Understanding responses to environments for the Prisoner's Dilemma: A machine learning approach

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A systematic literature review of the Prisoner's Dilemma.

The Prisoner's Dilemma is a well known game used since the 1950's as a framework for studying the emergence of cooperation; a topic of continuing interest for mathematical, social, biological and ecological sciences. The iterated version of the game, the Iterated Prisoner's Dilemma, attracted attention in the 1980's after the publication of the "The Evolution of Cooperation" and has been a topic of pioneering research ever since. The aim of this paper is to provide a systematic literature review on Prisoner's Dilemma related research. This is achieved by reviewing selected pieces of work and partition the literature into five different sections with each reviewing a different aspect of research. The questions answered in this manuscript are (1) what are the research trends in the field (2) what are the already existing results within the field.

2.1 Introduction

Based on the Darwinian principle of survival of the fittest cooperative behaviour should not be favoured, however, cooperation is plentiful in nature. A paradigm of understanding the emergence of these behaviours is a particular two player non-cooperative game called the Prisoner's Dilemma (PD), originally described in [64].

In the PD each player has two choices, to either be selfless and cooperate or to be selfish and defect. Each decision is made simultaneously and independently. The utility of each player is influenced by its own behaviour, and the behaviour of the opponent. Both players do better if they choose to cooperate than if both choose to defect. However, a player has the temptation to deviate as that player will receive a higher payoff than that of mutual cooperation. Players' payoffs are generally represented by (2.1). Both players receive a reward for mutual cooperation, R, and a payoff P for mutual defection. A player that defects while the other cooperates receives a payoff of T, whereas the cooperator receives S. The dilemma exists due to constraints (2.2) and (2.3).

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} \tag{2.1}$$

$$T > R > P > S \tag{2.2}$$

$$2R > T + S \tag{2.3}$$

Another common representation of the payoff matrix is given by (2.4), where b is the benefit of the altruistic behaviour and c it's its cost (constraints (2.2) and (2.3) still hold).

$$\begin{pmatrix} b-c & c \\ b & 0 \end{pmatrix} \tag{2.4}$$

Constraints (2.2-2.3) guarantee that it never benefits a player to cooperate, indeed mutual defection is a Nash equilibrium. However, when the game is studied in a manner where prior outcome matters, defecting is no longer necessarily the dominant choice.

The repeated form of the game is called the Iterated Prisoner's Dilemma (IPD) and theoretical works have shown that cooperation can emerge once players interact repeatedly. Arguably, the most important of these works is Robert Axelrod's "The Evolution of Cooperation" [33]. In his book Axelrod reports on a series of computer tournaments he organised. In these tournaments academics from several fields were invited to design computer strategies to compete. Axelrod's work showed that greedy strategies did very poorly in the long run whereas altruistic strategies did better. "The Evolution of Cooperation" is considered a milestone in the field but it is not the only one. On the contrary, the PD has attracted attention ever since the game's origins.

This manuscript presents a qualitative description of selected pieces of work. These have been separated into five sections, each reviewing a different aspect of research. The topics reviewed at each section are the following:

- section 2.2, **Origins of the Prisoner's Dilemma**.
- section 2.3, **Axelrod's tournaments and intelligent design of strategies**.
- section 2.4, **Evolutionary dynamics**
- section 2.5, **Structured strategies and training**.
- section 2.6, **Software**.

The aim of this work is to provide a concrete summary of the existing literature on the PD. This is done to provide a review which will allow the research community to understand overall trends in the field, and already existing results.

2.2 Origins of the prisoner's dilemma

The origin of the PD goes back to the 1950s in early experiments conducted at RAND [64] to test the applicability of games described in [143]. The game received its name later the same year. According to [186], Albert W. Tucker (the PhD supervisor of John Nash [142]), in an attempt to deliver the game with a story during a talk described the players as prisoners and the game has been known as the Prisoner's Dilemma ever since.

The early research on the IPD was limited. The only source of experimental results was through human subject research where pairs of participants simulated plays of the game. Human subject research had disadvantages. Humans could behave randomly and in several experiments both the size and the background of the individuals were different, thus comparing results of two or more studies became difficult.

The main aim of these early research experiments was to understand how conditions such as the gender of the participants [62, 123, 126], the physical distance between the participants [175], the effect of their opening moves [185] and even how the experimenter, by varying the tone of their voice and facial expressions [68], could influence the outcomes and subsequently the emergence of cooperation. An early figure that sought out to understand several of these conditions was the mathematical psychologist Anatol Rapoport. The results of his work are summarised in [163].

Rapoport was also interested in conceptualising strategies that could promote international cooperation. Decades later he would submit the winning strategy (Tit for Tat) of the first computer tournament, run by Robert Axelrod. In the next section these tournaments, and several strategies that were designed by researchers, such as Rapoport, are introduced.

2.3 Axelrod's tournaments and intelligently designed strategies

As discussed in Section 2.2, before 1980 a great deal of research was done in the field, however, as described in [31], the political scientist Robert Axelrod believed that there was no clear answer to the question of how to avoid conflict, or even how an individual should play the game. Combining his interest in artificial intelligence and political science Axelrod created a framework for exploring these questions using computer tournaments. Axelrod asked researchers to design a strategy with the purpose of wining an IPD tournament. This section covers Axelrod's original tournaments as well as research that introduced new intelligently designed strategies.

Axelrod's tournaments made the study of cooperation of critical interest. As described in [164], "Axelrod's "new approach" has been extremely successful and immensely influential in casting light on the conflict between an individual and the collective rationality reflected in the choices of a population whose members are unknown and its size unspecified, thereby opening a new avenue of research". In a collaboration with a colleague, Douglas Dion, Axelrod in [32] summarized a number of works that were immediately inspired from the "Evolution of Cooperation", and [100] gives a review of tournaments that have been conducted since the originals.

The first reported computer tournament took place in 1980 [27]. A total of 13 strategies were submitted, written in the programming languages Fortran or Basic. Each competed in a 200 turn match against all 12 opponents, itself and a player that played randomly (called **Random**). This type of tournament is referred to as a round robin. The tournament was repeated 5 times to get a more stable estimate of the scores for each pair of play. Each participant knew the exact number of turns and had access to the full history of each match. Furthermore, Axelrod performed a preliminary tournament and the results were known to the participants. This preliminary tournament is mentioned in [27] but no details were given. The payoff values used for equation (2.1) were $R = 3, P = 1, T = 5$ and $S = 0$. These values are commonly used in the literature and unless specified will be the values used in the rest of the works described here.

The winner of the tournament was determined by the total average score and not by the number of matches won. The strategy that was announced the winner was the strategy submitted by Rapoport, **Tit For Tat**. The success of Tit for Tat came as a surprise. It was not only the simplest submitted strategy, it would always cooperates on the first round and then mimic the opponent's previous move, but it had also won the tournament even though it could never beat any player it was interacting with.

In order to further test the results Axelrod performed a second tournament in 1980 [28]. The second tournament received much more attention and had a total of 62 entries. The participants knew the results of the previous tournament and the rules were similar with only a few alterations. The tournament was repeated 5 times and the length of each match was not known to the participants. Axelrod intended to use a fixed probability (refereed to as 'shadow of the future' [32]) of the game ending on the next move. However, 5 different number of turns were selected for each match 63, 77, 151, 308 and 401, such that the average length would be around 200 turns.

Nine of the original participants competed again in the second tournament. Two strategies that remained the same were Tit For Tat and **Grudger**. Grudger is a strategy that will cooperate as long as the opponent does not defect, submitted by James W. Friedman. The name Grudger was give to the strategy in [118], though the strategy goes by many names in the literature such as, Spite [36], Grim Trigger [35] and Grim [190]. New entries in the second tournament included **Tit for Two Tats** submitted by John Maynard Smith and **KPavlovC**. KPavlovC, is also known as Simpleton [163], introduced by Rapoport or just Pavlov [147]. The strategy is based on the fundamental behavioural mechanism win-stay, lose-shift. Pavlov is heavily studied in the literature and similarly to Tit for Tat it is used in tournaments today and has had many variants trying to build upon it's success, for example **PavlovD** and **Adaptive Pavlov** [114].

Despite the larger size of the second tournament none of the new entries managed to outperform the simpler designed strategy. The winner was once again Tit for Tat. Axelrod deduced the following guidelines for a strategy to perform well:

- The strategy would start of by cooperating.
- It would forgive it's opponent after a defection.
- It would always be provoked by a defection no matter the history.
- It was simple.

The success of Tit for Tat, however, was not unquestionable. Several papers showed that stochastic uncertainties severely undercut the effectiveness of reciprocating strategies and such stochastic uncertainties have to be expected in real life situations [134]. For example, in [138] it is proven that in an environment where **noise** (a probability that a player's move will be flipped) is introduced two strategies playing Tit for Tat receive the same average payoff as two Random players. Hammerstein, pointed out that if by mistake, one of two Tit for Tat players makes a wrong move, this locks the two opponents into a hopeless sequence of alternating defections and cooperations [174].

The poor performance of the strategy in noisy environments was also demonstrated in tournaments. In [39, 57] round robin tournaments with noise were performed, and Tit For Tat did not win. The authors concluded that to overcome the noise more generous strategies than Tit For Tat were needed. They introduced the strategies **Nice and Forgiving** and **OmegaTFT** respectively.

A second type of stochastic uncertainty is misperception, where a player's action is made correctly but it is recorded incorrectly by the opponent. In [195], a strategy called **Contrite Tit for Tat** was introduced that was more successful than Tit for Tat in such environments. The difference between the strategies was that Contrite Tit for Tat was not so fast to retaliate against a defection.

Several works extended the reciprocity based approach which has led to new strategies. For example Gradual [36] which was constructed to have the same qualities as those of Tit for Tat except one, **Gradual** had a memory of the game since the beginning of it. Gradual recorded the number of defections by the opponent and punished them with a growing number of defections. It would then enter a calming state in which it would cooperates for two rounds. In a tournament of 12 strategies, including both Tit for Tat and Pavlov, Gradual managed to outperformed them all. A strategy with the same intuition as Gradual is **Adaptive Tit for Tat** [188]. Adaptive Tit for Tat does not keep a permanent count of past defections, it maintains a continually updated estimate of the opponent's behaviour, and uses this estimate to condition its future actions. In the exact same tournament as in [36] with now 13 strategies Adaptive Tit for Tat ranked first.

Another extension of strategies was that of teams of strategies [55, 56, 169] that collude to increase one member's score. In 2004 Graham Kendall led the Anniversary Iterated Prisoner's Dilemma Tournament with a total of 223 entries. In this tournament participants were allowed to submit multiple strategies. A team from the University of Southampton submitted a total of 60 strategies [169]. All these were strategies that had been programmed with a recognition mechanism by default. Once the strategies recognised one another, one would act as leader and the other as a follower. The follower plays as a **Cooperator**, cooperates unconditionally and the leader would play as a **Defector** gaining the highest achievable score. The followers would defect unconditionally against other strategies to lower their score and help the leader. The result was that Southampton had the top three performers. Nick Jennings, who was part of the team, said that "We developed ways of looking at the Prisoner's Dilemma in a more realistic environment and we devised a way for computer agents to recognise and collude with one another despite the noise. Our solution beats the standard Tit For Tat strategy" [153].

2.3.1 Memory one Strategies

A set of strategies that have received a lot of attention in the literature are **memory one** strategies. In [148], Nowak and Sigmund proposed a structure for studying simple strategies that remembered only the previous turn, and moreover, only recorded the move of the opponent. These are called **reactive** strategies and they can be represented by using three parameters (y, p_1, p_2), where y is the probability to cooperate in the first move, and p_1 and p_2 the conditional probabilities to cooperate, given that the opponent's last move was a cooperation or a defection. For example Tit For Tat is a reactive strategy and it can be written as $(1, 1, 0)$. Another reactive strategy well known in the literature is **Generous Tit for Tat** [150].

In [149], Nowak and Sigmund extended their work to include strategies which consider the entire history of the previous turn to make a decision. These are called **memory one** strategies. If only a single turn of the game is taken into account and depending on the simultaneous moves of the two players there are only four possible states that the players could be in. These are:

- Both players cooperated, denoted as CC.
- First player cooperated while the second one defected, denoted as CD.
- First player defected while the second one cooperated, denoted as DC.
- Both players defected, denoted as DD.

Thus a memory one strategy can be denoted by the probabilities of cooperating after each state and the probability of cooperating in the first round, (y, p_1, p_2, p_3, p_4). For example Pavlov's memory one representation is $(1, 1, 0, 0, 1)$.

Memory one strategies made an impact when a specific set of memory one strategies were introduced called **Zero-determinant** (ZD) [159]. The American Mathematical Society's news section [91] stated that "the world of game theory is currently on fire" and in [180] it was stated that "Press and Dyson have fundamentally changed the viewpoint on the Prisoner's Dilemma". ZD are a set of extortionate strategies that can force a linear relationship between the long-run scores of both themselves and the opponent, therefore ensuring that the opponent will never do better than them.

Press and Dyson's suggested ZD strategies were the dominant family of strategies in the IPD. Moreover, they argued that memory is not beneficial. In [10, 83, 90, 89, 91, 104, 105, 113, 180] the effectiveness of ZD strategies is questioned. In [10], it was shown that ZD strategies are not evolutionary stable, and in [180] a more generous set of ZDs, the **Generous ZD**, were shown to outperform the more extortionate ZDs. Finally, in [83, 104, 105, 113], the 'memory does not benefit a strategy' statement was questioned. A set of more complex strategies, strategies that take in account the entire history set of the game, were trained and proven to be more stable than ZD strategies.

This section covered the original computer tournaments of Axelrod and the early success of Tit For Tat in these tournaments. Though Tit For Tat was considered to be the most robust basic strategy, reciprocity was found to not be enough in environments with uncertainties. There are at least two properties, that have been discussed in this section, for coping with such uncertainties; generosity and contrition. Generosity is letting a percentage of defections go unpunished, and contrition is lowering a strategy's readiness to defect following an opponent's defection.

In the later part of this section a series of new strategies which were built on the basic reciprocal approaches were presented, followed by the infamous memory one strategies, the zero-determinant strategies. Though the ZDs can be proven to be robust in pairwise interactions they were found to be lacking in evolutionary settings and in computer tournaments. Evolutionary settings and the emergence of cooperation under natural selection are covered in the next section.

2.4 Evolutionary dynamics

As yet, the emergence of cooperation has been discussed in the contexts of the one shot PD game and the IPD round robin tournaments. In the PD it is proven that cooperation will not emerge, furthermore, in a series of influential works Axelrod demonstrated that reciprocal behaviour favours cooperation when individuals interact repeatedly. But does natural selection favours cooperation? Understanding the conditions under which natural selection can favour cooperative behaviour is important in understanding social behaviour amongst intelligent agents [44].

Imagine a mixed population of cooperators and defectors where every time two individuals meet they play a game of PD. In such population the average payoff for defectors is always higher than cooperators. Under natural selection the frequency of defectors will steadily increase until cooperators become extinct. Thus natural selection favours defection in the PD (Figure 2.1). However, there are several mechanisms that allow the emergence of cooperation in an evolutionary context which will be covered in this section.

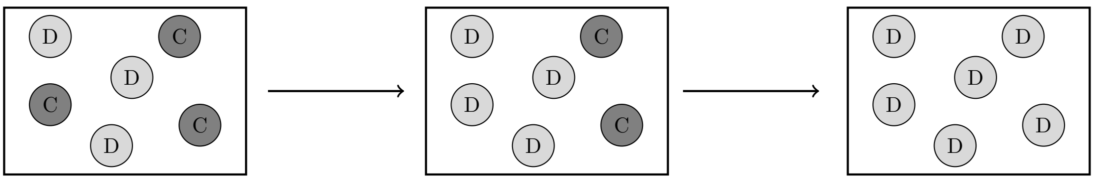


Figure 2.1: Natural selection favours defection in a mixed population of Cooperators and Defectors.

In the later sections of [28], Axelrod discusses an ecological tournament that he performed using the 62 strategies of the second tournament to understand the reproductive success of Tit for Tat. In his ecological tournament the prevalence of each type of strategy in each round was determined by that strategy's success in the previous round. The competition in each round would become stronger as weaker performers were reduced and eliminated. The ecological simulation concluded with a handful of nice strategies dominating the population whilst exploitative strategies had died off as weaker strategies were becoming extinct. This new result led Axelrod to study the IPD in an evolutionary context based on several of the approaches established by the biologist John M. Smith [177, 178, 179]. John M. Smith was a fundamental figure in evolutionary game theory and a participant of Axelrod's second tournament. Axelrod and the biologist William Donald Hamilton wrote about the biological applications of the evolutionary dynamics of the IPD [33] and won the Newcomb-Cleveland prize of the American

Association for the Advancement of Science.

In Axelrod's model [29] pairs of individuals from a population played the IPD. The number of interactions between the pairs were not fixed, but there was a probability defined as the importance of the future of the game w, where $0 < w < 1$, that the pair would interact again. In [29] it was shown that for a sufficient high w Tit For Tat strategies would become common and remain common because they were "collectively stable". Axelrod argued that collective stability implied evolutionary stability (ESS) and that when a collectively stable strategy is common in a population and individuals are paired randomly, no other rare strategy can invade. However, Boyd and Lorderbaum in [44] proved that if w, the importance of the future of the game, is large enough then no pure strategy is ESS because it can always be invaded by any pair of other strategies. This was also independently proven in [161].

All these conclusions were made in populations where the individuals could all interact with each other. In 1992, Nowak and May, considered a structured population where an individual's interactions were limited to its neighbours. More specifically, in [124] they explored how local interaction alone can facilitate population wide cooperation in a one shot PD game. The two deterministic strategies Defector and Cooperator, were placed onto a two dimensional square array where the individuals could interact only with the immediate neighbours. The number of immediate neighbours could be either, fourth, six or eight, as shown in Figure 2.2, where each node represents a player and the edges denote whether two players will interact. This topology is refereed to as spatial topology. Each cell of the lattice is occupied by a Cooperator or a Defector and at each generation step each cell owner interacts with its immediate neighbours. The score of each player is calculated as the sum of all the scores the player achieved at each generation. At the start of the next generation, each lattice cell is occupied by the player with the highest score among the previous owner and their immediate neighbours.

Local interactions proved that as long as small clusters of cooperators form, where they can benefit from interactions with other cooperators while avoiding interactions with defectors, global cooperation will continue. Thus, local interactions proved that even for the PD cooperation can emerge. Moreover in [155], whilst using the payoff matrix (2.4), it was shown that cooperation will evolve in a structured population as long as the benefit to cost ratio b/c is higher than the number of neighbours. In [199], graphs were a probability of rewiring ones connections was considered were studied. The rewire could be with any given node in the graphs and not just with immediate neighbours. Perc et al. concluded that "making new friends" may be an important activity for the successful evolution of cooperation, but also they must be selected carefully and one should keep their number limited.

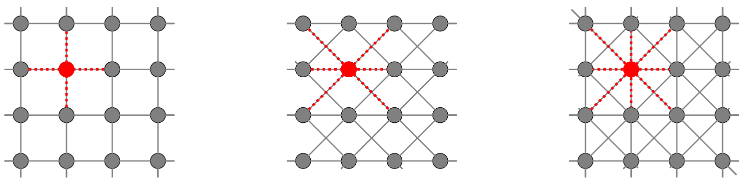


Figure 2.2: Spatial neighbourhoods

Another approach for increasing the likelihood of cooperation by increasing of assortative interactions among cooperative agents, include partner identification methods such as reputation [98, 151, 183], communication tokens [136] and tags [49, 82, 136, 166].

In this section evolutionary dynamics and the emergence of cooperation were reviewed. The following section focuses on strategy archetypes, training methods and strategies obtained from training.

2.5 Structured strategies and training

This section covers strategies that are different to that of intelligent design discussed in Section 2.3. These are strategies that have been **trained** using generic strategy archetypes. For example, in [30] Axelrod decided to explore deterministic strategies that took into account the last 3 turns of the game. As discussed in Section 2.3.1, for each turn there are 4 possible outcomes, CC, CD, DC, DD, thus for 3 turns there are a total of $4 \times 4 \times 4 = 64$ possible combinations. Therefore, the strategy can be defined by a series of 64 C's/D's, corresponding to each combination; this type of strategy is called a lookup table. This lookup table was then trained using a genetic algorithm [108]. During the training process random changes are made to a given lookup table. If the utility of the strategy has increased this change is kept, otherwise not.

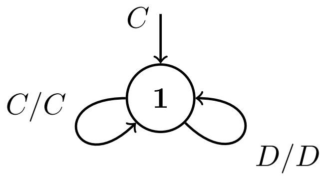
In 1996 John Miller considered finite state automata as an archetype [135], more specifically, Moore machines [140]. He used a genetic algorithm to train finite state machines in environments with noise. Miller's results showed that even a small difference in noise (from 1% to 3%) significantly changed the characteristics of the evolving strategies. The strategies he introduced were **Punish Twice**, **Punish Once for Two Tats** and **Punish Twice and Wait**. In [24] finite state automata and genetic algorithms were also used to introduce new strategies. In a series of experiments where the size of the population varied, there were two strategies frequently developed by the training process and more over they were developed only after the evolution had gone on for many generations. These were **Fortess3** and **Fortess4**. Following Miller's work in 1996, the first structured strategies based on neural networks that had be trained using a genetic algorithm was introduced in [85] by Harrald and Fogel. Harrald and Fogel considered a single layered neural network which had 6 inputs. These were the last 3 moves of the player and the opponent, similar to [30]. Neural networks have broadly been used to train IPD strategies since then with genetic algorithms [22, 50, 129] and particle swarm optimization [65].

In [83, 104] both genetic algorithm and particle swarm optimization were used to introduce a series of structured strategies based on lookup tables, finite state machines, neural networks, hidden Markov models [60] and Gambler. Hidden Markov models, are a stochastic variant of a finite state machine and Gamblers are stochastic variants of lookup tables. The structured strategies that arised from the training were put up against a large number of strategies in (1) a Moran process, which is an evolutionary model of invasion and resistance across time during which high performing individuals are more likely to be replicated and (2) a round robin tournament. In a round robin tournament which was simulated using the software [7] and the 200 strategies implemented within the software, the top spots were dominated by the trained strategies of all the archetypes. The top three strategies were **Evolved LookUp 2 2**

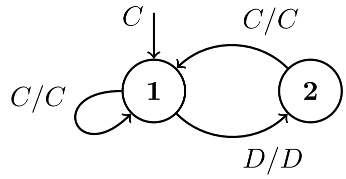
2, **Evolved HMM 5** and **Evolved FSM 16**.

In [104] it was demonstrated that these trained strategies would overtake the population in a Moran process. The strategies evolved an ability to recognise themselves by using a handshake. This recognition mechanism allowed the strategies to resist invasion by increasing the interactions between themselves, an approach similar to the one described in Section 2.4.

Throughout the different methods of training that have been discussed in this section, a spectrum of structured strategies can be found. Differentiating between strategies is not always straightforward. It is not obvious looking at a finite state diagram how a machine will behave, and many different machines, or neural networks can represent the same strategy. For example Figure 2.3 shows two finite automata and both are a representation of Tit for Tat.



(a) Tit for Tat as a finite state machine with 1 state.



(b) Tit for Tat as a finite state machine with 2 states.

Figure 2.3: Finite state machine representations of Tit for Tat. A machine consists of transition arrows associated with the states. Each arrow is labelled with A/R where A is the opponent's last action and R is the player's response. Finite state machines consist of a set of internal states. In (a) Tit for Tat finite state machine consists of 1 state and in (b) of 2.

To allow for identification of similar strategies a method called fingerprinting was introduced in [18]. The method of fingerprinting is a technique for generating a functional signature for a strategy [19]. This is achieved by computing the score of a strategy against a spectrum of opponents. The basic method is to play the strategy against a probe strategy with varying noise parameters. In [18] Tit for Tat is used as the probe strategy. In Figure 2.4 an example of Pavlov's fingerprint is given. Fingerprinting has been studied in depth in [19, 20, 21, 22]. Another type of fingerprinting is the transitive fingerprint [7]. The method represents the cooperation rate of a strategy against a set of opponents over a number of turns. An example of a transitive fingerprint is given in Figure 2.5.

This section covered structured strategies and training methods. In the following section software that has been developed with main aim simulating the IPD is presented.

2.6 Software

The research of the IPD heavily relies on software. This is to be expected as computer tournaments have become the main means of simulating the interactions in an IPD game. Many academic fields suffer from lack of source code availability and the IPD is not an exception. Several of the tournaments that have been discussed so far were generated using computer code, though not all of the source code is available. The code for Axelrod's original tournament is known to be lost and moreover for the second tournament the only source code available is the code for the 62 strategies (found on Axelrod's personal website [1]).

Several projects, however, are open, available and have been used as research tools or educa-

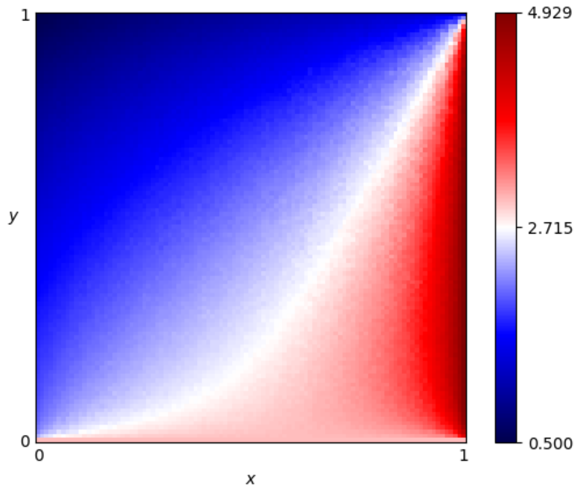


Figure 2.4: Pavlov fingerprinting with Tit for Tat used as the probe strategy. Figure was generated using [7].

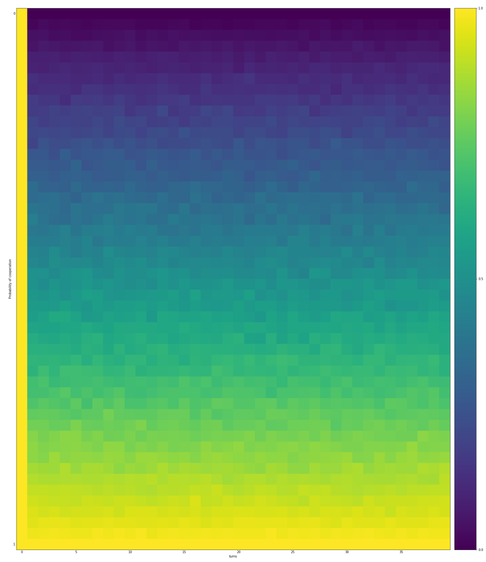


Figure 2.5: Transitive fingerprint of Tit for Tat against a set of 50 random opponents.

tional platforms over the years. Two research tools [4, 7] and two educational tools [2, 3] are briefly mentioned here. Both [4, 7] are open source projects. The "Game of Trust" [2] is an on-line, graphical user interface educational platform for learning the basics of game theory, the IPD and the notion of strategies. It attracted a lot of attention due to being "well-presented with scribble-y hand drawn characters" [92] and "a whole heap of fun" [103]. Finally [3] is a personal project written in PHP. It is a graphical user interface that offers a big collection of strategies and allows the user to try several matches and tournament configurations.

PRISON [4] is written in the programming language Java and a preliminary version was launched on 1998. It was used by its authors in several publications, such as [36], which introduced Gradual, and [37]. The project includes a good number of strategies from the literature but unfortunately the last update of the project dates back to 2004. Axelrod-Python [7] is a software used by [83, 104, 77, 193]. It is written in the programming language Python following best practice approaches [9, 40] and contains the largest collection of strategies, known to the author. The strategy list of the project has been cited by publications [13, 88, 144].

2.7 Conclusion

This manuscript presented a literature review on the Iterated Prisoner's Dilemma. The opening sections focused on research trends and published works of the field, followed by a presentation of research and educational software. More specifically, Section 2.2 covered the early years of research. This was when simulating turns of the game was only possible with human subject research. Following the early years, the pioneering tournaments of Axelrod were introduced in Section 2.3. Axelrod's work offered the field an agent based game theoretic framework to study the IPD. In his original papers he asked researchers to design strategies to test their performance with the new framework. The winning strategy of both his tournaments was Tit for Tat. The strategy however came with limitations which were explored by other researchers, and new intelligently designed strategies were introduced in order to surpass Tit for Tat with some contributions such as Pavlov and Gradual.

Soon researchers came to realise that strategies should not just do well in a tournament setting but should also be evolutionary robust. Evolutionary dynamic methods were applied to many works in the field, and factors under which cooperation emerges were explored, as described in Section 2.4. This was not done only for unstructured populations, where all strategies in the population can interact with each other, but also in population where interactions were limited to only strategies that were close to each other. In such topologies it was proven that even in the one shot game, cooperation can indeed emerge.

Evolutionary approaches can offer many insights in the study of the PD. In evolutionary settings strategies can learn to adapt and take over population by adjusting their actions; such algorithms can be applied so that evolutionarily robust strategies can emerge. Algorithms and structures used to train strategies in the literature were covered in Section 2.5. From these training methods several strategies are found, and to be able to differentiate between them fingerprinting was introduced. The research of best play and cooperation has been going on since the 1950s, and for simulating the game software has been developed along the way. This software has been briefly discussed in Section 2.6.

The study of the PD is still an ongoing field research where new variants and new structures of strategies are continuously being explored [154]. The game now serves as a model in a wide range of applications, for example in medicine and the study of cancer cells [14, 101], as well as in social situations and how they can be driven by rewards [58]. New research is still ongoing for example in evolutionarily dynamics on graphs [12, 87, 122].

Chapter 3

A bibliometric study of research topics, collaboration and influence in the field of the Iterated Prisoner's Dilemma

This manuscript explores the research topics and collaborative behaviour of authors in the field of the Prisoner's Dilemma using topic modelling and a graph theoretic analysis of the co-authorship network. The analysis identified five research topics in the Prisoner's Dilemma which have been relevant of the course of time. These are human subject research, biological studies, strategies, evolutionary dynamics on networks and modelling problems as a Prisoner's Dilemma game. Moreover, the results demonstrated the Prisoner's Dilemma is a field of continued interest, and although it is a collaborative field, it is not necessarily more collaborative than other scientific fields. The co-authorship network suggests that authors are focused on their communities and not many connections across the communities are made. The Prisoner Dilemma authors also do not influence or gain much information by their connections, unless they are connected to a "main" group of authors.

3.1 Introduction

The Prisoner's Dilemma (PD) is a well known game used since its introduction in the 1950's [64] as a framework for studying the emergence of cooperation; a topic of continued interest for mathematical, social [157], biological [187] and ecological [196] sciences. This manuscript presents a bibliometric analysis of 2,420 published articles on the Prisoner's Dilemma between 1951 and 2018. It presents the dominant topics in the PD publications, which have been identified using Latent Dirichlet Allocation [42], and it explores the changes in the dominant topics over time. The collaborative behaviour of the field is explored using the co-authorship network, and furthermore, the Latent Dirichlet Allocation topic analysis is combined with the co-authorship network analysis to assess the relative influence of authors in these topics. Assessing the collaborative behaviour of the field of collaboration itself is the main aim of this work.

As discussed in [198], bibliometrics (the statistical analysis of published works originally described by [160]) has been used to support historical assumptions about the development of fields [162], identify connections between scientific growth and policy changes [53], develop a quantitative understanding of author order [173], and investigate the collaborative structure of an interdisciplinary field [121]. Most academic research is undertaken in the form of collaborative effort and as [111] points out, it is rational that two or more people have the potential to do better as a group than individually. Indeed this is the very premise of the Prisoner's Dilemma itself. Collaboration in groups has a long tradition in experimental sciences and it has be proven to be productive according to [61]. The number of collaborations can be different between research fields and understanding how collaborative a field is not always an easy task. Several studies tend to consider academic citations as a measure for these things. A blog post published by Nature [146] argues that depending on citations can often be misleading because the true number of citations can not be known. Citations can be missed due to data entry errors, academics are influenced by many more papers than they actually cite and several of the citations are superficial.

A more recent approach to measuring collaborative behaviour, and to studying the development of a field is to use the co-authorship network, as described in [121]. The co-authorship network has many advantages as several graph theoretic measures can be used as proxies to explain author relationships. For example the average degree of a node corresponds to the average number of an authors' collaborators, and clustering coefficient corresponds to the extent that two collaborators of an author also collaborate with each other. In [121], the approach was applied to analyse the development of the field "evolution of cooperation", and in [198] to identify the subdisciplines of the interdisciplinary field of "cultural evolution" and investigate trends in collaboration and productivity between these subdisciplines. Moreover, [120] examined the long-term impact of co-authorship with established, highly-cited scientists on the careers of junior researchers. This paper builds upon the work done by [121] and [198], and extends their methodology. In [121, 198], a data set from a single source, Web of Science, is considered whereas the data set described here, archived at [74], has been collected from five sources.

Latent Dirichlet Allocation (LDA) is a topic modelling technique proposed in [42] as a generative probabilistic model for discovering underlying topics in collections of data. Applications of the technique include detection in image data [11, 52] and detection in video [145, 194]. Nevertheless, LDA has been applied by several works on publication data for identifying the topic structure of a subject area. In [96], it was applied to the publications on mathematical education of the journals "Educational Studies in Mathematics" and "Journal for Research in Mathematics Education" to identify the dominant topics that each journal was publishing on. The topics of the North American library and Information Science dissertations were studied chronologically in [182], and the main topic of the scientific content presented at EvoLang conferences was identified in [41]. In [41] the LDA approach is combined with clustering and a co-authorship network analysis. A clustering analysis is applied to the LDA topics, and the co-authorship network is analysed as a whole where the clusters are only used to differentiate between the authors' topics. In comparison, this works applies LDA to identify dominant topics in the Prisoner's Dilemma fields and analyses the networks corresponding to these topics individually.

The methodology used in this manuscript (including the data collection) is covered in Section 3.2 and a preliminary analysis of the data set is presented in Section 3.3. This manuscript makes usage of the methodology and data to address the following questions:

1. What are the research topics of the Prisoner's Dilemma?
2. Is one topic currently more in fashion?
3. How have the research topics changed over the years?
4. Is the Prisoner's Dilemma a collaborative field?
5. Are some topics more collaborative than others?
6. Are there authors which benefit more from their position in the network?

Results regarding questions 1-3 are presented in Section 3.4 and questions 4-6 are addressed in Section 3.5. The results are summarised in Section 4.5.

3.2 Methodology

Academic articles are accessible through scholarly databases. Several databases and collections today offer access through an open application protocol interface (API). An API allows users to query directly a journal's database and bypass the graphical user interface. Interacting with an API has two phases: requesting and receiving. The request phase includes composing a url with the details of the request. For example, `http://export.arxiv.org/api/query?search_query=abs:prisoner'sdilemma&max_results=1` represents a request message. The first part of the request is the address of the API. In this example the address corresponds to the API of arXiv. The second part of the request contains the search arguments. In this example it is requested that the word 'prisoners dilemma' exists within the article's title. The format of the request message is different from API to API. The receive phase includes receiving a number of raw metadata of articles that satisfies the request message. The raw metadata are commonly received in extensive markup language (xml) or Javascript object notation (json) formats [152]. Similarly to the request message, the structure of the received data differs from journal to journal.

The data collection is crucial to this study. To ensure that this study can be reproduced all code used to query the different APIs has been packaged as a Python library and is available online [75]. The software could be used for any type of projects similar to the one described here, documentation for it is available at: `http://arcas.readthedocs.io/en/latest/`. Project [75] allow users to collect articles from a list of APIs by specifying just a single keyword. Articles for which any of the terms "prisoner's dilemma", "prisoners dilemma", "prisoner dilemma", "prisoners evolution", "prisoner game theory" existed within the title, the abstract or the text are included in the analysis. Four prominent journals in the field and a preprint server were used as sources to collect data for this analysis:

- arXiv [132]; a repository of electronic preprints. It consists of scientific papers in the fields of mathematics, physics, astronomy, electrical engineering, computer science, quantitative biology, statistics, and quantitative finance, which all can be accessed online.
- PLOS [5]; a library of open access journals and other scientific literature under an open content license. It launched its first journal, PLOS Biology, in October 2003 and publishes seven journals, as of October 2015.
- IEEE Xplore Digital Library (IEEE) [95]; a research database for discovery and access to journal articles, conference proceedings, technical standards, and related materials on computer science, electrical engineering and electronics, and allied fields. It contains material published mainly by the Institute of Electrical and Electronics Engineers and other partner publishers.
- Nature [79]; a multidisciplinary scientific journal, first published on 4 November 1869. It was ranked the world's most cited scientific journal by the Science Edition of the 2010 Journal Citation Reports and is ascribed an impact factor of 40.137, making it one of the world's top academic journals.
- Springer [133]; a leading global scientific publisher of books and journals. It publishes close to 500 academic and professional society journals.

The data set has been archived and is available at [74]. Note that the latest data collection was performed on the 30$^{\text{th}}$ November 2018.

The relationship between the authors within a field will be modelled as a graph $G = (V_G, E_G)$ where V_G is the set of nodes and E_G is the set of edges. The set V_G represents the authors and an edge connects two authors if and only if those authors have written together. This co-authorship network is constructed using the main data set [74] and the open source package [81]. The PD network is denoted as G where the number of unique authors $|V(G)|$ is 4226 and $|E(G)|$ is 7642 . All authors' names were formatted as their first name and last name (i.e. Martin A. Nowak to Martin Nowak). This was done to avoid errors such as Martin A. Nowak and Martin Nowak being treated as a different person. There are some authors for which only their first initial was found. These entries are left as such.

The collaborativeness of the authors will be analysed using measures such as, isolated nodes, connected components, clustering coefficient, communities, modularity and average degree. These measures show the number of connections authors can have and how strongly connected these people are. The number of isolated nodes is the number of nodes that are not connected to another node, thus the number of authors that have published alone. The average degree denotes the average number of neighbours for each nodes, i.e. the average number of collaborations between the authors. A connected component is a maximal set of nodes such that each pair of nodes is connected by a path [59]. The number of connected components as well as the size of the largest connected component in the network are reported. The size of the largest connected component represents the scale of the central cluster of the entire network, as will be discussed in the analysis section. Clustering coefficient and modularity are also calculated. The clustering coefficient, defined as 3 times the number of triangles on the graph divided by the number of connected triples of nodes, is a local measure of the degree to which nodes in a graph tend to cluster together in a clique [59]. It shows to which extent the collaborators of an author also write together. In comparison, modularity is a global measure designed to measure the strength of division of a network into communities. The number of communities will be reported using the Clauset-Newman-Moore method [51]. Also the modularity index is calculated using the Louvain method described in [43]. The value of the modularity index can vary between $[-1, 1]$, a high value of modularity corresponds to a structure where there are dense connections between the nodes within communities but sparse connections between nodes in different communities. That means that there are many sub communities of authors that write together but not across communities. Two further points are aimed to be explored in this work, (1) which people control the flow of information; as in which people influence the field the most and (2) which are the authors that gain the most from the influence of the field. To measure these concepts centrality measures are going to be used. Centrality measures are often used to understand different aspects of social networks [112]. Two centrality measures have been chosen for this paper and these are closeness and betweenness centrality.

1. In networks some nodes have a short distance to a lot of nodes and consequently are able to spread information on the network very effectively. A representative of this idea is **closeness centrality**, where a node is seen as centrally involved in the network if it requires only few intermediaries to contact others and thus is structurally relatively independent. Closeness centrality is interpreted as influence. Authors with a high value of closeness centrality, are the authors that spread scientific knowledge easier on the network and they have high influence.

2. Another centrality measure is the **betweenness centrality**, where the determination of an author's centrality is based on the quotient of the number of all shortest paths between nodes in the network that include the node in question and the number of all shortest paths in the network. In betweenness centrality the position of the node matters. Nodes with a higher value of betweenness centrality are located in positions that a lot of information pass through, this is interpreted as the gain from the influence, thus these authors gain the most from their networks.

The articles contained in the data set ([74]) will be classified into research topics using LDA an unsupervised machine learning technique designed to summarize large collections of documents by a small number of conceptually connected topics or themes [42, 78]. The documents are the articles' abstracts and LDA was carried out using [165]. In LDA, each document/abstract is represented by a distribution over topics, and the topics themselves are represented by a distribution over words. More specifically, each topics is described by weights associated with words and each document by the probabilities of belonging to a specific topic. The probability of a document belonging to a topic is referred to as the percentage contribution denoted as c. For example the words and their associated weights for two topics A and B could be:

- Topic A: $0.039\times$"cooperation", $0.028\times$"study" and $0.026\times$"human".
- Topic B: $0.020\times$"cooperation", $0.028\times$"agents" and $0.026\times$"strategies".

The percentage contribution for a document with abstract "The study of cooperation in humans" has a $c_A = 0.039 + 0.028 + 0.026 = 0.093$ and $c_B = .020 + 0.0 + 0.0 = 0.020$. The topic to which a document is assigned to is based on the highest percentage contribution denoted as c^*.

For the given example the dominant topic is Topic A $c^* = c_A$. LAD requires that the number of topics is specified in advance before running the algorithm. The number of topics can be chosen using the coherence value [168] or through subjective minimisation of the overlapping keywords between two topics. Both these approaches will be used in this work.

Several of the approaches described in this section have previously been carried out in [41, 121, 182, 198], the novelty here is combining the approaches as well as applying them to a new data set. A preliminary analysis of the data set is presented in the following section.

3.3 Preliminary Analysis

The data set [74] consists of 2422 articles with unique titles. In case of duplicates the preprint version of an article (collected from arXiv) was dropped. Similarly to [121], 76 articles have not been collected from the aforementioned APIs but have been manually added because they are of interest. Examples of such papers include [64] the first publication on the PD, [155, 180] two well cited articles in the field, and a series of works from Robert Axelrod [27, 28, 30, 29, 166] a leading author of the field. A more detailed summary of the articles' provenance is given by Table 3.1. Only 3% of the data set consists of articles that were manually added and 27% of the articles were collected from arXiv. The average number of publications is also included in Table 3.1. Overall an average of 43 articles are published per year on the topic. The most significant contribution to this appears to be from arXiv with 11 articles per year, followed by Springer with 9 and PLOS with 8.

	Number of Articles	Percentage %	Year of first publication	Average number of publications per year
IEEE	294	12.14%	1973	5
Manual	76	3.14%	1951	1
Nature	436	18.00%	1959	8
PLOS	477	19.69%	2005	8
Springer	533	22.01%	1966	9
arXiv	654	27.00%	1993	11
Overall	2470	100.00%	1951	43

Table 3.1: Summary of [74] per provenance.

The data handled here is in fact a time series from the 1950s, the formulation of the game, until 2018 (Figure 3.1). Two observations can be made from Figure 3.1.

1. There is a steady increase of the number of publications since the 1980s and the introduction of computer tournaments [29] (work by Robert Axelrod).
2. There is a decrease in 2017-2018. This is due to our data set being incomplete. Articles that have been written in 2017-2018 have either not being published or were not retrievable by the APIs at the time of the last data collection.

These observations can be confirmed by studying the time series. Using [99], an exponential distribution is fitted to the data. The fitted model can be used to forecast the behaviour of the field for the next 5 years. Even though the time series has indicated a slight decrease, the model forecasts that the number of publications will keep increasing, thus demonstrating that the field of the PD continues to attract academic attention.

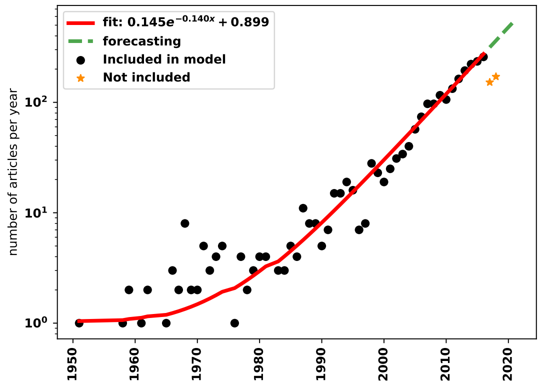


Figure 3.1: Number of articles published on the PD 1951-2018 (on a log scale), with a fitted exponential line, and a forecast for 2017-2022.

There are a total of 4226 authors in the data set ([74]) and several of these authors have had multiple publications collected from the data collection process. The highest number of articles collected for an author is 83 publications for Matjaz Perc. The distribution of the number of papers per author is given by Figure 3.2, and it can be seen that Matjaz Perc is an outlier. More specifically, most authors have 1 to 6 publications in the data set.

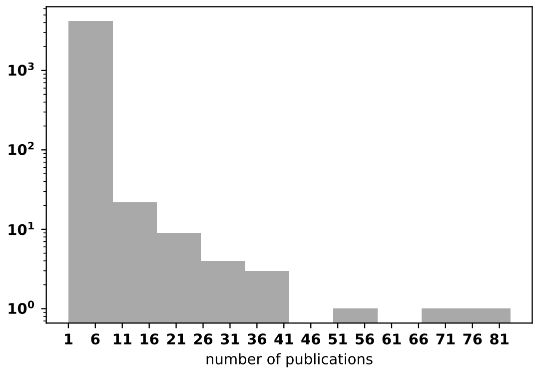


Figure 3.2: Distribution of number of papers per author (on a log scale).

The overall Collaboration Index (CI) or the average number of authors on multi-authored papers is 3.2, thus on average a non single author publication in the PD has 3 authors. This appears to be quite standard compared to other fields such as cultural evolution [198], Astronomy and Astrophysics, Genetics and Heredity, Nuclear and Particle Physics as reported by [128]. There are only a total of 545 publications with a single author, which corresponds to the 22% of the papers. It appears that academic publications tend to be undertaken in the form of collaborative effort, which is in line with the claim of [111]. From Figure 3.3 the trend of CI over the years is given. There are some peaks in the early years 1969 and 1980, however, a steady increase appears to happen after 2004. This could be an effect of better communication tools being introduced around that time which enabled more collaborations between researchers.

The collaborativeness of the authors is explored in more detail in Section 3.5 using the co-authorship network. The collaborative behaviour and relative influence of authors will also be explored in co-authorship networks which correspond to their publications research topics. These topics are presented in the next section.

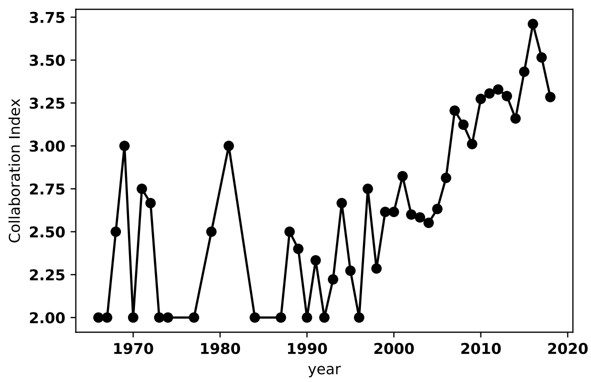


Figure 3.3: Collaboration index over time.

3.4 Research topics in the Prisoner's Dilemma research

In order to identify the topics which are being discussed in the field of the PD, the LDA algorithm implemented in [165] is applied to the abstracts of the data set. As mentioned before, the number of topics, which will be denoted as n, needs to be specified before running the algorithm. The appropriate number of topics is chosen based on the coherence value [168]. Figure 3.4 gives the coherence values of 18 models where $n \in \{2, 3, \dots, 19\}$, and it can be seen than the most appropriate number of topics is 6 with a coherence value of 0.418.

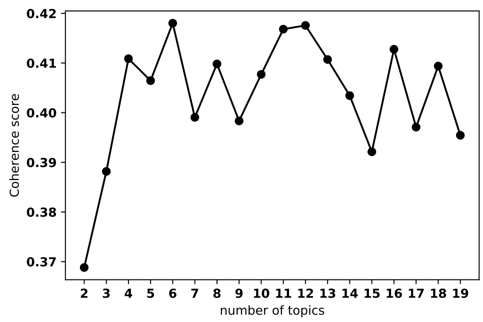


Figure 3.4: Coherence for LDA models over the number of topics.

An LDA model outputs an $N \times n$ matrix - N rows for N abstracts and n columns for n topics. The cells contain the percentage contributions for each topic for each abstract, c_i^j for $i \in \{1, 2, \dots, n\}$ for $j \in \{1, 2, \dots, N\}$. In essence, LDA maps every paper to a vector space of dimension the number of topics. In the case of 6 topics it is difficult to visualise the clustering of topics. To overcome this a dimensionality reduction approach called t-Distributed Stochastic Neighbour Embedding (t-SNE) [125] is applied to the LDA model outputs. More specifically, t-SNE is used to reduce the dimensions of each c^j from n to 2. Figure 3.5, gives the visualisation of LDA for $n = 6$. Each point represents a single document and its color corresponds to the topic with the highest percentage contribution. The documents which are clustered together have a similar percentage contribution distribution over the topics.

Even though the LDA model with $n = 6$ has the highest coherence value, Figure 3.5 shows that documents of the same topic are closer to documents from other topics than each other. For example the documents of topic 2 are divided into two clusters. The one cluster is closer to documents from topic 4 and the other has a few documents closer to topic 1. In the case of

$n = 6$ topic 4 appears to be on "evolution of cooperation on networks", and the papers from topic 2 surrounded from topic 4 include the articles "Evolutionary prisoner's dilemma game on hierarchical lattices" [191] and "Social evolution in structured populations" [54]. Publications that clearly also fit topic 4.

In comparison, 3.6 gives the visualisation of LDA $n = 5$ where the separation of the documents is more clear. Though several models, Figure 3.4, have a higher coherence value than the LDA model with $n = 5$, the separation of topics is not as clear for any model as it is for $n = 5$. Thus, $n = 5$ is chosen to carry out the analysis of this work, and moreover the LDA model for $n = 5$ has a coherence value 0.406 which is close to 0.418.

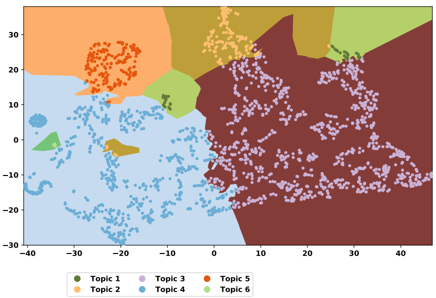


Figure 3.5: Visualisation of LDA with $n = 6$ on 2 dimensions.

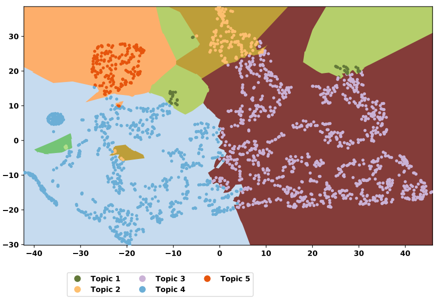


Figure 3.6: Visualisation of LDA with $n = 5$ on 2 dimensions.

What are the research topics of the Prisoner's Dilemma?

For $n = 5$ the articles are clustered and assigned to their dominant topic, based on the highest percentage contribution. The keywords associated with a topic, the most representative article of the topic (based on the percentage contribution) and its academic reference are given by Table 3.2. The topics are labelled as A, B, C, D and E, and more specifically:

- Based on the keywords associated with Topic A, and the most representative article, Topic A appears to be about **human subject research**. Several publications assigned to the topic study the PD by setting experiments and having human participants simulate the game instead of computer simulations. These articles include [131] which showed that prosocial behaviour increased with the age of the participants, [117] which studied the difference in cooperation between high-functioning autistic and typically developing children, [139] explored the gender effect in highschool students and [38] explored the effect of facial expressions of individuals.
- Though it is not immediate from the keywords associated with Topic B, investigating the papers assigned to the topic indicate that it is focused on **biological studies**. Papers assigned to the topic include papers which apply the PD to genetics [171, 176], to the study of tumours [15, 172] and viruses [187]. Other works include how phenotype affinity can affect the emergence of cooperation [197] and modelling bacterial communities as a spatial structured social dilemma.
- Based on the keywords and the most representative article Topic C appears to include publications on PD **strategies**. Publications in the topic include the introduction of new strategies [181], the search of optimality in strategies [34] and the training of strategies [97] with different representation methods. Moreover, publications that study the evolutionary

stability of strategies [10] and introduced methods of differentiating between them [19] are also assigned to C.

- The keywords associated with Topic D clearly show that the topic is focused on **evolutionary dynamics on networks**. Publications include [94] which explored the robustness of cooperation on networks, [192] which studied the effect of a strategy's neighbourhood on the emergence of cooperation and [48] which explored the fixation probabilities of any two strategies is spatial structures.
- The publication assigned to Topic E are on **modelling problems as a PD game**. Though Topic B is also concerned with problems being formulated as a PD, it includes only biological problems. In comparison, the problems in Topic E include decision making in operational research [156], information sharing among members in a virtual team [63], the measurement of influence in articles based on citations [93] and the price spikes in electric power markets [80], and not on biological studies.

Dominant Topic	Topic Keywords	Most Representative Article Title	Reference	# Documents	% Documents
A	social, behavior, human, study, experiment, cooperative, cooperation, suggest, find, behaviour	Facing Aggression: Cues Differ for Female versus Male Faces	[70]	496.0	0.2008
B	individual, group, good, show, high, increase, punishment, cost, result, benefit	Genomic and Gene-Expression Comparisons among Phage-Resistant Type-IV Pilus Mutants of Pseudomonas syringae pathovar phaseolicola	[176]	309.0	0.1251
C	game, strategy, player, agent, dilemma, play, payoff, state, prisoner, equilibrium	Fingerprinting: Visualization and Automatic Analysis of Prisoner's Dilemma Strategies	[176]	561.0	0.2271
D	cooperation, network, population, evolutionary, evolution, interaction, dynamic, structure, cooperator, study	Influence of initial distributions on robust cooperation in evolutionary Prisoner's Dilemma	[47]	556.0	0.2251
E	model, theory, base, system, problem, paper, propose, information, provide, approach	Gaming and price spikes in electric power markets and possible remedies	[80]	548.0	0.2219

Table 3.2: Keywords for each topic and the document with the most representative article for each topic.

Note that the whilst for the choice of 5 topics the actual clustering is not subjective (the algorithm is determining the output) the interpretation above is.

Five topics in the PD publications identified by the data set of this work are human subject research, biological studies, strategies, evolutionary dynamics on networks and modelling problems as a PD.

These 5 topics nicely summarise the PD research. They highlight the interdisciplinarity of the field; how it brings together applied modelling of real world situations (Topic B and E) and more theoretical notions such as evolutionary dynamics and optimality of strategies.

Is one topic currently more in fashion?

Figure 3.7 gives the number of articles per topic over time. The topics appear to have had a similar trend over the years, with topics B and D having a later start. Following the introduction of a topic the publications in that topic have been increasing. There is no decreasing trend in any of the topics. All the topics have been publishing for years and they still attract the interest of academics. Thus, **there does not seem to be any given topic more or less in**

fashion.

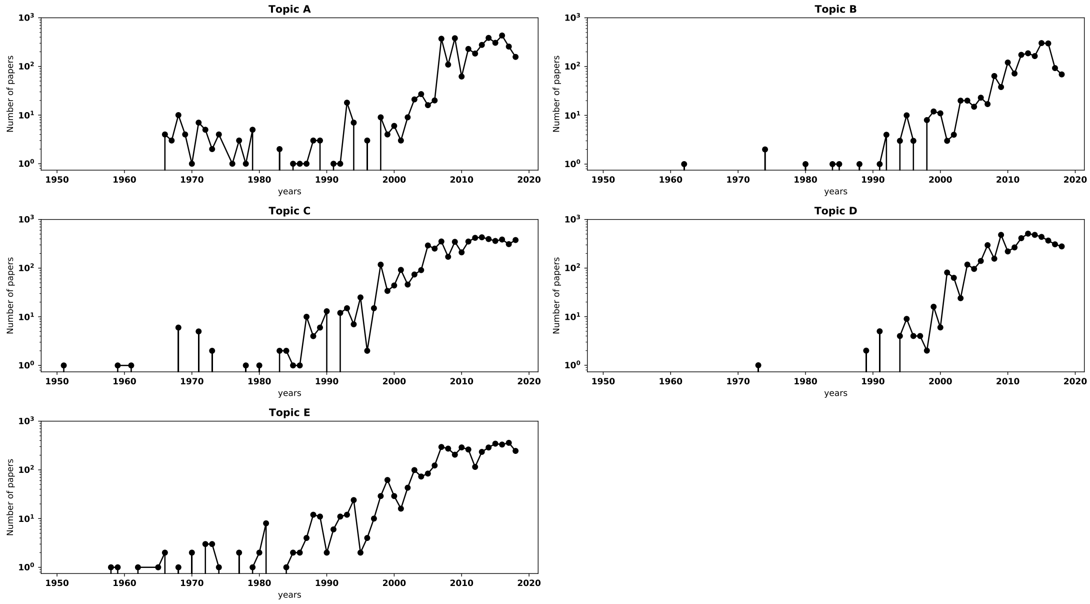


Figure 3.7: Number of articles per topic over the years (on a logged scale).

How do the research topics change over the years?

To gain a better understanding regarding the change in the topics over the years, LDA is applied to the cumulative data set over 8 time periods. These periods are 1951-1965, 1951-1973, 1951-1980, 1951-1988, 1951-1995, 1951-2003, 1951-2010, 1951-2018. The number of topics for each cumulative subset is chosen based on the coherence value and no objective approach is used. As a result, the period 1951-2018 has been assigned $n = 6$ which had the highest coherence value instead of 5. The chosen models for each period including the number of topics, their keywords and number of articles assigned to them are given by Table 3.3.

But how well do the five topics which were presented earlier fit the publications over time? This is answered by comparing the performance of three LDA models over the cumulative periods' publications. The three models are LDA models for the entire data set for n equal to 5, 6 and the optimal number of topics over time. For each model the c^* is estimated for each document in the cumulative data sets. The performance of the models are then compared based on:

$$\bar{c^*} \times n \tag{3.1}$$

where $\bar{c^*}$ is the median highest percentage contribution and n is the number of topics of a given period. A model with more topics will have more difficulty to assign papers. Thus, equation (refeq:ratio) is a measure of confidence in assigning a given paper to its topic weighted by the number of topics. The performances are given by Figure 3.8.

The five topics of the PD presented in this manuscript appear to always be less good at fitting the publications compared to the six topics of LDA $n = 6$. Moreover, there are less good than the topics of the optimal number of topics from 1951 to 1995. The difference in the performance values, equation (3.1), however are small. **The relevances of the five topics has been increasing over time, and though, the topics did not always fit the**

Period	Topic	Topic Keywords	Num of Documents	Percentage of Documents
1951-1965	1	problem, technology, divert, euler, subsystem, requirement, trace, technique, system, untried	3	0.375
1951-1965	2	interpret, requirement, programme, evolution, article, increase, policy, system, trace, technology	2	0.25
1951-1965	3	equipment, agency, conjecture, development, untried, programme, trend, technology, weapon, technique	1	0.125
1951-1965	4	variation, celebrated, trend, untried, change, involve, month, technique, subsystem, research	1	0.125
1951-1965	5	give, good, modern, trace, technique, ambiguity, problem, trend, technology, system	1	0.125
1951-1973	1	study, shock, cooperative, money, part, vary, investigate, good, receive, equipment	12	0.3243
1951-1973	2	cooperation, level, significantly, sequence, reward, provoke, descriptive, principal, display, argue	4	0.1081
1951-1973	3	player, make, effect, triad, experimental, motivation, dominate, hypothesis, instruction, trend	3	0.0811
1951-1973	4	ss, sex, male, female, dyad, design, suggest, college, factor, tend	3	0.0811
1951-1973	5	result, research, format, change, operational, analysis, relate, understanding, decision, money	2	0.0541
1951-1973	6	condition, give, high, treatment, conflict, cc, real, original, replication, promote	2	0.0541
1951-1973	7	group, competitive, show, interpret, scale, compete, escalation, free, variable, individualistic	2	0.0541
1951-1973	8	outcome, strategy, choice, type, pdg, difference, dummy, conclude, compare, consistent	2	0.0541
1951-1973	9	game, difference, pair, approach, behavior, person, weapon, occur, advantaged, differential	2	0.0541
1951-1973	10	response, present, dilemma, influence, cooperate, bias, point, amount, participate, factor	2	0.0541
1951-1973	11	trial, problem, previous, involve, prisoner, experiment, follow, tit, increase, initial	1	0.027
1951-1973	12	matrix, behavior, rational, black, model, research, broad, distance, complex, trace	1	0.027
1951-1973	13	play, finding, individual, noncooperative, white, nature, race, ratio, represent, prisoner	1	0.027
1951-1980	1	play, trial, group, follow, white, interpret, scale, black, trend, small	14	0.25
1951-1980	2	outcome, level, effect, type, dyad, vary, pdg, participate, understanding, arise	9	0.1607
1951-1980	3	game, strategy, cooperation, significant, difference, sentence, text, occur, differential, hypothesis	4	0.0714
1951-1980	4	male, female, find, result, sex, subject, experimental, situation, treatment, computer	4	0.0714
1951-1980	5	research, problem, influence, matrix, format, model, analysis, year, crime, equipment	4	0.0714
1951-1980	6	condition, dilemma, bias, free, attempt, book, year, dummy, prison, design	4	0.0714
1951-1980	7	variable, result, factor, individual, ability, triad, half, migration, change, investigate	3	0.0536
1951-1980	8	show, present, suggest, rational, compete, approach, characteristic, examine, person, conduct	3	0.0536
1951-1980	9	behavior, high, finding, relate, obtain, assistance, ratio, good, weapon, competition	3	0.0536
1951-1980	10	ss, shock, money, competitive, part, difference, pair, amount, man, information	3	0.0536
1951-1980	11	player, conflict, theory, decision, determine, produce, maker, cooperate, specialist, programming	2	0.0357
1951-1980	12	study, prisoner, make, response, experiment, noncooperative, standard, separate, conclude, initial	2	0.0357
1951-1980	13	give, cooperative, choice, cognitive, real, operational, set, subject, ascribe, concern	1	0.0179
1951-1988	1	trial, difference, find, choice, significant, competitive, effect, triad, interact, occur	24	0.2553
1951-1988	2	ss, shock, money, pair, response, part, high, tit, receive, amount	13	0.1383
1951-1988	3	suggest, paper, case, debate, view, achieve, framework, natural, assumption, finitely	10	0.1064
1951-1988	4	prisoner, dilemma, behavior, model, present, involve, person, increase, trust, experiment	8	0.0851
1951-1988	5	game, player, show, approach, repeat, previous, move, tat, related, include	8	0.0851
1951-1988	6	cooperation, level, mutual, equilibrium, standard, provide, information, human, real, question	6	0.0638
1951-1988	7	play, result, male, subject, female, cooperative, sex, experimental, treatment, computer	5	0.0532
1951-1988	8	research, study, variable, ability, factor, conflict, matrix, year, student, interpret	4	0.0426
1951-1988	9	problem, group, small, scale, social, issue, large, base, bias, party	4	0.0426
1951-1988	10	game, strategy, outcome, type, cooperate, ethical, pdg, explain, dependent, separate	4	0.0426
1951-1988	11	give, condition, individual, major, dyad, behaviour, produce, conflict, assistance, collectively	3	0.0319
1951-1988	12	situation, iterate, statement, rational, card, side, paradox, true, consequence, front	2	0.0213
1951-1988	13	inflation, hypothesis, rate, run, change, demand, nominal, cost, output, growth	2	0.0213
1951-1988	14	theory, make, analysis, decision, system, examine, work, soft, lead, hard	1	0.0106
1951-1995	1	strategy, population, evolution, iterate, tit, opponent, evolve, dynamic, set, tat	31	0.1732
1951-1995	2	game, repeat, assumption, rule, person, equilibrium, general, finitely, indefinitely, analyze	24	0.1341
1951-1995	3	inflation, long, rate, hypothesis, run, policy, cost, nominal, demand, programming	20	0.1117
1951-1995	4	condition, outcome, trial, find, difference, cooperation, experiment, level, significant, response	15	0.0838
1951-1995	5	rational, result, receive, statement, money, paradox, shock, iterate, consequence, common	14	0.0782
1951-1995	6	cooperation, show, competitive, high, probability, conflict, simulation, altruism, yield, natural	14	0.0782
1951-1995	7	prisoner, dilemma, give, point, defect, form, cooperator, increase, relate, ethical	10	0.0559
1951-1995	8	player, give, decision, provide, cooperative, game, previous, pair, determine, interact	9	0.0503
1951-1995	9	play, cooperate, result, male, subject, female, time, relationship, suggest, student	8	0.0447
1951-1995	10	problem, group, theory, good, approach, society, large, scale, issue, level	8	0.0447
1951-1995	11	study, situation, behaviour, computer, argue, change, implication, characteristic, real, associate	8	0.0447
1951-1995	12	model, paper, behavior, examine, present, mutual, expectation, develop, type, variable	7	0.0391
1951-1995	13	make, research, system, analysis, choice, work, base, relation, world, wide	6	0.0335
1951-1995	14	individual, social, behavior, standard, choose, evolutionary, partner, payoff, defection, small	5	0.0279
1951-2003	1	game, player, dilemma, prisoner, theory, give, paper, make, group, problem	151	0.4266
1951-2003	2	cooperation, result, play, show, cooperate, condition, cooperative, high, level, time	106	0.2994
1951-2003	3	strategy, model, agent, study, behavior, individual, population, evolutionary, state, player	97	0.274
1951-2010	1	model, theory, paper, base, make, present, problem, provide, human, decision	325	0.3454
1951-2010	2	game, strategy, player, agent, play, dilemma, system, behavior, show, state	322	0.3422
1951-2010	3	cooperation, network, study, population, individual, evolutionary, social, evolution, interaction, structure	294	0.3124
1951-2018	1	model, theory, system, base, paper, problem, propose, present, approach, provide	556	0.2251
1951-2018	2	behavior, social, human, decision, study, experiment, make, suggest, result, behaviour	482	0.1951
1951-2018	3	individual, group, good, social, punishment, level, cost, mechanism, dilemma, cooperative	428	0.1733
1951-2018	4	game, strategy, player, agent, play, dilemma, state, prisoner, payoff, equilibrium	380	0.1538
1951-2018	5	population, evolutionary, dynamic, model, selection, result, evolution, evolve, show, process	351	0.1421
1951-2018	6	cooperation, network, interaction, structure, study, evolution, find, behavior, cooperative, simulation	273	0.1105

Table 3.3: Topic modelling result for the cumulative data set over the periods

majority of published work over time, there were still papers being published on those topics.

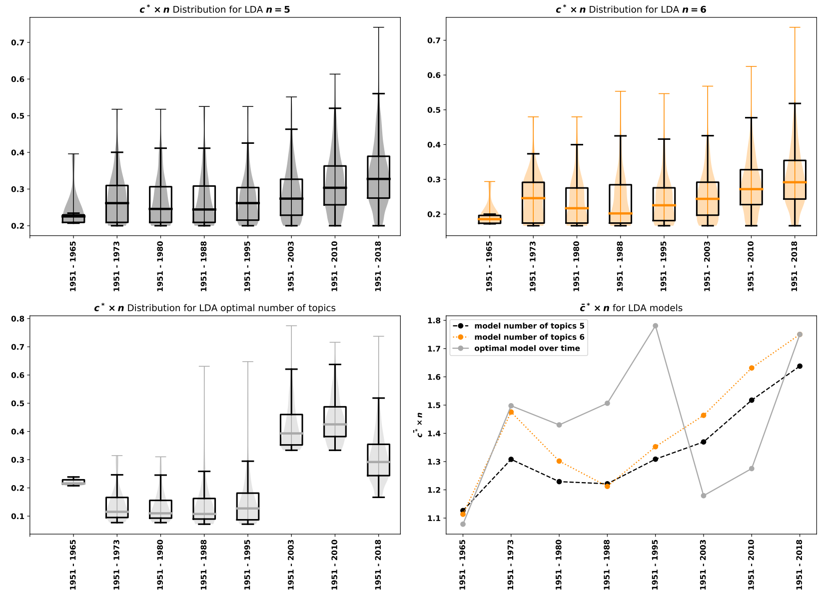


Figure 3.8: Maximum percentage contributions (c^*) over the time periods, for the LDA models for the entire data set for n equal to 5, 6 and the optimal number of topics over time.

In the following section the collaborative behaviour of authors in the field, and within the field's topics as were presented in this section, are explored using a network theoretic approach.

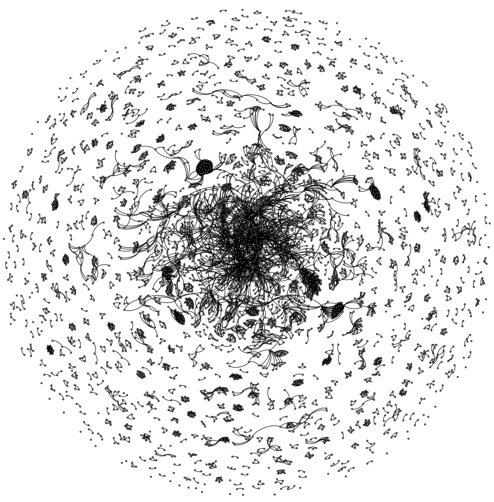
3.5 Analysis of co-authorship network

The collaborative behaviour of authors in the field of the PD is assessed using the co-authorship network, which as mentioned in Section 3.2 is denoted as G. There are a total of 947 connected components in G and the largest component has a size of 796 nodes. The largest connected component is going to be refereed to as the main cluster of the network and is denoted as $\bar{G}$. A graphical representation of both networks is shown in Figure 3.9 and a metrics summary is given by Table 3.4.

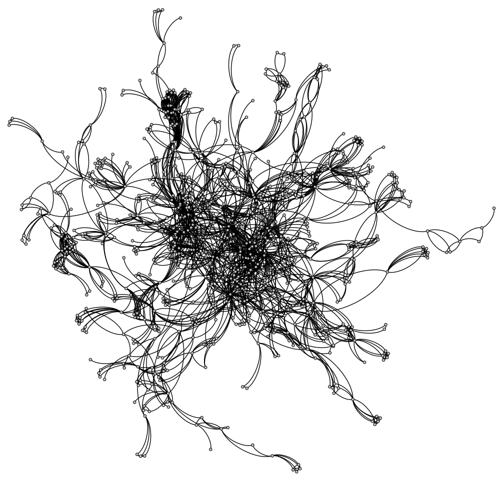
Is the Prisoner's Dilemma a collaborative field?

Based on Table 3.4 an author in G has on average 4 collaborators and a 70% probability of collaborating with a collaborator's co-author. An author of $\bar{G}$ on average is 7% more likely to write with a collaborator's co-author and on average has 2 more collaborators. Moreover, there are only 3.2 % of authors in the PD that has no connection to any other author.

How does this compare to other fields? Two more data sets for the topics "Price of Anarchy" and "Auction Games" have been collected in order to compare the collaborative behaviour of the PD to other game theoretic fields. A total of 3444 publications have been collected for Auction games and 748 for Price of Anarchy. Price of Anarchy is relatively a new field, with the first publication on the topic being [107] in 1999. This explains the small number of articles that have been retrieved. Both data sets have been archived and are available in [72, 73]. The networks for both data sets have been generated in the same way as G. A summary of the networks' metrics are given by Table 3.5.



(a) G the co-authorship network for the IPD.



(b) $\bar{G}$ the largest connected component of G.

Figure 3.9: A graphical representation of G and $\bar{G}$

The average degrees for the Price of Anarchy and for Auction games are lower than the PD's. In Auction games an author is more likely to have no collaborators, and in the Price of Anarchy there are almost no authors that are not connected to someone. This could be an effect of the field being introduced in more modern days. Overall, an author in the PD has on average more collaborators and there are less isolated authors compared to another well established game theoretic field. These results seem to indicate that the PD is a *relatively* collaborative field.

However, both G and $\bar{G}$ have a high modularity (larger than 0.84) and a large number of communities (967 and 25 respectively). A high modularity implies that authors create their own publishing communities but not many publications from authors from different communities occur. Thus, author tends to collaborate with authors in their communities but not many efforts are made to create new connections to other communities and spread the knowledge of the field across academic teams. The fields of both Price of Anarchy and Auction games also have high modularity, and that could indicate that is in fact how academic publications are.

Thus, **the PD is indeed a collaborative field but perhaps it is not more collaborative than other fields**, as there is no effort from the authors to write with people outside their community.

	# Nodes	# Edges	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
G	4011	7642	3.2	947	796	3.811	967	0.96491	0.701
$\bar{G}$	796	2214	0.0	1	796	5.563	25	0.84406	0.773

Table 3.4: Network metrics for G and $\bar{G}$ respectively.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
Auction Games	5165	7861	256	5.0	1272	1348	3.044	1294	0.957	0.622
Price of Anarchy	1155	1953	4	0.3	245	222	3.382	253	0.965	0.712

Table 3.5: Network metrics for auction games and price of anarchy networks respectively.

The evolution of the networks was also explored over time by constructing the network cumulatively over 51 periods. Except from the first period 1951-1966 the rest of the periods have a yearly interval (data for the years 1975 and 1982 were not retrieved by the collection data process). The metrics of each sub network are given in the Appendix 3.7.

The results, similarly to the results of [121], confirm that the networks grow over time and that the networks always had a high modularity. Since the first publications authors tend to write with people from their communities, and that is not an effect of a specific time period.

Are some topics more collaborative than other?

The networks corresponding to the topics of Section 3.3 have also been generated similarly to G. Note that authors with publications in more than one topic exist, and these authors are included in all the corresponding networks. A metrics' summary for all five topic networks is given by Table 3.6.

Topic B is the network with the highest average degree followed by Topic A. The topic with the smallest average degree, 2.5, is Topic C. In topics A and B the number of isolated nodes is very small $lessthan(0.2)$ compared to Topic E where the percentage of isolated nodes is approximately 6%. Moreover, in topics C and E an author is 10% more likely to collaborate with a collaborator's co-author. Thus, **topics "human subject research" and "biological studies" tend to be more collaborative than the topic of "strategies", and an authors in these are less likely to have at least one collaborator compared to the topic of "modelling problems as a PD"**.

"Evolutionary dynamics on networks" also appear to be a collaborative topic. In fact the network of the topic is a sub graph of $\bar{G}$, the main cluster of G and it will be demonstrated in the following section that authors in this network are more like to gain from the influence of the network compared to any other topic network.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
Topic A	1124	2137	15	1.3	264	56	3.802	265	0.983	0.759
Topic B	695	1382	13	1.9	157	80	3.977	158	0.950	0.773
Topic C	900	1141	41	4.6	281	29	2.536	281	0.981	0.636
Topic D	880	1509	17	1.9	174	312	3.430	183	0.918	0.701
Topic E	1045	1964	59	5.6	354	31	3.759	354	0.926	0.664

Table 3.6: Network metrics for topic networks.

Are there authors which benefit more from their position in the network?

There are two centrality measures reported in this work, closeness and betweenness centrality. Closeness centrality is a measure of how easy it is for an author to contact others, and consequently affect them; influence them. Thus closeness centrality here is a measure of influence. Betweenness centrality is a measure of how many paths pass through a specific node, thus the amount of information this person has access to. Betweenness centrality is used here as a measure of how much an author gains from the field. All centrality measure can have values ranging from 0 to 1. The influence and the amount of information an author has access to are used to explore which authors benefit more from their position.

For G and $\bar{G}$ the most central authors based on closeness and betweenness centralities are given by Table 3.7. The most central authors in G and $\bar{G}$ are the same. This implies that the results on centrality heavily rely on the main cluster (as expected). Matjaz Perc is an author with 83 publications in the data set and the most central authors based on both centrality measures. The most central authors are fairly similar between the two measures. The author that appear to be central based on one measure and not the other are Martin Nowak, Franz Weissing, Jianye Hao, Angel Sanchez and Valerio Capraro which have access to information due to their positioning but do not influence the network as much, and the opposite is true for Attila Szolnoki, Luo-Luo Jiang Sandro Meloni, Cheng-Yi Xia and Xiaojie Chen.

It is obvious that in G the centralities values are low which suggests that in the PD authors do not benefit from their positions. This could be an effect of information not flowing from one community to another as authors tend to write with people from their communities. Nevertheless, **there are authors that do benefit from their position, but these are only the authors connected to the main cluster**.

The centrality measures for the topic networks have also been estimated and are given in Tables 3.8-3.9. If information was flowing between the communities of the research topics then there would be an increase to the values of centralities for the sub networks. However, the only topic where authors gain from their positions are the authors of Topic D (topic on evolutionary

	G				$\bar{G}$			
	Name	Betweenness	Name	Closeness	Name	Betweenness	Name	Closeness
1	Matjaz Perc	0.015	Matjaz Perc	0.066	Matjaz Perc	0.373	Matjaz Perc	0.330
2	Zhen Wang	0.011	Long Wang	0.060	Zhen Wang	0.279	Long Wang	0.301
3	Long Wang	0.007	Yamir Moreno	0.059	Long Wang	0.170	Yamir Moreno	0.299
4	Martin Nowak	0.006	Attila Szolnoki	0.059	Martin Nowak	0.159	Attila Szolnoki	0.297
5	Angel Sanchez	0.004	Zhen Wang	0.059	Angel Sanchez	0.114	Zhen Wang	0.296
6	Yamir Moreno	0.004	Arne Traulsen	0.056	Yamir Moreno	0.110	Arne Traulsen	0.281
7	Arne Traulsen	0.004	Luo-Luo Jiang	0.055	Arne Traulsen	0.107	Luo-Luo Jiang	0.280
8	Franz Weissing	0.004	Sandro Meloni	0.055	Franz Weissing	0.101	Sandro Meloni	0.278
9	Jianye Hao	0.004	Cheng-Yi Xia	0.055	Jianye Hao	0.094	Cheng-Yi Xia	0.276
10	Valerio Capraro	0.004	Xiaojie Chen	0.055	Valerio Capraro	0.093	Xiaojie Chen	0.276

Table 3.7: 10 most central authors based on betweenness and closeness centralities for G and $\bar{G}$.

dynamics on network). From the list of names it is obvious that these authors are part of $\bar{G}$, and that the network of Topic D is a sub network of $\bar{G}$. This confirms the results. The people benefiting from their position in the co-authorship networks corresponding to research topics of the PD are only the people from the main cluster of G.

The fact that most authors of the main cluster are primarily publishing in evolutionary dynamics on networks indicates that publishing in this specific topic differs from the other topics covered in this manuscript. There appears to be more collaboration and influence in the publications on evolutionary dynamics and authors are more likely to gain from their position, though it is not clear as to why.

	Topic A		Topic B		Topic C		Topic D		Topic E	
	Name	Betweeness	Name	Betweeness	Name	Betweeness	Name	Betweeness	Name	Betweeness
1	David Rand	0.002	Long Wang	0.006	Daniel Ashlock	0.001	Matjaz Perc	0.064	Zengru Di	0.0
2	Valerio Capraro	0.001	Luo-Luo Jiang	0.005	Matjaz Perc	0.000	Luo-Luo Jiang	0.037	Jian Yang	0.0
3	Angel Sanchez	0.001	Martin Nowak	0.004	Karl Tuyls	0.000	Yamir Moreno	0.031	Yevgeniy Vorobeychik	0.0
4	Feng Fu	0.001	Matjaz Perc	0.003	Philip Hingston	0.000	Christoph Hauert	0.027	Otavio Teixeira	0.0
5	Martin Nowak	0.000	Attila Szolnoki	0.003	Eun-Youn Kim	0.000	Long Wang	0.024	Roberto Oliveira	0.0
6	Nicholas Christakis	0.000	Christian Hilbe	0.002	Wendy Ashlock	0.000	Zhen Wang	0.024	M. Nowak	0.0
7	Pablo Branas-Garza	0.000	Yamir Moreno	0.002	Attila Szolnoki	0.000	Han-Xin Yang	0.023	M. Harper	0.0
8	Toshio Yamagishi	0.000	Xiaojie Chen	0.002	Seung Baek	0.000	Martin Nowak	0.020	Xiao Han	0.0
9	James Fowler	0.000	Arne Traulsen	0.002	Martin Nowak	0.000	Angel Sanchez	0.017	Zhesi Shen	0.0
10	Long Wang	0.000	Zhen Wang	0.002	Thore Graepel	0.000	Zhihai Rong	0.016	Wen-Xu Wang	0.0

Table 3.8: 10 most central authors based on betweenness centrality for topics' networks.

	Topic A		Topic B		Topic C		Topic D		Topic E	
	Name	Closeness	Name	Closeness	Name	Closeness	Name	Closeness	Name	Closeness
1	David Rand	0.027	Long Wang	0.043	Karl Tuyls	0.022	Matjaz Perc	0.123	Stefanie Widder	0.029
2	Valerio Capraro	0.023	Matjaz Perc	0.041	Thore Graepel	0.019	Zhen Wang	0.109	Rosalind Allen	0.029
3	Jillian Jordan	0.022	Attila Szolnoki	0.040	Joel Leibo	0.018	Long Wang	0.107	Thomas Pfeiffer	0.029
4	Nicholas Christakis	0.021	Martin Nowak	0.040	Edward Hughes	0.017	Yamir Moreno	0.105	Thomas Curtis	0.029
5	James Fowler	0.020	Olivier Tenaillon	0.038	Matthew Phillips	0.017	Luo-Luo Jiang	0.104	Carsten Wiuf	0.029
6	Martin Nowak	0.020	Xiaojie Chen	0.038	Edgar Duenez-Guzman	0.017	Attila Szolnoki	0.103	William Sloan	0.029
7	Angel Sanchez	0.019	Bin Wu	0.038	Antonio Castaneda	0.017	Gyorgy Szabo	0.102	Otto Cordero	0.029
8	Gordon Kraft-Todd	0.019	Yanling Zhang	0.037	Iain Dunning	0.017	Xiaojie Chen	0.102	Sam Brown	0.029
9	Akihiro Nishi	0.019	Feng Fu	0.037	Tina Zhu	0.017	Guangming Xie	0.101	Babak Momeni	0.029
10	Anthony Evans	0.019	David Rand	0.037	Kevin Mckee	0.017	Lucas Wardil	0.101	Wenying Shou	0.029

Table 3.9: 10 most central authors based on closeness centrality for topics' networks.

The distributions of both centrality measures for all the networks of this work are given in the Appendix 3.8.2.

3.6 Conclusion

This manuscript has explored the research topics in the publications of the Iterated Prisoner's Dilemma, and moreover, the authors' collaborative behaviour and their influence in the research field. This was achieved by applying network theoretic approaches and a LDA algorithm to a total of 2422 publications. Both the software [75] and the data [75] have been archived and are available to be used by other researchers. In fact [75] has been used by [127] and [184].

The data collection and an introduction to the methodology used in this work were covered in Section 3.2. Section 3.3 covered an initial analysis of the data set which demonstrated that the PD is a field that continues to attract academic attention and publications. In Section 3.4 LDA was applied to the data set to identify topics on which researchers have been publishing. The LDA analysis showed that the data could be classified into 5 topics associated with human subject research, biological studies, strategies, evolutionary dynamics on networks and modelling problems as a PD. These topics summarize the field of the PD well, as they demonstrate its interdisciplinarity and applications to a variety of problems. A temporal analysis explored how relevant these topics have been over the course of time, and it revealed that even though there were not the necessarily always the most discussed topics they were still being explored by researchers.

The collaborative behaviour of the field was explored in Section 3.5 by constructing the co authorship network. It was concluded that the field is a collaborative field, where authors are likely to write with a collaborator's co-authors and on average an author has 4 co-authors, however it not necessarily more collaborative than other fields. The authors tend to collaborate with authors from one community, but not many authors are involved in multiple communities. This however might be an effect of academic research, and it might not be true just for the field of the PD. Exploring the influence of authors and their gain from being in the network of the field demonstrated that authors do not gain much, and the authors with influence are only the ones connected to the main cluster, to a "main" group of authors. This 'main" group of authors consists of authors publishing in evolutionary dynamics on networks. Thus, an author would be aiming to publish on this topic if they were interested in gaining from their position in the publications of the PD.

The study of the PD is the study of cooperation and investigating the cooperative behaviours of authors is what this work has aimed to achieve. Interesting areas of future work would include extending this analysis to more game theoretic sub fields, to evaluate whether the results remain the same.

3.7 Cumulative Networks Metrics

3.7.1 Collaborativeness metrics for cumulative graphs, $\tilde{G} \subseteq G$

Period	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
1951 - 1966	6	3	0	0.0	3	2	1.000	3	0.667	0.000
1951 - 1967	8	4	0	0.0	4	2	1.000	4	0.750	0.000
1951 - 1968	19	15	0	0.0	8	5	1.579	8	0.684	0.228
1951 - 1969	20	17	0	0.0	8	6	1.700	8	0.630	0.250
1951 - 1970	22	18	0	0.0	9	6	1.636	9	0.667	0.227
1951 - 1971	33	28	0	0.0	13	6	1.697	13	0.827	0.424
1951 - 1972	39	34	0	0.0	15	6	1.744	15	0.867	0.513
1951 - 1973	42	35	1	2.4	17	6	1.667	17	0.873	0.476
1951 - 1974	42	35	1	2.4	17	6	1.667	17	0.873	0.476
1951 - 1976	42	35	1	2.4	17	6	1.667	17	0.873	0.476
1951 - 1977	44	36	1	2.3	18	6	1.636	18	0.880	0.455
1951 - 1978	44	36	1	2.3	18	6	1.636	18	0.880	0.455
1951 - 1979	47	40	1	2.1	18	6	1.702	18	0.884	0.454
1951 - 1980	47	40	1	2.1	18	6	1.702	18	0.884	0.454
1951 - 1981	50	46	1	2.0	18	6	1.840	18	0.889	0.497
1951 - 1983	51	46	2	3.9	19	6	1.804	19	0.889	0.487
1951 - 1984	53	47	2	3.8	20	6	1.774	20	0.894	0.469
1951 - 1985	53	47	2	3.8	20	6	1.774	20	0.894	0.469
1951 - 1986	53	47	2	3.8	20	6	1.774	20	0.894	0.469
1951 - 1987	56	48	3	5.4	22	6	1.714	22	0.898	0.443
1951 - 1988	62	52	4	6.5	25	6	1.677	25	0.909	0.449
1951 - 1989	75	62	5	6.7	31	6	1.653	31	0.926	0.424
1951 - 1990	79	64	5	6.3	33	6	1.620	33	0.930	0.403
1951 - 1991	87	69	6	6.9	37	6	1.586	37	0.937	0.400
1951 - 1992	95	72	10	10.5	42	6	1.516	42	0.941	0.367
1951 - 1993	106	81	12	11.3	47	6	1.528	47	0.947	0.366
1951 - 1994	124	95	16	12.9	56	6	1.532	56	0.955	0.394
1951 - 1995	135	102	17	12.6	61	6	1.511	61	0.960	0.384
1951 - 1996	142	105	18	12.7	65	6	1.479	65	0.962	0.365
1951 - 1997	155	115	20	12.9	71	6	1.484	71	0.966	0.392
1951 - 1998	191	140	21	11.0	87	6	1.466	87	0.973	0.367
1951 - 1999	221	169	25	11.3	99	6	1.529	99	0.977	0.397
1951 - 2000	250	195	27	10.8	110	6	1.560	110	0.979	0.418
1951 - 2001	287	235	30	10.5	125	7	1.638	125	0.977	0.419
1951 - 2002	335	278	36	10.7	146	7	1.660	146	0.979	0.428
1951 - 2003	381	310	40	10.5	168	7	1.627	168	0.982	0.413
1951 - 2004	437	370	40	9.2	185	10	1.693	185	0.983	0.424
1951 - 2005	532	476	41	7.7	214	19	1.789	214	0.985	0.458
1951 - 2006	640	603	43	6.7	246	22	1.884	246	0.987	0.486
1951 - 2007	793	877	46	5.8	283	25	2.212	283	0.985	0.532
1951 - 2008	948	1170	50	5.3	318	33	2.468	319	0.985	0.558
1951 - 2009	1108	1442	54	4.9	356	71	2.603	358	0.982	0.573
1951 - 2010	1300	1936	66	5.1	402	133	2.978	405	0.965	0.592
1951 - 2011	1560	2375	79	5.1	472	157	3.045	475	0.970	0.613
1951 - 2012	1837	2865	80	4.4	534	209	3.119	537	0.969	0.634
1951 - 2013	2149	3420	93	4.3	603	322	3.183	609	0.965	0.644
1951 - 2014	2481	3971	103	4.2	683	399	3.201	694	0.962	0.658
1951 - 2015	2938	4877	110	3.7	765	504	3.320	779	0.965	0.675
1951 - 2016	3469	6532	114	3.3	850	613	3.766	863	0.964	0.696
1951 - 2017	3735	7072	119	3.2	895	706	3.787	912	0.964	0.700
1951 - 2018	4011	7642	128	3.2	947	796	3.811	967	0.966	0.701

3.7.2 Collaborativeness metrics for cumulative graphs' main clusters, $\tilde{G} \subseteq \bar{G}$

Periods	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
1951 - 1966	2	1	0	0.0	1	2	1.000	1	0.000	0.000
1951 - 1967	2	1	0	0.0	1	2	1.000	1	0.000	0.000
1951 - 1968	5	8	0	0.0	1	5	3.200	1	0.000	0.867
1951 - 1969	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1970	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1971	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1972	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1973	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1974	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1976	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1977	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1978	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1979	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1980	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1981	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1983	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1984	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1985	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1986	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1987	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1988	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1989	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1990	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1991	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1992	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1993	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1994	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1995	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1996	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1997	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1998	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 1999	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 2000	6	10	0	0.0	1	6	3.333	2	0.020	0.833
1951 - 2001	7	21	0	0.0	1	7	6.000	1	0.000	1.000
1951 - 2002	7	21	0	0.0	1	7	6.000	1	0.000	1.000
1951 - 2003	7	21	0	0.0	1	7	6.000	1	0.000	1.000
1951 - 2004	10	13	0	0.0	1	10	2.600	2	0.376	0.553
1951 - 2005	19	28	0	0.0	1	19	2.947	3	0.544	0.730
1951 - 2006	22	35	0	0.0	1	22	3.182	4	0.527	0.720
1951 - 2007	25	39	0	0.0	1	25	3.120	5	0.558	0.686
1951 - 2008	33	62	0	0.0	1	33	3.758	4	0.623	0.736
1951 - 2009	71	148	0	0.0	1	71	4.169	6	0.697	0.698
1951 - 2010	133	387	0	0.0	1	133	5.820	7	0.726	0.749
1951 - 2011	157	465	0	0.0	1	157	5.924	8	0.727	0.725
1951 - 2012	209	611	0	0.0	1	209	5.847	11	0.733	0.737
1951 - 2013	322	892	0	0.0	1	322	5.540	12	0.780	0.743
1951 - 2014	399	1109	0	0.0	1	399	5.559	15	0.794	0.742
1951 - 2015	504	1368	0	0.0	1	504	5.429	24	0.811	0.751
1951 - 2016	613	1677	0	0.0	1	613	5.471	21	0.819	0.761
1951 - 2017	706	1935	0	0.0	1	706	5.482	29	0.830	0.772
1951 - 2018	796	2214	0	0.0	1	796	5.563	25	0.845	0.773

3.8 Centrality Measures Distributions

3.8.1 Distributions for G and $\bar{G}$

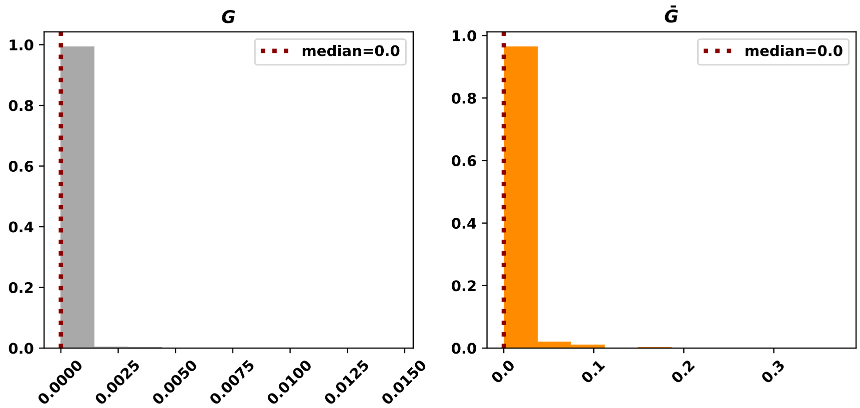


Figure 3.10: Distributions of betweenness centrality in G and $\bar{G}$

3.8.2 Distributions for Topic Networks

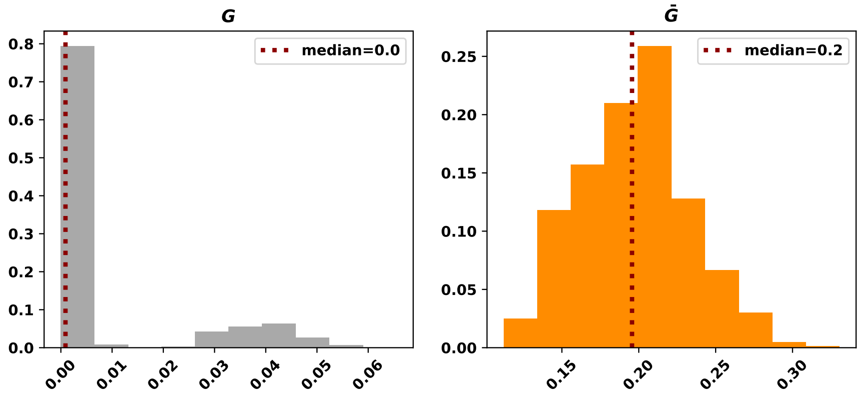


Figure 3.11: Distributions of closeness centrality in G and $\bar{G}$

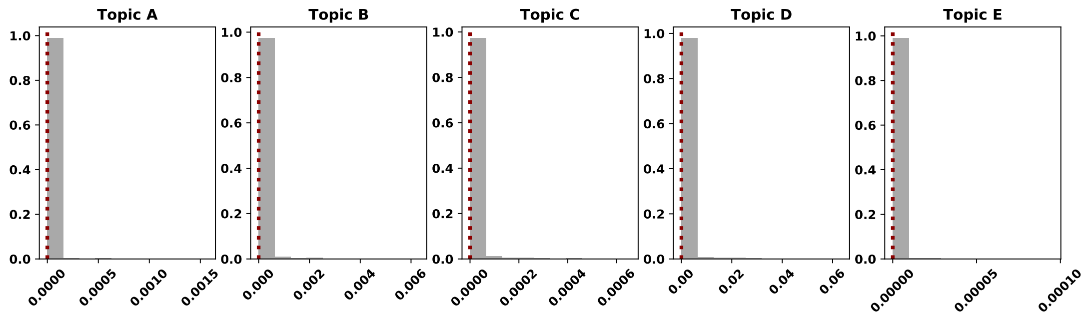


Figure 3.12: Distributions of betweenness centrality in topics' networks.

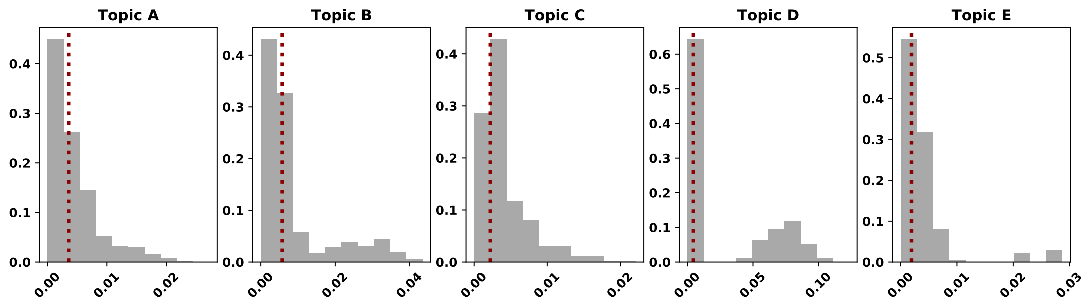


Figure 3.13: Distributions of closeness centrality in topics' networks.

Chapter 4

A meta analysis of tournaments and an evaluation of performance in the Iterated Prisoner's Dilemma.

The research reported in this Chapter has lead in a manuscript, entitled:
"Properties of winning Iterated Prisoner's Dilemma strategies"
Available at: `https://arxiv.org/abs/2001.05911`
Associated data set: [71, 76]
Axerod-Python library version: 3.0.0

The manuscript's abstract is the following:

Researchers have explored the performance of Iterated Prisoner's Dilemma strategies for decades: from the celebrated performance of Tit for Tat, to the introduction of the zero-determinant strategies, to the use of sophisticated learning structures such as neural networks, many new strategies have been introduced and tested in a variety of tournaments and population dynamics. Typical results in the literature, however, rely on performance against a small number of somewhat arbitrarily selected strategies in a small number of tournaments, casting doubt on the generalizability of conclusions. We analyze a large collection of 195 strategies in 45606 tournaments, present the top performing strategies across multiple tournament types, and distill their salient features. The results show that there is not yet a single strategy that performs well in diverse Iterated Prisoner's Dilemma scenarios, nevertheless there are several properties that heavily influence the best performing strategies. This refines the properties described by R. Axelrod in light of recent and more diverse opponent populations to: be nice, be provocable and generous, be a little envious, be clever, and adapt to the environment. More precisely, we find that strategies perform best when their probability of cooperation matches the total tournament population's aggregate cooperation probabilities, or a proportion thereof in the case of noisy and probabilistically ending tournaments, and that the manner in which a strategy

achieves the ideal cooperation rate is crucial. The features of high performing strategies help cast some light on why strategies such as Tit For Tat performed historically well in tournaments and why zero-determinant strategies typically do not fare well in tournament settings.

The differences between the Chapter and the manuscript include ....

4.1 Introduction

As stated in Chapter 1 conceptualising strategies and understanding the best way of playing the game has been of interest to the scientific community since the formulation of the game. In Chapter 2 it was established that following the computer tournaments of Axelrod in the 1980's, a strategy's performance in a round robin computer tournament became a common evaluation technique for newly designed strategies.

The winner of both of Axelrod's tournaments [27, 28] was the simple strategy Tit For Tat and Axelrod concluded that the strategy's robustness was due to four properties, which he adapted into four suggestions on doing well in an IPD:

- Do not be envious by striving for a payoff larger than the opponent's payoff
- Be "nice"; Do not be the first to defect
- Reciprocate both cooperation and defection; Be provocable to retaliation and forgiveness
- Do not be too clever by scheming to exploit the opponent

Forgiveness is a strategy's ability to go from a DC to C aiming to achieve mutual cooperation again, the only way Tit For Tat would end up in DC, in environments without noise, is if it had received a defection and then retaliated. Subsequently, Tit For Tat would forgive an opponent that apologises (in a DC round) by returning to cooperation, since mutual cooperation is better than mutual defection. As a result of the strategy's strong performance in both tournaments, and moreover in a series of evolutionary experiments [29], Tit For Tat was often claimed to be a highly robust (and sometimes the most robust) strategy for the IPD.

A large collection of works were discussed in Chapter 2 which introduced a broad collection of strategies, and new strategies and competitions are published frequently, as established in Chapter 3. The question, however, still remains the same: what is the best way to play the game?

Compared to the works reviewed in Chapter 2, where typically a few selected or introduced strategies are evaluated on a small number of tournaments and/or small number of opponents, this Chapter evaluates the performance of 195 strategies in 45606 tournaments. Furthermore, a large portion of these strategies are drawn from the known and named strategies in IPD literature, including many previous tournament winners, in contrast to other work that may have randomly generated many essentially arbitrary strategies (typically restrained to a class such as memory-one strategies, or those of a certain structural form such as finite state machines or deterministic memory two strategies). Additionally, our tournaments come in a number of variations including standard tournaments emulating Axelrod's original tournaments, tournaments with noise, probabilistic match length, and both noise and probabilistic match length.

Additionally, the analysis of this Chapter considers tournament variations including standard tournaments, tournaments with noise, probabilistic match length, and both noise and probabilistic match length. This diversity of strategies and tournament types yields new insights and tests earlier claims in alternative settings against known powerful strategies.

The later part of the Chapter evaluates the impact of features on the performance of the strategies using modern machine learning techniques. These features include measures regarding a strategy's behaviour and measures regarding the tournaments. The outcomes reinforce the discussion started by Axelrod, and conclude that the properties of a successful strategy in the IPD are:

- ~~Do not be envious~~ Be a little bit envious
- Be "nice" in non-noisy environments or when game lengths are longer
- Reciprocate both cooperation and defection appropriately; Be provocable in tournaments with short matches, and generous when matches are longer
- ~~Do not be too clever~~ It's ok to be clever
- Adapt to the environment; Adjust to the mean population cooperation

The rest of the Chapter is structured as follows:

- section 4.2 covers the different tournament types and the data collection which are made possible due to APL.
- section 4.3 focuses on the best performing strategies for each type of tournament and overall.
- section 4.4,explores the traits which contribute to a good performance.

4.2 Data collection

The data set generated for this Chapter was created with APL version 3.0.0. APL allows for different types of IPD computer tournaments to be simulated and contains a large list of strategies. Most of these are strategies described in the literature with a few exceptions of strategies that have been contributed specifically to the package. A list of the strategies is given in the Appendix A.1. Although APL features several tournament types, only standard, noisy, probabilistic ending, and noisy probabilistic ending tournaments are considered here.

Standard tournaments are tournaments similar to that of Axelrod's tournaments [27]. There are N strategies which all play an iterated game of n number of turns against each other. Note that self-interactions are not included. Similarly, *noisy tournaments* have N strategies and n number of turns, but at each turn there is a probability p_n that a player's action will be flipped. *Probabilistic ending tournaments*, are of size N and after each turn a match between strategies ends with a given probability p_e. Finally, *noisy probabilistic ending* tournaments have both a noise probability p_n and an ending probability p_e. For smoothing the simulated results a tournament is repeated for k number of times. This was allowed to vary in order to evaluate the effect of smoothing. The winner of each tournament is based on the median score a strategy achieved and not by the number of wins.

The process of collecting tournament results is described by Algorithm 1. For each trial a random size N is selected, and from the 195 strategies a random list of N strategies is chosen. For the given list of strategies a standard, a noisy, a probabilistic ending and a noisy probabilistic ending tournament are performed and repeated k times. The parameters for the tournaments, as well as the number of repetitions, are selected once for each trial. The parameters and their respective minimum and maximum values are given by Table 4.1.

parameter	parameter explanation	min value	max value
N	number of strategies	3	195
k	number of repetitions	10	100
n	number of turns	1	200
p_n	probability of flipping action at each turn	0	1
p_e	probability of match ending in the next turn	0	1

Table 4.1: Data collection; parameters' values

Algorithm 1: Data collection Algorithm

foreach *seed* $\in [0, 11420]$ **do**

  $N \leftarrow$ randomly select integer $\in [N_{min}, N_{max}]$;

  players $\leftarrow$ randomly select N players;

  $k \leftarrow$ randomly select integer $\in [k_{min}, k_{max}]$;

  $n \leftarrow$ randomly select integer $\in [n_{min}, n_{max}]$;

  $p_n \leftarrow$ randomly select float $\in [p_{n\,min}, p_{n\,max}]$;

  $p_e \leftarrow$ randomly select float $\in [p_{e\,min}, p_{e\,max}]$;

  result standard $\leftarrow$ Axelrod.tournament(players, n, k);

  result noisy $\leftarrow$ Axelrod.tournament(players, n, p_n, k);

  result probabilistic ending $\leftarrow$ Axelrod.tournament(players, p_e, k);

  result noisy probabilistic ending $\leftarrow$ Axelrod.tournament(players, p_n, p_e, k);

return *result standard, result noisy, result probabilistic ending, result noisy probabilistic ending*;

A total of 11400 trials of Algorithm 1 have been run. For each trial the results for 4 different tournaments were collected, thus a total of 45606 (11400 × 4) tournament results have been retrieved. Each tournament outputs a result summary in the form of Table 4.2. Each strategy has participated on average in 5154 tournaments of each type. The strategy with the maximum participation in each tournament type is Inverse Punisher with 5639 entries. The strategy with the minimum entries is EvolvedLookerUp 1 1 1 which was selected in 4693 trials.

A result summary (Table 4.2) has N rows because each row contains information for each strategy that participated in the tournament. The information includes the strategy's rank, median score, the rate with which the strategy cooperated (C_r), its match win count, and the probability that the strategy cooperated in the opening move. Moreover, the probabilities of a strategy being in any of the four states (CC, CD, DC, DD), and the rate of which the strategy cooperated after each state. The *normalised rank* is a feature that has been manually added to the result summary. The rank R of a given strategy can vary between 0 (first) and $N - 1$ (last), and thus the normalised rank, denoted as r, is calculated as a strategy's rank divided by $N - 1$.

						Rates							
Rank	Name	Median score	Cooperation rating (C_r)	Win	Initial C	CC	CD	DC	DD	CC to C	CD to C	DC to C	DD to C
0	EvolvedLookerUp2 2 2	2.97	0.705	28.0	1.0	0.639	0.066	0.189	0.106	0.836	0.481	0.568	0.8
1	Evolved FSM 16 Noise 05	2.875	0.697	21.0	1.0	0.676	0.020	0.135	0.168	0.985	0.571	0.392	0.07
2	PSO Gambler 1 1 1	2.874	0.684	23.0	1.0	0.651	0.034	0.152	0.164	1.000	0.283	0.000	0.136
3	PSO Gambler Mem1	2.861	0.706	23.0	1.0	0.663	0.042	0.145	0.150	1.000	0.510	0.000	0.122
4	Winner12	2.835	0.682	20.0	1.0	0.651	0.031	0.141	0.177	1.000	0.441	0.000	0.462
...	...	...	...	...	...	...	...	...	...	...	...	...	...

Table 4.2: Output result of a single tournament.

4.3 Top ranked strategies

The performance of each strategy is evaluated in four tournament types, as presented in section 4.2, followed by an evaluation of their performance over all the 45606 simulated tournaments. Each strategy participated in multiple tournaments of the same type (on average 5154). For example Tit For Tat participated in a total of 5114 tournaments of each type. The strategy's normalised rank distribution in these is given in Figure 4.1. A value of $r = 0$ corresponds to a strategy winning the tournament where a value of $r = 1$ corresponds to the strategy coming last. Because of the strategies' multiple entries their performance is evaluated based on the *median normalised rank* denoted as $\bar{r}$.

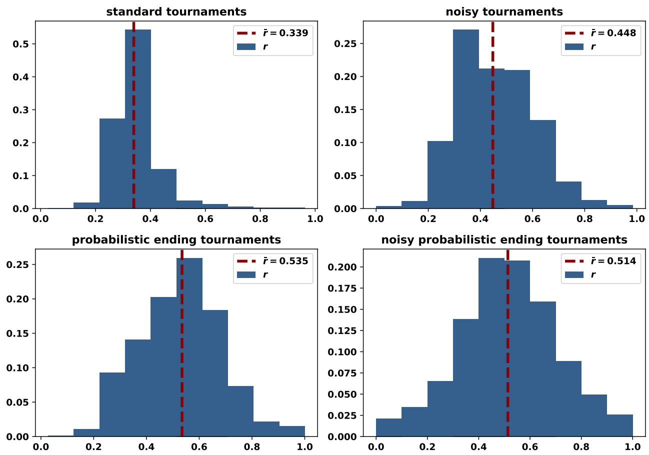
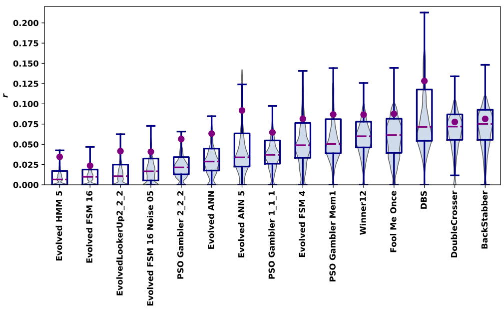


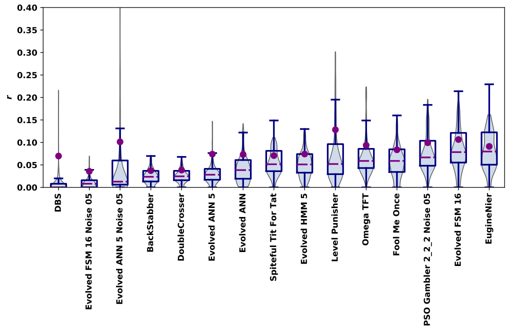
Figure 4.1: Tit For Tat's r distribution in tournaments. Lower values of r correspond to better performances. The best performance of the strategy has been in standard tournaments where it achieved a $\bar{r}$ of 0.34.

The top 15 strategies for each tournament type based on $\bar{r}$ are given in Table 4.3. The data collection process was designed such that the probabilities of noise and ending of the match varied between 0 and 1. However, commonly used values for these probabilities are values less than 0.1. Thus, Table 4.3 also includes the top 15 strategies in noisy tournaments with $p_n < 0.1$ and probabilistic ending tournaments with $p_e < 0.1$. The r distributions for the top ranked strategies of Table 4.3 are given by Figure 4.2.

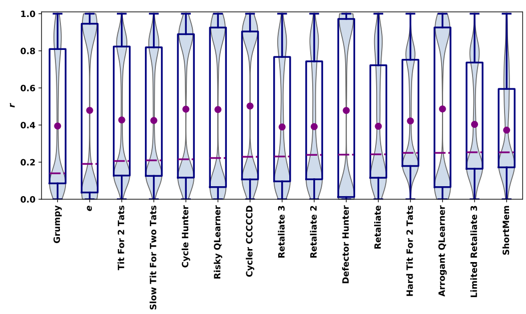
In standard tournaments 10 out of the 15 top strategies were introduced in [84]. These are strategies based on finite state automata (FSM), hidden Markov models (HMM), artificial neural networks (ANN), lookup tables (LookerUp) and stochastic lookup tables (Gambler) that



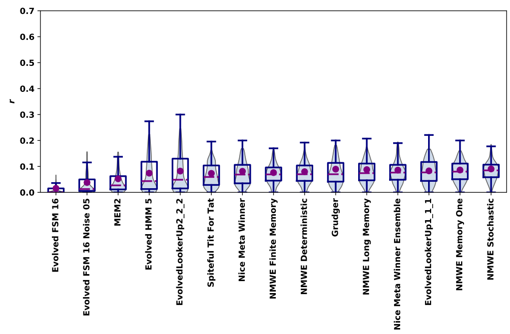
(a) r distributions of top 15 strategies in standard tournaments.



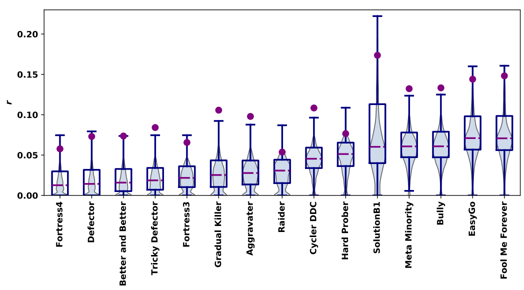
(b) r distributions of top 15 strategies in noisy tournaments with $p_n < 0.1$.



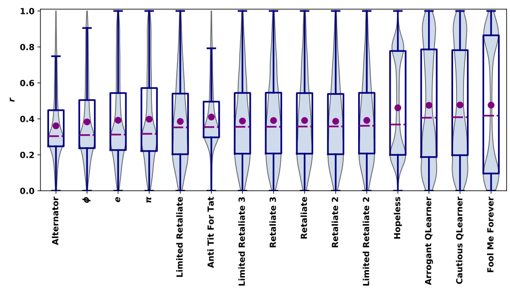
(c) r distributions of top 15 strategies in noisy tournaments.



(d) r distributions of top 15 strategies in 1139 probabilistic ending tournaments with $p_e < 0.1$.



(e) r distributions of top 15 strategies in probabilistic ending tournaments.



(f) r distributions of top 15 strategies in noisy probabilistic ending tournaments.

Figure 4.2: r distributions of the top 15 strategies in different environments. A lower value of $\bar{r}$ corresponds to a more successful performance. A strategy's r distribution skewed towards zero indicates that the strategy ranked highly in most tournaments it participated in. Most distributions are skewed towards zero except the distributions with unrestricted noise, supporting the conclusions from Table 4.3.

	Standard		Noisy		Noisy ($p_n < 0.1$)		Probabilistic ending		Probabilistic ending ($p_e < 0.1$)		Noisy probabilistic ending	
	Name	$\bar{r}$	Name	$\bar{r}$	Name	$\bar{r}$	Name	$\bar{r}$	Name	$\bar{r}$	Name	$\bar{r}$
0	Evolved HMM 5	0.007	Grumpy	0.140	DBS	0.000	Fortress4	0.013	Evolved FSM 16	0.000	Alternator	0.304
1	Evolved FSM 16	0.010	e	0.194	Evolved FSM 16 Noise 05	0.008	Defector	0.014	Evolved FSM 16 Noise 05	0.013	ϕ	0.310
2	EvolvedLookerUp2 2 2	0.011	Tit For 2 Tats	0.206	Evolved ANN 5 Noise 05	0.013	Better and Better	0.016	MEM2	0.027	e	0.312
3	Evolved FSM 16 Noise 05	0.017	Slow Tit For Two Tats	0.210	BackStabber	0.024	Tricky Defector	0.019	Evolved HMM 5	0.044	π	0.317
4	PSO Gambler 2 2 2	0.021	Cycle Hunter	0.215	DoubleCrosser	0.025	Fortress3	0.022	EvolvedLookerUp2 2 2	0.049	Limited Retaliate	0.353
5	Evolved ANN	0.029	Risky QLearner	0.222	Evolved ANN 5	0.028	Gradual Killer	0.025	Spiteful Tit For Tat	0.060	Anti Tit For Tat	0.354
6	Evolved ANN 5	0.034	Retaliate 3	0.229	Evolved ANN	0.038	Aggravater	0.028	Nice Meta Winner	0.068	Limited Retaliate 3	0.356
7	PSO Gambler 1 1 1	0.037	Cycler CCCCCD	0.235	Spiteful Tit For Tat	0.051	Raider	0.031	NMWE Finite Memory	0.069	Retaliate 3	0.356
8	Evolved FSM 4	0.049	Retaliate 2	0.239	Evolved HMM 5	0.051	Cycler DDC	0.045	NMWE Deterministic	0.070	Retaliate	0.357
9	PSO Gambler Mem1	0.050	Defector Hunter	0.240	Level Punisher	0.052	Hard Prober	0.051	Grudger	0.070	Retaliate 2	0.358
10	Winner12	0.060	Retaliate	0.242	Omega TFT	0.059	SolutionB1	0.060	NMWE Long Memory	0.074	Limited Retaliate 2	0.361
11	Fool Me Once	0.061	Hard Tit For 2 Tats	0.250	Fool Me Once	0.059	Meta Minority	0.061	Nice Meta Winner Ensemble	0.076	Hopeless	0.368
12	DBS	0.071	Limited Retaliate 3	0.253	PSO Gambler 2 2 2 Noise 05	0.067	Bully	0.061	EvolvedLookerUp1 1 1	0.077	Arrogant QLearner	0.407
13	DoubleCrosser	0.072	ShortMem	0.253	Evolved FSM 16	0.078	EasyGo	0.071	NMWE Memory One	0.080	Cautious QLearner	0.409
14	BackStabber	0.075	Limited Retaliate	0.257	EugineNier	0.080	Fool Me Forever	0.071	Winner12	0.085	Fool Me Forever	0.418

Table 4.3: Top performances for each tournament type based on $\bar{r}$. The results of each type are based on 11420 unique tournaments. The results for noisy tournaments with $p_n < 0.1$ are based on 1151 tournaments, and for probabilistic ending tournaments with $p_e < 0.1$ on 1139. The top ranks indicate that trained strategies perform well in a variety of environments, but so do simple deterministic strategies. The normalised medians are close to 0 for most environments, except environments with noise not restricted to 0.1 regardless of the number of turns. Noisy and noisy probabilistic ending tournaments have the highest medians.

have been trained using reinforcement learning algorithms (evolutionary and particle swarm algorithms). They have been trained to perform well against a subset of the strategies in APL in a standard tournament, thus their performance in the specific setting was anticipated although still noteworthy given the random sampling of tournament participants. DoubleCrosser, BackStabber and Fool Me Once, are strategies not from the literature but from the APL. DoubleCrosser is an extension of BackStabber and both strategies make use of the number of turns because they are set to defect on the last two rounds. It should be noted that these strategies can be characterised as "cheaters" because the source code of the strategies allows them to know the number of turns in a match (unless the match has a probabilistic ending). These strategies were expected to not perform as well in tournaments where the number of turns is not specified. Finally, Winner 12 [130] and DBS [26] are both from the literature. DBS is a strategy specifically designed for noisy environments, however, it ranks highly in standard tournaments as well. Similarly the fourth ranked player, Evolved FSM 16 Noise 05, was trained for noisy tournaments yet performs well in standard tournaments. Figure 4.2a shows that these strategies typically perform well in any standard tournament in which they participate.

In the case of noisy tournaments with smaller noise $p_n < 0.1$ the top performing strategies include strategies specifically designed for noisy tournaments. These are DBS, Evolved FSM 16 Noise 05, Evolved ANN 5 Noise 05, PSO Gambler 2 2 2 Noise 05 and Omega Tit For Tat [102]. Omega Tit For Tat, another strategy designed to break the deadlocking cycles of CD and DC that Tit For Tat can fall into in noisy environments, places 10th. The rest of the top ranks are occupied by strategies which performed well in standard tournaments and deterministic strategies such as Spiteful Tit For Tat [4], Level Punisher [8], Eugine Nier [158].

In contrast, the performance of the top ranked strategies in noisy environments when $p_n \in [0, 1]$ is bimodal. The top strategies include strategies which decide their actions based on the cooperation to defection ratio, such as ShortMem [46], Grumpy [7] and e [7], and the Retaliate strategies which are designed to defect if the opponent has tricked them more often than a given percentage of the times that they have done the same. The bimodality of the r distributions is explained by Figure 4.3 which demonstrates that the top 6 strategies were highly ranked due to the their performance in tournaments with $p_n > 0.5$, and that in tournaments with $p_n < 0.5$ they performed poorly. At a noisy level of 0.5 or greater, mostly cooperative strategies become mostly defectors and vice versa.

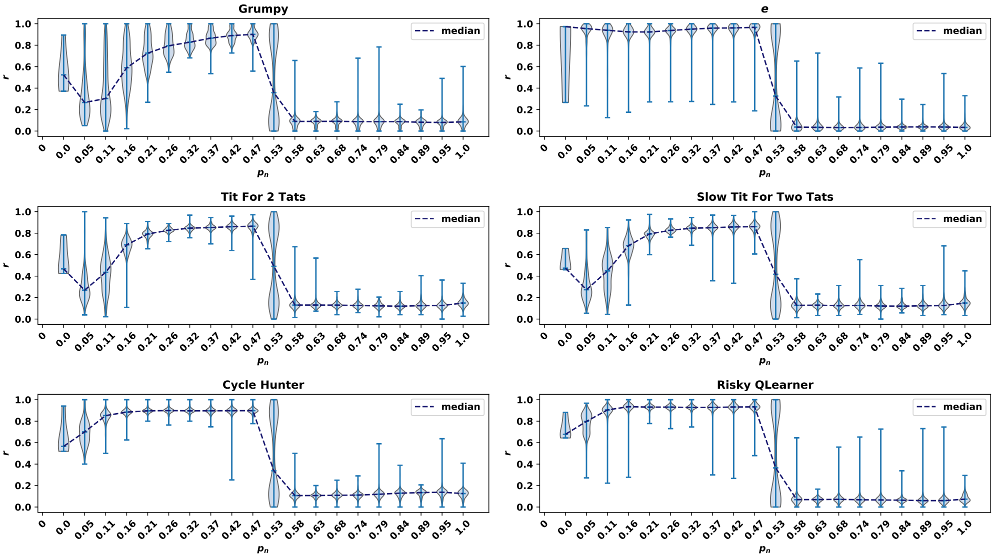


Figure 4.3: Normalised rank r distributions for top 6 strategies in noisy tournaments over the probability of noisy (p_n).

The most effective strategies in probabilistic ending tournaments with $p_e < 0.1$ are a series of ensemble Meta strategies, trained strategies which performed well in standard tournaments, and Grudger [7] and Spiteful Tit for Tat [4]. The Meta strategies [7] utilize a team of strategies and aggregate the potential actions of the team members into a single action in various ways. Figure 4.2d indicates that these strategies performed well in any probabilistic ending tournament.

In probabilistic ending tournaments with $p_e \in [0, 1]$ the top ranks are mostly occupied by defecting strategies such as Better and Better, Gradual Killer, Hard Prober (all from [7]), Bully (Reverse Tit For Tat) [141] and Defector, and a series of strategies based on finite state automata introduced by Daniel Ashlock and Wendy Ashlock: Fortress 3, Fortress 4 (both introduced in [23]), Raider [25] and Solution B1 [25]. The success of defecting strategies in probabilistic ending tournaments is due to larger values of p_e which lead to shorter matches (the expected number of rounds is $1/p_e$), so the impact of the PD being iterated is subdued. This is captured by the Folk Theorem [67] as defecting strategies do better when the likelihood of the game ending in the next turn increases. This is demonstrated by Figure 4.4, which gives the distributions of r for the top 6 strategies in probabilistic ending tournaments over p_e.

The top performances in tournaments with both noise and a probabilistic ending and the top performances over the entire data set have the largest median values compared to the top rank strategies of the other tournament types, Figure 4.2f and Figure 4.5. The $\bar{r}$ for the top strategy is approximately at 0.3, indicating that the most successful strategy can on average just place in the top 30% of the competition.

On the whole, the analysis of this section has shown that:

- In standard tournaments the dominating strategies were strategies that had been trained

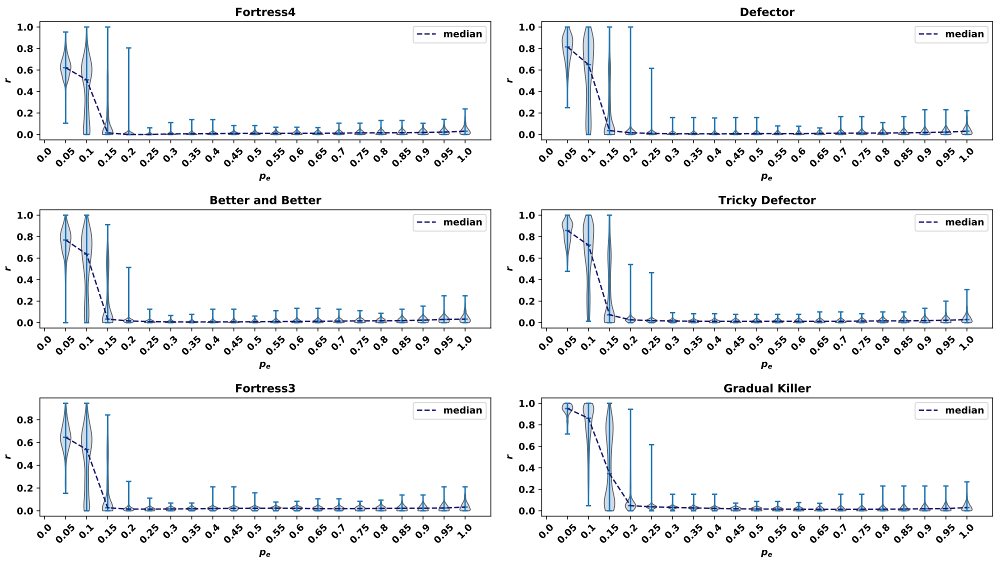


Figure 4.4: Normalised rank r distributions for top 6 strategies in probabilistic ending tournaments over p_e. The 6 strategies start of with a high median rank, however, their ranked decreased as the the probability of the game ending increased and at the point of $p_e = 0.1$.

Name	$\bar{r}$
Limited Retaliate 3	0.286
Retaliate 3	0.297
Retaliate 2	0.302
Limited Retaliate 2	0.304
Limited Retaliate	0.311
Retaliate	0.317
BackStabber	0.324
DoubleCrosser	0.331
Nice Meta Winner	0.350
PSO Gambler 2 2 2 Noise 05	0.351
Grudger	0.352
NMWE Memory One	0.357
Evolved HMM 5	0.358
Nice Meta Winner Ensemble	0.359
Forgetful Fool Me Once	0.359

Table 4.4: Top performances over all the tournaments. The top ranks include strategies that have been previously mentioned. The set of Retaliate strategies occupy the top spots followed by BackStabber and DoubleCrosser. The distributions of the Retaliate strategies have no statistical difference. PSO Gambler and Evolved HMM 5 are trained strategies introduced in [84] and Nice Meta Winner and NMWE Memory One are strategies based on teams. Grudger is a strategy from R. Axelrod's original tournament and Forgetful Fool Me Once is based on the same approach as Grudger.

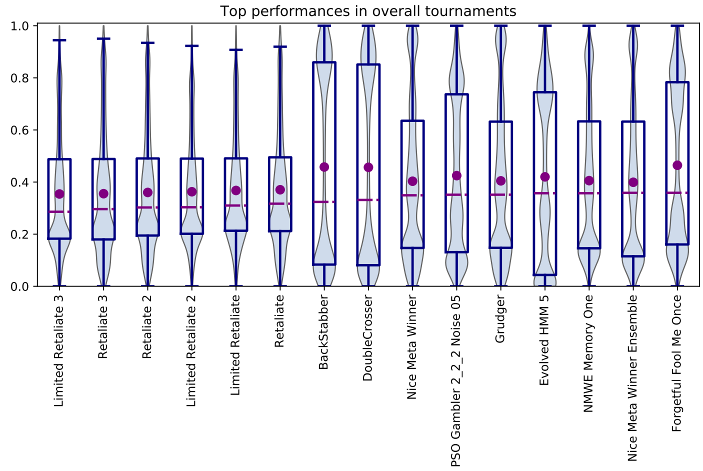


Figure 4.5: r distributions for best performed strategies in the data set [71]. A lower value of $\bar{r}$ corresponds to a more successful performance.

using reinforcement learning techniques.

- In noisy environments where the noise probability strictly less than 0.1 was considered, the successful strategies were strategies specifically designed or trained for noisy environments.
- In probabilistic ending tournaments most of the highly ranked strategies were defecting strategies and trained finite state automata, all by the authors of [23, 25]. These strategies ranked high due to their performance in tournaments where the probability of the game ending after each turn was bigger than 0.1.
- In probabilistic tournaments with p_e less than 0.1 the highly ranked strategies were strategies based on the behaviour of others.
- From the collection of strategies considered here, no strategy can be consistently successful in noisy environments, except if the value of noise is constrained to less than a 0.1.

Though there is not a single strategy that repeatably outranks all others in any of the distinct tournament types, or even across the tournament types, there are specific types of strategies have been repeatably ranked in the top ranks. These have been strategies that have been trained, strategies that retaliate, and strategies that would adapt their behaviour based on preassigned rules to achieve the highest outcome. These results contradict some of R. Axelrod's suggestions, and more specifically, the suggestions 'Do not be clever' and 'Do not be envious'. The features and properties contributing a strategy's success are further explored in section 4.4.

4.4 Evaluation of performance

This section examines the performance of the strategies based on features of strategies described in Table 4.5. These features are measures regarding a strategy's behaviour from the tournaments the strategies competed in as well as intrinsic properties such as whether a strategy is deterministic or stochastic.

feature	feature explanation	source	value type	min value	max value
stochastic	If a strategy is stochastic	strategy classifier from APL	boolean	Na	Na
makes use of game	If a strategy makes used of the game information	strategy classifier from APL	boolean	Na	Na
makes use of length	If a strategy makes used of the number of turns	strategy classifier from APL	boolean	Na	Na
memory usage	The memory size of a strategy divided by the number of turns	memory size from APL	float	0	1
SSE	A measure of how far a strategy is from ZD behaviour	method described in [105]	float	0	1
max cooperating rate ($C_{\max}$)	The biggest cooperating rate in a given tournament	result summary	float	0	1
min cooperating rate ($C_{\min}$)	The smallest cooperating rate in a given tournament	result summary	float	0	1
median cooperating rate (C_{median})	The median cooperating rate in a given tournament	result summary	float	0	1
mean cooperating rate (C_{mean})	The mean cooperating rate in a given tournament	result summary	float	0	1
C_r / $C_{\max}$	A strategy's cooperating rate divided by the maximum	result summary	float	0	1
$C_{\min}$ / C_r	A strategy's cooperating rate divided by the minimum	result summary	float	0	1
C_r / C_{median}	A strategy's cooperating rate divided by the median	result summary	float	0	1
C_r / C_{mean}	A strategy's cooperating rate divided by the mean	result summary	float	0	1
C_r	The cooperating ratio of a strategy	result summary	float	0	1
CC to C rate	The probability a strategy will cooperate after a mutual cooperation	result summary	float	0	1
CD to C rate	The probability a strategy will cooperate after being betrayed by the opponent	result summary	float	0	1
DC to C rate	The probability a strategy will cooperate after betraying the opponent	result summary	float	0	1
DD to C rate	The probability a strategy will cooperate after a mutual defection	result summary	float	0	1
p_n	The probability of a player's action being flip at each interaction	trial summary	float	0	1
n	The number of turns	trial summary	integer	1	200
p_e	The probability of a match ending in the next turn	trial summary	float	0	1
N	The number of strategies in the tournament	trial summary	integer	3	195
k	The number of repetitions of a given tournament	trial summary	integer	10	100

Table 4.5: The features which are included in the performance evaluation analysis. Stochastic, makes use of length and makes use of game are APL classifiers that determine whether a strategy is stochastic or deterministic, whether it makes use of the number of turns or the game's payoffs. The memory usage is calculated as the number of turns the strategy considers to make an action (which is specified in the APL) divided by the number of turns. The SSE (introduced in [105]) shows how close a strategy is to behaving as a ZDs, and subsequently, in an extortionate way. The method identifies the ZDs closest to a given strategy and calculates the algebraic distance between them, defined as SSE. More details on the measure are presented in Chapter **??**. A SSE value of 1 indicates no extortionate behaviour at all whereas a value of 0 indicates that a strategy is behaving as a ZDs. The rest of the features considered are the CC to C, CD to C, DC to C, and DD to C rates as well as cooperating ratio of a strategy, the minimum (C_{min}), maximum (C_{max}), mean (C_{mean}) and median (C_{median}) cooperating ratios of each tournament.

The memory usage of strategies is the number of rounds of play used by the strategy divided by the number of turns in each match. For example, Winner12 uses the previous two rounds of play, and if participating in a match with 100 turns its memory usage would be 2/100. For strategies with an infinite memory size, for example Evolved FSM 16 Noise 05, memory usage is equal to 1. Note that for tournaments with a probabilistic ending the number of turns was not collected, so the memory usage feature is not used for probabilistic ending tournaments.

The correlation coefficients between the features of Table 4.5 the median score and the median normalised rank are given by Table 4.6. The correlation coefficients between all features of Table 4.5 have been calculated and a graphical representation can be found in the Appendix B.

	Standard		Noisy		Probabilistic ending		Noisy probabilistic ending		Overall	
	r	median score	r	median score	r	median score	r	median score	r	median score
CC to C rate	-0.501	0.501	0.414	-0.504	0.408	-0.323	0.260	0.022	0.108	0.081
CD to C rate	0.226	-0.199	0.456	-0.330	0.320	-0.017	0.205	-0.220	0.281	-0.177
C_r	-0.323	0.384	0.711	-0.678	0.714	-0.832	0.579	-0.135	0.360	-0.124
C_r / C_{max}	-0.323	0.381	0.616	-0.551	0.714	-0.833	0.536	-0.116	0.395	-0.265
C_r / C_{mean}	-0.331	0.358	0.731	-0.740	0.721	-0.861	0.649	-0.621	0.428	-0.439
C_r / C_{median}	-0.331	0.353	0.652	-0.669	0.712	-0.852	0.330	-0.466	0.294	-0.405
C_r / C_{min}	0.109	-0.080	-0.358	0.250	-0.134	0.150	-0.368	0.113	0.000	0.280
C_{max}	-0.000	0.049	0.000	0.023	-0.000	0.046	0.000	-0.004	-0.000	0.553
C_{mean}	-0.000	0.229	-0.000	0.271	0.000	0.200	0.000	0.690	-0.000	0.544
C_{median}	0.000	0.209	-0.000	0.240	-0.000	0.187	-0.000	0.673	0.000	-0.250
C_{min}	0.000	0.084	0.000	-0.017	-0.000	0.007	-0.000	0.041	-0.161	-0.190
DC to C rate	0.127	-0.100	0.509	-0.504	-0.018	0.033	0.341	-0.016	0.173	-0.088
DD to C rate	0.412	-0.396	0.533	-0.436	-0.103	0.176	0.378	-0.263	0.237	-0.239
N	0.000	-0.009	-0.000	0.002	-0.000	0.003	-0.000	0.001	-0.000	-0.001
k	0.000	-0.002	-0.000	0.003	-0.000	0.001	-0.000	-0.008	0.000	-0.001
n	0.000	-0.125	-0.000	-0.024	-	-	-	-	0.000	-0.074
p_e	-	-	-	-	0.000	0.165	0.000	-0.058	0.000	0.055
p_n	-	-	-0.000	0.207	-	-	-0.000	-0.650	-0.000	-0.256
Make use of game	-0.003	-0.022	0.025	-0.082	-0.053	-0.108	0.013	-0.016	-0.004	-0.053
Make use of length	-0.158	0.124	0.005	-0.123	-0.025	-0.090	0.014	-0.016	-0.041	-0.026
SSE	0.473	-0.452	0.463	-0.337	-0.156	0.223	0.305	-0.259	0.233	-0.167
memory usage	-0.082	0.095	-0.007	-0.017	-	-	-	-	-0.053	0.046
stochastic	0.006	-0.024	0.022	-0.026	0.002	-0.130	0.021	-0.013	0.013	-0.048

Table 4.6: Correlations between the features of Table 4.5 and the normalised rank and the median score.

In standard tournaments the features CC to C, C_r, C_r/C_{max} and the cooperating ratio compared to C_{median} and C_{mean} have a moderately negative effect on the normalised rank (smaller rank is better), and a moderate positive on the median score. The SSE error and the DD to C rate have the opposite effects. Thus, in standard tournaments behaving cooperatively corresponds to a more successful performance. Even though being nice generally pays off that does not hold against defective strategies. Being more cooperative after a mutual defection, that is not retaliating, is associated to lesser overall success in terms of normalised rank. Figure 4.6 confirms that the winners of standard tournaments always cooperate after a mutual cooperation and almost always defect after a mutual defection.

Compared to standard tournaments, in both noisy and in probabilistic ending tournaments the higher the rates of cooperation the lower a strategy's success and median score. A strategy would want to cooperate less than both the mean and median cooperator in such settings. In probabilistic ending tournaments the correlation coefficients have larger values, indicating a stronger effect. Thus a strategy will be punished more by its cooperative behaviour in prob-

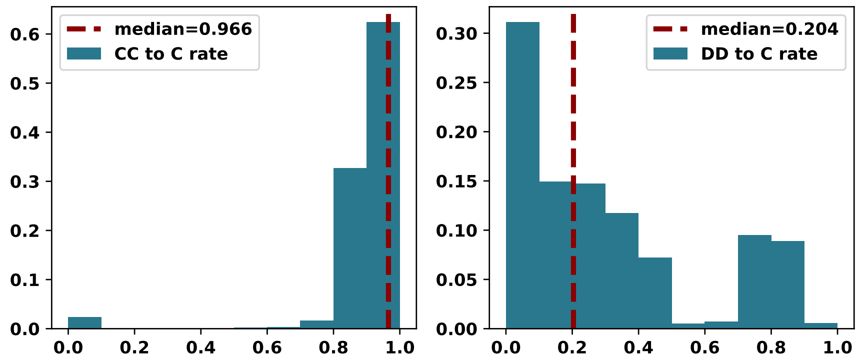


Figure 4.6: Distributions of CC to C and DD to C for the winners in standard tournaments.

abilistic ending environments, supporting the results of section 4.4 as well. The distributions of the C_r of the winners in both tournaments are given by Figure 4.7. It confirms that the winners in noisy tournaments cooperated less than 35% of the time and in probabilistic ending tournaments less than 10%. In noisy probabilistic ending tournaments and over all the tournaments' results, the only features that had a moderate effect are $C_r/C_{\text{mean}}, C_r/C_{\text{max}}$ and C_r. In such environments cooperative behaviour appears to be punished less than in noisy and probabilistic ending tournaments.

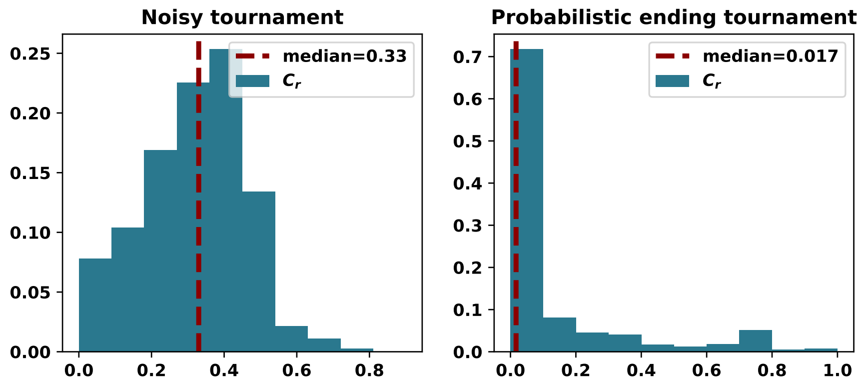


Figure 4.7: C_r distributions of the winners in noisy and in probabilistic ending tournaments.

A multivariate linear regression has been fitted to model the relationship between the features and the normalised rank. Based on the graphical representation of the correlation matrices given in Appendix B several of the features are highly correlated and have been removed before fitting the linear regression model. The features included are given by Table 4.7 alongside their corresponding p values in the distinct tournaments and their regression coefficients.

A multivariate linear regression has also be fitted on the median score. The coefficients and p values of the features can be found in Appendix C.1. The results of the two methods are in agreement.

The feature C_r/C_{mean} has a statistically significant effect across all models and a high regression coefficient. It has both a positive and negative impact on the normalised rank depending on the environment. For standard tournaments, Figure 4.8 gives the distributions of several features for the winners of standard tournaments. The C_r/C_{mean} distribution of the winner is also given in Figure 4.8. A value of $C_r/C_{\text{mean}} = 1$ implies that the cooperating ratio of the winner was

	Standard		Noisy		Probabilistic ending		Noisy probabilistic ending		Overall	
	R adjusted: 0.541		R adjusted: 0.639		R adjusted: 0.587		R adjusted: 0.577		R adjusted: 0.242	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
CC to C rate	-0.042	0.000	-0.007	0.000	0.017	0.000	0.111	0.0	-0.099	0.0
CD to C rate	0.297	0.000	-0.068	0.000	0.182	0.000	0.023	0.0	0.129	0.0
C_r / C_{max}	-	-	1.856	0.000	-	-	1.256	0.0	-	-
C_r / C_{mean}	-0.468	0.000	-0.577	0.000	0.525	0.000	-0.120	0.0	0.300	0.0
C_{max}	-0.071	0.000	-	-	-0.022	0.391	1.130	0.0	-	-
C_{mean}	0.118	0.000	-2.558	0.000	-0.023	0.001	-1.489	0.0	-	-
C_{min}	-0.161	0.000	-1.179	0.000	-0.170	0.000	-	-	-	-
C_{min} / C_r	0.057	0.000	-0.320	0.000	0.125	0.000	-	-	-0.103	0.0
DC to C rate	0.198	0.000	0.040	0.000	-0.030	0.000	0.022	0.0	0.064	0.0
k	0.000	0.319	0.000	0.020	0.000	0.002	0.000	0.0	-	-
n	0.000	0.000	-	-	-	-	-	-	-	-
p_e	-	-	-	-	0.000	0.847	-0.083	0.0	-	-
p_n	-	-	-0.048	0.000	-	-	-	-	-	-
SSE	0.258	0.000	0.153	0.000	-0.041	0.000	0.100	0.0	0.056	0.0
constant	0.697	0.000	1.522	0.000	-0.057	0.019	-0.472	0.0	0.178	0.0
memory usage	-0.010	0.000	-0.000	0.035	-	-	-	-	-	-

Table 4.7: Results of multivariate linear regressions with r as the dependent variable. R squared is reported for each model.

the same as the mean cooperating ratio of the tournament, and in standard tournaments, the median is 1. Therefore, an effective strategy in standard tournaments was the mean cooperator of its respective tournament.

The distributions of SSE and CD to C rate for the winners of standard tournaments are also given in Figure 4.8. The SSE distributions for the winners indicate that the strategy behaved in a ZD way in several tournaments, however, not constantly. The winners participated in matches where they did not try to extortionate their opponents. Furthermore, the CD to C distribution indicates that if a strategy were to defect against the winners the winners would reciprocate on average with a probability of 0.5.

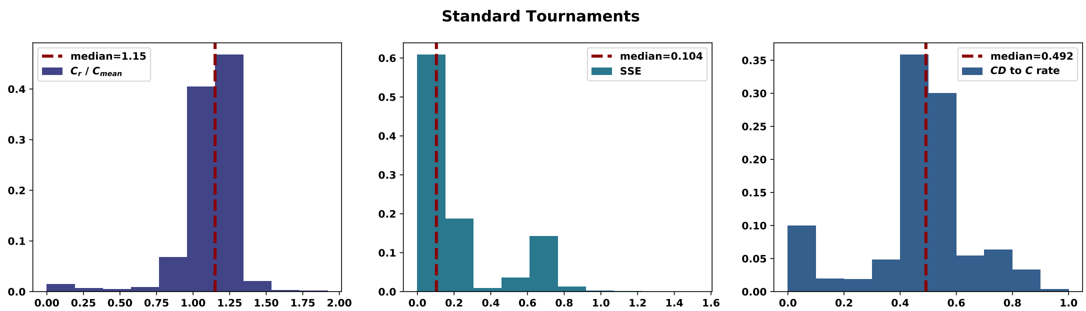


Figure 4.8: Distributions of C_r/C_{mean}, SSE and CD to C ratio for the winners of standard tournaments. A value of $C_r/C_{\text{mean}} = 1$ imply that the cooperating ratio of the winner was the same as the mean cooperating ratio of the tournament. An SSE distribution skewed towards 0 indicates a extortionate behaviour by the strategy.

Similarly for the rest of the different tournaments types, and the entire data set the distributions of C_r/C_{mean}, SSE and CD to C ratio are given by Figures 4.9, 4.11, 4.12 and 4.13.

Based on the C_r/C_{mean} distributions the successful strategies have adapted differently to the mean cooperator depending on the tournament type. In noisy tournaments where the median of the distribution is at 0.67, and thereupon the winners cooperated 67% of the time the mean cooperator did. In tournaments with noise and a probabilistic ending the winners cooperated 60%, whereas in settings that the type of the tournament can vary between all the types the winners cooperated 67% of the time the mean cooperator did. Lastly, in probabilistic ending tournaments above more defecting strategies prevail (section 4.3), and this result is reflected here.

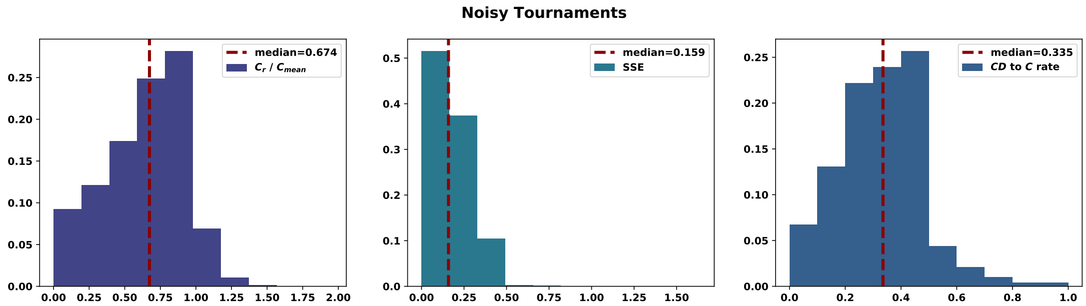
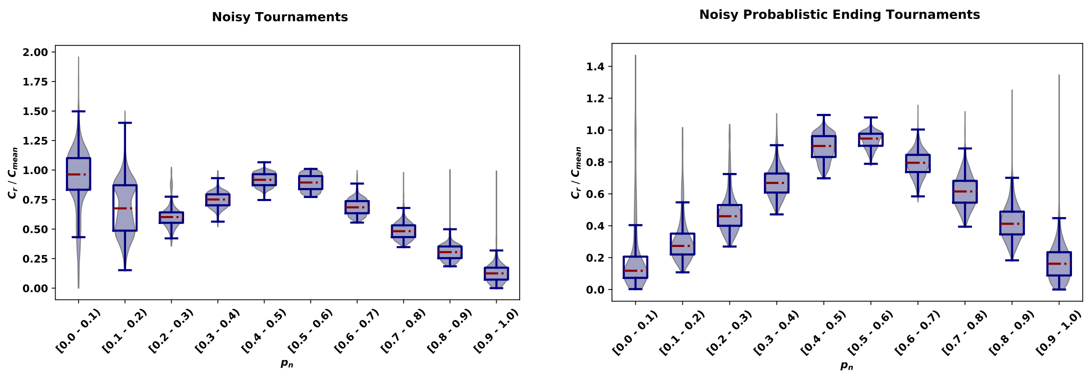


Figure 4.9: Distributions of C_r/C_{mean}, SSE and CD to C ratio for the winners of noisy tournaments.

The probability of noise has been observed to substantially affect optimal behaviour. Figure 4.10 gives the ratio C_r/C_{mean} for the winners in tournaments with noise, over the probability of noise. From Figure 4.10a it is clear that the cooperating only 67% of the time the mean cooperator did is optimal only when $p_n \in [0.2, 0.4)$ and $p_n \in [0.6, 0.7]$. In environments with $p_n < 0.1$ the winners want to be close to the mean cooperator, similarly to standard tournaments, and as the probability of noise is exceeding 0.5 (where the game is effectively inverted) strategies should aim to be less and less cooperative.

Figure 4.10 gives C_r/C_{mean} for the winners over p_n in tournaments with noise and a probabilistic ending. The optimal proportions of cooperations are different now that the number of turns is not fixed, successful strategies want to be more defecting that the mean cooperator, that only changes when p_n approaches 0.5. Figure 4.10 demonstrates how the adjustments to C_r/C_{mean} change over the noise in the to the environment, and thus supports how important adapting to the environment is for a strategy to be successful.

The distributions of the SSE across the tournament types suggest that successful strategies exhibit some extortionate behaviour, but not constantly. ZDs are a set of strategies that are often envious as they try to exploit their opponents. The winners of the tournaments considered in this work are envious, but not as much as many ZDs. Though the exact interactions between the matches have not been recorded here, the work of [84] which introduced the trained strategies that appeared in the top ranked strategies of Section 4.3 did. In [84] it was shown that clever strategies managed to achieve mutual cooperation with stronger strategies whilst exploiting the weaker strategies. This could explain the clever winners of our analysis, and would explain the SSE distributions. This could also be the reason why ZDs fail to appear in the tops ranks – they try to exploit all opponents and cannot actively adapt back to mutual cooperation against stronger strategies, which requires more depth of memory. Note that ZDs also tend to perform poorly in population games for a similar reason: they attempt to exploit other players using ZDs, failing to form a cooperative subpopulation [106]. This makes them



(a) C_r/C_{mean} distribution for winners in noisy tournaments over p_n.

(b) C_r/C_{mean} distribution for winners in noisy probabilistic ending tournaments over p_n.

Figure 4.10: C_r/C_{mean} distributions over intervals of p_n. These distributions model the optimal proportion of cooperation compared to C_{mean} as a function of (p_n).

good invaders but poor resisters of invasion.

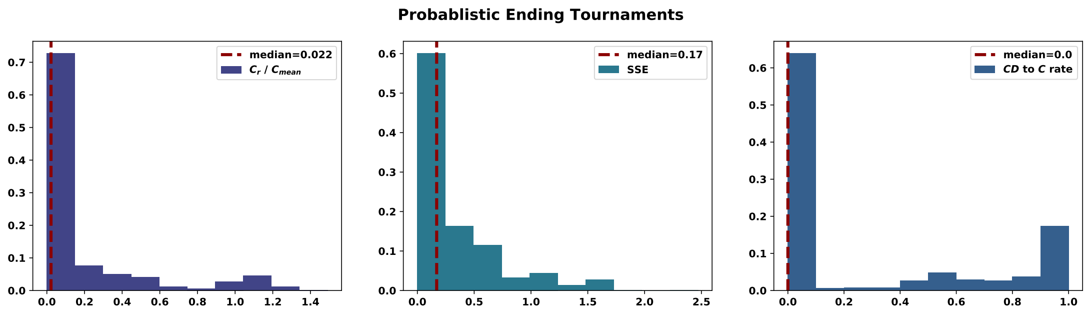


Figure 4.11: Distributions of C_r/C_{mean}, SSE and CD to C ratio for the winners of probabilistic ending tournaments.

The distributions of the CD to C rate evaluate the behaviour of a successful strategy after its opponent has defected against it. In standard tournaments it was observed that a successful strategy reciprocates with a probability of 0.5, and in a setting that the type of the tournament can vary between all the examined types a winning strategy would reciprocate on average with a probability of 0.58. In tournaments with noise a strategy is less likely to cooperate following a defection compared to standard tournaments, and in probabilistic ending tournaments a strategy will reciprocate a defection. This leads to adjusting the recommendation of being provocable to defections made by Axerlod. A strategy should be provocable in tournaments with short matches, but in the rest of the settings a strategy should be more generous.

Further statistically significant features with strong effects include C_r/C_{min}, C_r/C_{max}, C_{min} and C_{max}. These add more emphasis on how important it is for a strategy to adapt to its environment. Finally, the features number of turns, repetitions and the probabilities of noise and the game ending had no significant effects based on the multivariate regression models.

A third method that evaluates the importance of the features in Table 4.5 using clustering and random forests can be found in the Appendix C.2. The results uphold the outcomes of the correlation and multivariate regression. It also evaluates the effects of the classifiers stochastic, make use of game, and make use of length which have not been evaluated by the methods above because there are binary variables. The results imply that they have no significant effect on a

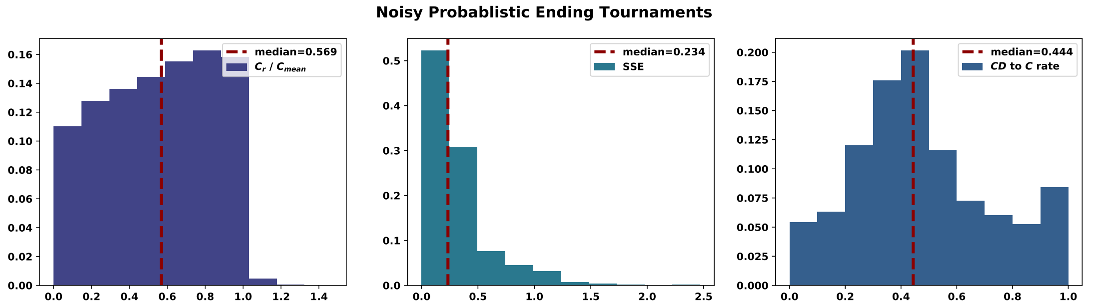


Figure 4.12: Distributions of C_r/C_{mean}, SSE and CD to C ratio for the winners of noisy probabilistic ending tournaments.

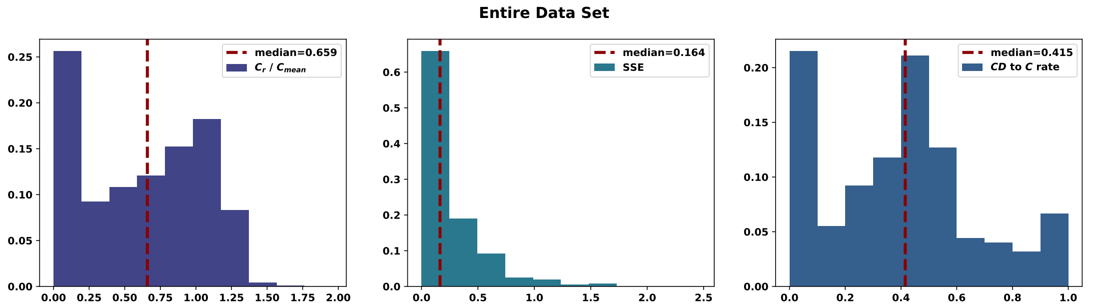


Figure 4.13: Distributions of C_r/C_{mean}, SSE and CD to C ratio for the winners over the tournaments of the entire data set.

strategy's performance.

4.5 Chapter Summary

This Chapter explored the performance of 195 strategies of the IPD in 45606 computer tournaments. The collection of computer tournaments presented here is the largest and most diverse collection in the literature. The 195 strategies are drawn from the APL and include strategies from the IPD literature. The computer tournaments include tournaments of four different types.

So what is the best way of playing the IPD? And is there a single dominant strategy for the IPD?

There was not a single strategy within the collection of the 195 strategies that managed to perform well in all the tournaments variations it competed in. Even if on average a strategy ranked highly in a specific environment this did not guarantee its success over the different tournament types. Nevertheless, in sections 4.3 and 4.4 examined the best performing strategies across various tournament types and analysed their salient features. It was demonstrated that there are properties associated with the success of strategies which in fact contradict the originally suggested properties of Axelrod [29].

It was showed that complex or **clever** strategies can be effective, whether trained against a corpus of possible opponents or purposely designed to mitigate the impact of noise such as the DBS strategy. Moreover, it was found that some strategies designed or trained for noisy environments were also highly ranked in noise-free tournaments which reinforces the idea that strategies' complexity/cleverness is not necessarily a liability, rather it can confer adaptability to a more diverse set of environments. It was also showed that while the type of exploitation attempted by ZDs is not typically effective in standard tournaments, **envious** strategies capable of both exploiting and not their opponents can be highly successful. Based on the results of [84] this could be because they are selectively exploiting weaker opponents while mutually cooperating with stronger opponents. Highly noisy or tournaments with short matches also favoured envious strategies. These environments mitigated the value of being nice. Uncertainty enables exploitation, reducing the ability of maintaining or enforcing mutual cooperation, while triggering grudging strategies to switch from typically cooperating to typically defecting.

The features analysis of the best performing strategies demonstrated that a strategy should reciprocate, as suggested by Axelrod, but it should relax its readiness to do so and be more **generous**. For noisy environments this is inline with the results of [39, 57, 138, 174], however, it was also showed that generosity pays off even in standard settings, and that in fact the only setting a strategy would want to be too provocable is when the matches are not long. Forgiveness as defined by Axerlod was not explored in this Chapter. This was mainly because the two round states were not recorded during the data collection. This could be a topic of future work that examines the impact of considering more rounds of history. The features analysis also concluded that there is a significant importance in **adapting to the environment**, and more specifically, to the mean cooperator. In standard tournaments a strategy would aim to be the mean cooperator while in noisy tournaments the best performing players cooperate at a lower rate than the tournament population on average. Moreover, the manner in which a strategy achieves a given cooperation rate relative to the tournament population average is important.

This could potentially explain the early success of Tit For Tat. Tit For Tat naturally achieves a cooperation rate near C_{mean} by virtue of copying its opponent's last move while also minimizing instances where it is exploited by an opponent (cooperating while the opponent defects), at least in non-noisy tournaments. It could also explain why Tit For N Tats does not fare well for $N > 1$ – it fails to achieve the proper cooperation ratio by tolerating too many defections.

Similarly, the results could suggest an explanation regarding the intuitively unexpected effectiveness of memory-one strategies historically. Given that among the important features associated with success are the relative cooperation rate to the population average and the four memory-one probabilities of cooperating conditional on the previous round of play, these features can be optimized by a memory-one strategy such as TFT. Usage of more history becomes valuable when there are exploitable opponent patterns. This is indicated by the importance of SSE as a feature, showing that the first-approximation provided by a memory-one strategy is no longer sufficient.

These results highlight a central idea in evolutionary game theory in this context: the fitness landscape is a function of the population (where fitness in this case is tournament performance). While that may seem obvious now, it shows why historical tournament results on small or arbitrary populations of strategies have so often failed to produce generalizable results.

Overall, the five properties successful strategies need to have in a IPD competition based on the analysis that has been presented in this manuscript are:

- Be “nice” in non-noisy environments or when game lengths are longer
- Be provocable in tournaments with short matches, and generous when matches are longer
- Be a little bit envious
- Be clever
- Adapt to the environment (including the population of strategies).

In this Chapter optimal behaviour was explored whilst considering a collection of pre defined strategies. Chapter **??** thought it considers environments of only a specific set of IPD strategies, it estimates the exact best responses to those environments.

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Appendix A

List of Strategies

A.1 List of strategies considered in Chapter 4

The strategies considered in Chapter 4, which are from APL version 3.0.0.

1. ϕ [7]
2. π [7]
3. e [7]
4. ALLCorALLD [7]
5. Adaptive [115]
6. Adaptive Pavlov 2006 [102]
7. Adaptive Pavlov 2011 [115]
8. Adaptive Tit For Tat: 0.5 [189]
9. Aggravater [7]
10. Alexei [158]
11. Alternator [29, 137]
12. Alternator Hunter [7]
13. Anti Tit For Tat [90]
14. AntiCycler [7]
15. Appeaser [7]
16. Arrogant QLearner [7]
17. Average Copier [7]
18. Backstabber [7]
19. Better and Better [4]
20. Bully [141]
21. Calculator [4]
22. Cautious QLearner [7]
23. Champion [28]
24. CollectiveStrategy [116]
25. Contrite Tit For Tat [195]
26. Cooperator [29, 137, 159]
27. Cooperator Hunter [7]
28. Cycle Hunter [7]
29. Cycler CCCCCD [7]
30. Cycler CCCD [7]
31. Cycler CCCDCD [7]
32. Cycler CCD [137]
33. Cycler DC [7]
34. Cycler DDC [137]
35. DBS [26]
36. Davis [27]
37. Defector [29, 137, 159]
38. Defector Hunter [7]
39. Double Crosser [7]
40. Desperate [190]
41. DoubleResurrection [8]
42. Doubler [4]
43. Dynamic Two Tits For Tat [7]
44. EasyGo [115, 4]
45. Eatherley [28]
46. Eventual Cycle Hunter [7]
47. Evolved ANN [7]
48. Evolved ANN 5 [7]
49. Evolved ANN 5 Noise 05 [7]
50. Evolved FSM 16 [7]
51. Evolved FSM 16 Noise 05 [7]
52. Evolved FSM 4 [7]
53. Evolved HMM 5 [7]

54. EvolvedLookerUp1 1 1 [7]
55. EvolvedLookerUp2 2 2 [7]
56. Eugine Nier [158]
57. Feld [27]
58. Firm But Fair [66]
59. Fool Me Forever [7]
60. Fool Me Once [7]
61. Forgetful Fool Me Once [7]
62. Forgetful Grudger [7]
63. Forgiver [7]
64. Forgiving Tit For Tat [7]
65. Fortress3 [23]
66. Fortress4 [23]
67. GTFT [69, 147]
68. General Soft Grudger [7]
69. Gradual [36]
70. Gradual Killer [4]
71. Grofman[27]
72. Grudger [27, 35, 36, 190, 115]
73. GrudgerAlternator [4]
74. Grumpy [7]
75. Handshake [167]
76. Hard Go By Majority [137]
77. Hard Go By Majority: 10 [7]
78. Hard Go By Majority: 20 [7]
79. Hard Go By Majority: 40 [7]
80. Hard Go By Majority: 5 [7]
81. Hard Prober [4]
82. Hard Tit For 2 Tats [180]
83. Hard Tit For Tat [6]
84. Hesitant QLearner[7]
85. Hopeless [190]
86. Inverse [7]
87. Inverse Punisher [7]
88. Joss [27, 180]
89. Knowledgeable Worse and Worse [7]
90. Level Punisher [8]
91. Limited Retaliate 2 [7]
92. Limited Retaliate 3 [7]
93. Limited Retaliate [7]
94. MEM2 [119]
95. Math Constant Hunter [7]
96. Meta Hunter Aggressive [7]
97. Meta Hunter [7]
98. Meta Majority [7]
99. Meta Majority Finite Memory [7]
100. Meta Majority Long Memory [7]
101. Meta Majority Memory One [7]
102. Meta Minority [7]
103. Meta Mixer [7]
104. Meta Winner [7]
105. Meta Winner Deterministic [7]
106. Meta Winner Ensemble [7]
107. Meta Winner Finite Memory [7]
108. Meta Winner Long Memory [7]
109. Meta Winner Memory One [7]
110. Meta Winner Stochastic [7]
111. NMWE Deterministic [7]
112. NMWE Finite Memory [7]
113. NMWE Long Memory [7]
114. NMWE Memory One [7]
115. NMWE Stochastic [7]
116. Naive Prober [115]
117. Negation [6]
118. Nice Average Copier [7]
119. Nice Meta Winner [7]
120. Nice Meta Winner Ensemble [7]
121. Nydegger [27]
122. Omega TFT [102]
123. Once Bitten [7]
124. Opposite Grudger [7]
125. PSO Gambler 1 1 1 [7]
126. PSO Gambler 2 2 2 [7]
127. PSO Gambler 2 2 2 Noise 05 [7]
128. PSO Gambler Mem1 [7]
129. Predator [23]
130. Prober [115]

131. Prober 2 [4]
132. Prober 3 [4]
133. Prober 4 [4]
134. Pun1 [23]
135. Punisher [7]
136. Raider [25]
137. Random Hunter [7]
138. Random: 0.5 [27, 189]
139. Remorseful Prober [115]
140. Resurrection [8]
141. Retaliate 2 [7]
142. Retaliate 3 [7]
143. Retaliate [7]
144. Revised Downing [27]
145. Ripoff [19]
146. Risky QLearner [7]
147. SelfSteem [46]
148. ShortMem [46]
149. Shubik [27]
150. Slow Tit For Two Tats [7]
151. Slow Tit For Two Tats 2 [4]
152. Sneaky Tit For Tat [7]
153. Soft Go By Majority [29, 137]
154. Soft Go By Majority 10 [7]
155. Soft Go By Majority 20 [7]
156. Soft Go By Majority 40 [7]
157. Soft Go By Majority 5 [7]
158. Soft Grudger [115]
159. Soft Joss [4]
160. SolutionB1 [17]
161. SolutionB5 [17]
162. Spiteful Tit For Tat [4]
163. Stalker [46]
164. Stein and Rapoport [27]
165. Stochastic Cooperator [10]
166. Stochastic WSLS [7]
167. Suspicious Tit For Tat [36, 90]
168. TF1 [7]
169. TF2 [7]
170. TF3 [7]
171. Tester [28]
172. ThueMorse [7]
173. ThueMorseInverse [7]
174. Thumper [19]
175. Tit For 2 Tats (**Tf2T**) [29]
176. Tit For Tat (**TfT**) [27]
177. Tricky Cooperator [7]
178. Tricky Defector [7]
179. Tullock [27]
180. Two Tits For Tat (**2TfT**) [29]
181. VeryBad [46]
182. Willing [190]
183. Win-Shift Lose-Stay (**WShLSt**) [115]
184. Win-Stay Lose-Shift (**WSLS**) [109, 147, 180]
185. Winner12 [130]
186. Winner21 [130]
187. Worse and Worse[4]
188. Worse and Worse 2[4]
189. Worse and Worse 3[4]
190. ZD-Extort-2 v2 [110]
191. ZD-Extort-2 [180]
192. ZD-Extort-4 [7]
193. ZD-GEN-2 [110]
194. ZD-GTFT-2 [180]
195. ZD-SET-2 [110]

Appendix B

Correlation coefficients of strategies features

A graphical representation of the correlation coefficients for the features of Table 4.5, Chapter 4.

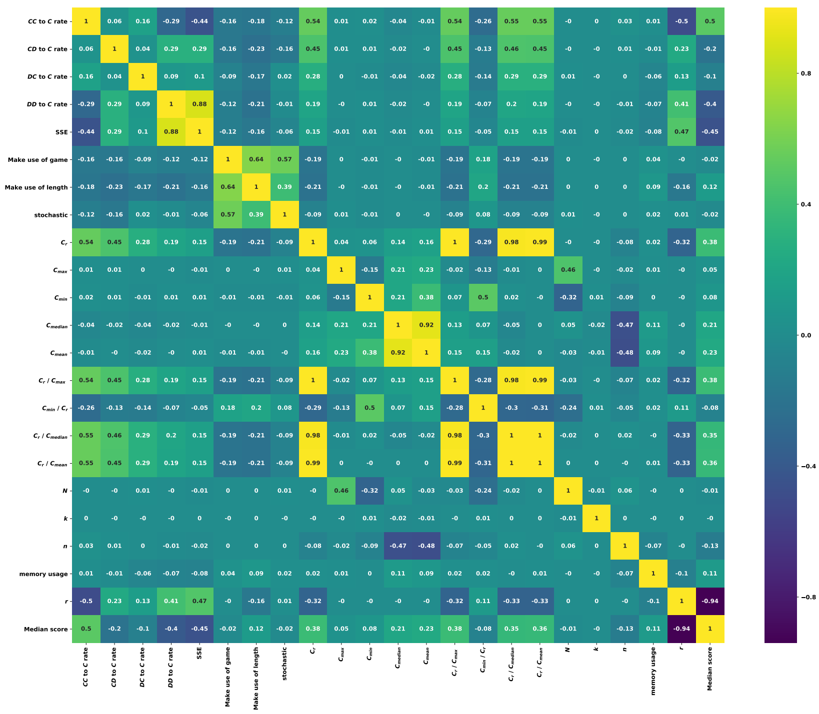


Figure B.1: Correlation coefficients of features in Table 4.5 for standard tournaments

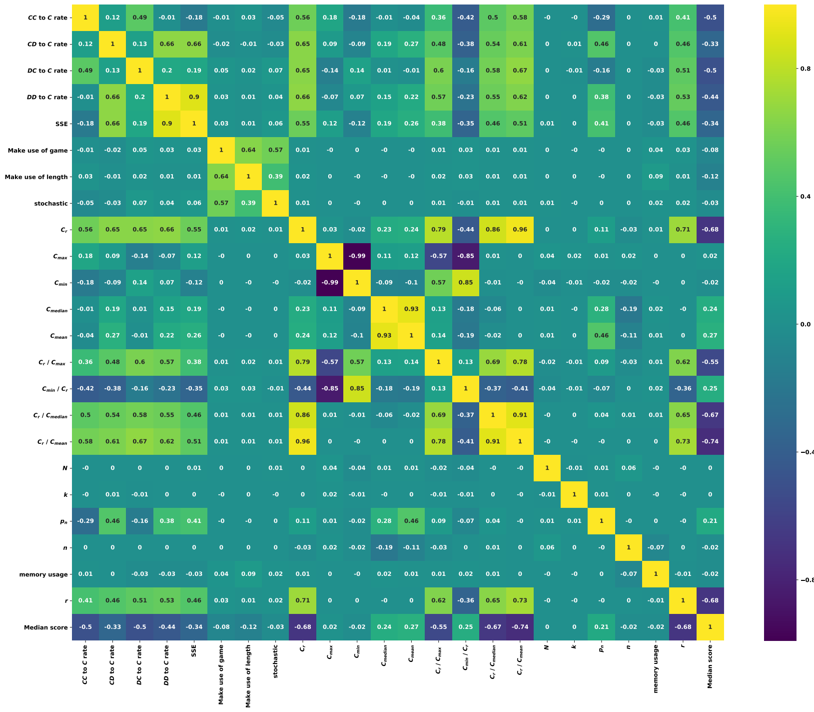


Figure B.2: Correlation coefficients of features in Table 4.5 for noisy tournaments

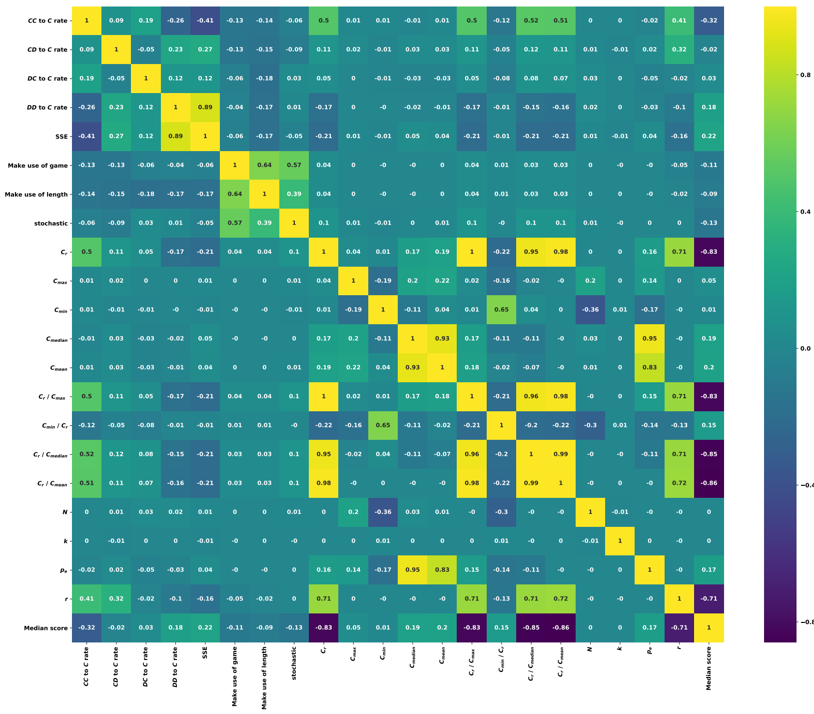


Figure B.3: Correlation coefficients of features in Table 4.5 for probabilistic ending tournaments

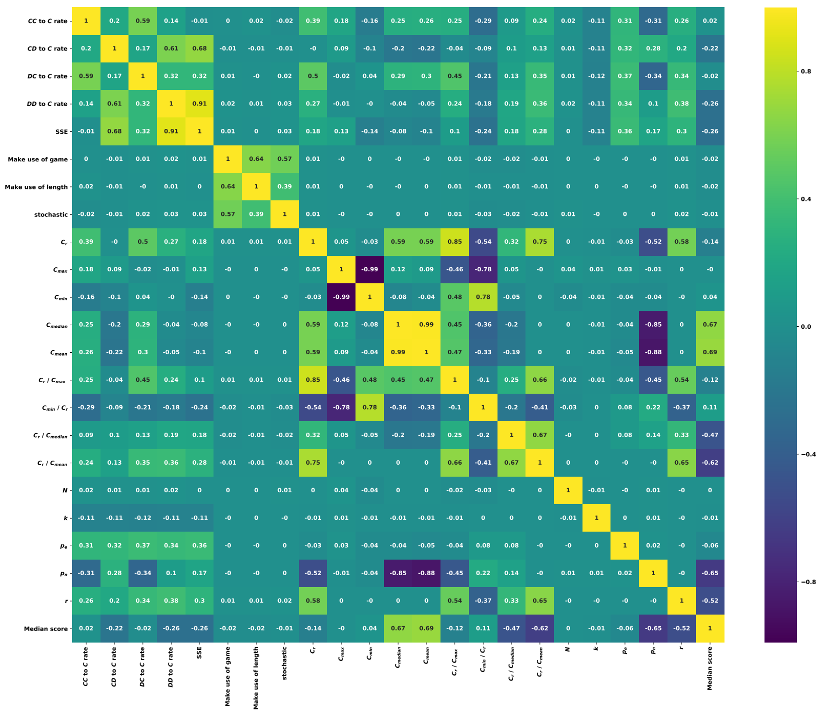


Figure B.4: Correlation coefficients of features in Table 4.5 for noisy probabilistic ending tournaments

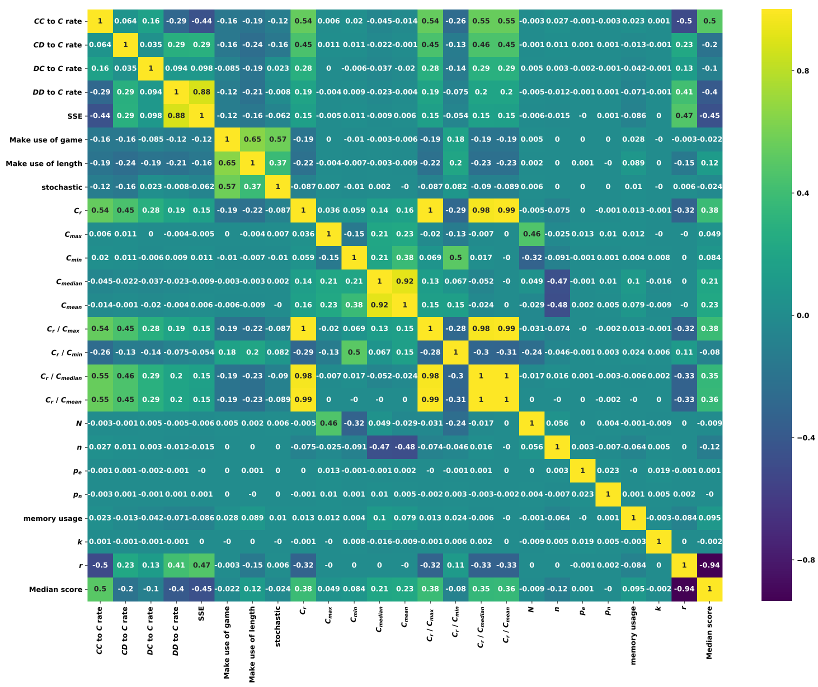


Figure B.5: Correlation coefficients of features in Table 4.5 for data set

Appendix C

Further Analysis on Features Importance

C.1 Multivariate linear regressions on median score

A multivariate linear regression has also been fitted to model the relationship between the features and the median score. The features included are given by Table C.1 alongside their corresponding p values in the distinct tournaments and their regression coefficients.

	Standard		Noisy		Probabilistic ending		Noisy probabilistic ending		Overall	
	R adjusted: 0.576		R adjusted: 0.679		R adjusted: 0.816		R adjusted: 0.930		R adjusted: 0.318	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
CC to C rate	0.043	0.000	-0.380	0.000	0.224	0.000	0.078	0.0	0.308	0.0
CD to C rate	-0.325	0.000	0.124	0.000	0.060	0.000	0.073	0.0	-0.014	0.0
C_r / C_{max}	-	-	-1.044	0.000	-	-	-1.251	0.0	-	-
C_r / C_{mean}	0.553	0.000	-0.101	0.000	-1.136	0.000	-0.089	0.0	-0.665	0.0
C_{max}	0.059	0.000	-	-	-0.044	0.086	-1.396	0.0	-	-
C_{mean}	1.837	0.000	3.078	0.000	1.506	0.000	3.645	0.0	-	-
C_{min}	0.156	0.000	1.528	0.000	0.311	0.000	-	-	-	-
C_{min} / C_r	-0.049	0.000	-0.378	0.000	-0.204	0.000	-	-	-0.257	0.0
DC to C rate	-0.204	0.000	0.074	0.000	0.066	0.000	0.066	0.0	0.007	0.0
k	-0.000	0.853	-0.000	0.987	0.000	0.008	0.000	0.0	-	-
n	-0.000	0.000	-	-	-	-	-	-	-	-
p_e	-	-	-	-	0.025	0.000	-0.095	0.0	-	-
p_n	-	-	0.124	0.000	-	-	-	-	-	-
SSE	-0.294	0.000	-0.319	0.000	0.055	0.000	0.010	0.0	-0.015	0.0
constant	0.925	0.000	1.536	0.000	2.466	0.000	2.299	0.0	2.924	0.0
memory usage	0.010	0.000	-0.004	0.000	-	-	-	-	-	-

Table C.1: Results of multivariate linear regressions with the median score as the dependent variable. R squared is reported for each model.

C.2 Evaluation based on clustering and random forest.

The final method to evaluate the features importance in a strategy's success is a combination of a clustering task and a random forest algorithm. Initially the performances are clustered into different clusters based on them being successful or not. The performances are clustered into successful and unsuccessful clusters based on 4 different approaches. More specifically:

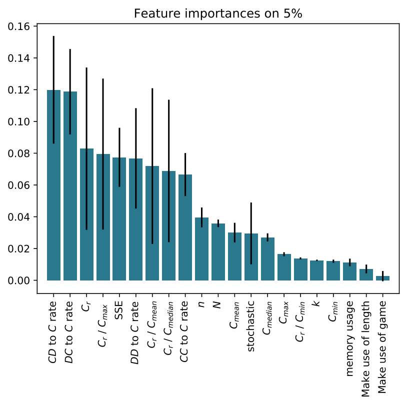
- **Approach 1:** The performances are divided into two clusters based on whether their performance was in the top 5% of their respective tournaments. Thus, whether r was smaller or larger than 0.05.
- **Approach 2:** The performances are divided into two clusters based on whether their performance was in the top 25% of their respective tournaments. Thus, whether r was smaller or larger than 0.25.
- **Approach 3:** The performances are divided into two clusters based on whether their performance was in the top 50% of their respective tournaments. Thus, whether r was smaller or larger than 0.50.
- **Approach 4:** The performances are clustered based on their normalised rank and their median score by a $k-$means algorithm [16]. The number of clusters is not deterministically chosen but it is based on the silhouette coefficients [170].

Once the performances have been assigned to a cluster for each approach a random forest algorithm [45] is applied. The problem is a supervised problem where the random forest algorithm predicts the cluster to which a performance has been assigned to using the features of Table 4.5. The random forest models are trained on a training set of 70% of the tournaments results. The accuracy of each model based on R^2 and the number of clusters for each tournament type (because in the case of Approach 4 it is not deterministically chosen) are given by Table C.2. The out of the bag error (OOB) [86] has also been calculated. The models fit well, and a high value of both the accuracy measures on the test data and the OOB error indicate that the model is not over fitting.

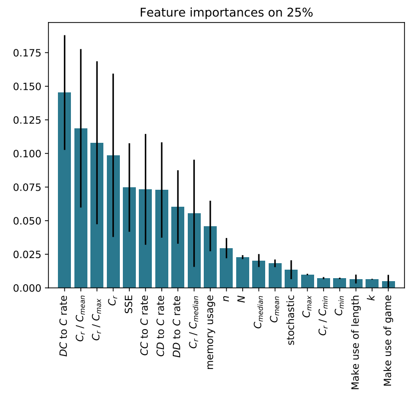
The importance that the features of Table 4.5 had on each random forest model are given by Figures C.1, C.2, C.3, C.4 and C.5. These show that the classifiers stochastic, make use of game and make use of length have no significant effect, and several of the features that are highlighted by the importance are inline with the correlation results. Moreover, the smoothing parameter k appears to no have a significant effect either. The most important features based on the random forest analysis were C_r/C_{median} and C_r/C_{mean}.

Tournament type	Clustering Approach	Number of clusters	R^2 training data	R^2 test data	R^2 OOB score
standard	Approach 1	2	0.998831	0.987041	0.983708
	Approach 2	2	0.998643	0.978626	0.969202
	Approach 3	2	0.998417	0.985217	0.976538
	Approach 4	2	0.998794	0.990677	0.982959
noisy	Approach 1	2	0.997786	0.972229	0.968332
	Approach 2	2	0.997442	0.963254	0.955219
	Approach 3	2	0.997152	0.953164	0.940528
	Approach 4	3	0.996923	0.950728	0.935444
probabilistic ending	Approach 1	2	0.997909	0.981490	0.978120
	Approach 2	2	0.997883	0.973492	0.967150
	Approach 3	2	0.990448	0.890068	0.875822
	Approach 4	2	0.999636	0.995183	0.992809
noisy probabilistic ending	Approach 1	2	0.995347	0.957846	0.952353
	Approach 2	2	0.992813	0.909346	0.898613
	Approach 3	2	0.990579	0.824794	0.806540
	Approach 4	4	0.989465	0.841652	0.824052
over 45606 tournaments	Approach 1	2	0.997271	0.972914	0.969198
	Approach 2	2	0.996323	0.951194	0.940563
	Approach 3	2	0.993707	0.906941	0.891532
	Approach 4	3	0.993556	0.913335	0.898453

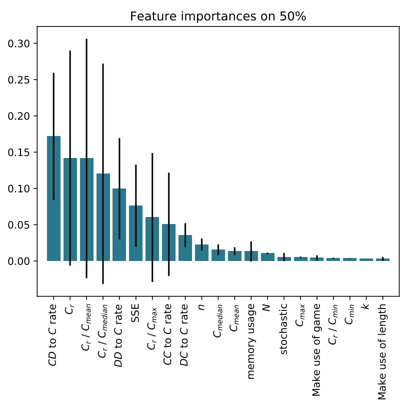
Table C.2: Accuracy metrics for random forest models.



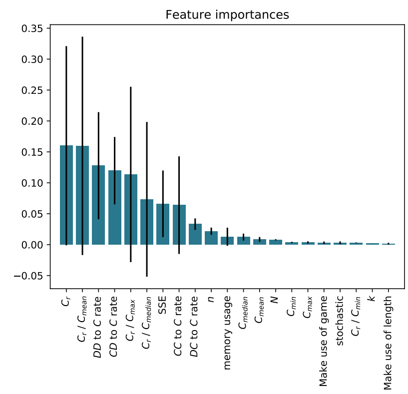
(a) Importance of features for clusters on 5% performance.



(b) Importance of features for clusters on 25% performance.

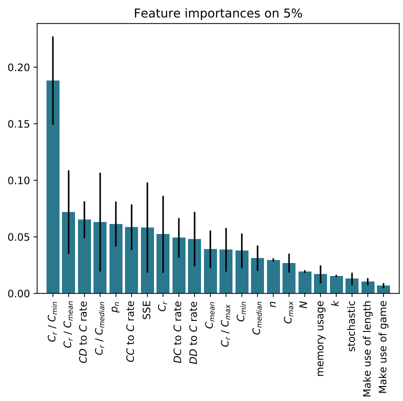


(c) Importance of features for clusters on 50% performance.

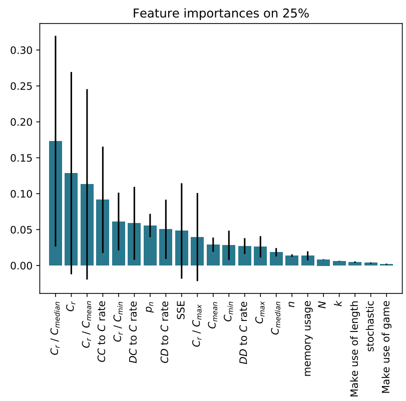


(d) Importance of features for clusters based on $k$means algorithm.

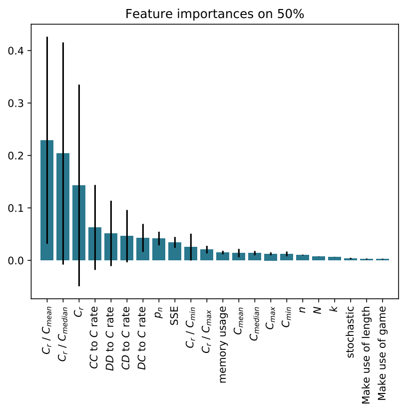
Figure C.1: Importance of features in standard tournaments for different clustering methods.



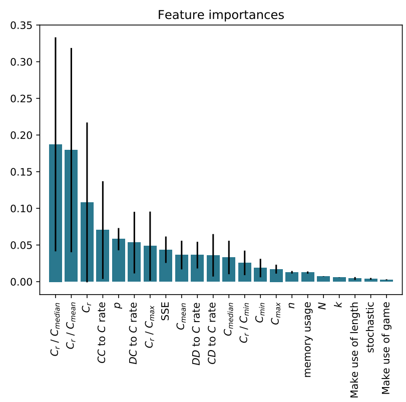
(a) Importance of features for clusters on 5% performance.



(b) Importance of features for clusters on 25% performance.

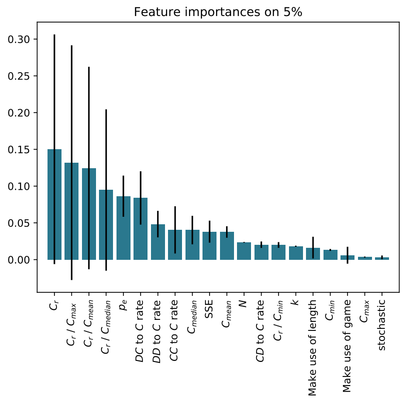


(c) Importance of features for clusters on 50% performance.

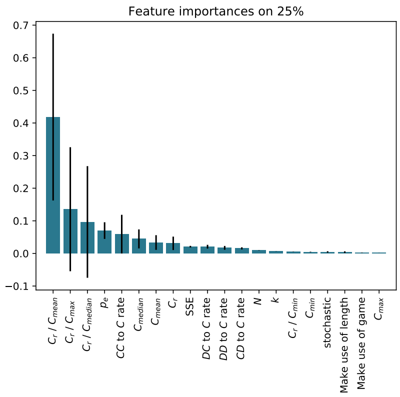


(d) Importance of features for clusters based on $k$means algorithm.

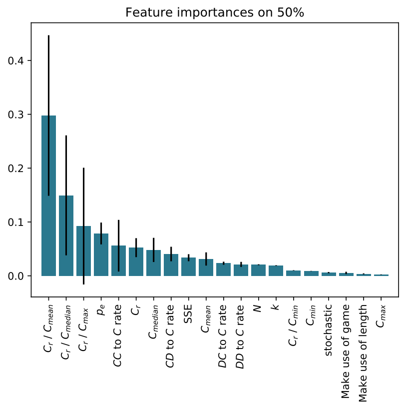
Figure C.2: Importance of features in noisy tournaments for different clustering methods.



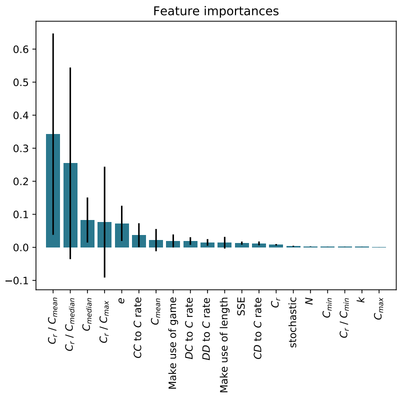
(a) Importance of features for clusters on 5% performance.



(b) Importance of features for clusters on 25% performance.

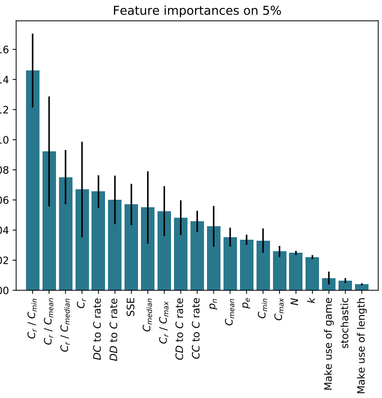


(c) Importance of features for clusters on 50% performance.

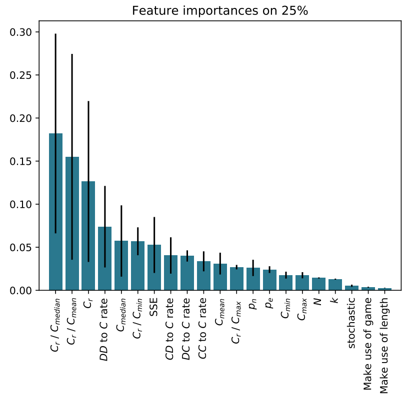


(d) Importance of features for clusters based on $k$means algorithm.

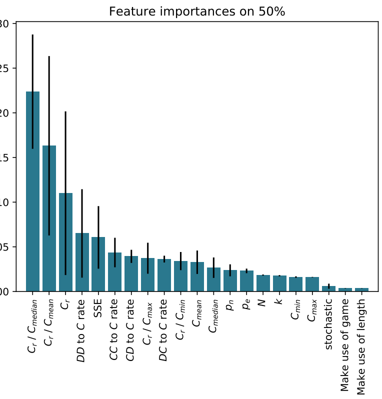
Figure C.3: Importance of features in probabilistic ending tournaments for different clustering methods.



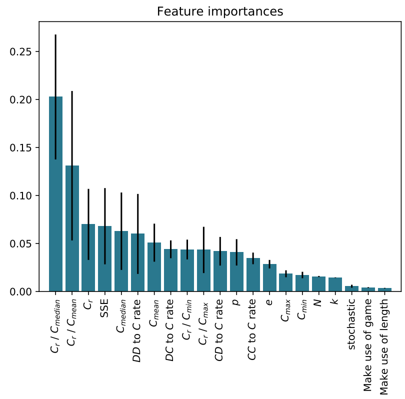
(a) Importance of features for clusters on 5% performance.



(b) Importance of features for clusters on 25% performance.

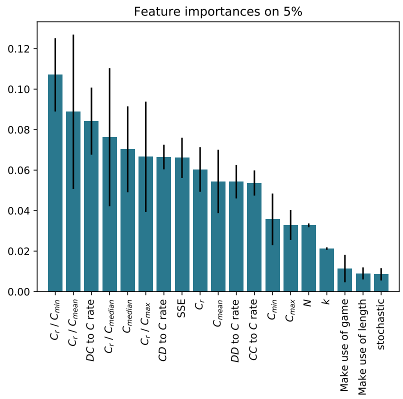


(c) Importance of features for clusters on 50% performance.

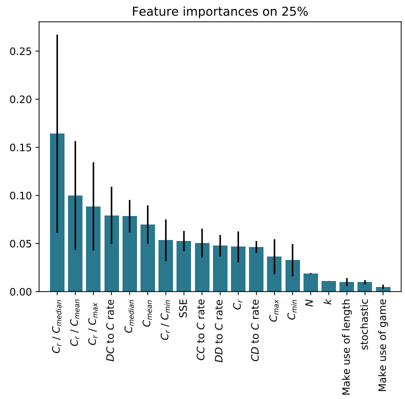


(d) Importance of features for clusters based on $k$means algorithm.

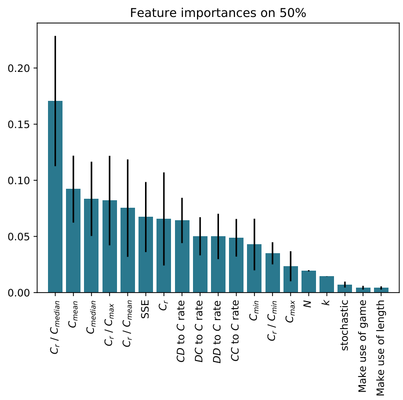
Figure C.4: Importance of features in noisy probabilistic ending tournaments for different clustering methods.



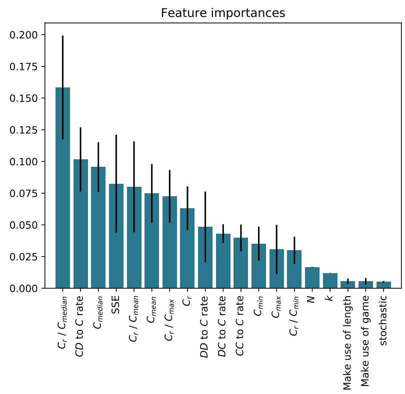
(a) Importance of features for clusters on 5% performance.



(b) Importance of features for clusters on 25% performance.



(c) Importance of features for clusters on 50% performance.



(d) Importance of features for clusters based on $k$means algorithm.

Figure C.5: Importance of features over all the tournaments for different clustering methods.