Understanding responses to environments for the Prisoner's Dilemma: A machine learning approach

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Month 2020

Submitted in partial fulfillment of
the requirements for the degree of
Doctor of Philosophy.

School of Mathematics
Ysgol Mathemateg

Executive Summary

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Chapter 1

Introduction

Chapter 2

A systematic literature review of the Prisoner's Dilemma.

2.1 Introduction

This Chapter provides a systematic literature on the Prisoner's Dilemma. It partitions the literature in five different sections each reviewing a different aspect of research. The Chapter is structured as follows:

- section 2.2 presents the origin of the Prisoner's Dilemma and reviews the early publications in the field and the use of human subject research.
- section 2.3 presents the pioneering computer tournaments of Robert Axelrod and reviews IPD strategies of intelligent design.
- section 2.4 review research on evolutionary dynamics.
- section 2.5 defines structured strategies in the IPD, the notion of training and discusses related papers.
- section 2.6 reports several pieces of software used for simulating the PD game.

The aim of this Chapter is to provide a concrete summary of the existing literature on the PD and contribute to research question 1.

2.2 Origins of the prisoner's dilemma

The origin of the PD goes back to the 1950s in early experiments conducted at RAND [69] to test the applicability of games described in [152]. The game received its name later the same year. According to [197], Albert W. Tucker (the PhD supervisor of John Nash [151]), in an attempt to deliver the game with a story during a talk described the players as prisoners and the game has been known as the Prisoner's Dilemma ever since.

The early research on the IPD was limited. The only source of experimental results was through human subject research where pairs of participants simulated plays of the game. Human subject research had disadvantages. Humans could behave randomly and in several experiments both the size and the background of the individuals were different, thus comparing results of two or more studies became difficult.

The main aim of these early research experiments was to understand how conditions such as the gender of the participants [67, 131, 134], the physical distance between the participants [183], the effect of their opening moves [196] and even how the experimenter, by varying the tone of their voice and facial expressions [73], could influence the outcomes and subsequently the emergence of cooperation. An early figure that sought out to understand several of these conditions was the mathematical psychologist Anatol Rapoport. The results of his work are summarised in [171].

Rapoport was also interested in conceptualising strategies that could promote international cooperation. Decades later he would submit the winning strategy (Tit for Tat) of the first computer tournament, run by Robert Axelrod. These tournaments and several strategies that were designed by researchers, such as Rapoport, are introduced in the following section.

2.3 Axelrod's tournaments and intelligently designed strategies

As discussed in Section 2.2, before 1980 a great deal of research was done in the field, however, as described in [32], the political scientist Robert Axelrod believed that there was no clear answer to the question of how to avoid conflict, or even how an individual should play the game. Combining his interest in artificial intelligence and political science Axelrod created a framework for exploring these questions using computer tournaments.

Axelrod's tournaments made the study of cooperation of critical interest. As described in [172], "Axelrod's "new approach" has been extremely successful and immensely influential in casting light on the conflict between an individual and the collective rationality reflected in the choices of a population whose members are unknown and its size unspecified, thereby opening a new avenue of research". In a collaboration with a colleague, Douglas Dion, Axelrod in [33] summarized a number of works that were immediately inspired from the "Evolution of Cooperation", and [108] gives a review of tournaments that have been conducted since the originals.

The first reported computer tournament took place in 1980 [28]. A total of 13 strategies were submitted, written in the programming languages Fortran or Basic. Each competed in a 200 turn match against all 12 opponents, itself and a player that played randomly (called *Random*). This type of tournament is referred to as a round robin. The tournament was repeated 5 times to get a more stable estimate of the scores for each pair of play. Each participant knew the exact number of turns and had access to the full history of each match. Furthermore, Axelrod performed a preliminary tournament and the results were known to the participants. This preliminary tournament is mentioned in [28] but no details were given.

The winner of the tournament was determined by the total average score and not by the number of matches won. The strategy that was announced the winner was the strategy submitted by Rapoport, *Tit For Tat*. The success of Tit for Tat came as a surprise. It was not only the simplest submitted strategy, it would always cooperates on the first round and then mimic the opponent's previous move, but it had also won the tournament even though it could never beat any player it was interacting with.

In order to further test the results Axelrod performed a second tournament in 1980 [29]. The second tournament received much more attention and had a total of 62 entries. The participants knew the results of the previous tournament and the rules were similar with only a few alterations. The tournament was repeated 5 times and the length of each match was not known to the participants. Axelrod intended to use a fixed probability (refereed to as 'shadow of the future' [33]) of the game ending on the next move. However, 5 different number of turns were selected for each match 63, 77, 151, 308 and 401, such that the average length would be around 200 turns.

Nine of the original participants competed again in the second tournament. Two strategies that remained the same were Tit For Tat and *Grudger*. Grudger is a strategy that will cooperate as long as the opponent does not defect, submitted by James W. Friedman. The name Grudger was give to the strategy in [126], though the strategy goes by many names in the literature such as, Spite [37], Grim Trigger [36] and Grim [202]. New entries in the second tournament included *Tit for Two Tats* submitted by John Maynard Smith and *KPavlovC*. KPavlovC, is also known as Simpleton [171], introduced by Rapoport or just Pavlov [156]. The strategy is based on the fundamental behavioural mechanism win-stay, lose-shift. Pavlov is heavily studied in the literature and similarly to Tit for Tat it is used in tournaments today and has had many variants trying to build upon it's success, for example *PavlovD* and *Adaptive Pavlov* [124].

Despite the larger size of the second tournament none of the new entries managed to outperform the simpler designed strategy. The winner was once again Tit for Tat. Axelrod deduced the following guidelines for a strategy to perform well:

- The strategy would start of by cooperating.
- It would forgive it's opponent after a defection.
- It would always be provoked by a defection no matter the history.
- It was simple.

The success of Tit for Tat, however, was not unquestionable. Several papers showed that stochastic uncertainties severely undercut the effectiveness of reciprocating strategies and such stochastic uncertainties have to be expected in real life situations [142]. For example, in [147] it is proven that in an environment where *noise* (a probability that a player's move will be flipped) is introduced two strategies playing Tit for Tat receive the same average payoff as two Random players. Hammerstein, pointed out that if by mistake, one of two Tit for Tat players makes a wrong move, this locks the two opponents into a hopeless sequence of alternating defections and cooperations [182].

The poor performance of the strategy in noisy environments was also demonstrated in tournaments. In [41, 62] round robin tournaments with noise were performed, and Tit For Tat did not win. The authors concluded that to overcome the noise more generous strategies than Tit For Tat were needed. They introduced the strategies *Nice and Forgiving* and *OmegaTFT* respectively.

A second type of stochastic uncertainty is misperception, where a player's action is made correctly but it is recorded incorrectly by the opponent. In [207], a strategy called *Contrite*

Tit for Tat was introduced that was more successful than Tit for Tat in such environments. The difference between the strategies was that Contrite Tit for Tat was not so fast to retaliate against a defection.

Several works extended the reciprocity based approach which has led to new strategies. For example Gradual [37] which was constructed to have the same qualities as those of Tit for Tat except one, *Gradual* had a memory of the game since the beginning of it. Gradual recorded the number of defections by the opponent and punished them with a growing number of defections. It would then enter a calming state in which it would cooperates for two rounds. In a tournament of 12 strategies, including both Tit for Tat and Pavlov, Gradual managed to outperformed them all. A strategy with the same intuition as Gradual is *Adaptive Tit for Tat* [199]. Adaptive Tit for Tat does not keep a permanent count of past defections, it maintains a continually updated estimate of the opponent's behaviour, and uses this estimate to condition its future actions. In the exact same tournament as in [37] with now 13 strategies Adaptive Tit for Tat ranked first.

Another extension of strategies was that of teams of strategies [60, 61, 177] that collude to increase one member's score. In 2004 Graham Kendall led the Anniversary Iterated Prisoner's Dilemma Tournament with a total of 223 entries. In this tournament participants were allowed to submit multiple strategies. A team from the University of Southampton submitted a total of 60 strategies [177]. All these were strategies that had been programmed with a recognition mechanism by default. Once the strategies recognised one another, one would act as leader and the other as a follower. The follower plays as a *Cooperator*, cooperates unconditionally and the leader would play as a *Defector* gaining the highest achievable score. The followers would defect unconditionally against other strategies to lower their score and help the leader. The result was that Southampton had the top three performers. Nick Jennings, who was part of the team, said that "We developed ways of looking at the Prisoner's Dilemma in a more realistic environment and we devised a way for computer agents to recognise and collude with one another despite the noise. Our solution beats the standard Tit For Tat strategy" [162].

2.3.1 Memory one Strategies

A set of strategies that have received a lot of attention in the literature are *memory one* strategies. In [157], Nowak and Sigmund proposed a structure for studying simple strategies that remembered only the previous turn, and moreover, only recorded the move of the opponent. These are called *reactive* strategies and they can be represented by using three parameters (y, p_1, p_2), where y is the probability to cooperate in the first move, and p_1 and p_2 the conditional probabilities to cooperate, given that the opponent's last move was a cooperation or a defection. For example Tit For Tat is a reactive strategy and it can be written as $(1, 1, 0)$. Another reactive strategy well known in the literature is *Generous Tit for Tat* [159].

In [158], Nowak and Sigmund extended their work to include strategies which consider the entire history of the previous turn to make a decision. These are called *memory one* strategies. If only a single turn of the game is taken into account and depending on the simultaneous moves of the two players there are only four possible states that the players could be in. These are:

- Both players cooperated, denoted as CC.

- First player cooperated while the second one defected, denoted as CD.
- First player defected while the second one cooperated, denoted as DC.
- Both players defected, denoted as DD.

Thus a memory one strategy can be denoted by the probabilities of cooperating after each state and the probability of cooperating in the first round, (y, p_1, p_2, p_3, p_4). For example Pavlov's memory one representation is $(1, 1, 0, 0, 1)$.

Memory one strategies made an impact when a specific set of memory one strategies were introduced called *Zero-determinant* (ZD) [167]. The American Mathematical Society's news section [97] stated that "the world of game theory is currently on fire" and in [189] it was stated that "Press and Dyson have fundamentally changed the viewpoint on the Prisoner's Dilemma". ZD are a set of extortionate strategies that can force a linear relationship between the long-run scores of both themselves and the opponent, therefore ensuring that the opponent will never do better than them.

Press and Dyson's suggested ZD strategies were the dominant family of strategies in the IPD. Moreover, they argued that memory is not beneficial. In [10, 89, 96, 95, 97, 113, 114, 123, 189] the effectiveness of ZD strategies is questioned. In [10], it was shown that ZD strategies are not evolutionary stable, and in [189] a more generous set of ZDs, the *Generous ZD*, were shown to outperform the more extortionate ZDs. Finally, in [89, 113, 114, 123], the 'memory does not benefit a strategy' statement was questioned. A set of more complex strategies, strategies that take in account the entire history set of the game, were trained and proven to be more stable than ZD strategies.

This section covered the original computer tournaments of Axelrod and the early success of Tit For Tat in these tournaments. Though Tit For Tat was considered to be the most robust basic strategy, reciprocity was found to not be enough in environments with uncertainties. There are at least two properties, that have been discussed in this section, for coping with such uncertainties; generosity and contrition. Generosity is letting a percentage of defections go unpunished, and contrition is lowering a strategy's readiness to defect following an opponent's defection.

In the later part of this section a series of new strategies which were built on the basic reciprocal approaches were presented, followed by the infamous memory one strategies, the zero-determinant strategies. Though the ZDs can be proven to be robust in pairwise interactions they were found to be lacking in evolutionary settings and in computer tournaments. Evolutionary settings and the emergence of cooperation under natural selection are covered in the next section.

2.4 Evolutionary dynamics

As yet, the emergence of cooperation has been discussed in the contexts of the one shot PD game and the IPD round robin tournaments. In the PD it is proven that cooperation will not emerge, furthermore, in a series of influential works Axelrod demonstrated that reciprocal behaviour favours cooperation when individuals interact repeatedly. But does natural selection favours cooperation? Understanding the conditions under which natural selection can favour cooperative behaviour is important in understanding social behaviour amongst intelligent agents [47].

Imagine a mixed population of cooperators and defectors where every time two individuals meet they play a game of PD. In such population the average payoff for defectors is always higher than cooperators. Under natural selection the frequency of defectors will steadily increase until cooperators become extinct. Thus natural selection favours defection in the PD (Figure 2.1). However, there are several mechanisms that allow the emergence of cooperation in an evolutionary context which will be covered in this section.

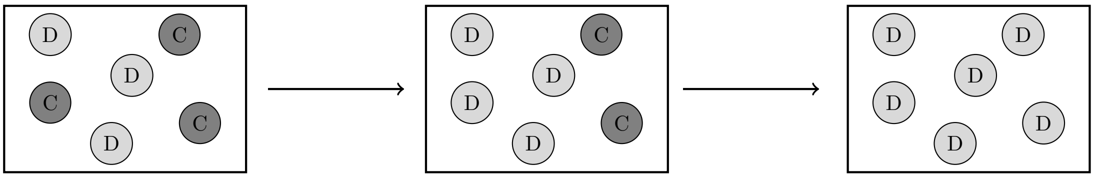

Figure 2.1: Natural selection favours defection in a mixed population of Cooperators and Defectors.

In the later sections of [29], Axelrod discusses an ecological tournament that he performed using the 62 strategies of the second tournament to understand the reproductive success of Tit for Tat. In his ecological tournament the prevalence of each type of strategy in each round was determined by that strategy's success in the previous round. The competition in each round would become stronger as weaker performers were reduced and eliminated. The ecological simulation concluded with a handful of nice strategies dominating the population whilst exploitative strategies had died off as weaker strategies were becoming extinct. This new result led Axelrod to study the IPD in an evolutionary context based on several of the approaches established by the biologist John M. Smith [185, 186, 187]. John M. Smith was a fundamental figure in evolutionary game theory and a participant of Axelrod's second tournament. Axelrod and the biologist William Donald Hamilton wrote about the biological applications of the evolutionary dynamics of the IPD [34] and won the Newcomb-Cleveland prize of the American Association for the Advancement of Science.

In Axelrod's model [30] pairs of individuals from a population played the IPD. The number of interactions between the pairs were not fixed, but there was a probability defined as the importance of the future of the game w, where $0 < w < 1$, that the pair would interact again. In [30] it was shown that for a sufficient high w Tit For Tat strategies would become common and remain common because they were "collectively stable". Axelrod argued that collective stability implied evolutionary stability (ESS) and that when a collectively stable strategy is common in a population and individuals are paired randomly, no other rare strategy can invade. However, Boyd and Lorderbaum in [47] proved that if w, the importance of the future of the game, is large enough then no pure strategy is ESS because it can always be invaded by any pair of other strategies. This was also independently proven in [169].

All these conclusions were made in populations where the individuals could all interact with each other. In 1992, Nowak and May, considered a structured population where an individual's interactions were limited to its neighbours. More specifically, in [132] they explored how local interaction alone can facilitate population wide cooperation in a one shot PD game. The two deterministic strategies Defector and Cooperator, were placed onto a two dimensional square array where the individuals could interact only with the immediate neighbours. The number of immediate neighbours could be either, fourth, six or eight, as shown in Figure 2.2, where each node represents a player and the edges denote whether two players will interact. This topology is refereed to as spatial topology. Each cell of the lattice is occupied by a Cooperator or a Defector and at each generation step each cell owner interacts with its immediate neighbours. The score of each player is calculated as the sum of all the scores the player achieved at each generation. At the start of the next generation, each lattice cell is occupied by the player with the highest score among the previous owner and their immediate neighbours.

Local interactions proved that as long as small clusters of cooperators form, where they can benefit from interactions with other cooperators while avoiding interactions with defectors, global cooperation will continue. Thus, local interactions proved that even for the PD cooperation can emerge. Moreover in [164], whilst using the donation game (Eq (**??**)), it was shown that cooperation will evolve in a structured population as long as the benefit to cost ratio b/c is higher than the number of neighbours. In [211], graphs were a probability of rewiring ones connections was considered were studied. The rewire could be with any given node in the graphs and not just with immediate neighbours. Perc et al. concluded that "making new friends" may be an important activity for the successful evolution of cooperation, but also they must be selected carefully and one should keep their number limited.

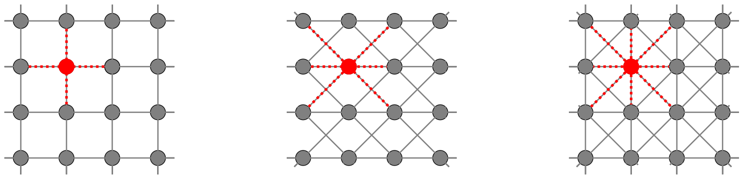

Figure 2.2: Spatial neighbourhoods

Another approach for increasing the likelihood of cooperation by increasing of assortative interactions among cooperative agents, include partner identification methods such as reputation [104, 160, 193], communication tokens [144] and tags [53, 88, 144, 174].

In this section evolutionary dynamics and the emergence of cooperation were reviewed. The following section focuses on strategy archetypes, training methods and strategies obtained from training.

2.5 Structured strategies and training

This section covers strategies that are different to that of intelligent design discussed in Section 2.3. These are strategies that have been through a *training* process using generic strategy archetypes. For example, in [31] Axelrod decided to explore deterministic strategies that took into account the last 3 turns of the game. As discussed in Section 2.3.1, for each turn there are 4 possible outcomes, CC, CD, DC, DD, thus for 3 turns there are a total of $4 \times 4 \times 4 = 64$ possible combinations. Therefore, the strategy can be defined by a series of 64 C's/D's, corresponding to each combination; this type of strategy is called a lookup table. This lookup table was then trained using a genetic algorithm [118]. During the training process random changes are made to a given lookup table. If the utility of the strategy has increased this change is kept, otherwise not.

In 1996 John Miller considered finite state automata as an archetype [143], more specifically, Moore machines [149]. He used a genetic algorithm to train finite state machines in environments with noise. Miller's results showed that even a small difference in noise (from 1% to 3%) significantly changed the characteristics of the evolving strategies. The strategies he introduced were *Punish Twice*, *Punish Once for Two Tats* and *Punish Twice and Wait*. In [24] finite state automata and genetic algorithms were also used to introduce new strategies. In a series of experiments where the size of the population varied, there were two strategies frequently developed by the training process and more over they were developed only after the evolution had gone on for many generations. These were *Fortess3* and *Fortess4*. Following Miller's work in 1996, the first structured strategies based on neural networks that had be trained using a genetic algorithm was introduced in [91] by Harrald and Fogel. Harrald and Fogel considered a single layered neural network which had 6 inputs. These were the last 3 moves of the player and the opponent, similar to [31]. Neural networks have broadly been used to train IPD strategies since then with genetic algorithms [22, 54, 137] and particle swarm optimization [70].

In [89, 113] both genetic algorithm and particle swarm optimization were used to introduce a series of structured strategies based on lookup tables, finite state machines, neural networks, hidden Markov models [65] and Gambler. Hidden Markov models, are a stochastic variant of a finite state machine and Gamblers are stochastic variants of lookup tables. The structured strategies that arised from the training were put up against a large number of strategies in (1) a Moran process, which is an evolutionary model of invasion and resistance across time during which high performing individuals are more likely to be replicated and (2) a round robin tournament. In a round robin tournament which was simulated using the software [8] and the 200 strategies implemented within the software, the top spots were dominated by the trained strategies of all the archetypes. The top three strategies were *Evolved LookUp 2 2 2*, *Evolved HMM 5* and *Evolved FSM 16*.

In [113] it was demonstrated that these trained strategies would overtake the population in a Moran process. The strategies evolved an ability to recognise themselves by using a handshake. This recognition mechanism allowed the strategies to resist invasion by increasing the interactions between themselves, an approach similar to the one described in Section 2.4.

Throughout the different methods of training that have been discussed in this section, a spectrum of structured strategies can be found. Differentiating between strategies is not always straightforward. It is not obvious looking at a finite state diagram how a machine will behave, and many different machines, or neural networks can represent the same strategy. For example Figure 2.3 shows two finite automata and both are a representation of Tit for Tat.

To allow for identification of similar strategies a method called fingerprinting was introduced in [18]. The method of fingerprinting is a technique for generating a functional signature for a strategy [19]. This is achieved by computing the score of a strategy against a spectrum of opponents. The basic method is to play the strategy against a probe strategy with varying noise parameters. In [18] Tit for Tat is used as the probe strategy. In Figure 2.4 an example of Pavlov's fingerprint is given. Fingerprinting has been studied in depth in [19, 20, 21, 22]. Another type of fingerprinting is the transitive fingerprint [8]. The method represents the

(a) Tit for Tat as a finite state machine with 1 state.

(b) Tit for Tat as a finite state machine with 2 states.

Figure 2.3: Finite state machine representations of Tit for Tat. A machine consists of transition arrows associated with the states. Each arrow is labelled with A/R where A is the opponent's last action and R is the player's response. Finite state machines consist of a set of internal states. In (a) Tit for Tat finite state machine consists of 1 state and in (b) of 2.

cooperation rate of a strategy against a set of opponents over a number of turns. An example of a transitive fingerprint is given in Figure 2.5.

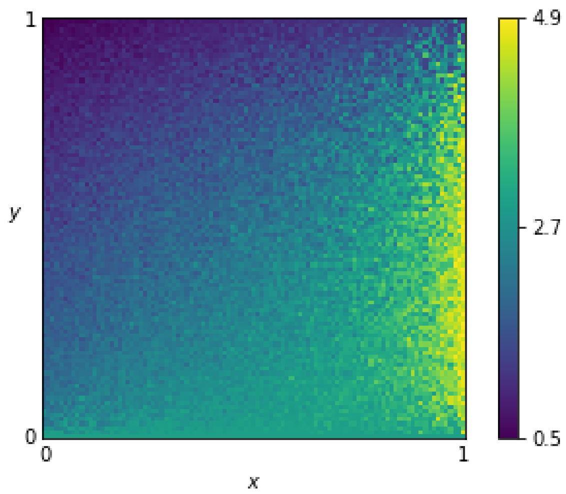

Figure 2.4: Pavlov fingerprinting with Tit for Tat used as the probe strategy. Figure was generated using [8].

This section covered structured strategies and training methods. In the following section software that has been developed with main aim simulating the IPD is presented.

2.6 Software

The research of the IPD heavily relies on software. This is to be expected as computer tournaments have become the main means of simulating the interactions in an IPD game. Many academic fields suffer from lack of source code availability and the IPD is not an exception. Several of the tournaments that have been discussed so far were generated using computer code, though not all of the source code is available. The code for Axelrod's original tournament is known to be lost and moreover for the second tournament the only source code available is the code for the 62 strategies (found on Axelrod's personal website [1]).

Several projects, however, are open, available and have been used as research tools or educational platforms over the years. Two research tools [4, 8] and two educational tools [2, 3] are

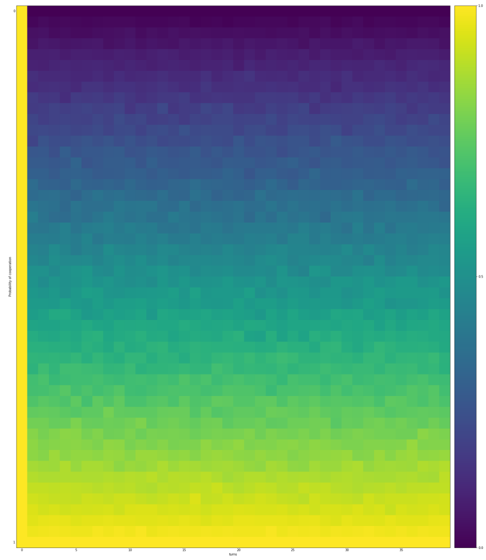

Figure 2.5: Transitive fingerprint of Tit for Tat against a set of 50 random opponents.

briefly mentioned here. Both [4, 8] are open source projects. The "Game of Trust" [2] is an on-line, graphical user interface educational platform for learning the basics of game theory, the IPD and the notion of strategies. It attracted a lot of attention due to being "well-presented with scribble-y hand drawn characters" [98] and "a whole heap of fun" [112]. Finally [3] is a personal project written in PHP. It is a graphical user interface that offers a big collection of strategies and allows the user to try several matches and tournament configurations.

PRISON [4] is written in the programming language Java and a preliminary version was launched on 1998. It was used by its authors in several publications, such as [37], which introduced Gradual, and [38]. The project includes a good number of strategies from the literature but unfortunately the last update of the project dates back to 2004. Axelrod-Python [8] is a software used by [89, 113, 83, 205]. It is written in the programming language Python following best practice approaches [9, 42] and contains the largest collection of strategies, known to the author. The strategy list of the project has been cited by publications [13, 94, 153].

2.7 Summary

This Chapter presented a literature review on the Iterated Prisoner's Dilemma. The opening sections focused on research trends and published works of the field, followed by a presentation of research and educational software. More specifically, Section 2.2 covered the early years of research. This was when simulating turns of the game was only possible with human subject research. Following the early years, the pioneering tournaments of Axelrod were introduced in Section 2.3. Axelrod's work offered the field an agent based game theoretic framework to study the IPD. In his original papers he asked researchers to design strategies to test their performance with the new framework. The winning strategy of both his tournaments was Tit for Tat. The strategy however came with limitations which were explored by other researchers, and new intelligently designed strategies were introduced in order to surpass Tit for Tat with some contributions such as Pavlov and Gradual.

Soon researchers came to realise that strategies should not just do well in a tournament setting but should also be evolutionary robust. Evolutionary dynamic methods were applied to many works in the field, and factors under which cooperation emerges were explored, as described in Section 2.4. This was not done only for unstructured populations, where all strategies in the population can interact with each other, but also in population where interactions were limited to only strategies that were close to each other. In such topologies it was proven that even in the one shot game, cooperation can indeed emerge.

Evolutionary approaches can offer many insights in the study of the PD. In evolutionary settings strategies can learn to adapt and take over population by adjusting their actions; such algorithms can be applied so that evolutionarily robust strategies can emerge. Algorithms and structures used to train strategies in the literature were covered in Section 2.5. From these training methods several strategies are found, and to be able to differentiate between them fingerprinting was introduced. The research of best play and cooperation has been going on since the 1950s, and for simulating the game software has been developed along the way. This software has been briefly discussed in Section 2.6.

The study of the PD is still an ongoing field research where new variants and new structures of strategies are continuously being explored [163]. The game now serves as a model in a wide range of applications, for example in medicine and the study of cancer cells [14, 109], as well as in social situations and how they can be driven by rewards [63]. New research is still ongoing for example in evolutionarily dynamics on graphs [12, 93, 130].

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