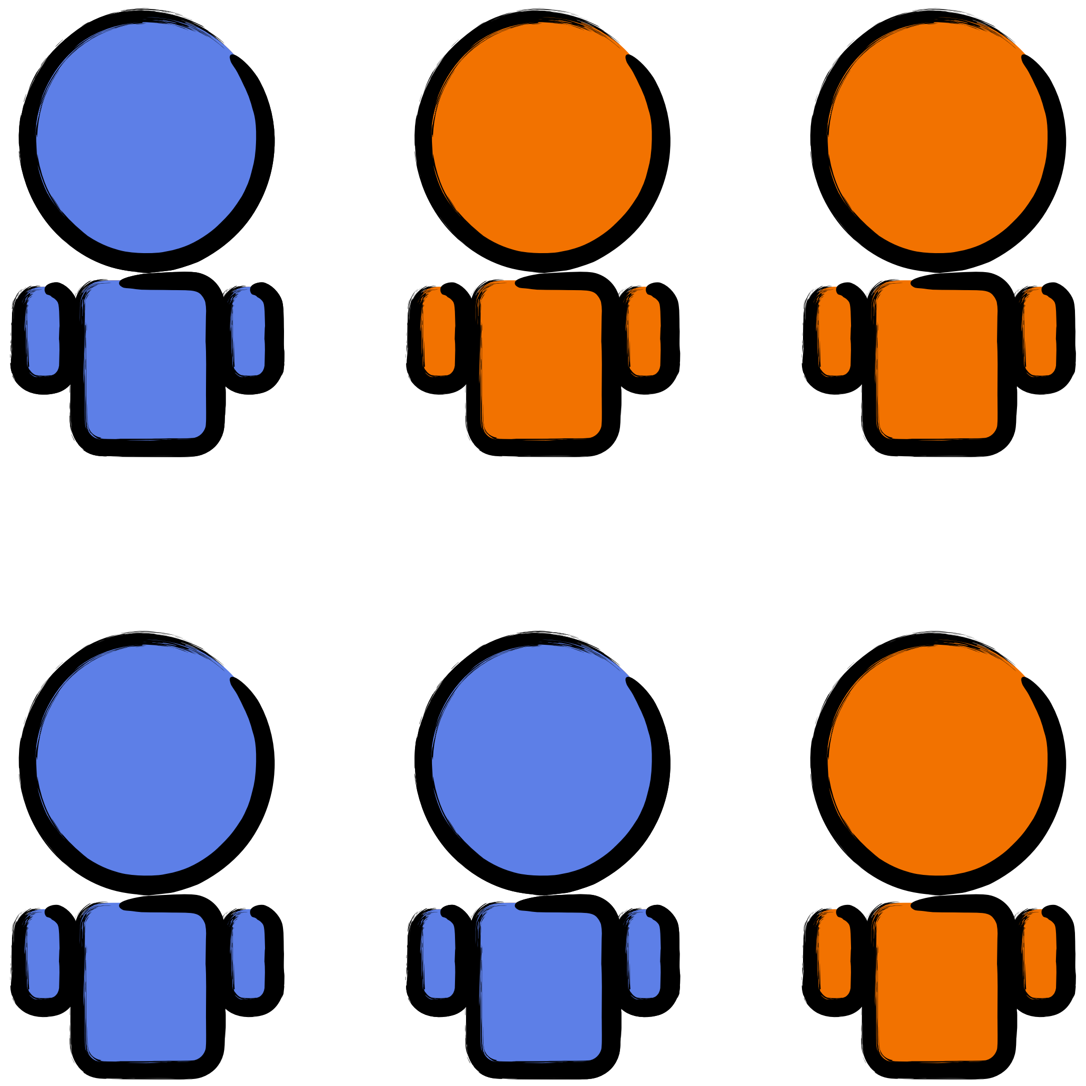
Limit of weak selection



What's the probability that Blue fixes in the population?

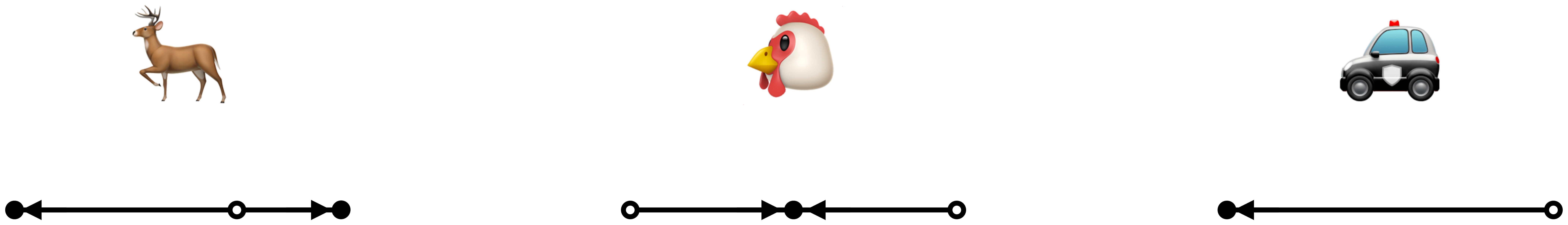
Pairwise comparison process

Limit of weak selection

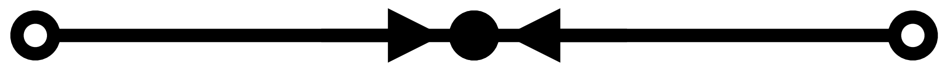


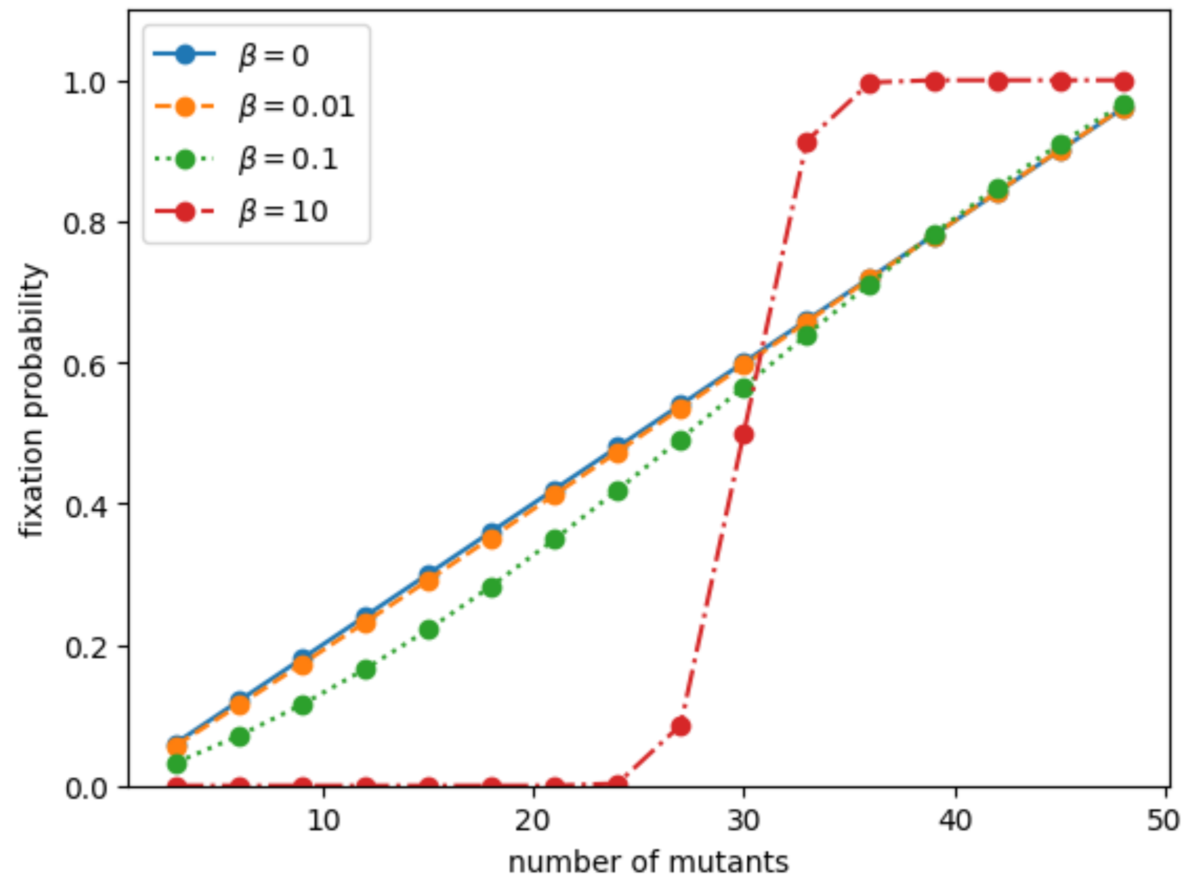
What's the probability that Blue fixes in the population?

Pairwise comparison process

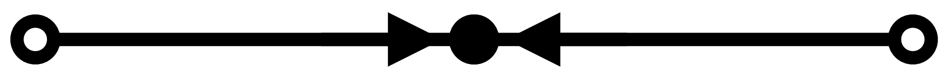


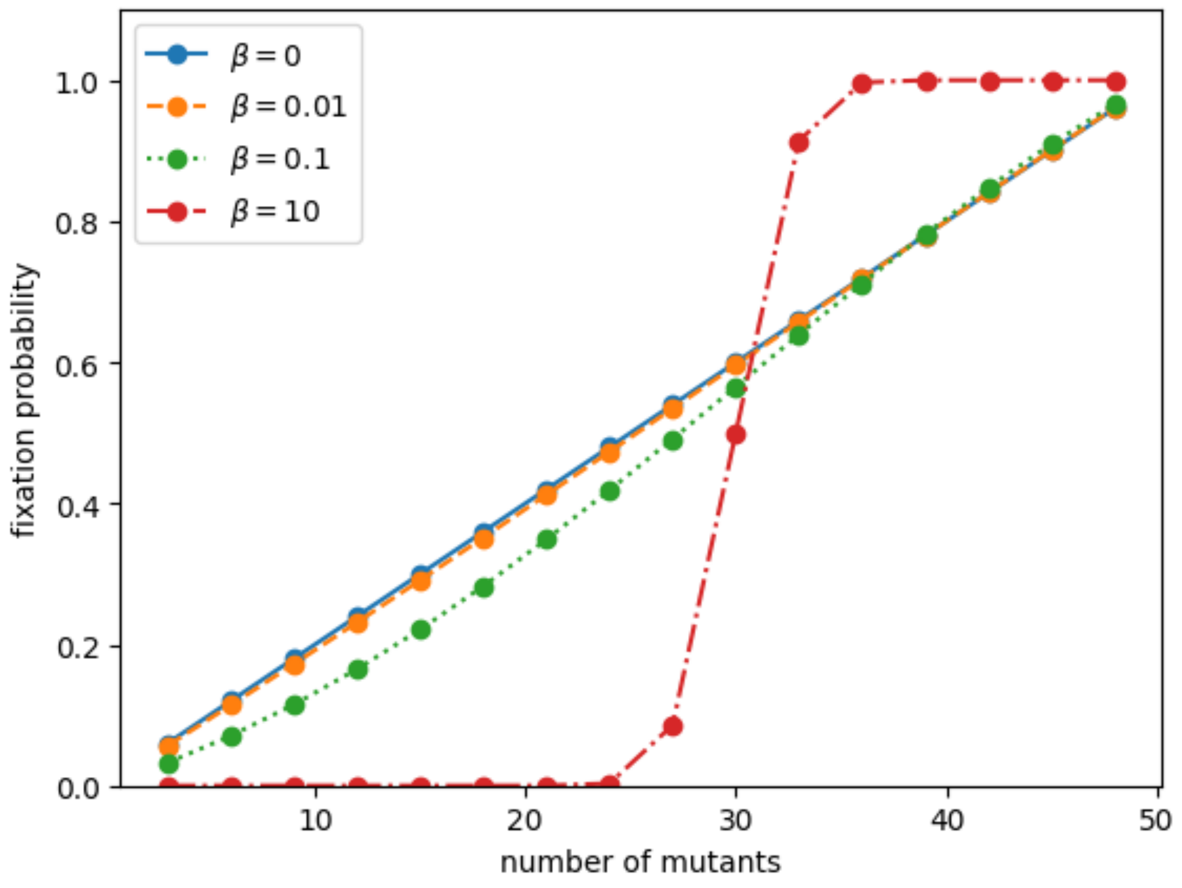
Pairwise comparison process

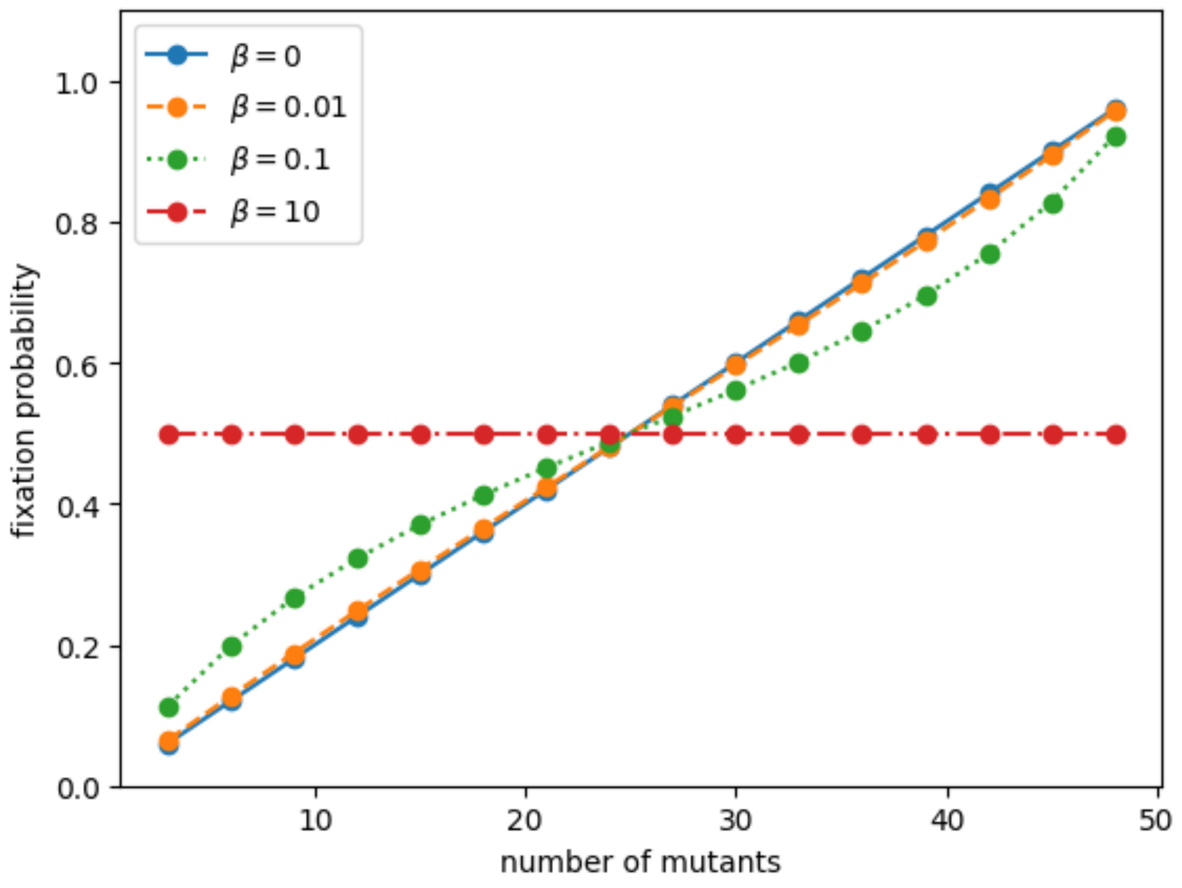




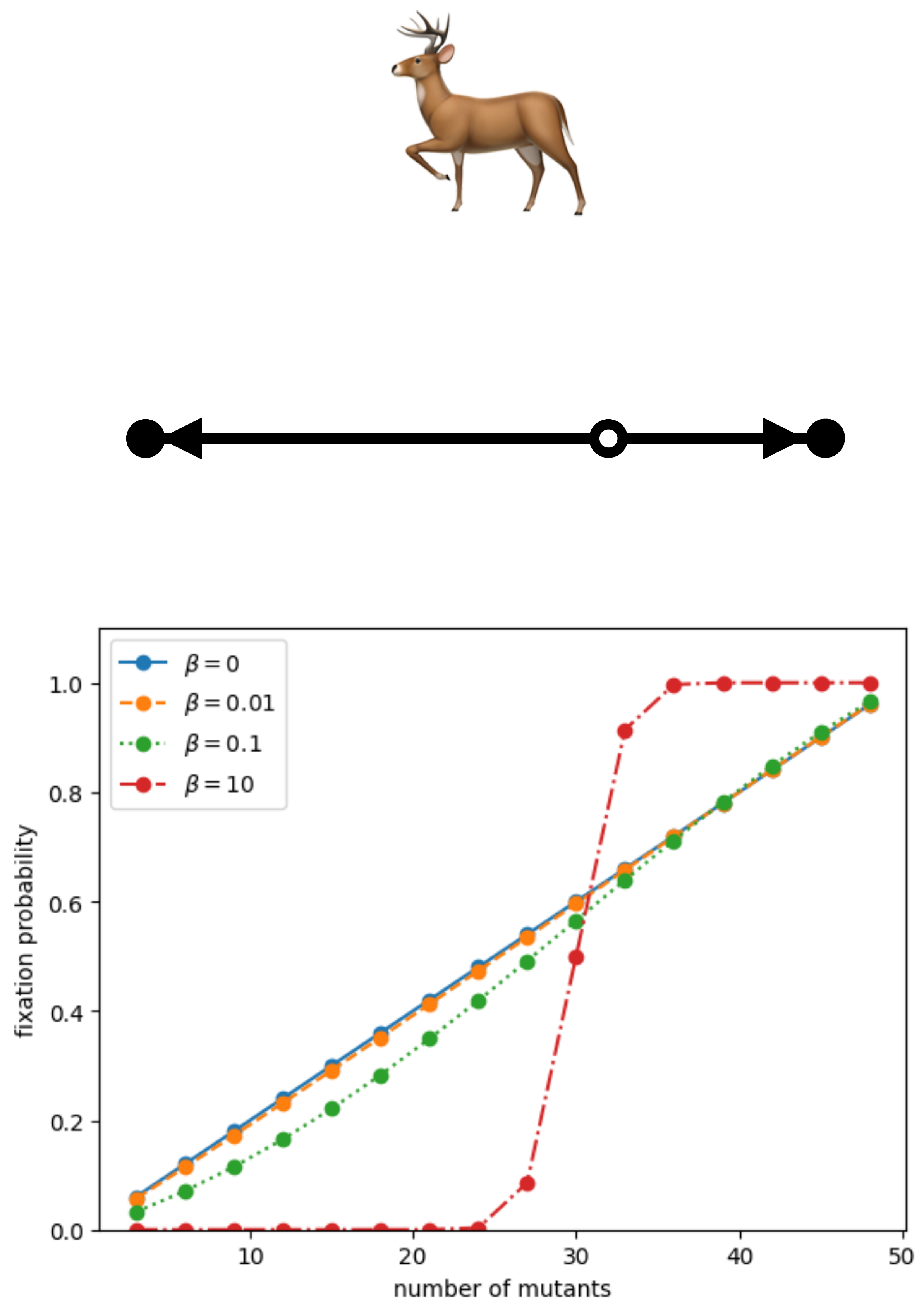
Pairwise comparison process

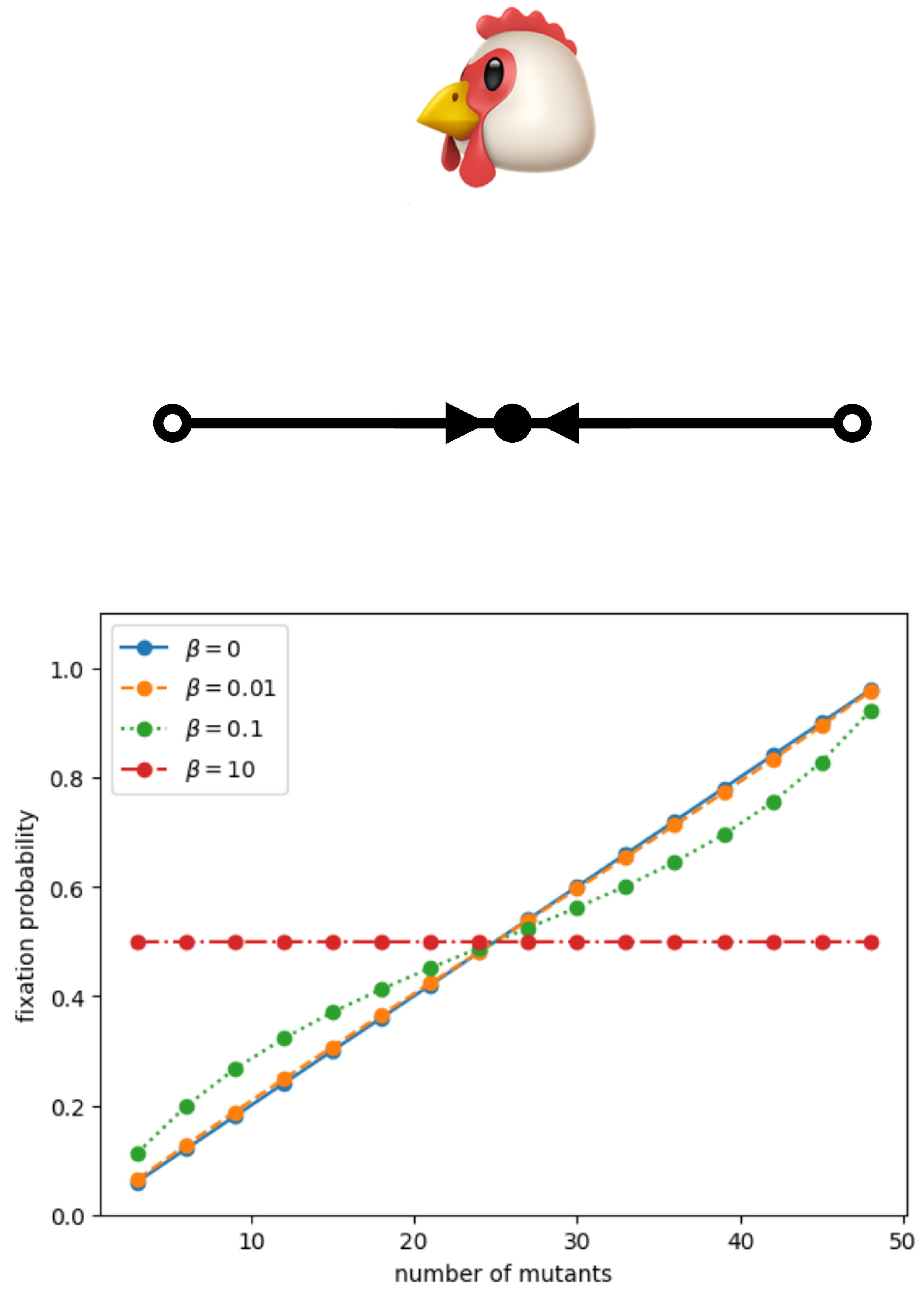


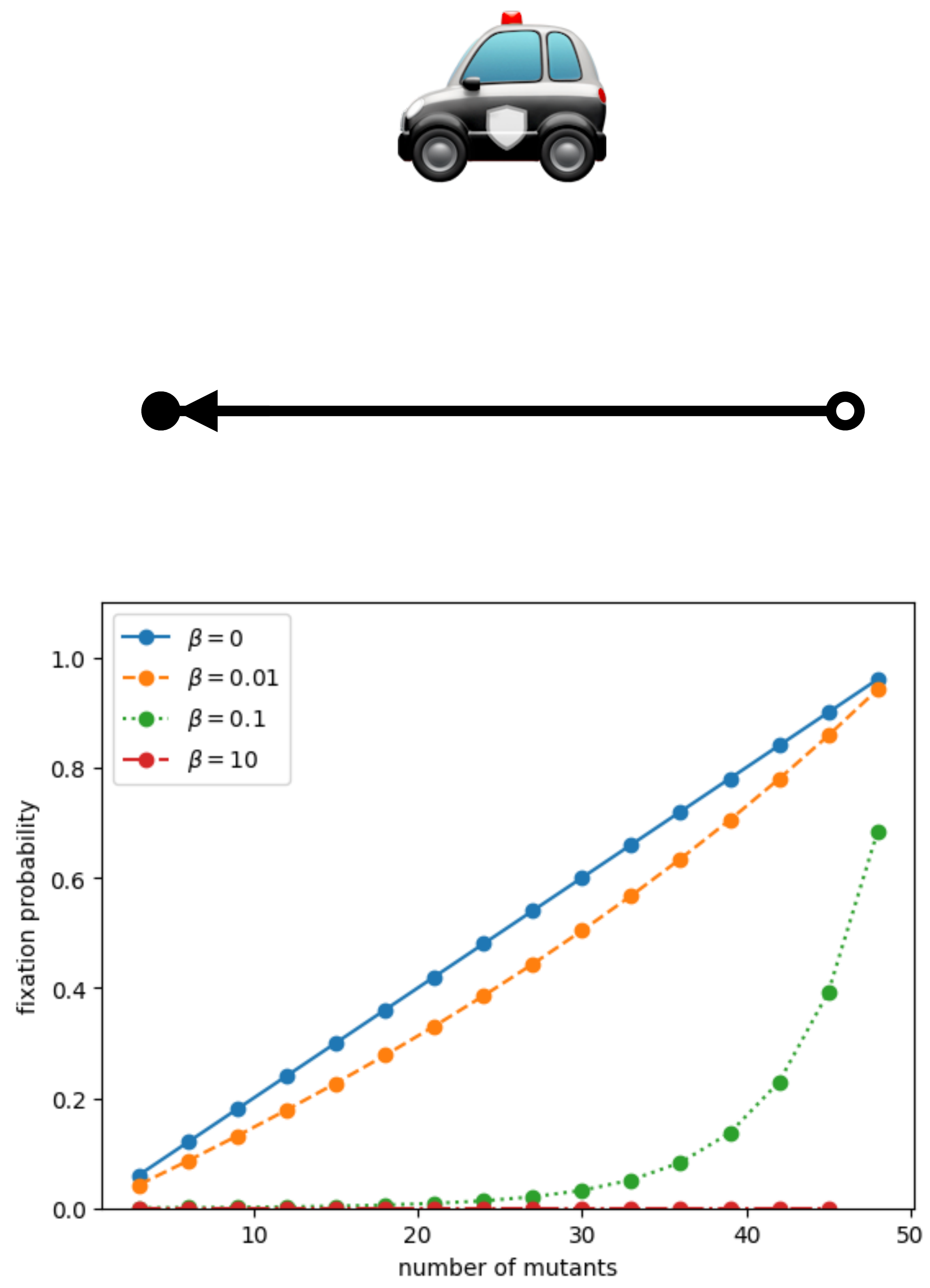




Pairwise comparison process





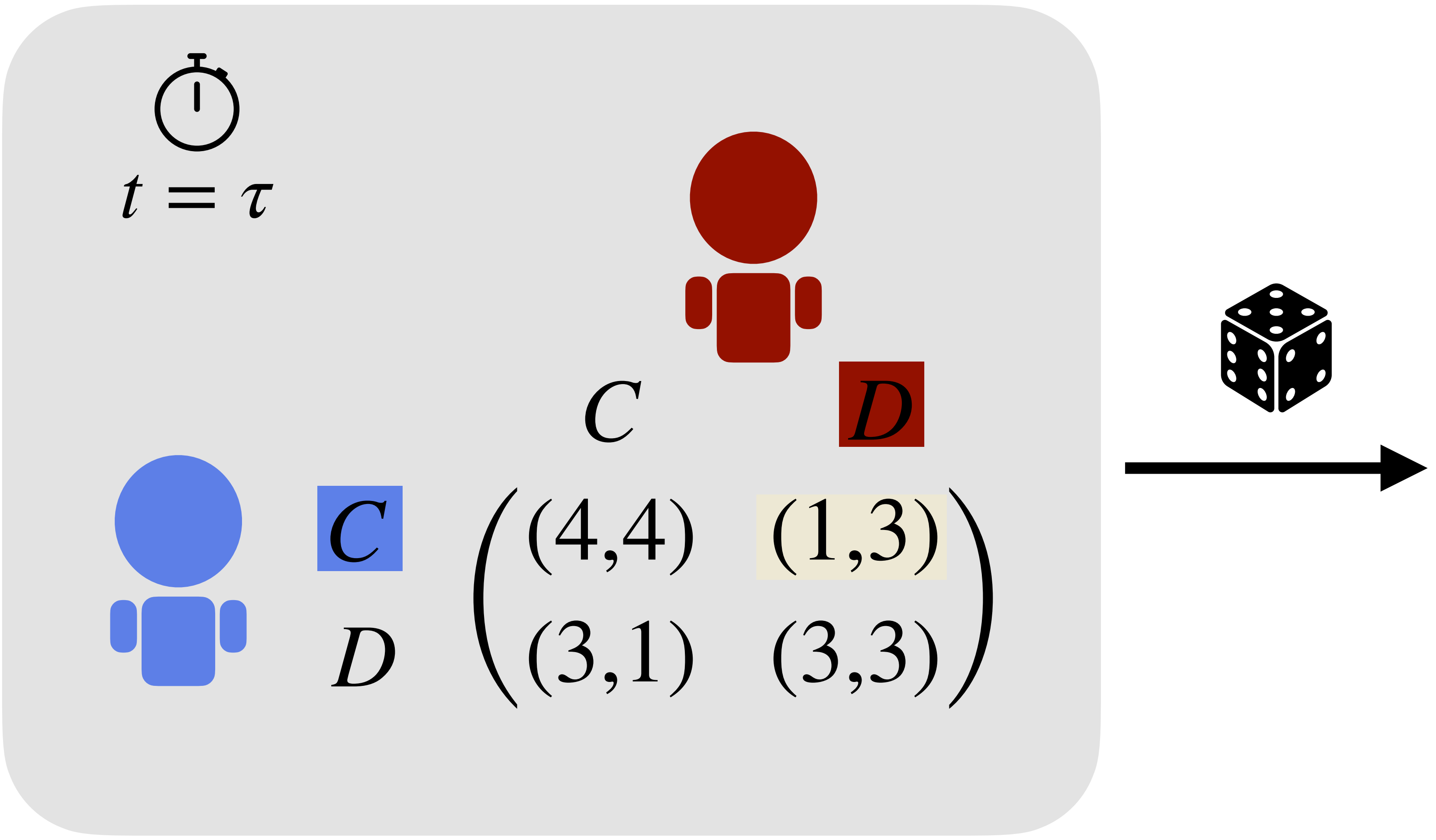


Introspection dynamics

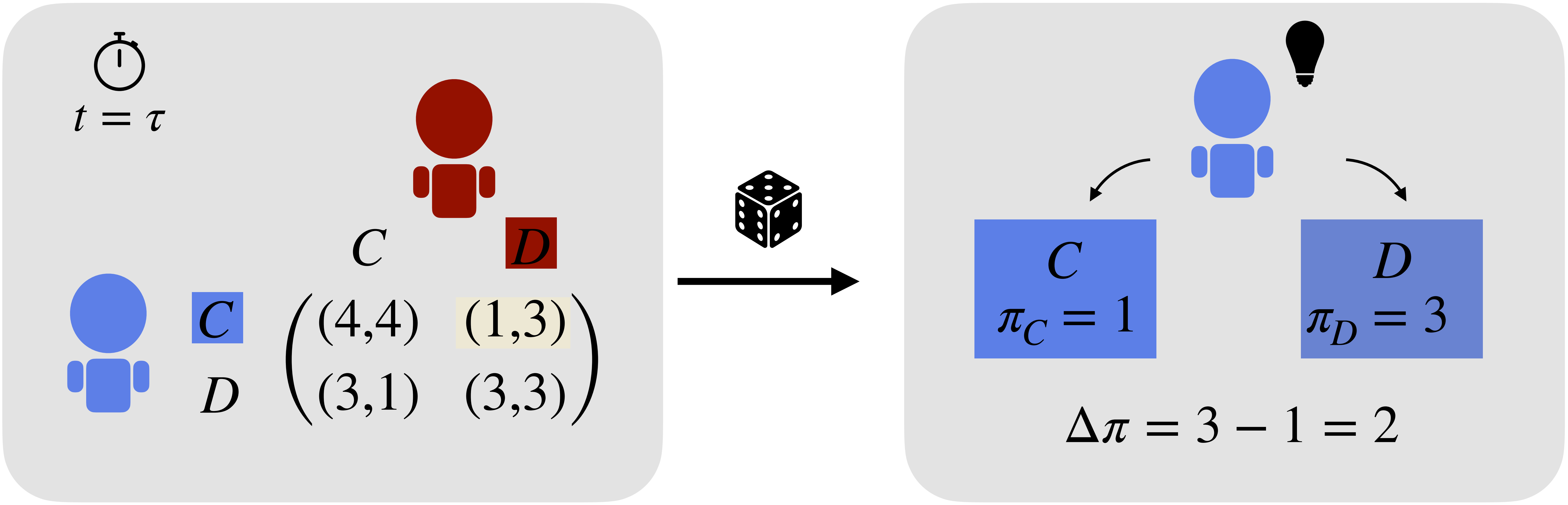
$t = \tau$

$$\begin{array}{ccc} & C & \boxed{D} \\ \boxed{C} & \multicolumn{2}{c}{\multirow{2}{*}{$\begin{pmatrix} (4,4) & (1,3) \\ (3,1) & (3,3) \end{pmatrix}$}} \\ D & & \end{array}$$

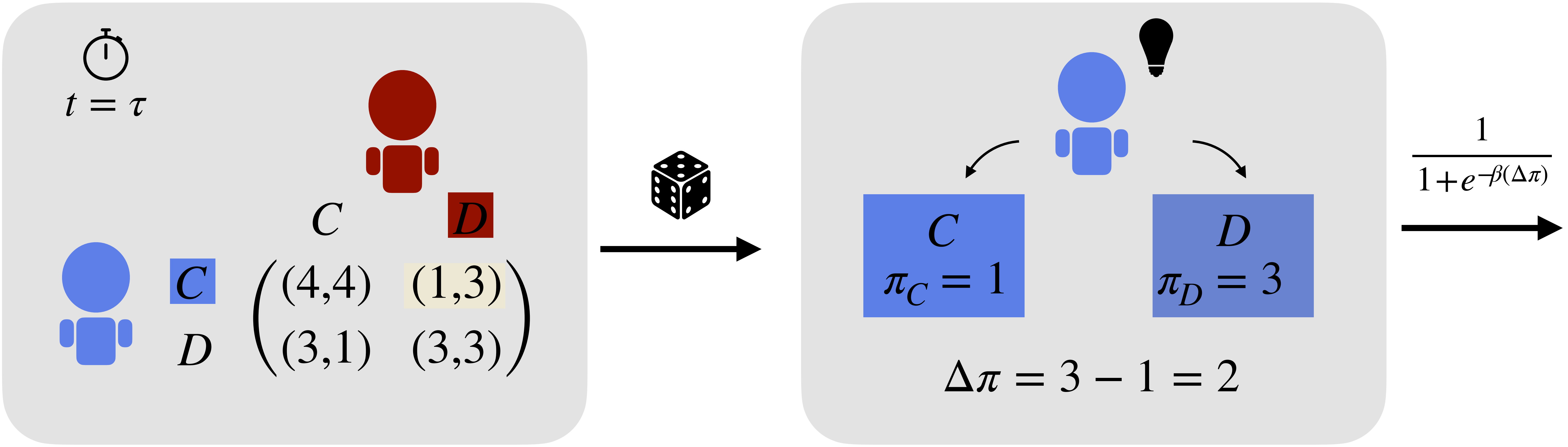
Introspection dynamics



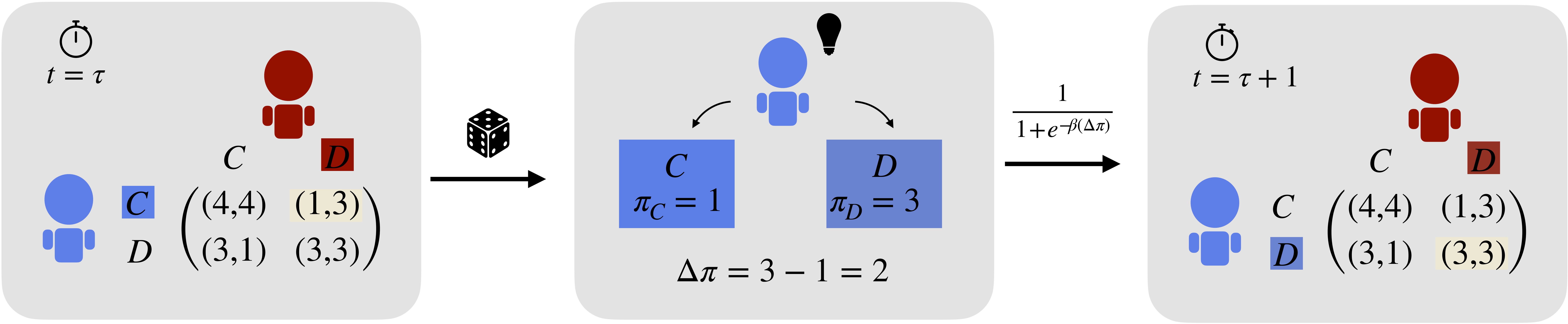
Introspection dynamics



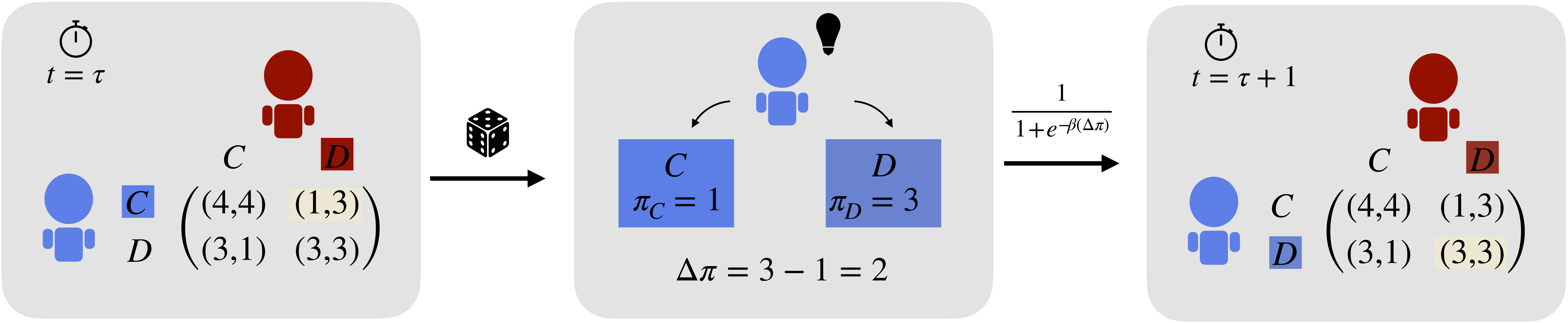
Introspection dynamics



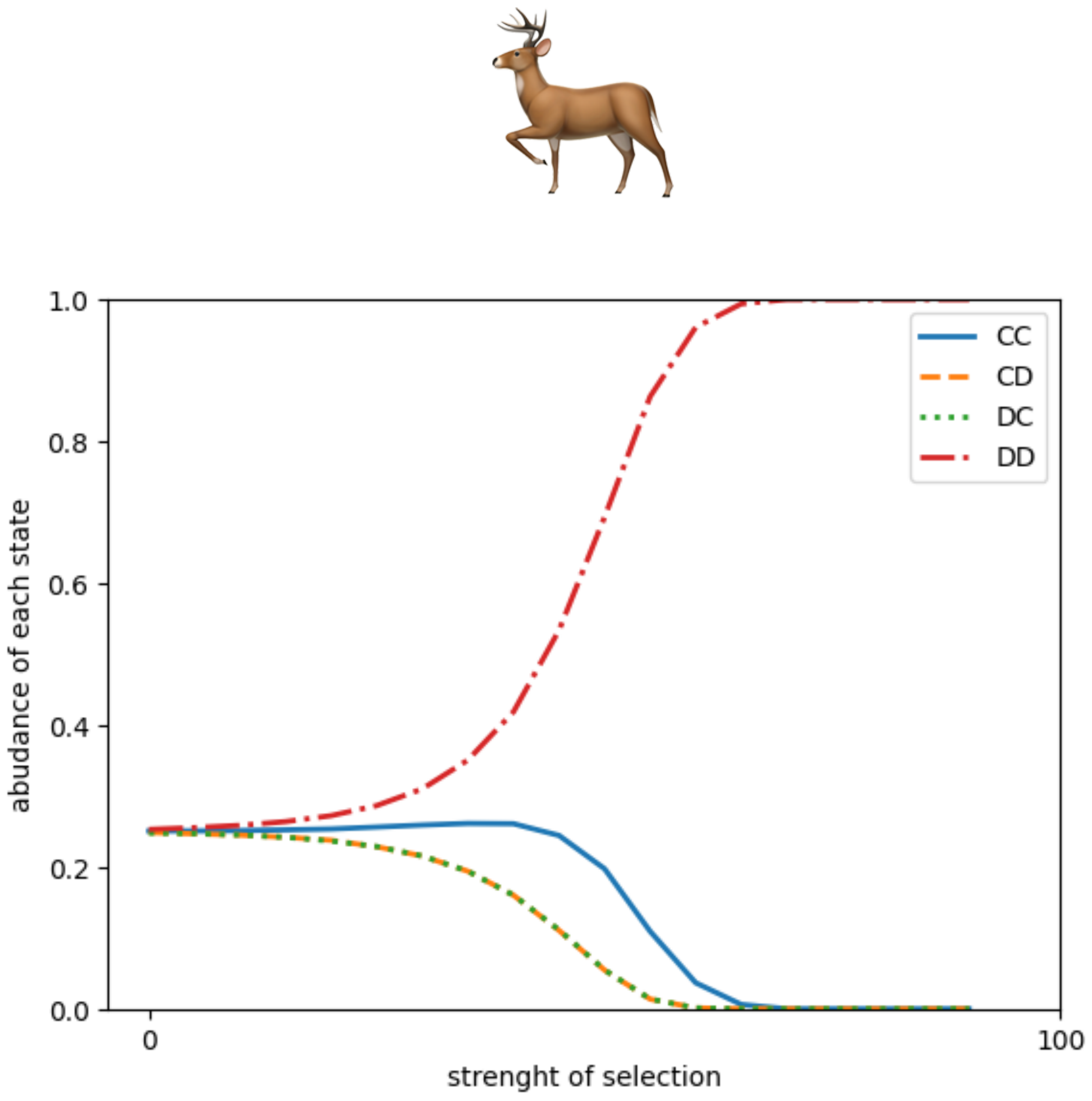
Introspection dynamics

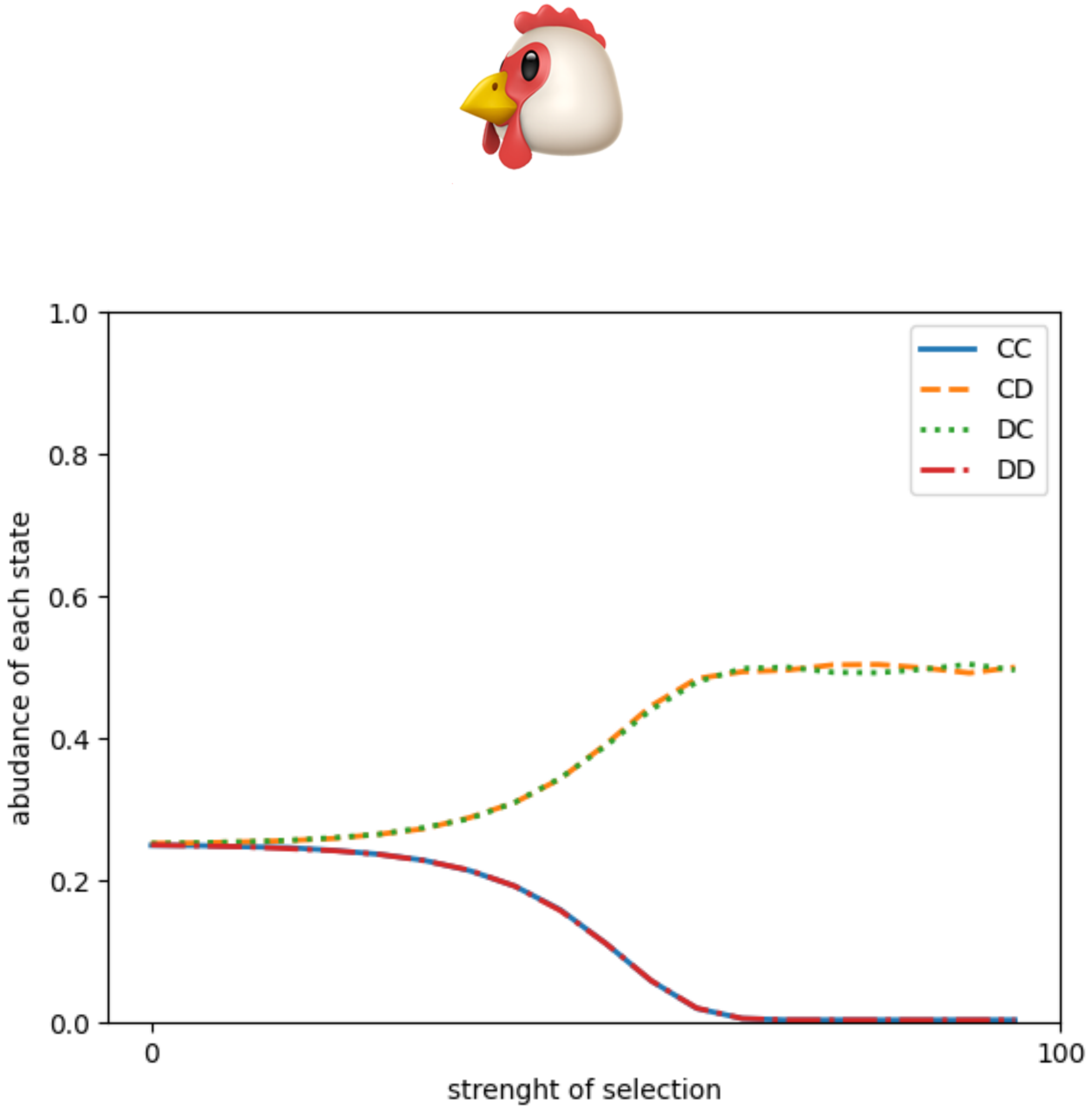


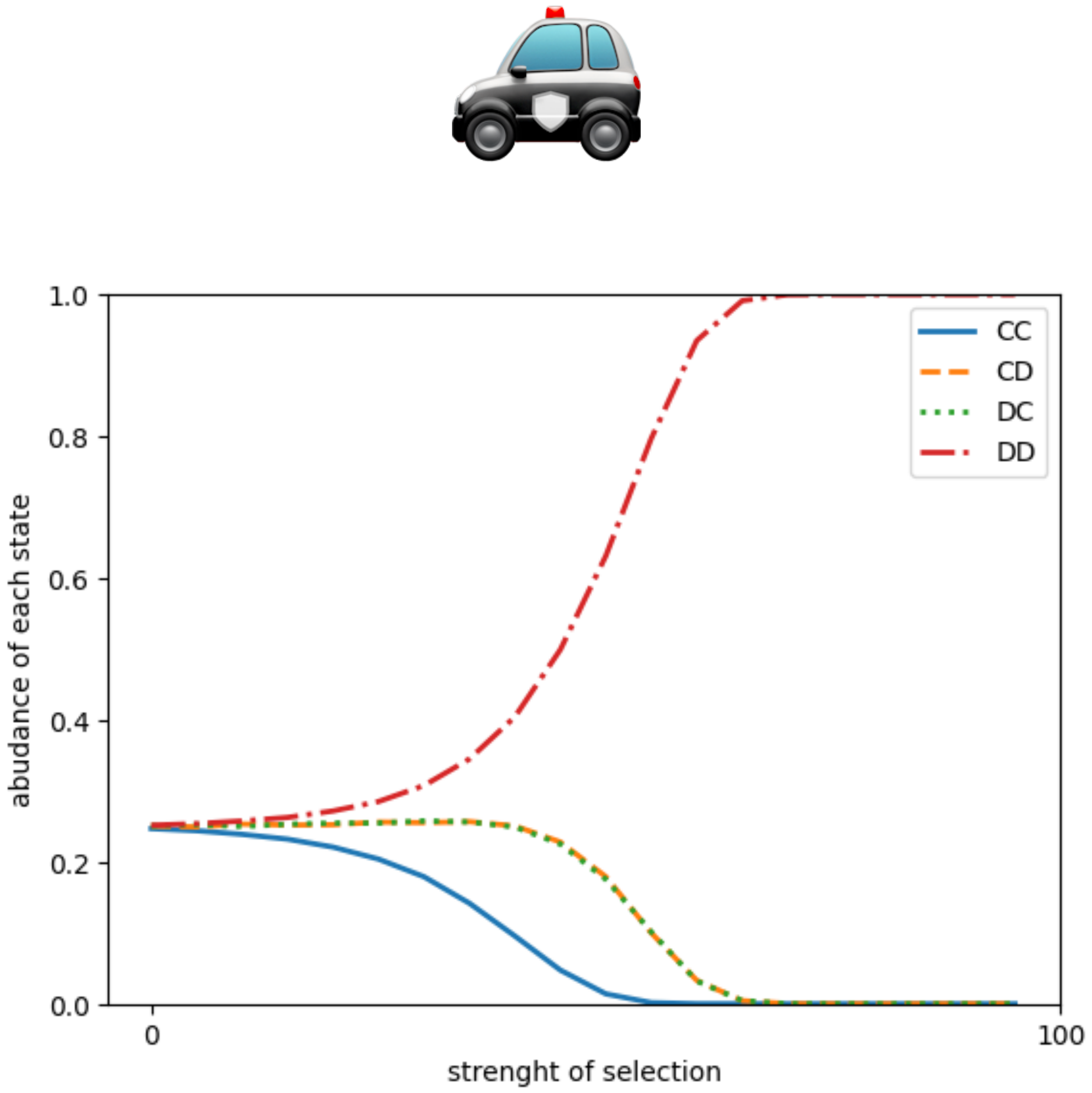
Introspection dynamics



Introspection dynamics

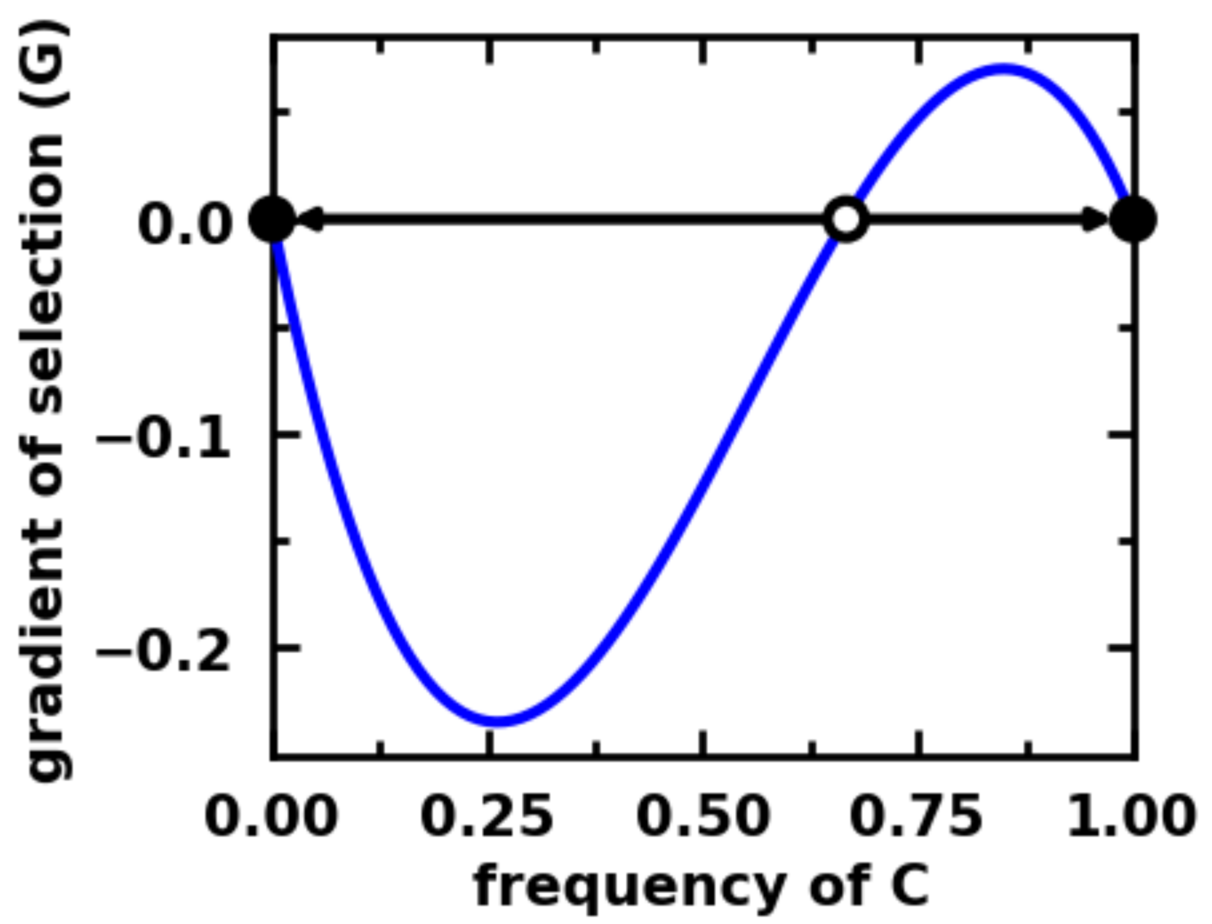




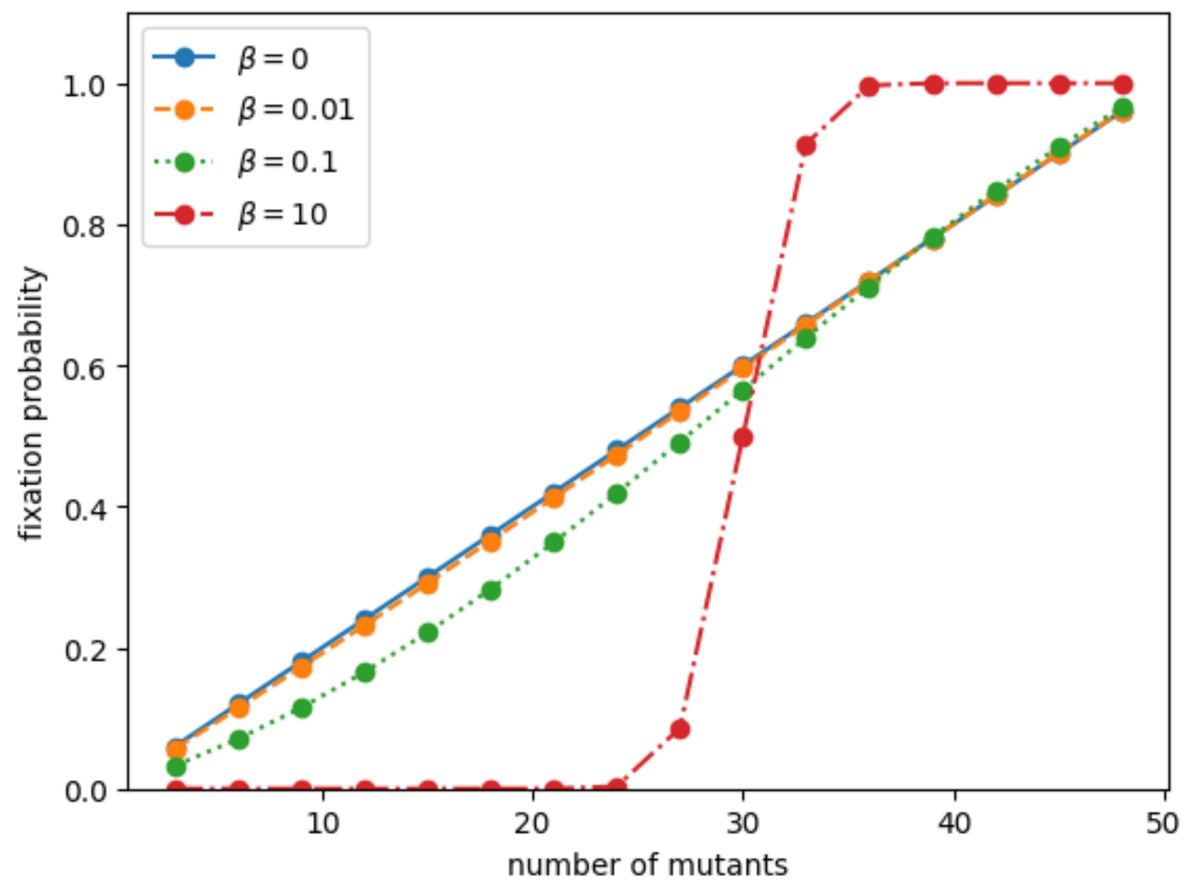


Learning dynamics

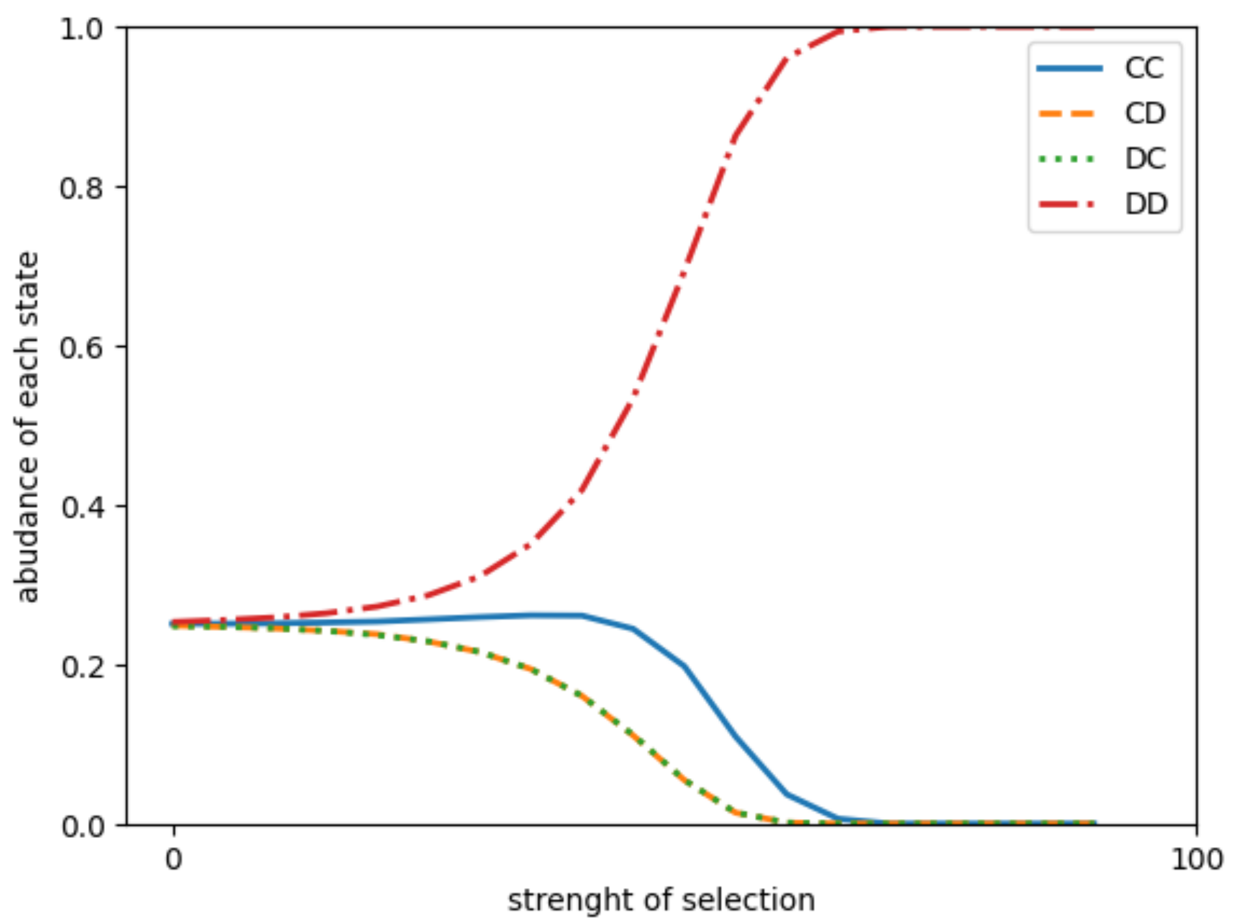
Replicator dynamics



Pairwise comparison process

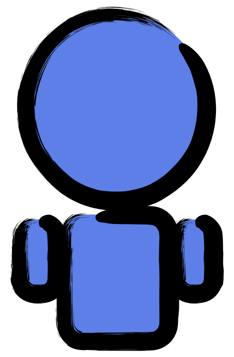
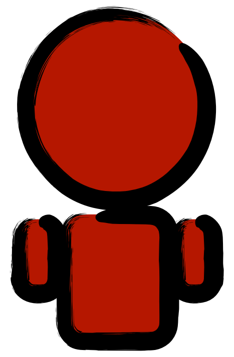


Introspection dynamics



3. Repeated Games

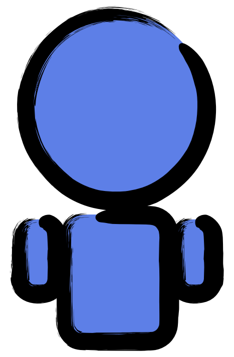
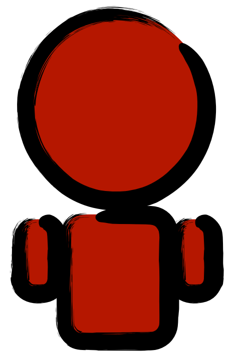
Repeated prisoner's dilemma

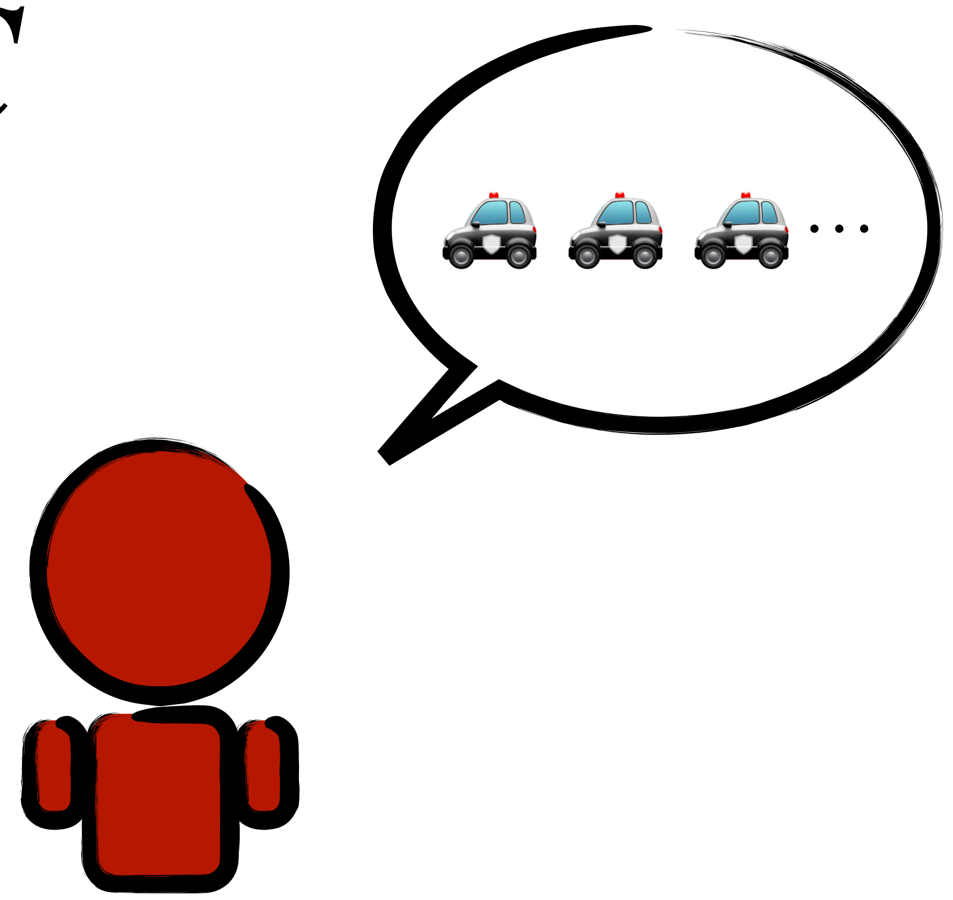
	1	2	3		$n-1$	n
	$\begin{array}{c@{}c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} r & s \\ t & p \end{pmatrix} \end{array}$	$\begin{array}{c@{}c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} r & s \\ t & p \end{pmatrix} \end{array}$	$\begin{array}{c@{}c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} r & s \\ t & p \end{pmatrix} \end{array}$	$\cdots$	$\begin{array}{c@{}c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} r & s \\ t & p \end{pmatrix} \end{array}$	$\begin{array}{c@{}c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} r & s \\ t & p \end{pmatrix} \end{array}$
	C	D	C		C	C
	D	C	C		D	C

Repeated Prisoner's Dilemma

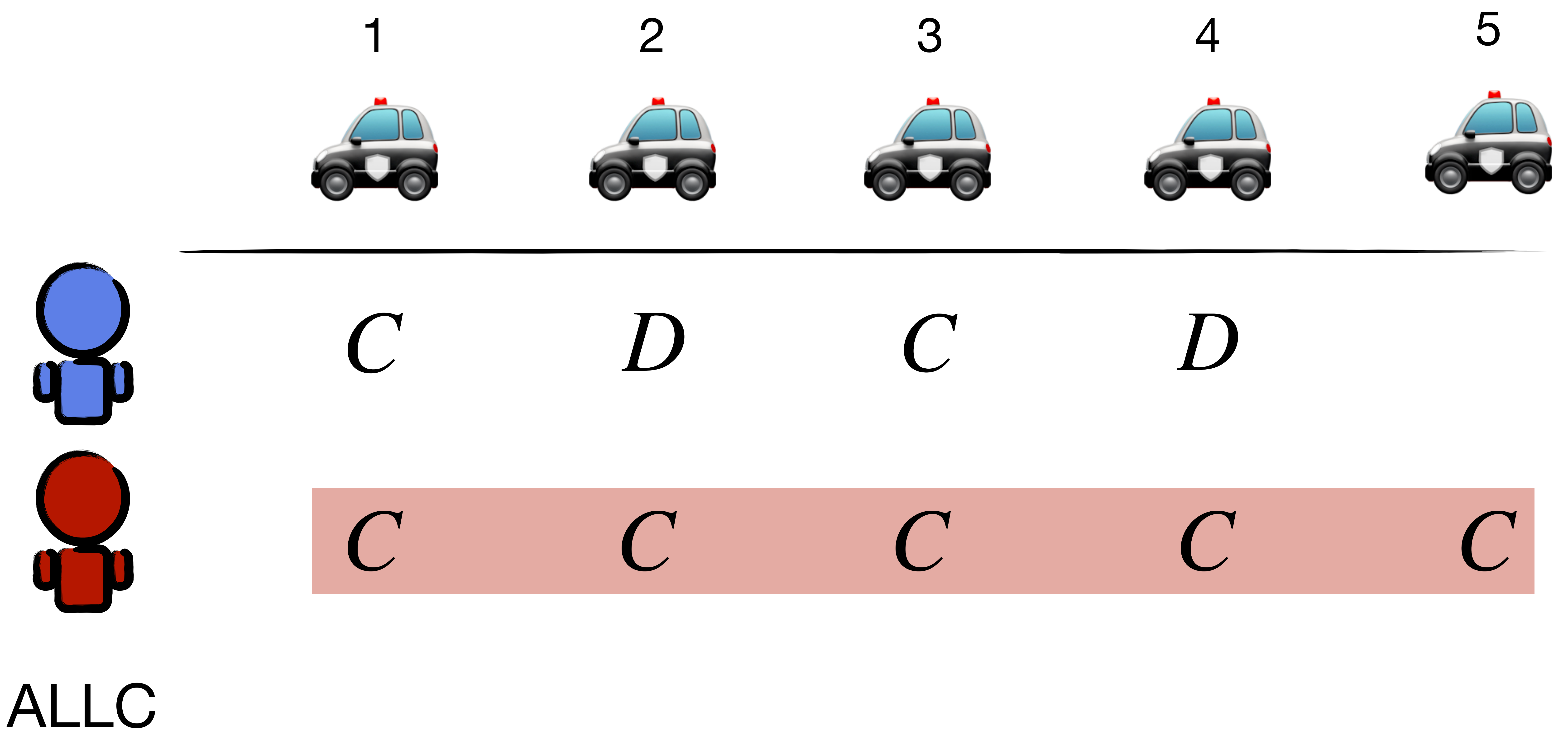
	1	2	3		$n-1$	n
				$\dots$		
	C	D	C		C	C
	D	C	C		D	C

Repeated Prisoner's Dilemma

	1	2	3	…	$n-1$	n
	C	D	C		C	C
	D	C	C		D	C



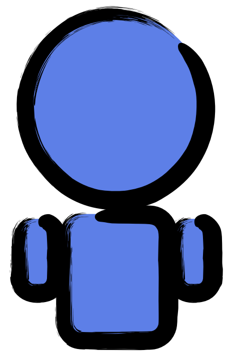
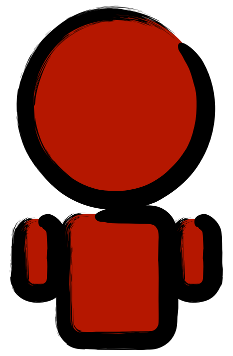
Strategies



Strategies

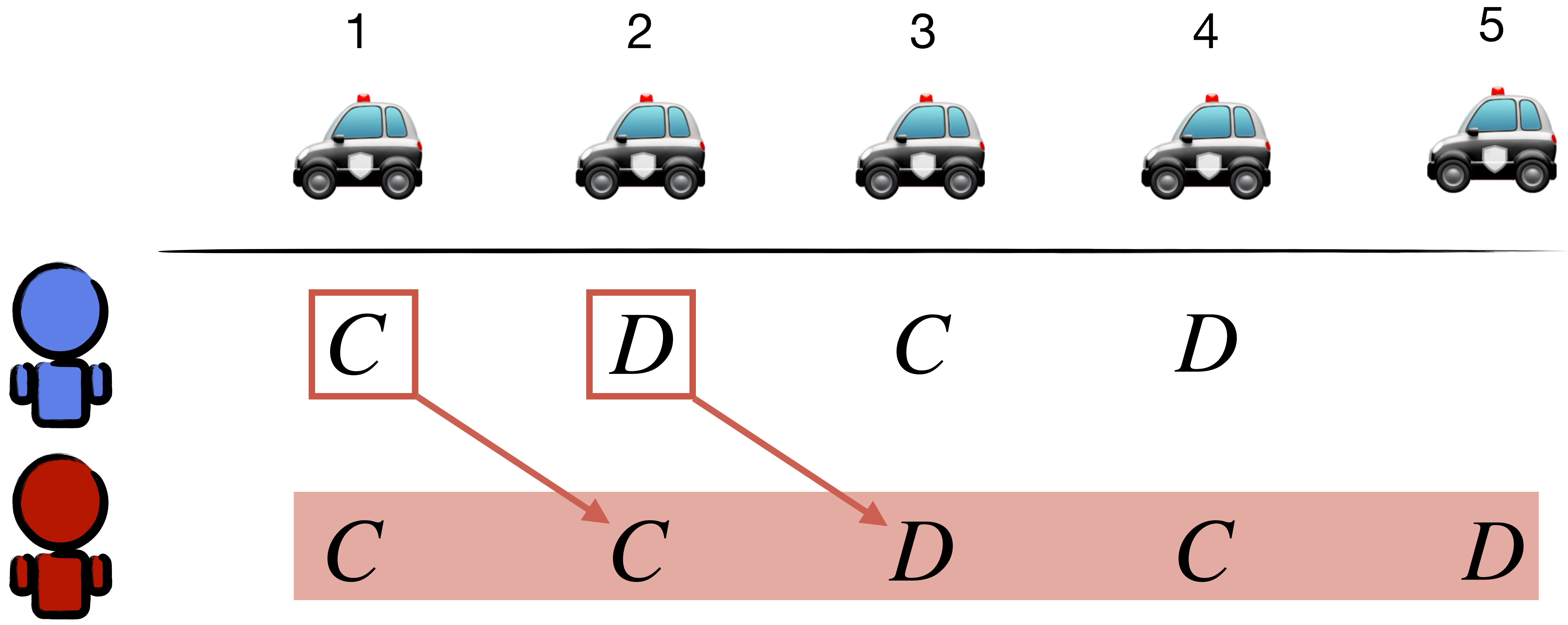
	1	2	3	4	5
	C	D	C	D	
ALLD	D	D	D	D	D

Strategies

	1	2	3	4	5
	🚓	🚓	🚓	🚓	🚓
	C	D	C	D	
	C	D	C	C	D

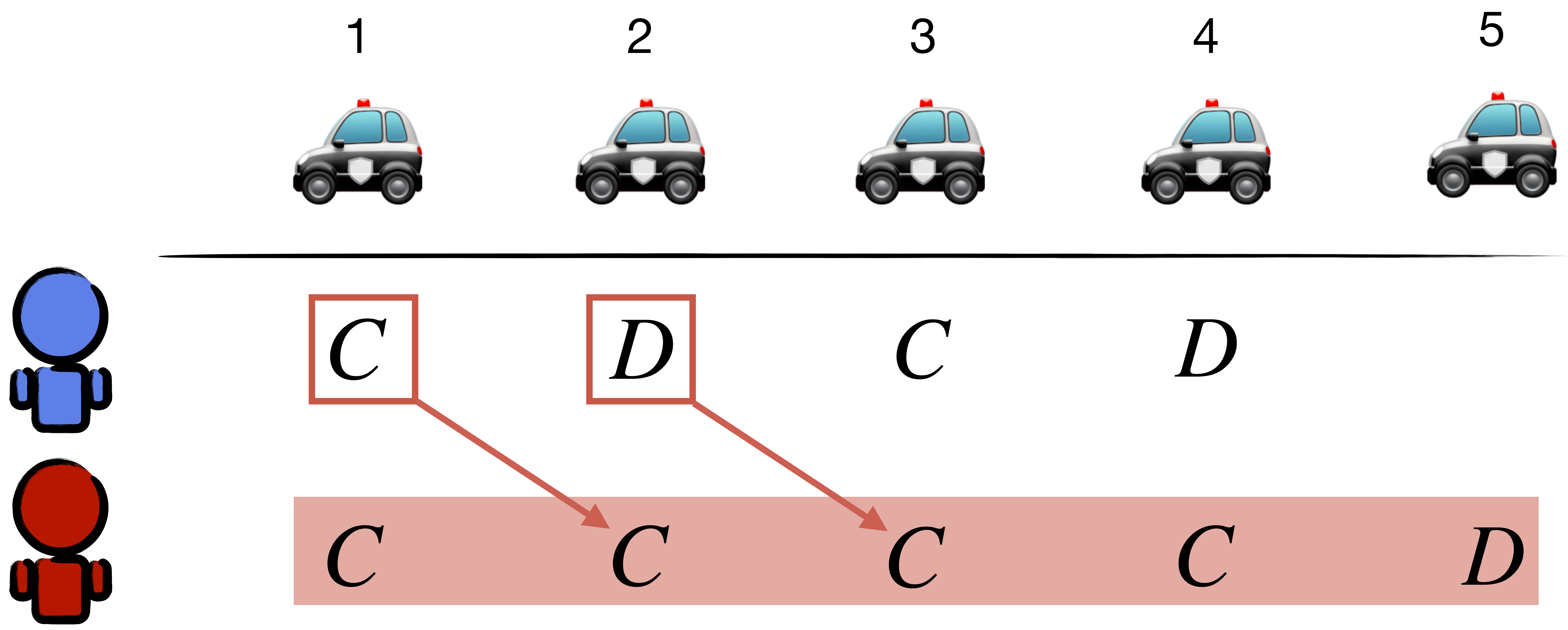
Random $\frac{1}{2}$

Strategies



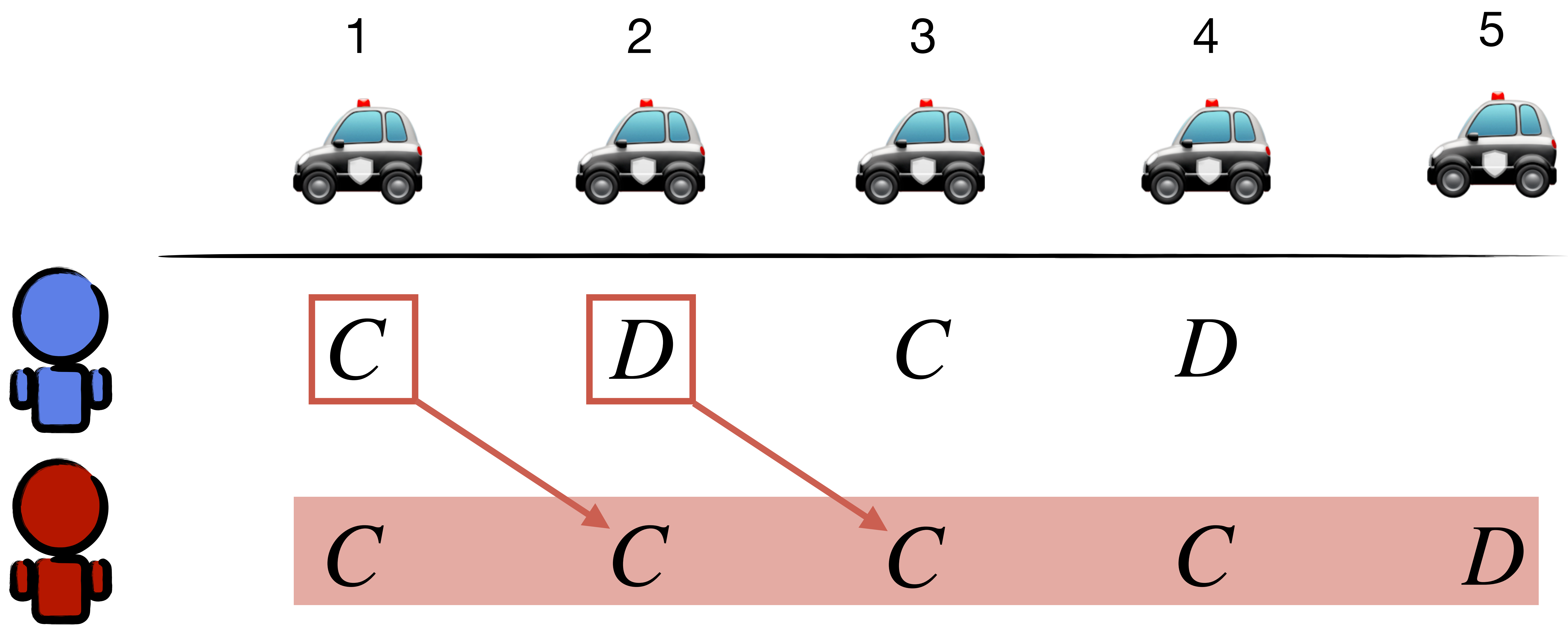
Tit For Tat

Strategies



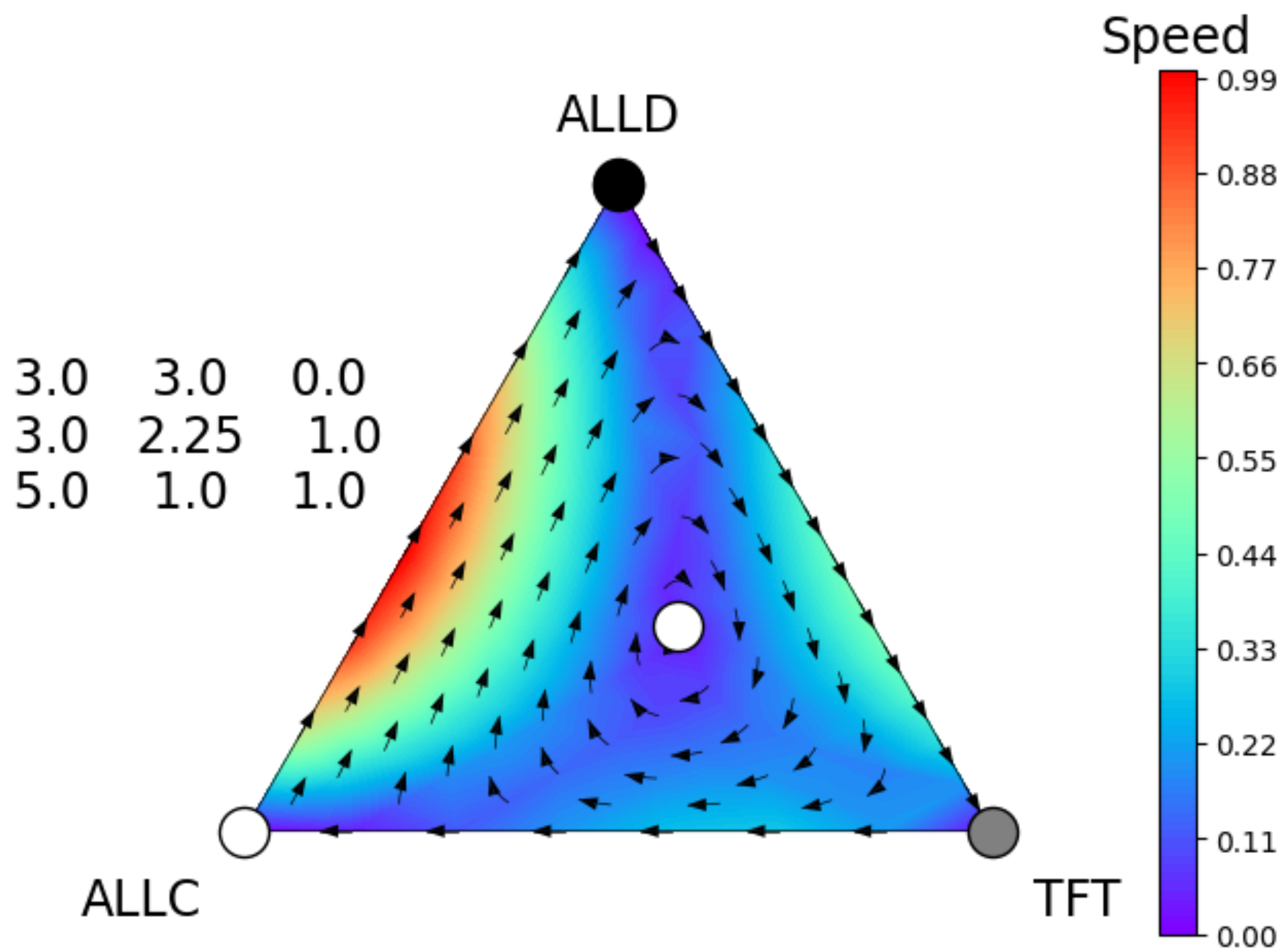
Generous Tit For Tat

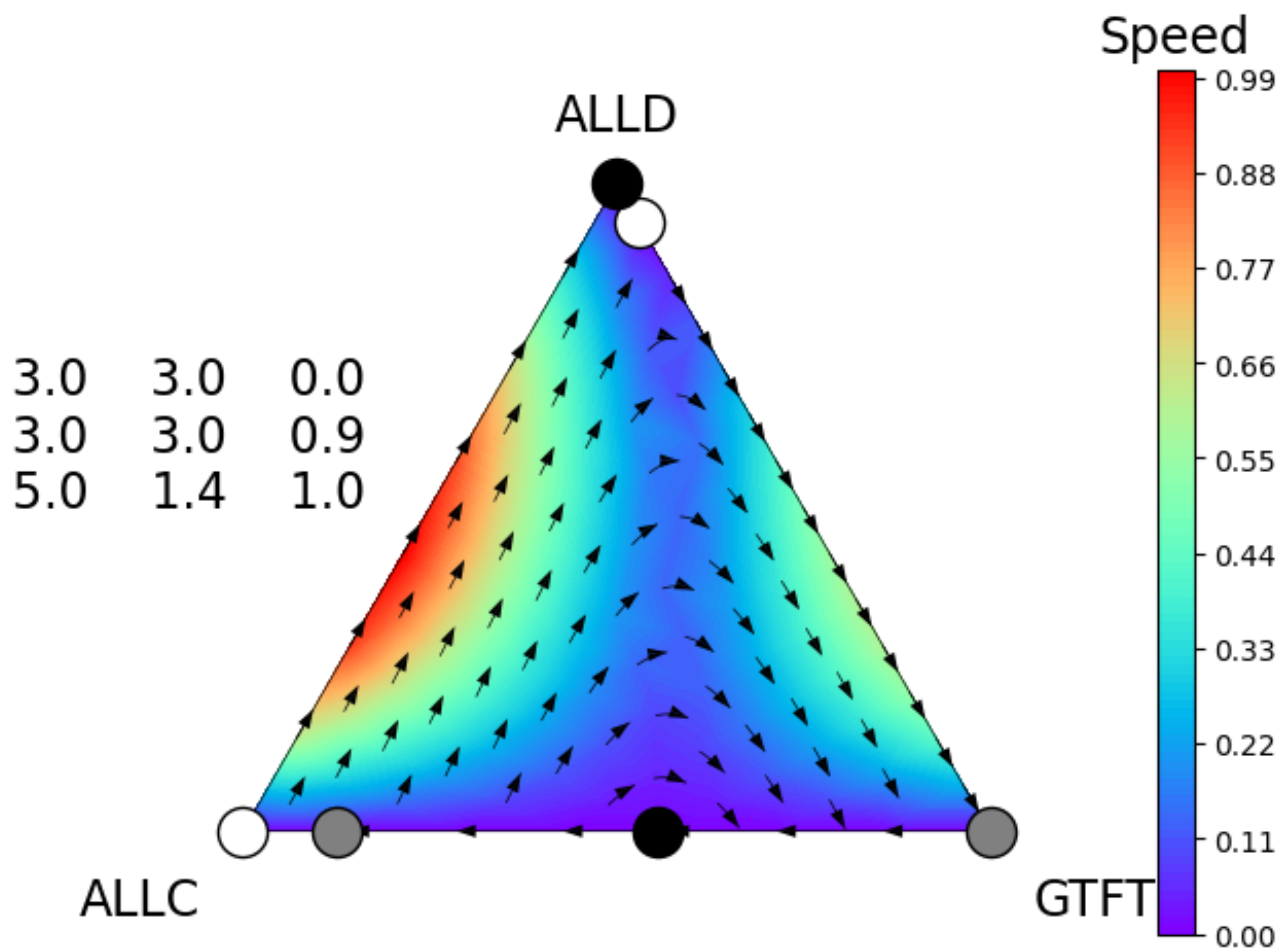
Strategies



Generous Tit For Tat

Replicator dynamics

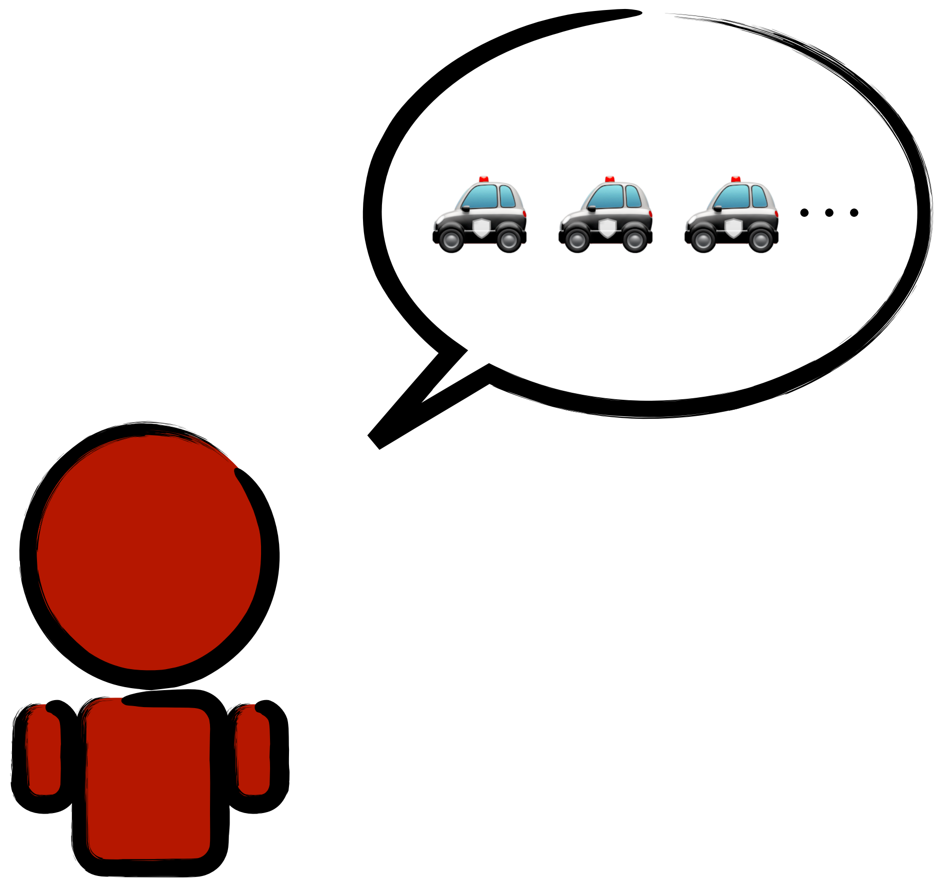




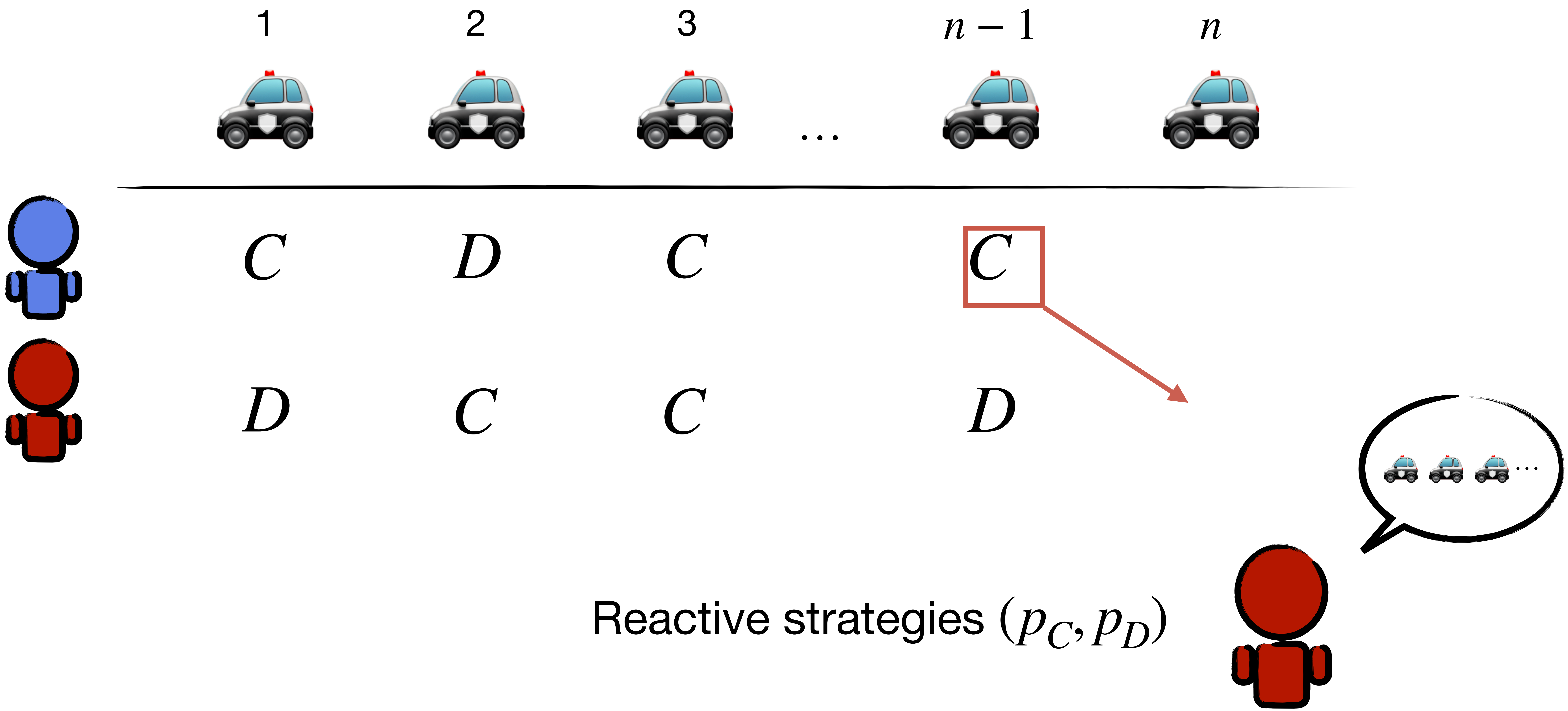
Strategies

	1	2	3	$\dots$	$n-1$	n
	C	D	C		C	
	D	C	C		D	

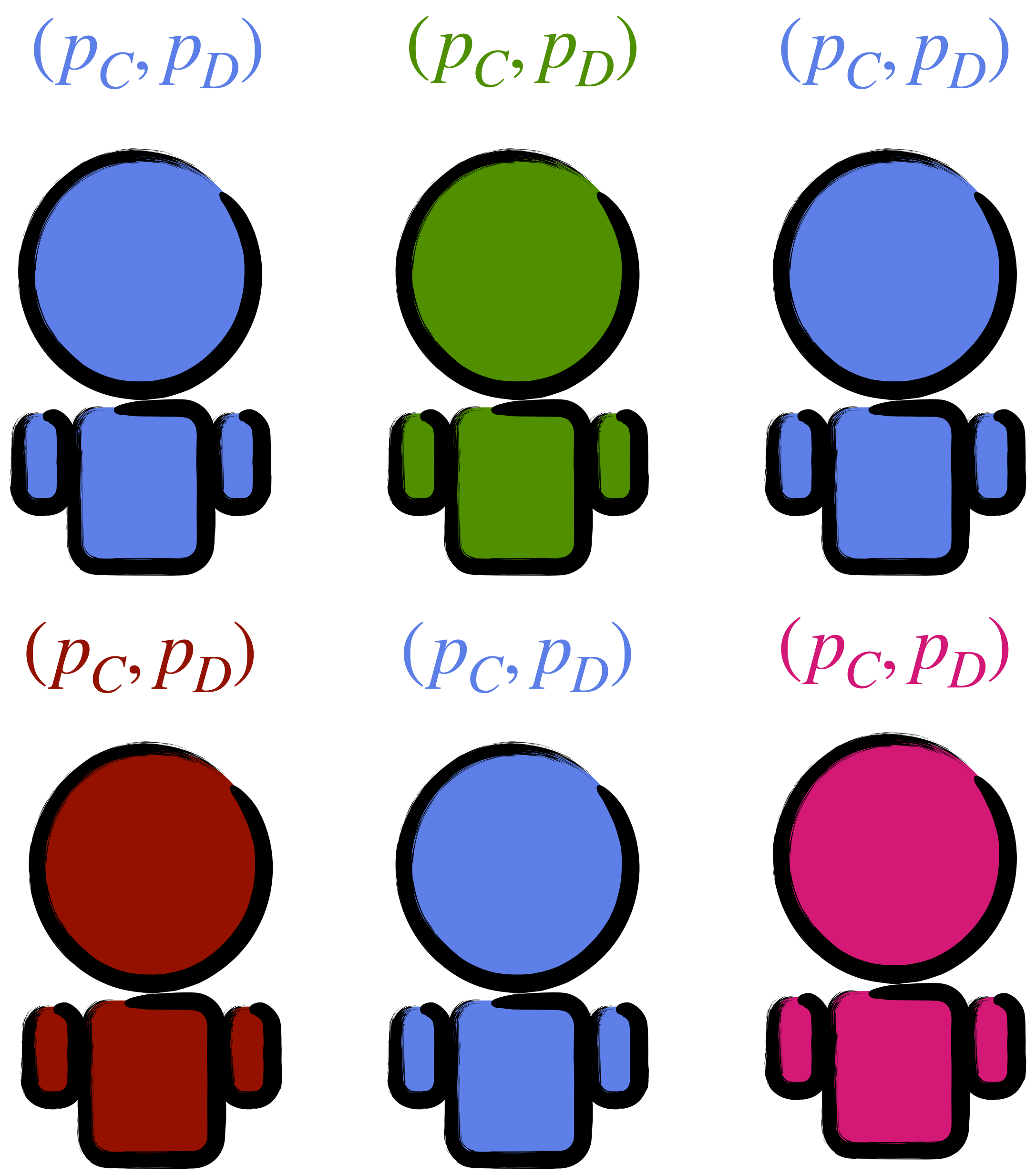
	1	2	3	$\ldots$	$n-1$	n
	C	D	C		C	
	D	C	C		D	



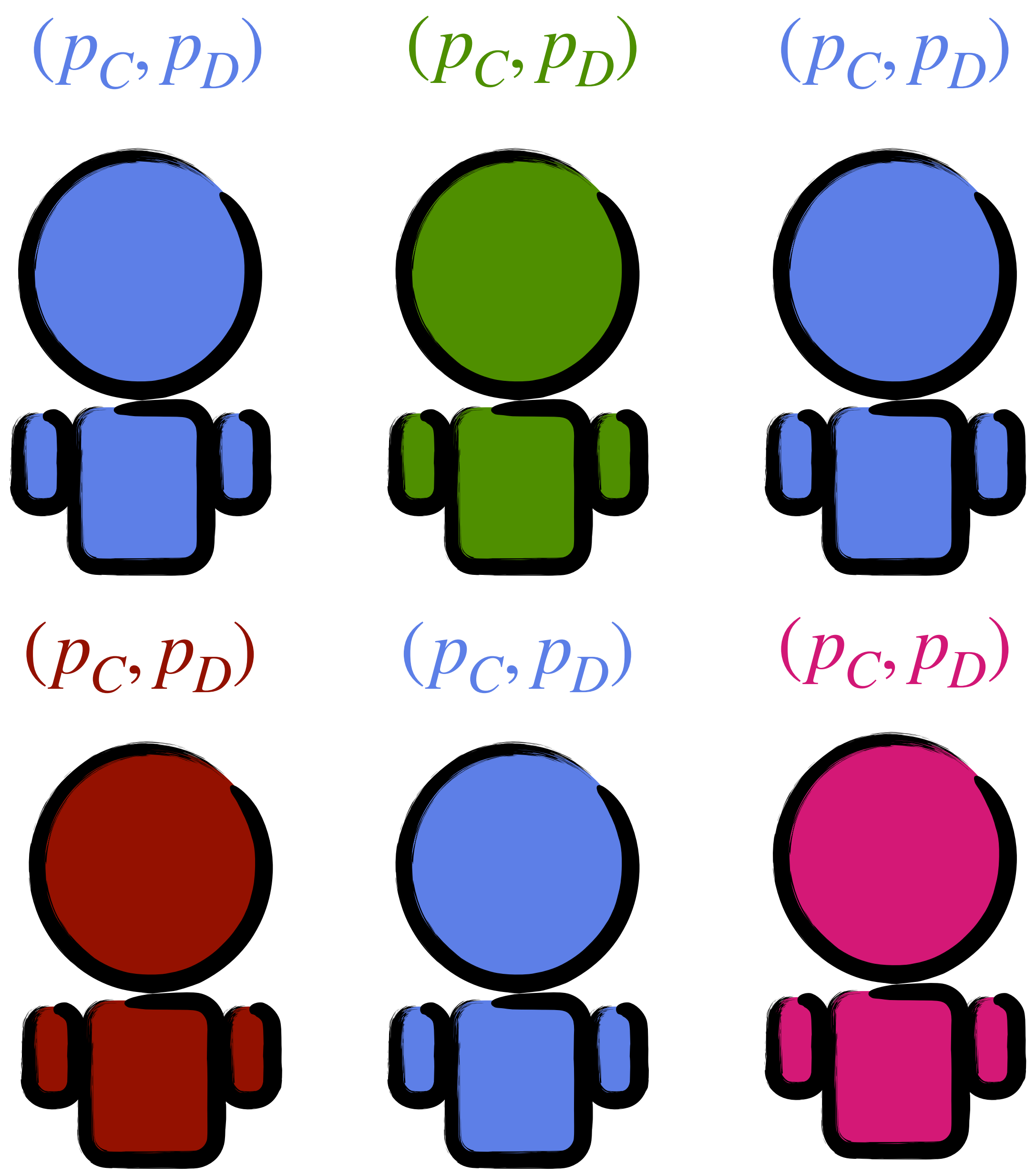
Strategies

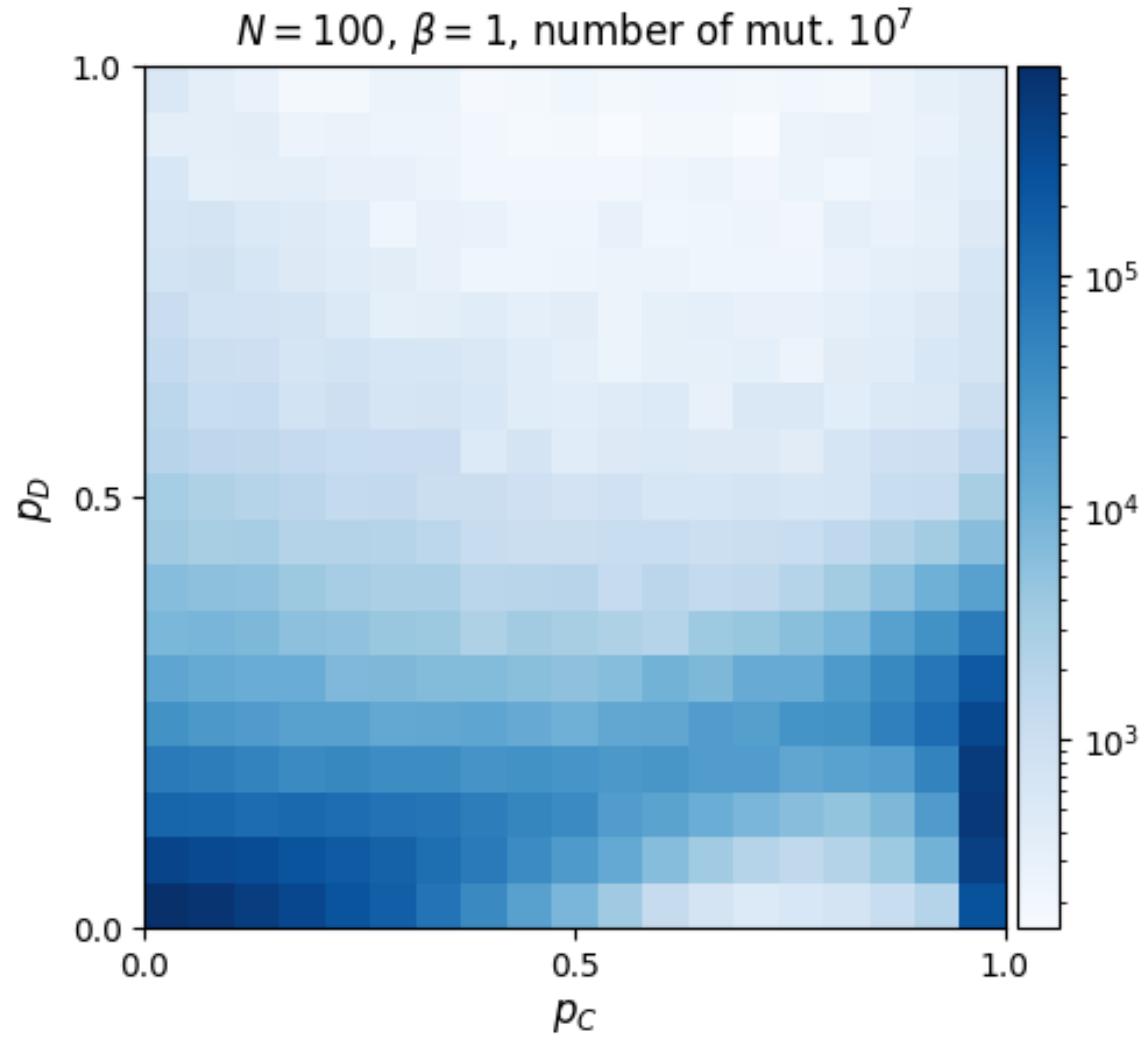


Pairwise comparison process

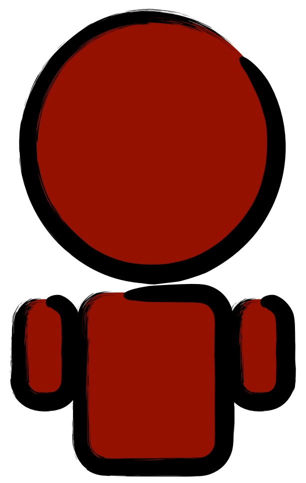


Pairwise comparison process

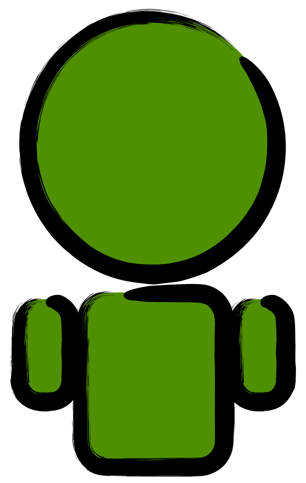




Introspection dynamics

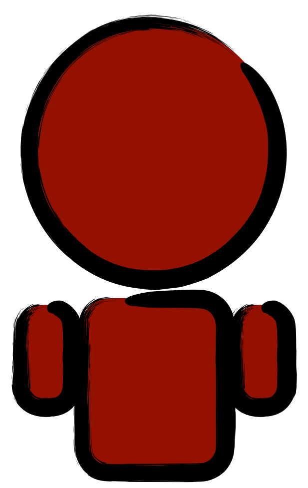


(p_C, p_D)

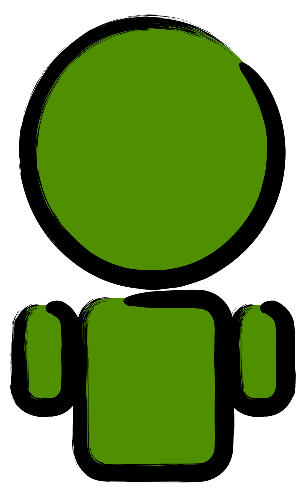


(p_C, p_D)

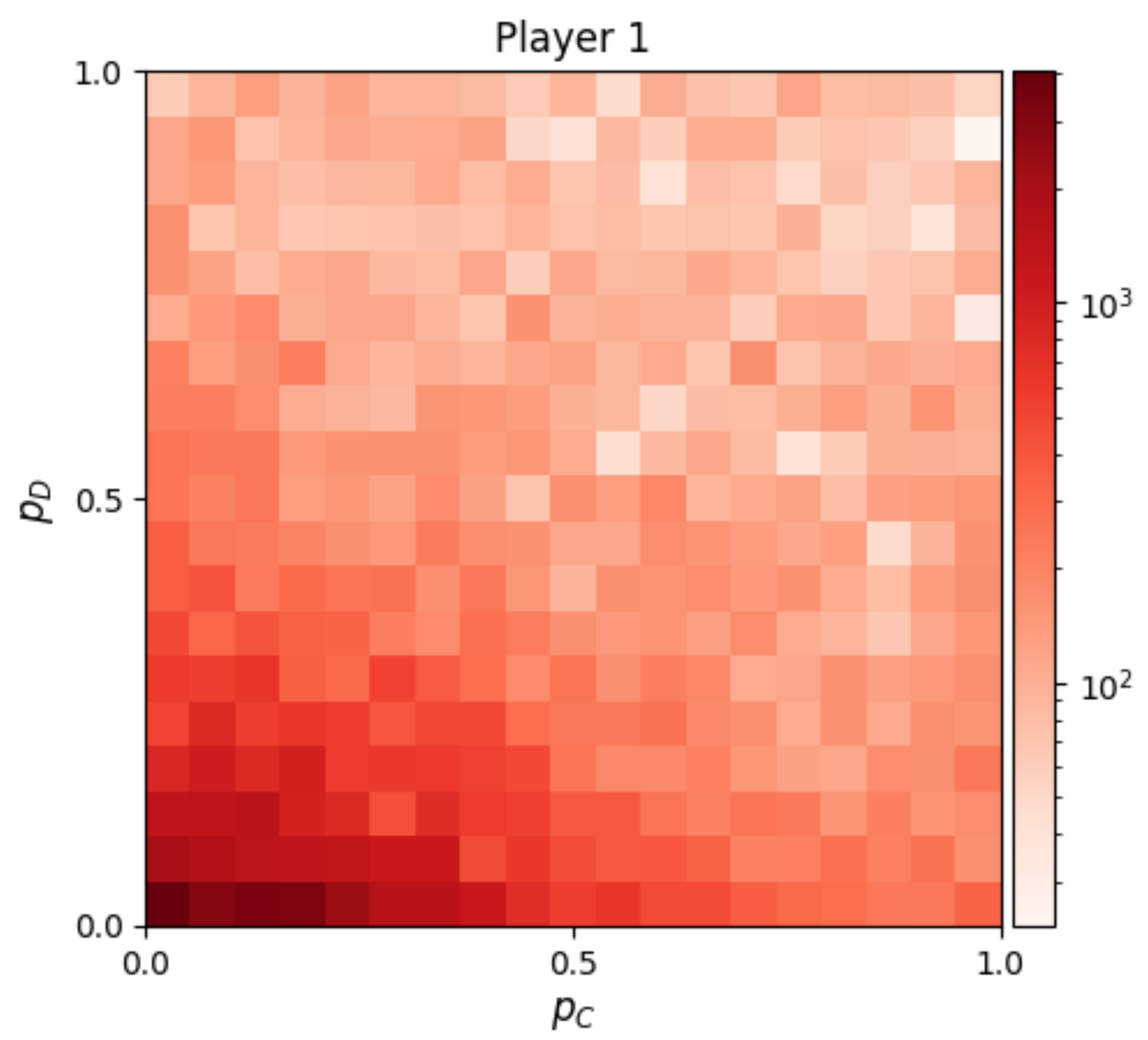
Introspection dynamics

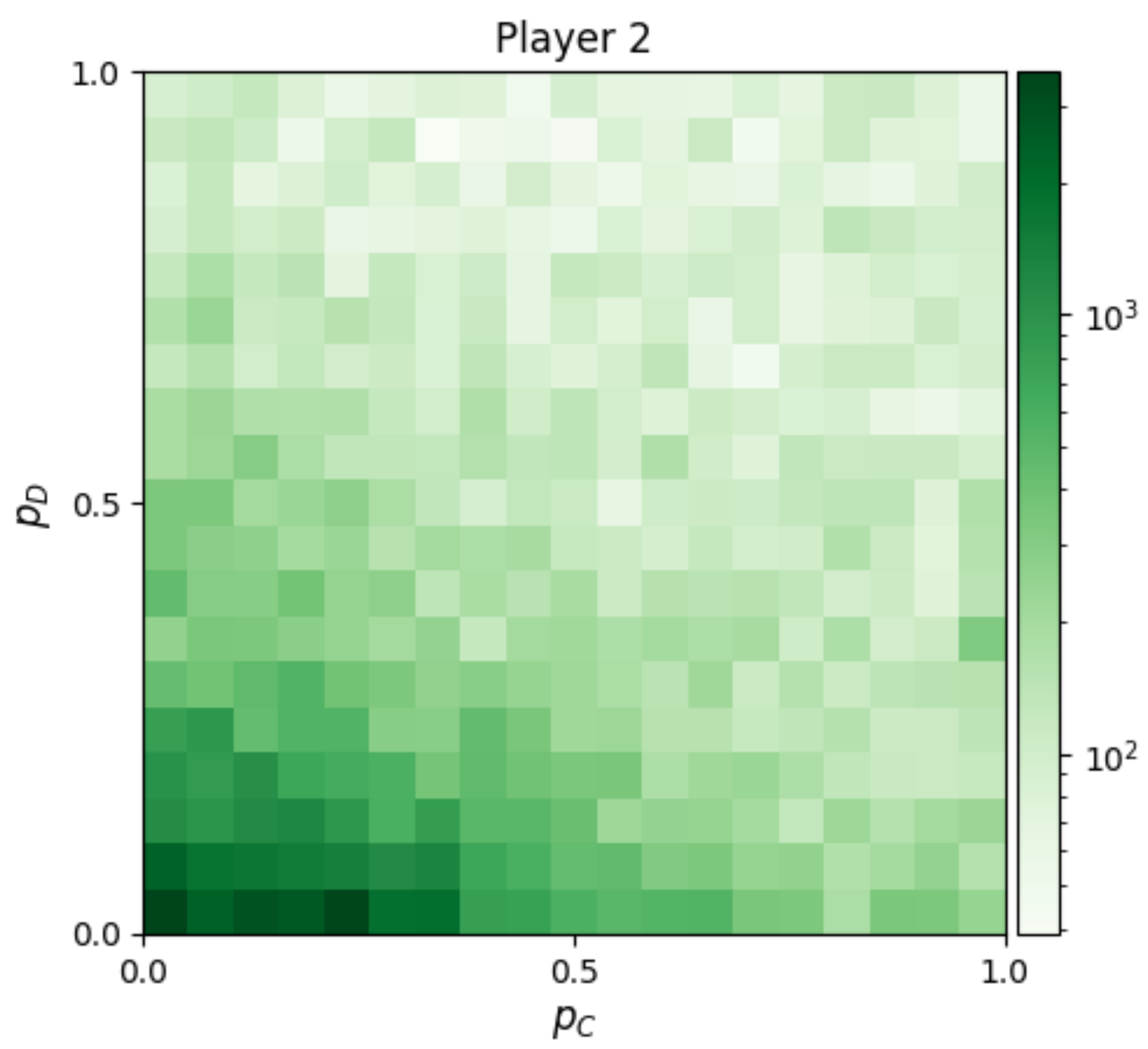


(p_C, p_D)



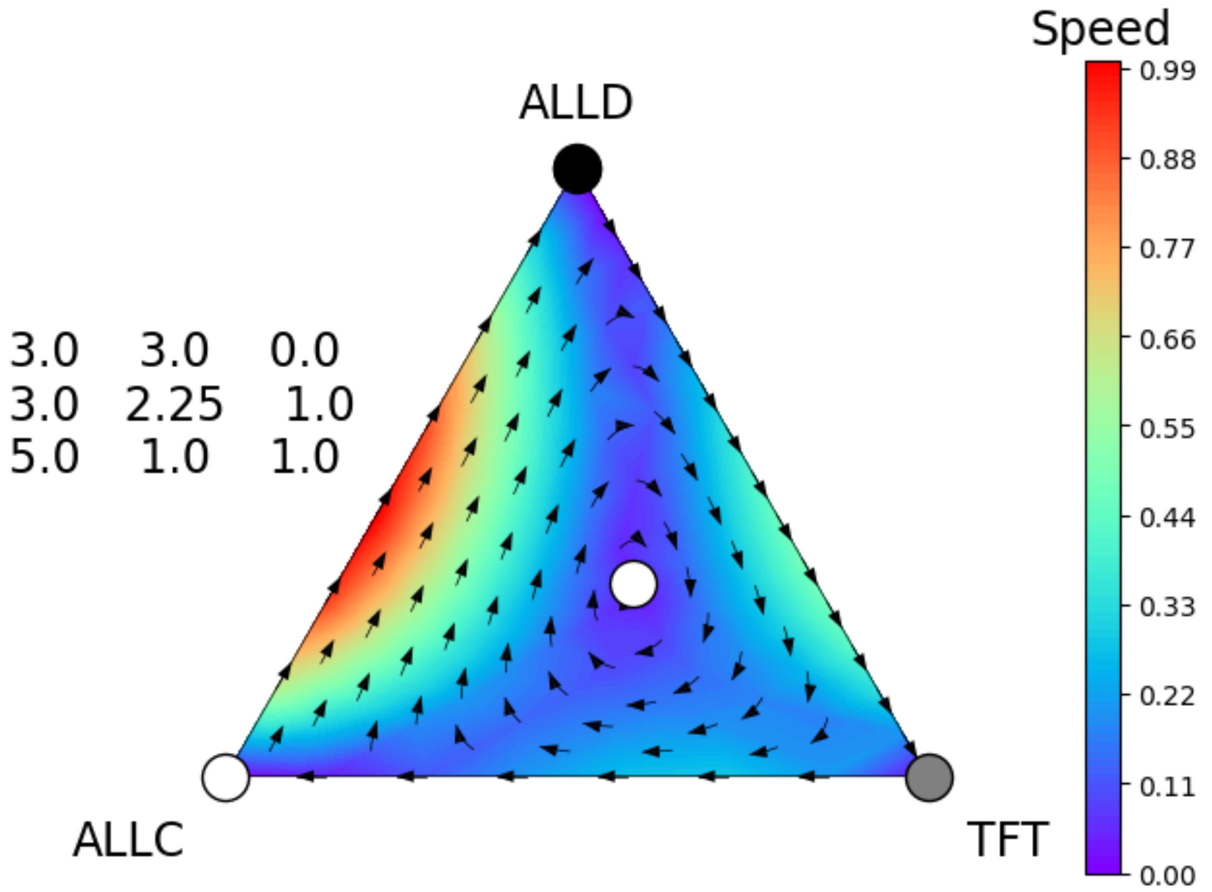
(p_C, p_D)



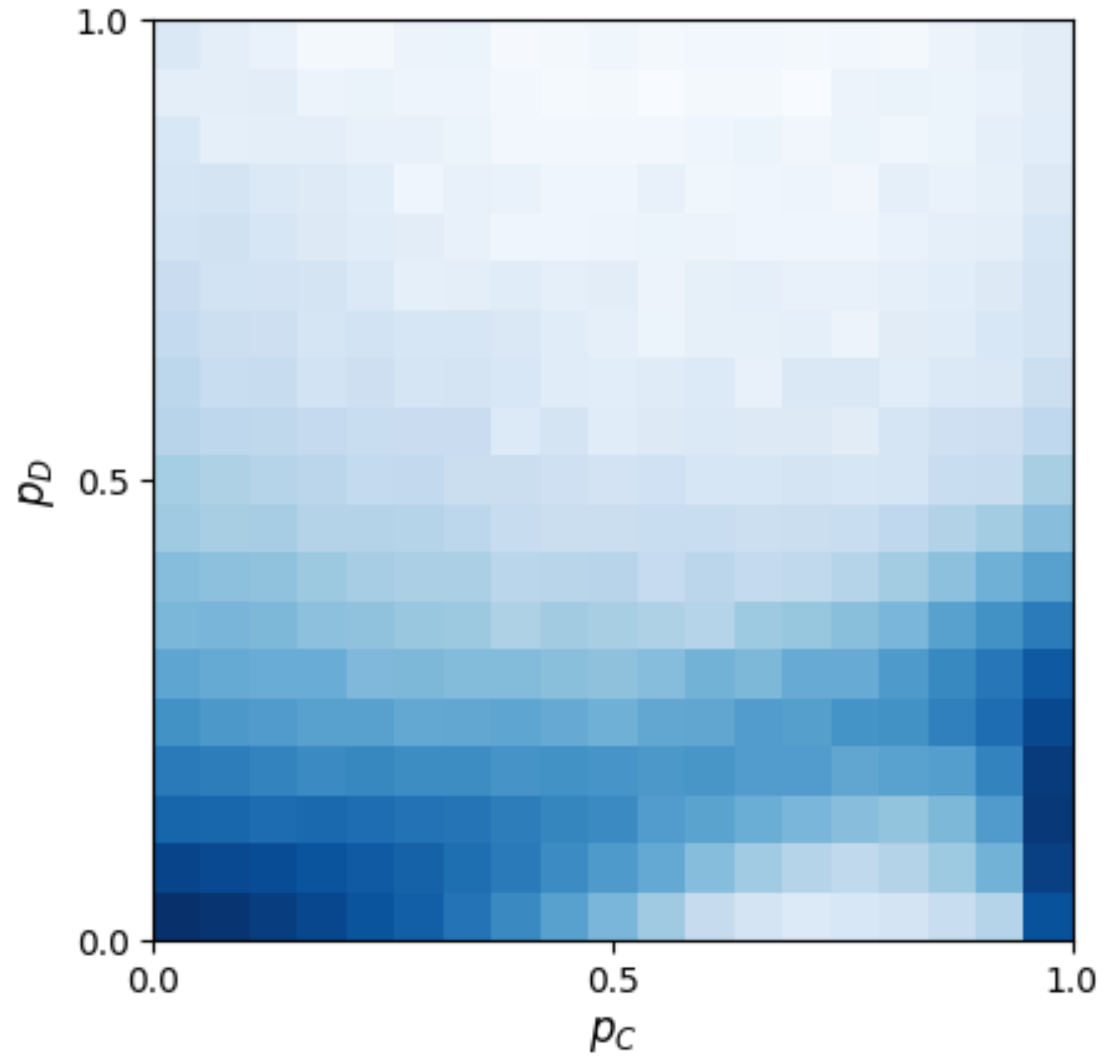


Learning dynamics

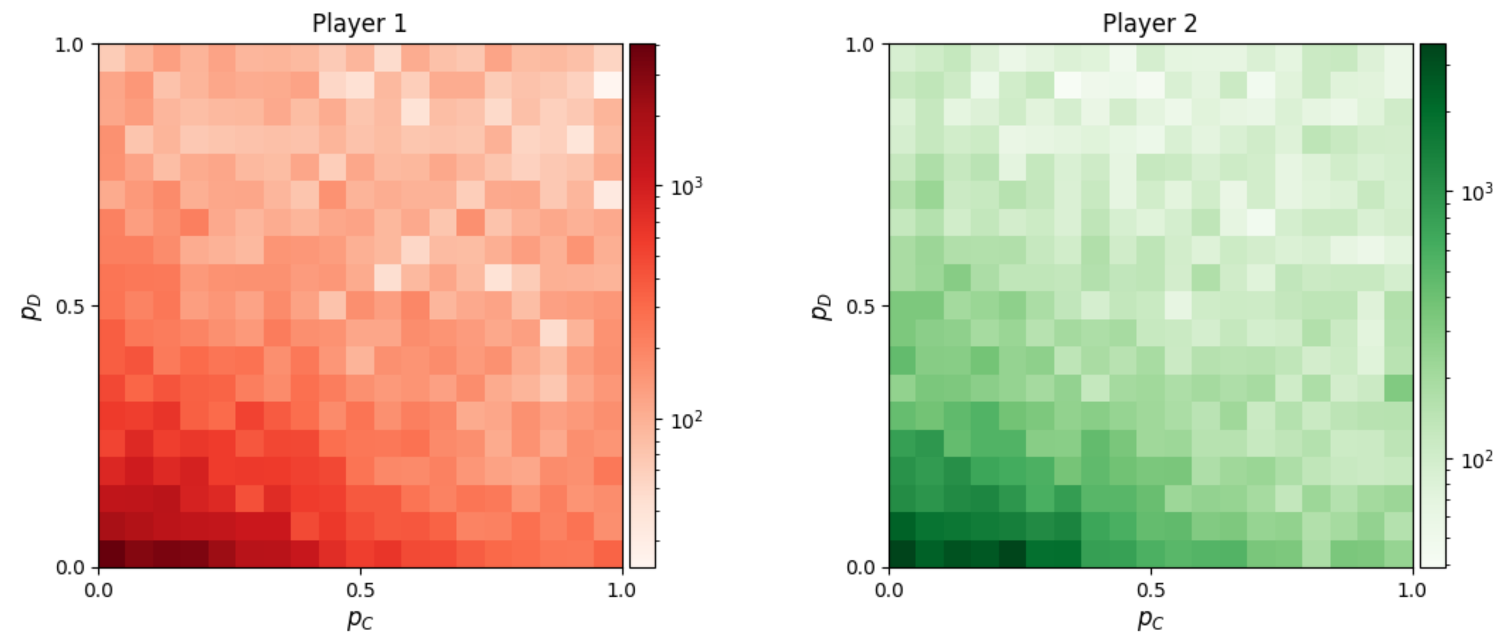
Replicator dynamics



Pairwise comparison process



Introspection dynamics



We focused on 2×2 normal form games

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (r, r) & (s, t) \\ (t, s) & (p, p) \end{pmatrix} \end{array}$$

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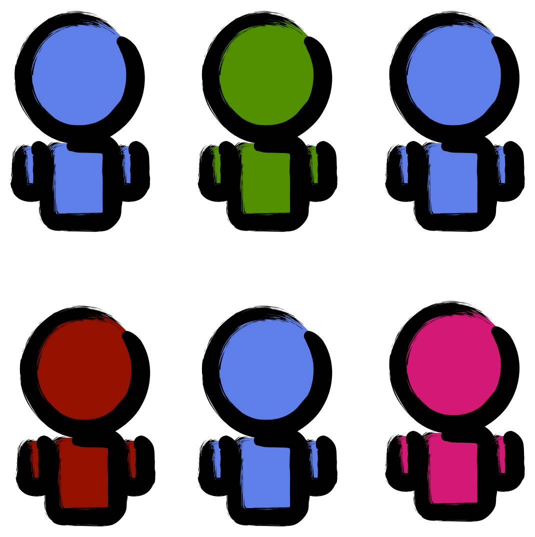
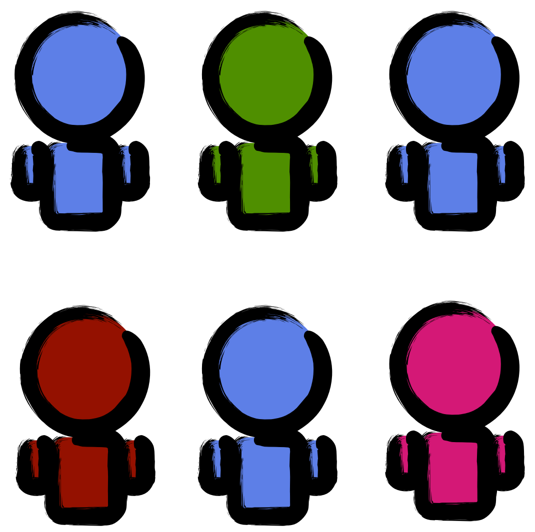
We covered a few learning processes



We focused on 2×2 normal form games

$$
\begin{array}{ccc}
 & C & D \\
\begin{array}{c} C \\ D \end{array} & \multicolumn{2}{c}{\begin{pmatrix} (r, r) & (s, t) \\ (t, s) & (p, p) \end{pmatrix}}
\end{array}
$$

We covered a few learning processes



We covered a few open source packages

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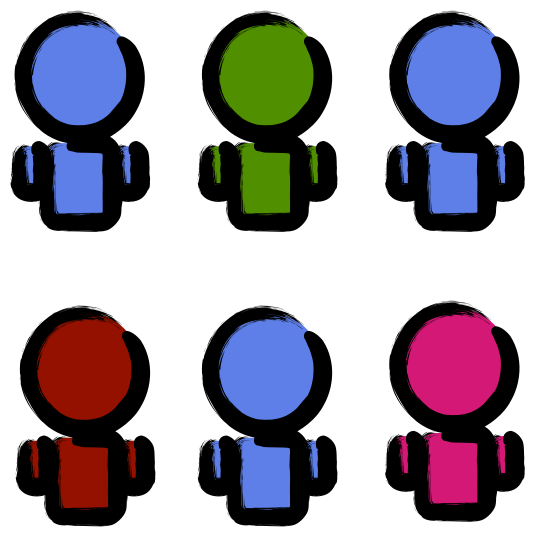

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drvinceknight/Nashpy

mirzaevinom/egtplot

Socrats/EGTTools/

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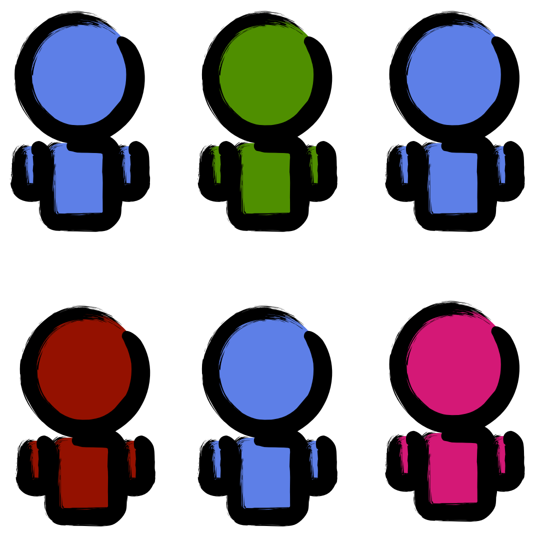

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Material will be available online:

Nikoleta-v3/learning-in-games

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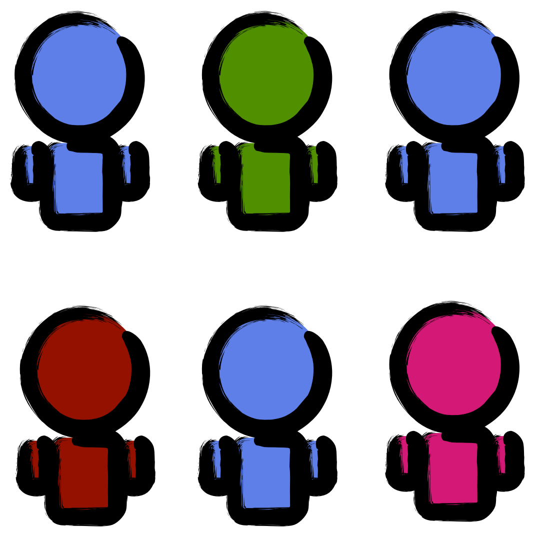

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Thank you!