A systematic literature review of the Prisoner's Dilemma; collaboration and influence.

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Abstract

The Prisoner's Dilemma is a well known game used since the 1950's as a framework for studying the emergence of cooperation; a topic of continuing interest for mathematical, social, biological and ecological sciences. The iterated version of the game, the Iterated Prisoner's Dilemma, attracted attention in the 1980's after the publication of the "The Evolution of Cooperation" and has been a topic of pioneering research ever since.

The aim of this paper is to provide a systematic literature review on the Prisoner's Dilemma. This is achieved in two ways. Firstly, we review selected pieces of work and partition the literature timeline into five different sections with each reviewing a different aspect of it's research. Secondly, we analyse a comprehensive data set of articles on the Prisoner's Dilemma collected from five different prominent journals.

The questions answered in this manuscript are (1) what are the research trends in the field, (2) what are the already existing results within the field, (3) how collaborative is the field and (4) do authors influence the field more compared to other fields.

1 Introduction

Based on the Darwinian principle of survival of the fittest cooperative behaviour should not be favoured, however, cooperation is plentiful in nature. The golden paradigm of understanding the emergence of these behaviours is a particular two player non-cooperative game called the Prisoner's Dilemma (PD), originally described in [46].

In the PD each player has two choices, to either be selfless and cooperate or to be selfish and choose to defect. Each decision is made simultaneously and independently. The utility of each player is influenced by its own behaviour, and the behaviour of the opponent. Both players do better if they choose to cooperate than if both choose to defect. However, a player has the temptation to deviate as that player will receive a higher payoff than that of a mutual cooperation. Players' payoffs are generally represented by (1). Both players receive a reward for mutual cooperation, R, and a payoff P for mutual defection. A player that defects while the other cooperates receives a payoff of T, whereas the cooperator receives S. The dilemma exists due to constraints (2) and (3).

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} \tag{1}$$

$$T > R > P > S \tag{2}$$

$$2R > T + S \tag{3}$$

Another common representation of the payoff matrix is given by (4), where b is the benefit of the altruistic behaviour and c it's its cost (constraints (2) and (3) still hold).

$$\begin{pmatrix} b-c & c \\ b & 0 \end{pmatrix} \tag{4}$$

Constraints (2), (3) and rational behaviour guarantee that it never benefits a player to cooperate, indeed mutual defections is a Nash equilibrium. However, when the game is studied in a manner where prior outcome matters, defecting is no longer necessarily the dominant choice.

The repeated form of the game is called the Iterated Prisoner's Dilemma (IPD) and theoretical works have shown that cooperation can emerge once players interact repeatedly. Arguably, the most important of these works has been Robert Axelrod's "The Evolution of Cooperation" [24]. In his book Axelrod reports on a series of computer tournaments he organised. In these tournaments academics from several fields were invited to design computer strategies to compete. Axelrod's work showed that greedy strategies did very poorly in the long run whereas altruistic strategies did better. "The Evolution of Cooperation" is considered a milestone in the field but it is not the only one. On the contrary, the PD has attracted attention ever since the game's origins.

In Section 2 of this manuscript, a qualitative description of selected pieces of work will be presented. These have been separated into five sections, each reviewing a different aspect of research. The topics reviewed at each section are the following:

- Section 2.1, the **origins of the Prisoner's Dilemma**.
- Section 2.2, **Axelrod's tournaments and intelligent design of strategies**.
- Section 2.3, **Evolutionary dynamics**
- Section 2.4, **Structured strategies and training**.
- Section 2.5, **Software**.

The aim of Section 2 is to provide a concrete summary of the existing literature on the PD. This is done to provide a review which will allow the research community to understand overall trends in the field, and already existing results.

In Section 3 a comprehensive data set of literature regarding the PD, and collected from the following sources, is presented and analysed. The data set has been archived and is available at [42].

- arXiv [82]; a repository of electronic preprints. It consists of scientific papers in the fields of mathematics, physics, astronomy, electrical engineering, computer science, quantitative biology, statistics, and quantitative finance, which all can be accessed online.
- PLOS [5]; a library of open access journals and other scientific literature under an open content license. It launched its first journal, PLOS Biology, in October 2003 and publishes seven journals, as of October 2015.
- IEEE Xplore Digital Library (IEEE) [63]; a research database for discovery and access to journal articles, conference proceedings, technical standards, and related materials on computer science, electrical engineering and electronics, and allied fields. It contains material published mainly by the Institute of Electrical and Electronics Engineers and other partner publishers.
- Nature [51]; a British multidisciplinary scientific journal, first published on 4 November 1869. It was ranked the world's most cited scientific journal by the Science Edition of the 2010 Journal Citation Reports and is ascribed an impact factor of 40.137, making it one of the world's top academic journals.
- Springer [83]; a leading global scientific publisher of books and journals. It publishes close to 500 academic and professional society journals.

The aim of the analysis is to review the amount of published academic articles as well as to measure and explore the collaborations within the field.

2 Timeline

In this section literature regarding the PD is reviewed. The review starts from the formulation of the game and covers publications all the way to today.

2.1 Origins of the prisoner's dilemma

The origin of the PD goes back to the 1950s in early experiments conducted at RAND [46] to test the applicability of games described in [91]. The game received it's name later the same year. According to [123], Albert W. Tucker (the PhD supervisor of John Nash [90]), in an attempt to deliver the game with a story during a talk described the players as prisoners and the game has been known as the Prisoner's Dilemma ever since.

The early research on the PD had been very constrained. The only source of experimental results was through human subject research where pairs of participants simulated rounds of the game. Human subject research had disadvantages. Humans could behave randomly and in several experiments both the size and the background of the individuals were different, thus comparing results of two or more studies became difficult.

The main aim of these early research experiments had been to understand how conditions such as the gender of the participants [45, 78, 80], the physical distance between the participants [114], the effect of their opening moves [121] and even how the experimenter, by varying the tone of their voice and facial expressions [48], could influence the outcomes and subsequently the emergence of cooperation. An early figure that sought out to understand several of these conditions was the mathematical psychologist Professor Anatol Rapoport. The results of his work are summarised in [107], written alongside Albert M. Chammah.

Rapoport was also interested in conceptualising strategies that could promote international cooperation. Decades later he would submit the winning strategy (Tit for Tat) of the first computer tournament, run by Robert Axelrod. Though human subject research is still used today [122] they have largely been replaced by computer tournaments. In the next section these tournaments, and several strategies that were designed by researchers, such as Rapoport, are introduced.

2.2 Axelrod's tournaments and intelligently designed strategies

As discussed in Section 2.1, before 1980 a great deal of research was done in the field, however, as described in [22], the political scientist Robert Axelrod believed that there was no clear answer to the question of how to avoid conflict, or even how an individual should play the game. Combining his interest in artificial intelligence and political science Axelrod created a framework for exploring these questions using computer tournaments. Axelrod asked researchers to design a strategy with the purpose of wining an IPD tournament. These strategies were constructed and not designed through an undirected process (such as in Section 2.4), and here they are referred to as strategies of intelligent design. This section covers Axelrod's original tournaments as well as research that introduced new intelligently designed strategies.

Axelrod's tournaments made the study of cooperation of critical interest. As described in [108], "Axelrod's new approach has been extremely successful and immensely influential in casting light on the conflict between an individual and the collective rationality reflected in the choices of a population whose members are unknown and its size unspecified, thereby opening a new avenue of research". In a collaboration with a colleague, Douglas Dion, Axelrod in [23] summarized a number of works that were immediately inspired from the "Evolution of Cooperation", and [66] gives a review of tournaments that have been conducted since the originals.

The first reported computer tournament took place in 1980 [18]. A total of 13 strategies were submitted, written in the programming languages Fortran or Basic. Each competed in a 200 turn match against all 12 opponents, itself and a player that played randomly (called **Random**). This type of tournament is referred to as a round robin.

The tournament was run only once, each participant knew the exact length of the matches and had access to the full history of each match. Furthermore, Axelrod performed a preliminary tournament and the results were known to the participants. This preliminary tournament is mentioned in [18] but no details were given. The payoff values used for equation (1) were $R = 3, P = 1, T = 5$ and $S = 0$. These values are commonly used in the literature and unless specified will be the values used in the rest of the works described here.

The winner of the tournament was determined by the total average score and not by the number of matches won. The strategy that was announced the winner was the strategy submitted by Rapoport, **Tit For Tat**. The success of Tit for Tat came as a surprise. It was not only the simplest submitted strategy, it would always cooperates on the first round and then mimic the opponent's previous move, but it had also won the tournament even though it could never beat any player it was interacting with.

In order to further test the results Axelrod performed a second tournament in 1980 [19]. The second tournament received much more attention and had a total of 62 entries. The participants knew the results of the previous tournament and the rules were similar with only a few alterations. The tournament was repeated 5 times and the length of each match was not known to the participants. Axelrod intended to use a fixed probability (refereed to as 'shadow of the future' [23]) of the game ending on the next move. However, 5 different match lengths were selected for each match 63, 77, 151, 308 and 401, such that the average length would be around 200 turns.

Nine of the original participants competed again in the second tournament. Two strategies that remained the same were Tit For Tat and **Grudger**. Grudger is a strategy that will cooperate as long as the opponent does not defect, submitted by James W. Friedman. The name Grudger was give to the strategy in [75], though the strategy goes by many names in the literature such as, Spite [27], Grim Trigger [25] and Grim [125]. New entries in the second tournament included **Tit for Two Tats** submitted by John Maynard Smith and **KPavlovC**. KPavlovC, is also known as Simpleton [107], introduced by Rapoport or just Pavlov [94]. The strategy is based on the fundamental behavioural mechanism win-stay, lose-shift. Pavlov is heavily studied in the literature and similarly to Tit for Tat it's used in tournaments perform until today and has had many variants trying to build upon it's success, for example **PavlovD** and **Adaptive Pavlov** [74].

Despite the larger size of the second tournament none of the new entries managed to outperform the simpler designed strategy. The winner was once again Tit for Tat. Axelrod deduced that the performance of the strategy was because:

- The strategy would start of by cooperating.
- It would forgive it's opponent after a defection.
- It would always be provoked by a defection no matter the history.
- It was simple.
- As soon as the opponents identified that they were playing Tit for Tat, they would choose to cooperate for the rest of the game.

The success of Tit for Tat, however, was not unquestionable. Several papers showed that stochastic uncertainties severely undercut the effectiveness of reciprocating strategies and such stochastic uncertainties have to be expected in real life situations [85]. For example, in [88] it's proven that in an environment where **noise** (a probability that a player's move will be flipped) is introduced two strategies playing Tit for Tat receive the same average payoff as two Random players. Hammerstein, pointed out that if by mistake, one of two Tit for Tat players makes a wrong move, this locks the two opponents into a hopeless sequence of alternating defections and cooperations [113].

The poor performance of the strategy in noisy environments was also demonstrated in tournaments. In [29, 38] round robin tournaments with noise were performed, and Tit For Tat did not win either. The authors concluded that to overcome the noise error more generous strategies than Tit For Tat were needed. They introduced the strategies **Nice and Forgiving** and **OmegaTFT** respectively. A second type of stochastic uncertainty is misperception, where a player's action is made correctly but it's recorded incorrectly by the opponent. In [128], a strategy called **Contrite Tit for Tat** was introduced that was more successful than Tit for Tat in such environments. The difference between the strategies was that Contrite Tit for Tat was not so fast to retaliate a defection.

Several works extended the reciprocity based approach which has led to new strategies. For example Gradual [27]. Gradual was constructed to have the same qualities as those of Tit for Tat except one, **Gradual** had a memory of the game since the beginning of it. Gradual recorded the number of defections by the opponent and punished them with a growing number of defections. It would then enter a calming state in which it would cooperates for two rounds. In a tournament of 12 strategies, including both Tit for Tat and Pavlov, Gradual managed to outperformed them all. A strategy with the same intuition as Gradual is **Adaptive Tit for Tat** [124]. Adaptive Tit for Tat does not keep a permanent count of past defections, it maintains a continually updated estimate of the opponents behaviour, and uses this estimate to condition its future actions. In the exact same tournament as in [27] with now 13 strategies Adaptive Tit for Tat ranked first.

Another extension of strategies was that of teams of strategies [36, 37, 110] that collude to increase one member's score. In 2004 Graham Kendall led the Anniversary Iterated Prisoner's Dilemma Tournament with a total of 223 entries. In this tournament participants were allowed to submit multiple strategies. A team from the University of Southampton submitted a total of 60 strategies [110]. All these were strategies that had been programmed with a recognition mechanism by default. Once the strategies recognised one another, one would act as leader and the other as a follower. The follower plays as a **Cooperator**, cooperates unconditionally and the leader would play as a **Defector** gaining the highest achievable score. The followers would defect unconditionally against other strategies to lower their score and help the leader. The result was that Southampton had the top three performers. Nick Jennings, who was part of the team, said that "We developed ways of looking at the Prisoner's Dilemma in a more realistic environment and we devised a way for computer agents to recognise and collude with one another despite the noise. Our solution beats the standard Tit For Tat strategy" [100].

2.2.1 Memory one Strategies

A set of intelligent design strategies that have received a lot of attention in the literature are **memory one** strategies. In [95], Nowak and Sigmund proposed a structure for studying simple strategies that remembered only the previous turn, and moreover, only recorded the move of the opponent. These are called **reactive** strategies and they can be represented by using three parameters (y, p_1, p_2), where y is the probability to cooperate in the first move, and p_1 and p_2 the conditional probabilities to cooperate, given that the opponent's last move was a cooperation or a defection. For example Tit For Tat is a reactive strategy and it can be written as $(1, 1, 0)$. Another reactive strategy well known in the literature is **Generous Tit for Tat** [97].

In [96], Nowak and Sigmund extended their work to include strategies which consider the entire history of the previous turn to make a decision. These are called **memory one** strategies. If only a single turn of the game is taken into account and depending on the simultaneous moves of the two players there are only four possible states that the players could be in. These are ,CC, CD, DC, DD, thus a memory one strategy is denoted by the probabilities of cooperating after each state and the probability of cooperating in the first round, (y, p_1, p_2, p_3, p_4). For example Pavlov's memory one representation is $(1, 1, 0, 0, 1)$.

Memory one strategies made an impact when a specific set of memory one strategies were introduced called **Zero-determinant** (ZD) [103]. The American Mathematical Society's news section [60] stated that "the world of game theory is currently on fire" and in [119] it was stated that "Press and Dyson have fundamentally changed the viewpoint on the Prisoner's Dilemma". ZD are a set of extortionate strategies that can force a linear relationship between the long-run scores of both themselves and the opponent, therefore ensuring that the opponent will never do better than them.

Press and Dyson's suggested the ZD were the dominant family of strategies in the IPD. Moreover, they argued that memory is not beneficial. In [7, 58, 59, 60, 69, 73, 119] the effectiveness of ZD strategies is questioned. In [7], it was shown that ZD strategies are not evolutionary stable, and in [119] a more generous set of ZDs, the **Generous ZD**, were shown to outperform the more extortionate ZDs. Finally, in [73], the 'memory does not benefit a strategy' statement was questioned. A set of more complex strategies, strategies that take in account the entire history set of the game, were trained and proven to be more stable than ZD strategies.

This section covered the original computer tournaments of Axelrod and the early success of Tit For Tat in these tournaments. Though Tit For Tat was considered to be the most robust basic strategy, reciprocity was found to not be enough in environments with uncertainties. There are at least two properties, that have been discussed in this section, for coping with such uncertainties; generosity and contrition. Generosity is letting a percentage of defections go unpunished, and contrition, is lowering a strategy's readiness to defect following an opponent's defection.

In the later part of this section a series of new strategies which were build on the basic reciprocal approachs were presented, followed by the infamous memory one strategies, the zero-determinant. Though the ZDs can be proven to be robust in pairwise interactions they were found to be lacking in evolutionary settings and in computer tournaments. Evolutionary settings and the emergence of cooperation under natural selection are covered in the next section.

2.3 Evolutionary dynamics

As yet, the emergence of cooperation has been discussed in the contexts of the one shot PD game and the IPD round robin tournaments. In the PD it is proven that cooperation will not emerge, furthermore, in a series of influencing works Axelrod demonstrated that reciprocal behaviour favours cooperation when individuals interact repeatedly. But does natural selection favours cooperation? Understanding the conditions under which natural selection can favour cooperative behaviour is important in understanding social behaviour amongst humans and other mammals [31].

Imagine a mixed population of cooperators, C, and defectors, D, where every time two individuals meet they play a game of PD. In such population the average payoff for defectors is always higher than cooperators. Under natural selection the frequency of defectors will steadily increase until cooperators become extinct. Thus natural selection favours defection in the PD (Figure 1). However, there several mechanisms that allow the emergence of cooperation in an evolutionary context which will be cover in this section.

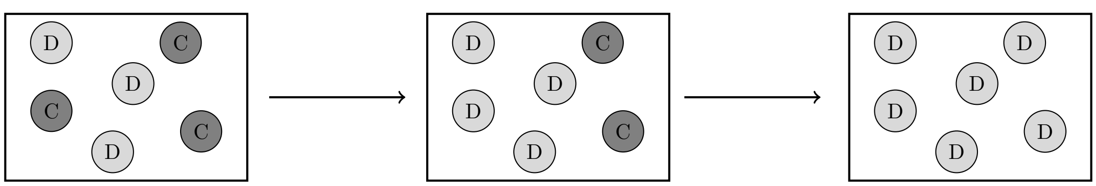


Figure 1: Natural selection favours defection in a mixed population of Cooperators and Defectors.

In the later sections of [19], Axelrod discusses an ecological tournament that he performed using the 62 strategies of the second tournament to understand the reproductive success of Tit for Tat. In his ecological tournament the prevalence of each type of strategy in each round was determined by that strategy's success in the previous round. The competition in each round would become stronger as weaker performers were reduced and eliminated. The ecological simulation concluded with a handful of nice strategies dominating the population whilst exploitative strategies had died off as weaker strategies were becoming extinct. This new result led Axelrod to study the IPD in an evolutionary context based on several of the approaches established by the biologist John M. Smith [116, 117, 118]. A fundamental figure in evolutionary game theory and a participant of Axelrod's second tournament. Axelrod and the biologist William Donald Hamilton wrote about the biological applications of the evolutionary dynamics of the IPD [24] and won the Newcomb-Cleveland prize of the American Association for the Advancement of Science.

In Axelrod's model [20] pairs of individuals were paired from a population and played the IPD. The number of interactions between the pairs were not fixed, but there was a probability defined as the importance of the future of the game w, where $0 < w < 1$, that the pair would interact again. In [20] it was shown that for a sufficient high w Tit For Tat strategies would become common and remain common because they were "collectively stable". Axelrod argued that collective stability implied evolutionary stability (ESS) and that when a collectively stable strategy is common in a population and individuals are paired randomly, no other rare strategy can invade. However, Boyd and Lorderbaum in [31] proved that if w, the importance of the future of the game, is large enough then no pure strategy is ESS because it can always be invaded by any pair of other strategies. This was also independently proven in [105].

All these conclusions were made in populations where the individuals could all interact with each other. In 1992, Nowak and May, considered a structured population where an individual's interactions were limited to it's neighbours. More specifically, in [79] they explored how local interaction alone can facilitate population wide cooperation in a one shot PD game. The two deterministic strategies Defector and Cooperator, were placed onto a two dimensional square array where the individuals could interact only with the immediate neighbours. The number of immediate neighbours could be either, fourth, six or eight, as shown in Figure 2, where each node represents a player and the edges denote whether two players will interact. This topology is refereed to as spatial topology. Each cell of the lattice is occupied by a Cooperator or a Defector and at each generation step each cell owner interacts with its immediate neighbours. The score of each player is calculated as the sum of all the scores the player achieved at each generation. At the start of the next generation, each lattice cell is occupied by the player with the highest score among the previous owner and their immediate neighbours.

Local interactions proved that as long as small clusters of cooperators form, where they can benefit from interactions with other cooperators while avoiding interactions with defectors, global cooperation will continue. Thus, local interactions proved that even for the PD cooperation can emerge. Moreover in [102], whilst using the payoff matrix (4), it was that shown cooperation will evolve in a structured population as long as the benefit to cost ratio b/c is higher to the number of neighbours. In [130], graphs were a probability of rewiring ones connections was in place were studied. The rewire could be with any given node in the graphs and not just with imitate neighbours. Perc et al concluded that "making new friends" may be an important activity for the successful evolution of cooperation, but also they must be selected carefully and one should keep their number limited.

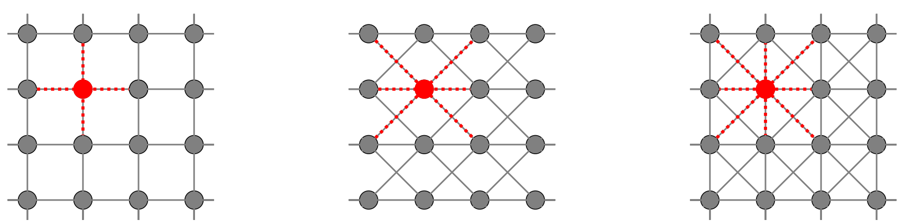


Figure 2: Spatial neighbourhoods

Another approach for increasing the likelihood of cooperation by increasing of assortative interactions among cooperative agents, include partner identification methods such as reputation [64, 98, 120], communication tokens [87] and tags [32, 53, 87, 109].

In this section evolutionary dynamics and the emergence of cooperation were reviewed. The following section focuses on strategy archetypes, training methods and strategies obtained from training.

2.4 Structured strategies and training

This section covers strategies that are different to that of intelligent design discussed in Section 2.2. These are strategies that have been **trained** using generic strategy archetypes. For example, in [21] Axelrod decided to explore deterministic strategies that took into account the last 3 turns of the game. As discussed in Section 2.2.1, for each turn there are 4 possible outcomes, CC, CD, DC, DD, thus for 3 turns there are a total of $4 \times 4 \times 4 = 64$ possible combinations. Therefore, the strategy can be defined by a series of 64 C's/D's, corresponding to each combination; a lookup table (a generic strategy archetype). This lookup table was then trained using a genetic algorithm. During the training process random changes are made to a given lookup table. If the utility of the strategy has increased this change is kept, otherwise not. The strategies obtained by these training process are referred here as structured strategies, and several structured strategies as well as archetypes and training methods are covered in this section.

In 1996 John Miller considered finite state automata as an archetype [86], more specifically, Moore machines [89]. He used a genetic algorithm to train finite state machines in environments with noise. Miller's results showed that even a small difference in noise (from 1% to 3%) significantly changed the characteristics of the evolving strategies. The strategies he introduced were **Punish Twice**, **Punish Once for Two Tats** and **Punish Twice and Wait**. In [17] finite state automata and genetic algorithms were also used to introduce new strategies. In a series of experiments where the size of the population varied, there were two strategies frequently developed by the training process and more over they were developed only after the evolution had gone on for many generations. These were **Fortess3** and **Fortess4**. Following Miller's work in 1996, the first structured strategies based on neural networks that had be trained using a genetic algorithm was introduced in [55] by Harrald and Fogel. Harrald and Fogel considered a single layered neural network which had 6 inputs. These were the last 3 moves of the player and the opponent, similar to [21]. Neural networks have broadly been used to train IPD strategies since then with genetic algorithms [16, 33, 81] and particle swarm optimization [47].

In [54, 69] both genetic algorithm and particle swarm optimization were used to introduce a series of structured strategies based on lookup tables, finite state machines, neural networks, hidden Markov models [43] and a Gambler. Hidden Markov models, are a stochastic variant of a finite state machine and Gamblers are stochastic variants of lookup tables. The structured strategies that arised for the training were put up against a large number of strategies in (1) a Moran process, which is an evolutionary model of invasion and resistance across time during which high performing individuals are more likely to be replicated and (2) a round robin tournament. In a round robin tournament which was simulated using the software [6] and the 200 strategies implemented within the software, the top spots were dominated by the trained strategies of all the archetypes. More specifically, the top three strategies had been **Evolved LookUp 2 2 2**, **Evolved HMM 5** and **Evolved FSM 16**. Moreover, [69] demonstrated that these trained strategies would overtake the population in a Moran process. The strategies evolved an ability to recognise themselves by using a handshake. This recognition mechanism allowed the strategies to resist invasion by increasing the interactions between themselves, an approach described in Section 2.3.

Throughout the different methods of training that have been discussed in this section, a spectrum of structured strategies can be found. Differentiating between strategies is not always an easy task. It is not obvious looking at a finite state diagram how a machine will behave, and many different machines, or neural networks can represent the same strategy. For example Figure 3 shows two finite automata and both are a representation of Tit for Tat.

(a) Tit for Tat as a finite state machine with 1 state. (b) Tit for Tat as a finite state machine with 2 states.

Figure 3: Finite state machine representations of Tit for Tat. Finite state machine consist of a set of internal states. In (a) Tit for Tat finite state machine consists of 1 state and in (b) of 2. A machine also consists of transitions arrows associated with the states. Each arrow is labelled with A/R where A is the opponent's last action and R is the player's response.

To allow for easier identification of similar strategies a method called fingerprinting was introduced in [12]. The method of fingerprinting is a technique for generating a functional signature for a strategy [13]. This is achieved by computing the score of a strategy against a spectrum of opponents. The basic method is to play the strategy against a probe strategy with varying noise parameters. In [12] Tit for Tat is used as the probe strategy. In Figure 4 an example of Pavlov's fingerprint is given. Fingerprinting has been studied in depth in [13, 14, 15, 16].

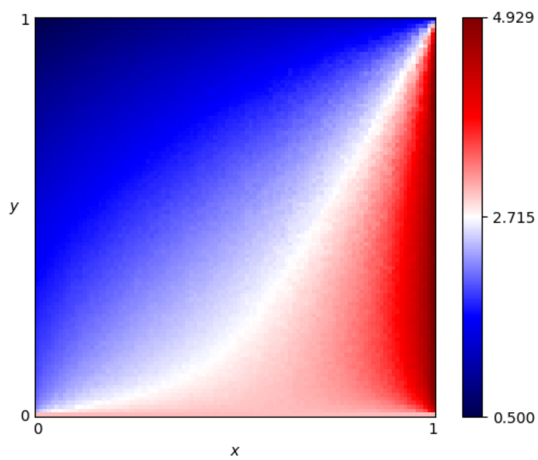


Figure 4: Pavlov fingerprinting with Tit for Tat used as the probe strategy. Figure was generated using [6].

This section covered structured strategies and training methods. In the following section software that has been developed with main aim simulating the IPD is presented.

2.5 Software

The research of the IPD heavily relays on software. This is to be expected as the pioneer computer tournaments have become the main means of simulating the interactions in an IPD game. Many academic fields suffer from the lack of source code availability and the PD is not an exception. Several of the tournaments that have been discussed so far were generated using computer code, though not all of the source code was made available by the authors. The code for Axelrod's original tournament is known to be lost and moreover for the second tournament the only source code available is the code for the 62 strategies (found on Axelrod's personal website [1]).

Several projects, however, are open, available and have been used as research tools or educational platforms over the years. Two research tools are briefly mentioned here [4, 6] and two educational tools [2, 3]. Both [4, 6] are open source projects used as research tools. PRISON is written in the programming language Java and preliminary version was launched on 1998. It was used by it's authors in several publications, such as [27], which introduced Gradual, and [28]. The project includes a good number of strategies from the literature but unfortunately the last update of the project dates back in 2004. Axelrod-Python is a software used by [54, 69, 50, 127]. It is written in the programming language Python following best practice approaches and contains the largest collection of strategies, known to the author. The strategy list of the project has been cited by publications [10, 57, 92].

The "Game of Trust" [2] is an on-line, graphical user interface educational platform for learning the basics of game theory, the IPD and the notion of strategies. It attracted a lot of attention due to being "well-presented with scribble-y hand drawn characters" [61] and "a whole heap of fun" [68]. Finally [3] is a personal project written in PHP. It's graphical user interface that offers a big collection of strategies and allows the user to try several matches and tournament configurations.

2.6 Conclusion and Contemporary research

This section of the paper served as a review of publications on the PD. This review has partitioned the literature into five different sections, focusing on different aspects of the research. Section 2.1 covered the early years of research. This was when simulating turns of the game was only possible with human subject research. Following the early years, the pioneer tournaments of Axelord were introduced in Section 2.2. Axelrod's work offered the field an agent based game theoretic framework to study the IPD. In his original papers he asked researchers to design strategies to test their performance with the new framework. The winning strategy of both his tournaments was Tit for Tat. The strategy however came with limitations which were explored by other researchers, and new strategies of intelligent design were introduced in order to surpass Tit for Tat with some contributions such as Pavlov and Gradual.

Soon researchers came to realise that strategies should not just do well in a tournament setting but should also be evolutionary robust. Evolutionary dynamics methods were applied to many works in the field, and factors under which cooperation emerges were explored, as described in Section 2.3. This was not done only for unstructured populations, where all strategies in the population can interact with each other, but also in population where interactions were limited to only strategies that were close to each other. In such topologies it was proven that even in the one shot game cooperation can indeed emerge.

Evolutionary approaches can offer many insights in the study of the PD. In evolutionary settings strategies can learn to adapt and take over population by adjusting their actions; such algorithms can be applied so that evolutionarily robust strategies can emerge. Algorithms and structures used to train strategies in the literature were covered in Section 2.4. From these training methods several strategies can emerge, and to be able to differentiate between strategies the method fingerprinting was introduced. The research of best play and cooperation has been going on since the 1950s, and for simulating the game software has been developed along the way. Few research and educational software have been briefly discussed in Section 2.5.

The study of the PD is still an ongoing field of pioneer and innovating research where new variants and new structures of strategies are continuously being explored [101]. The game now serves as a model in a wide range of applications, for example in medicine and the study of cancer cells [11, 67], as well as in social situations and how they can be driven by rewards [39]. New research is still ongoing on the topics/trend covered in each section of this literature review, for example on evolutionarily dynamics on graphs [8, 56, 77].

A large scale of articles has been covered in each of the corresponding sections of this review. This literature doe not pretend to have covered all the publications in the field. It will soon in the following section that the field has had many publications, exceeding 3000 articles. However, several important milestones of the field have been presented here.

3 Analysing a large corpus of articles

The focus of Section 2 has been to review academic publications on the topic of the IPD. Whilst in Section 2 several publications of specific interest were covered and the literature was manually partitioned in different sections, in the second part of this paper the publications are analysed using a large dataset of articles.

In Section 3.1 some background research on bibliometrics is discussed. The data collection process is covered in Section 3.2 and a preliminary analysis of the data is conducted in Section 3.3. In Section 3.4, the methodology which will be used to analyse the author relationships is presented, that is graph theoretical methods used to ascertain the level of collaborative nature of the field and identify influence. This type of analysis has been carried out in [76]. The novelty here is to consider approaches not considered in [76] such as the centrality measures of the network, and apply them to a new dataset. A further comparison of the results are made, relative to two other sub fields of game theory: auction games [84] and the price of anarchy [111] and a temporal analysis. Finally in Section 3.5, the results of the analysis are presented.

3.1 Background

As discussed in [129], bibliometrics (the statistical analysis of published works originally described by [104]) has been used to support historical assumptions about the development of fields [106], identify connections between scientific growth and policy changes [35], develop a quantitative understanding of author order [112], and investigate the collaborative structure of an interdisciplinary field [76]. Most academic research is undertaken in the form of collaborative effort and as [71] points out, it is rational that two or more people have the potential to do better as a group than individually. Collaboration in groups has a long tradition in experimental sciences and it has be proven to be productive according to [44]. The number of collaborations can be very different between research fields and understanding how collaborative a field is not always an easy task. Several studies tend to consider academic citations as a measure for these things. A blog post published by Nature [93] argues that depending on citations can often be misleading because the true number of citations can not be known. Citations can be missed due to data entry errors, academics are influenced by many more papers than they actually cite and several of the citations are superficial.

A more recent approach to measure collaborative behaviour is to use the co authorship network, as described in [76]. Using this approach has many advantages as several graph theoretic measures can be used as proxies to explain authors relationship. In [76], they analyse the development of the field "evolution of cooperation" using this approach. The topic "evolution of cooperation" is a multidisciplinary field which also includes a large number of publications on the PD. This paper builds upon the work done by [76] and extends their methodology. Though in [76], they considered a data set from a single source, Web of Science, the data set described here has been collected from five different sources. Moreover, the collaborative results of the analysis are compared to those of two different sub fields.

Co authorship networks have also been used in [129] for classifying topics of an interdisciplinary field. This was done using centrality measures, which will be covered in Section 3.2, here centrality measures are used in order to understand the influence an author can have and can receive by being part of an academic group. Furthermore, in [9] they look at the relationship between research impact and five classes of diversity: ethnicity, discipline, gender, affiliation, and academic age. These characteristics of the authors are not being captured here. In future work these characteristics would be included in the analysis.

3.2 Data Collection

Academic articles are accessible through scholarly databases. Several databases and collections today offer access through an open application protocol interface (API). An API allows users to query directly a journal's database and bypass the graphical user interface. Interacting with an API has two phases: requesting and receiving. The request phase includes composing a url with the details of the request. For example, `http://export.arxiv.org/api/query?search_query=abs:prisoner'sdilemma&max_results=1` represents a request message. The first part of the request is the address of the API. In this example the address corresponds to the API of arXiv. The second part of the request contains the search arguments. In this example it is requested that the word 'prisoners dilemma' exists within the article's title. The format of the request message is different from API to API. The receive phase includes receiving a number of raw metadata of articles that satisfies the request message. The raw metadata are commonly received in extensive markup language (xml) or Javascript object notation (json) formats [99]. Similarly to the request message, the structure of the received data differs from journal to journal.

The data collection is crucial to this study. To ensure that this study can be reproduced all code used to query the different APIs has been packaged as a Python library and is available online [49]. The software could be used for any type of projects similar to the one described here, documentation for it is available at: `http://arcas.readthedocs.io/en/latest/`. Project [49] allow users to collect articles from a list of APIs by specifying just a single keyword. Four prominent journals in the field and a pre print server were used as sources to collect data for this analysis: PLOS, Nature, IEEE, Springer and arXiv.

The following series of search terms were used to identify relevant articles:

- "prisoner's dilemma",
- "prisoners dilemma",
- "prisoner dilemma",
- "prisoners evolution",
- "prisoner game theory"

and articles for which any of these terms existed within the title, the abstract or the text are included in the analysis. More specifically, 23% of article considered here were included because any of the above terms existed within the abstract, 50% within the main text and 27% within the title. As will be described in Section 3.3, two other game theoretic sub fields auction games and the price of anarchy were also considered in this work. For collecting data on these sub fields the search terms used were "auction game theory" and "price of anarchy". The three data sets are archived and available at [40, 41, 42]. Note that the latest data collection was performed on 30$^{\text{th}}$November 2018.

3.3 Preliminary Analysis

A summary of each of the three data sets used is presented in this section. The three data sets are:

- The main data set which contains articles on the prisoner's dilemma [42].
- A data set which contains article on auction games [40].
- A data set which contains articles on the price of anarchy [41].

The main data set is archived at [42]. It consists of 3077 articles with unique titles. In case of duplicates the preprint version of an article (collected from arXiv) was dropped. Of these 3077 article, 77 have not been collected from the aforementioned APIs. These articles were of specific interest and manually added to the dataset throughout the writing of Section 2. A similar approach was used in [76] where a number of articles of interest where manually added to the data set. A more detailed summary of the articles' provenance is given by Table 1. Only 3% of the data set consists of articles that were manually added and 34% of the articles were collected from arXiv.

provenance	# of Articles	Percentage%
Manual	77	2.50%
IEEE	295	9.59%
PLOS	482	15.66%
Springer	572	18.59%
Nature	673	21.87%
arXiv	1056	34.32%

Table 1: Articles' provenance for the main data set [42].

The average number of publications was calculated for the entire dataset and for each provenance. The average number of publications is denoted as, $\mu_P = \frac{N_A}{N_Y}$, where N_A is the total number of articles and N_Y is the years of publication. The years of publication is calculated as the range between the collection date and the first published article, for each provenance, within the data. These averages are summarised in Table 2. Overall an average of 49 articles are published per year on the topic. The most significant contribution to this appears to be from arXiv with 16 articles per year, followed by Nature with 10 and Springer with 9.

	Average Yearly publication
IEEE	5.0
PLOS	8.0
Springer	9.0
Nature	11.0
arXiv	16.0
Overall	49.0

Table 2: Average yearly publication (μ_P) for main data set [42].

Though the average publication offers insights about the publications of the fields, it remains a constant number. The data handled here is a time series from 1950, when the game was introduced, until 2018 (Figure 5). Two observations can be made from Figure 5.

1. There is a steady increase to the number of publications since the 1980s and the introduction of computer tournaments.
2. There is a decrease in 2017-2018. This is due to our data set being incomplete. Articles that have been written in 2017-2018 have either not being published or were not retrievable by the APIs by the time of writing this manuscript.

These observations can be confirmed by studying the time series. Using [65], an exponential distribution is fitted to the data from 1980-2016. The exponential fit demonstrates that since 1980 there has been an increase in the number of publications until 2016 (Figure 6). The fitted model can also be used to forecast the behaviour of the field for the next 5 years. The forecasted periods are plotted in Figure 7. The time series has indicated a slight decrease however the model forecasts that the number of publications will keep increasing, thus indicating that the field of the iterated prisoner's dilemma still attracts academic attention.

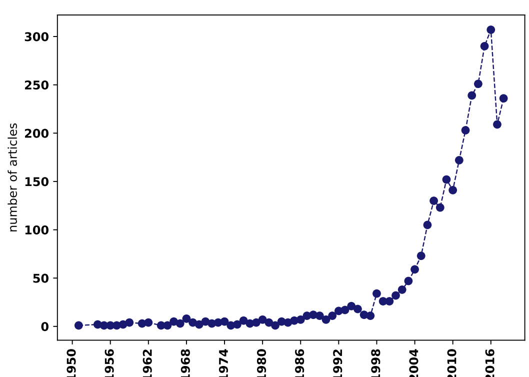


Figure 5: Line plot; # of articles published on the PD 1950-2019.

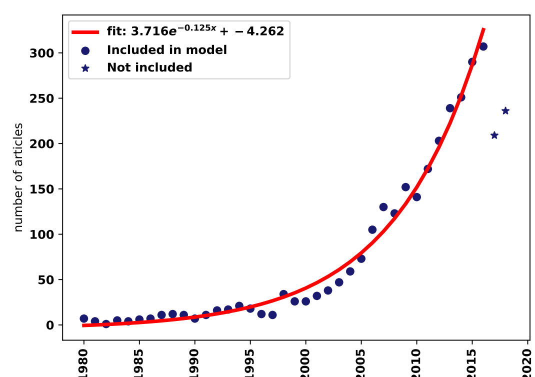


Figure 6: Scatter plot; # of articles published on the PD 1980-2019.

To allow for a comparative analysis two sub fields of game theory have been chosen for this work; auction games and the price of anarchy.

- Auction theory is a branch of game theory which researches the properties of auction markets. Game theory has been used for years to study auctions and the behaviour of bidders [115]. The earliest entry in our data set [40] goes back to 1974 (Figure 8). Note that no articles have been added manually for auction games.

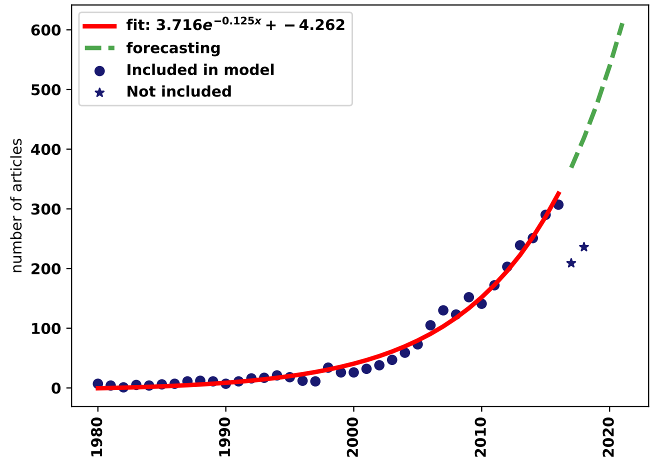


Figure 7: Forecast for 2017-2022.

- Price of Anarchy is a concept in game theory which measures how the efficiency of a system degrades due to selfish behaviour of it's agents. There is a variety of such measures however the price of anarchy has attracted a lot of attention since it's informal introduction in 1999 by [70]. Note that [70] has been manually added to the date set [41] and it's the first entry (Figure 9).

A summary of both data sets, in comparison to that of [42], is given by Table 3.

	Num. Articles	Num. Authors	Manual (%)	PLOS (%)	Nature (%)	Springer (%)	IEEE (%)	arXiv (%)	Av. Yearly Publication
Prisoner's Dilemma	3077	5772	2.5	15.66	21.87	18.59	9.59	34.32	49.0
Auction Games	3444	5362	-	-	5.89	37.63	7.46	51.36	93.0
Price of Anarchy	748	1316	0.13	1.74	24.73	37.97	30.61	8.82	39.0

Table 3: Measures of all three data sets.

The IPD and auction theory are popular topics and have been studied for decades. A large number of articles have been collected for both topics, 3077 and 3444 respectively. Though, auction games have a larger number of articles, the IPD has almost 300 more authors.

Auction games have an overall average yearly publication of 93 articles per year compared to the PD with 49 per year. The 50% of articles for auction games have been collected from the pre print server arXiv and no articles have been published in PLOS.

Compared to these two topics the price of anarchy is a fairly recent one. Only a total of 747 articles have been collected, however it has a large number of 1229 authors. On average each paper has two authors. It has an overall average yearly publication rate of 39 articles and the biggest contribution has been made to Springer (37%).

3.4 Methodology

The relationship between the authors within a field will be modelled as a graph $G = (V_G, E_G)$ where V_G is the set of nodes and E_G is the set of edges. The set V_G represents the authors and an edge connects two authors if and only if those authors have written together. The co authorship network is constructed using the main data set described in Section 3.3 and the open source package [52]. The PD network is denoted as G_1 where the number of unique authors $|V(G_1)|$ is 5772 and $|E(G_1)|$ is 10397. All authors' names were formatted as their last name and first initial (i.e. Martin A. Nowak to Martin Nowak). This was done to avoid errors such as Martin A. Nowak and Martin Nowak, being treated as a different person.

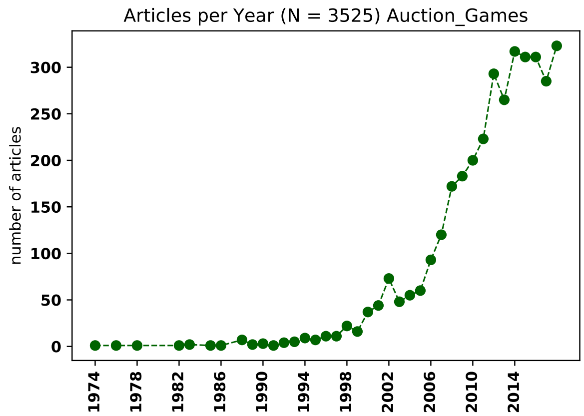


Figure 8: Line plot; # articles published on auction games 1974-2018.

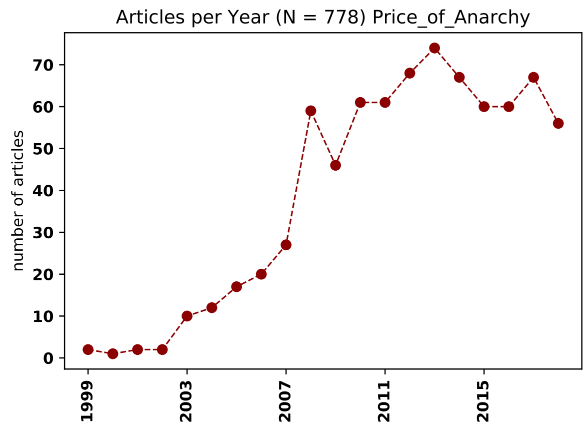


Figure 9: Line plot; # articles published on the price of anarchy.

Collaborativeness, will be analysed using measures such as, isolated nodes, connected components, clustering coefficient, communities, modularity and average degree. These measures show the number of connections authors can have and how strongly connected these people are. The number of isolated nodes is the number of nodes that are not connected to another node, thus the number of authors that have published alone. The average degree denotes the average number of neighbours for each nodes, i.e. the average number of collaborations between the authors.

A connected component is a maximal set of nodes such that each pair of nodes is connected by a path. The number of connected components as well as the size of the largest connected component in the network are reported. The size of the largest connected component represents the scale of the central cluster of the entire network, as it will discussed in the analysis section. Clustering coefficient and modularity and are also calculated. The clustering coefficient, defined as 3 times the number of triangle on the graph divided by the number of connected triples of nodes, is a local measure of the degree to which nodes in a graph tend to cluster together in a clique. It is precisely the probability that the collaborators of an author also write together.

In comparison, modularity is a global measure designed to measure the strength of division of a network into communities. The number of communities will be reported using the Clauset-Newman-Moore method [34]. Also the modularity index is calculated using the Louvain method described in [30]. The value of the modularity index can vary between $[-1, 1]$, a high value of modularity corresponds to a structure where there are dense connections between the nodes within communities but sparse connections between nodes in different communities. That means that authors in the network are mainly connected co-authors that they all have written together, and not to several different collaborators.

Networks are commonly dominated by one person who controls information flow and people that receive a great amount of information due to their position. Two further points are aimed to be explored in this work, (1) which people control the flow; as in which people influence the field the most and (2) which are the authors that gain the most from the influence of the field. To measure these concepts graph theoretic metrics, more specifically centrality measures are going to be used. Centrality measures are often used to understand different aspects fo social networks [72]. Two centrality measures have been chosen for this paper and these are closeness and betweenness centrality.

1. In networks some nodes have a short distance to a lot of nodes and consequently are able to spread information on the network very effectively. A representative of this idea is **closeness centrality**, where a node is seen as centrally involved in the network if it requires only few intermediaries to contact others and thus is structurally relatively independent. Here, this is interpreted as influence. Authors with a high value of closeness centrality, are the authors that spread scientific knowledge easier on the network and they have high influence.

2. Another centrality measure is the **betweenness centrality**, where the determination of an author's centrality

is based on the quotient of the number of all shortest paths between nodes in the network that include the node in question and the number of all shortest paths in the network. In betweenness centrality the position of the node matters. Nodes with a higher value of betweenness centrality are located in positions that a lot of information pass through them, this is interpreted as the gain from the influence, thus these authors gain the most from their networks.

In the next section all the metrics discussed here are calculated for the data sets in order to provide insights into the field.

3.5 Analysis of co authorship network

As mentioned previously, G_1 denotes the co authorship network of the IPD. A graphical representation is given by Figure 10a. It is evident that the network is disjoint, which is only natural as many authors write academic articles on their own. More specifically, a total of 176 authors, have had single author publications, which corresponds to the 3.3 (%) of authors in G_1.

There are a total of 1356 connected components and the largest one has a size of 815 nodes. The largest connected component is shown in Figure 10b and is going to be refereed to as the main cluster of the network. There are total of 1369 communities in G_1. The network has a clustering coefficient of 0.708, thus authors are 70% likely to write with a collaborator's co author and the degree distribution, Figure 11, shows that the average degree is approximately 4. Thus authors are on average connected to 4 other authors, however there are authors with far more connections, the largest one being 58.

In [76] the collaborative metrics for the "evolution of cooperation" co authorship network were reported. Though their network is of smaller size (number of nodes $3670 < 5394$), the collaborative metrics are fairly similar between the two graphs (clustering coeff. 0.632 and modularity 0.950 close to 0.977), indicating that for the same multidisciplinary field the same remarks can be made from a different co authors network. How do these compare to other fields and more specifically to other fields of game theory?

The auction games network G_2, and the price of anarchy network G_3 are given by Figures 10c and 10e and their respective largest cluster in Figures 10d and 10f. As stated before G_3 is the smallest network. G_2 network appears to be very similar to G_1, however it's main cluster is larger in size.

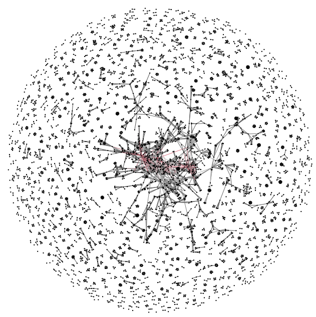
A summary of the collaborative metrics for all three co authorship networks is given by Table 4 and the degree distribution of all three networks is shown in Figure 11.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
Prisoner's Dilemma	5394	10397	176	3.3	1356	815	3.855	1369	0.977	0.708
Auction Games	5165	7861	256	5.0	1272	1348	3.044	1294	0.958	0.622
Price of Anarchy	1155	1953	4	0.3	245	222	3.382	253	0.965	0.712

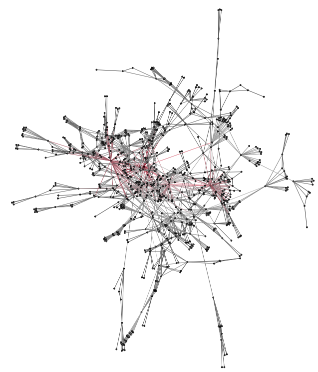
Table 4: Network metrics for G_1, G_2, G_3.

Using Table 4 and Figure 11 the following remarks can be made:

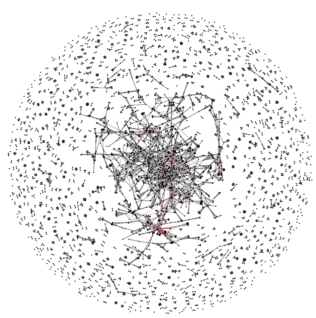
- Comparing to another well studied topic, auction games, the field of the IPD appears to be more collaborative. Due to the value of the average degree, authors in G_1 are known to have on average almost one more collaboration than G_2. A slightly lower cluster coefficient ($.622 < .702$) of auction games indicate that it is less likely for authors in G_2 to collaborate with a co author.
- Regarding the price of anarchy, the measures indicate that the field is not as mature as the other two sub fields. There are no isolated authors, which is more of an indication of the time the field has been active. As a more recent field there had been better communication tools that enable more collaborations between researches. The average degree as well as the clustering coefficient (clustering coeff.= 0.713) of G_3 is comparable to those of the IPD.



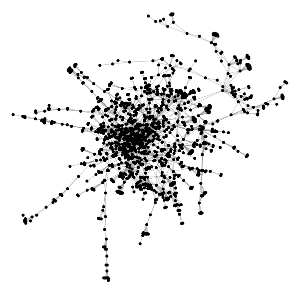
(a) G_1 network.



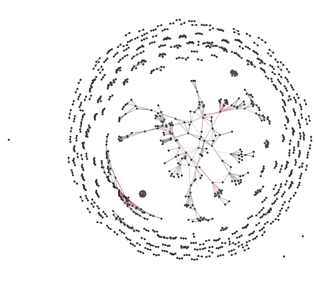
(b) G_1 largest connected component, $\bar{G}_1$.



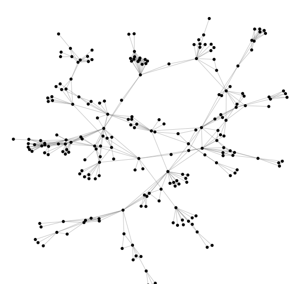
(c) G_2 network.



(d) G_2 largest connected component, $\bar{G}_2$.



(e) G_3 network.



(f) G_3 largest connected component, $\bar{G}_3$.

Figure 10: Graphical representations of G_1, G_2, G_3 and their respective main clusters.

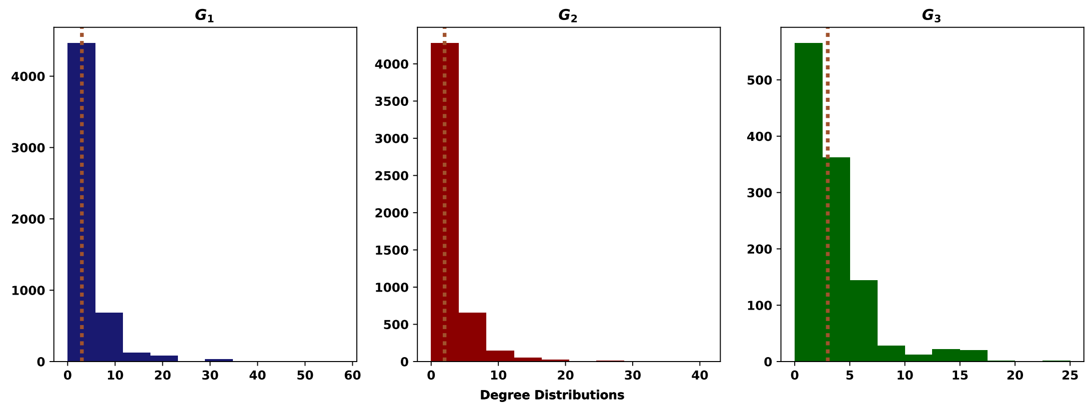


Figure 11: Degree distribution for networks G_1, G_2 and G_3. The descriptive statistics for each of the distribution are for G_1: mean = 3.85, median = 3, std = 4.25. For G_2: mean = 3.04, median = 2, std = 3.01 and for G_3: mean = 3.38, median = 3, std = 2.90. It is clear that these distributions are not normally distributed, which has also been verified using a statistical test. Moreover, the statistical difference of the medians has been tested using a Kruskal Wallis test. The medians of G_1 and G_3 are not significantly different, however they are are significantly large than that of G_2.

These results can be extended to the main clusters of each network, as shown in Table 5. The metrics' values are fairly similar and the size of G_2's main cluster does not appear to gave any significant effect; all the same conclusions are made.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
Prisoner's Dilemma	815	2300	0	0.0	1	815	5.644	30	0.854	0.775
Auction Games	1348	3158	0	0.0	1	1348	4.685	32	0.860	0.699
Price of Anarchy	222	521	0	0.0	1	222	4.694	13	0.824	0.711

Table 5: Network metrics for largest components, $\bar{G}_1, \bar{G}_2, \bar{G}_3$.

The change of the networks over time is also studied by constructing the network cumulatively using a year interval. A total of 64 sub graphs over 64 periods, starting in 1950, were created and all the collaborative metrics for each sub graph have been calculated. Note that years 1952 and 1953 have no publications in our data set. The metrics of each network for each period are given by Table 6. Similar to the results of [76], it can been observed that the network G_1 grows over time and that the network always had a high value of modularity.

To better assess the change over time for each metric they have been plotted in Figure 12. The number of nodes, connected components and the size of largest component have been normalised such that the trend between the three networks can be compared.

- In Figure 12a the normalised number of nodes, which is calculated by dividing by the total number of nodes in each respective network, is shown. A steep increase in the size of all three networks is spotted soon after 2000. This could indicate that more data have been available in the sources used in this work following the year 2000. It is however, definitely not a effect of a single field, as it is true for all three sub fields considered here. The sudden increase following the year 2000, is also reported by the number of connected components and the size of the main cluster, Figures 12c, 12d. A connected components represents at least one publication which means that indeed more articles are being gathered from 2000 onwards.
- Auction games have been throughout of time less collaborative compared to the IPD. The average degree (Figure 12b) and the clustering coefficient (Figure 12e) of the cumulative sub graphs have been lower than that of G_1. The only exception is during the years 2001-2008. For these year auction games appear to have had a more collaborative environment.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
1954 - 1950	3	0	3	100.0	3	1	0.000	-	-	0.000
1954 - 1955	2	0	2	100.0	2	1	0.000	-	-	0.000
1955 - 1956	3	0	3	100.0	3	1	0.000	-	-	0.000
1956 - 1957	4	0	4	100.0	4	1	0.000	-	-	0.000
1957 - 1958	6	0	6	100.0	6	1	0.000	-	-	0.000
1958 - 1959	7	0	7	100.0	7	1	0.000	-	-	0.000
1959 - 1961	7	0	7	100.0	7	1	0.000	-	-	0.000
1961 - 1962	8	0	8	100.0	8	1	0.000	-	-	0.000
1962 - 1964	9	0	9	100.0	9	1	0.000	-	-	0.000
1964 - 1965	10	0	10	100.0	10	1	0.000	-	-	0.000
1965 - 1966	17	3	11	64.7	14	2	0.353	14	0.666667	0.000
1966 - 1967	21	4	13	61.9	17	2	0.381	17	0.75	0.000
1967 - 1968	32	15	13	40.6	21	5	0.938	21	0.684444	0.135
1968 - 1969	36	17	16	44.4	24	6	0.944	24	0.629758	0.139
1969 - 1970	39	18	17	43.6	26	6	0.923	26	0.666667	0.128
1970 - 1971	51	28	18	35.3	31	6	1.098	31	0.826531	0.275
1971 - 1972	58	34	19	32.8	34	6	1.172	34	0.866782	0.345
1972 - 1973	59	35	18	30.5	34	6	1.186	34	0.873469	0.339
1973 - 1974	59	35	18	30.5	34	6	1.186	34	0.873469	0.339
1974 - 1975	60	35	19	31.7	35	6	1.167	35	0.873469	0.333
1975 - 1976	60	35	19	31.7	35	6	1.167	35	0.873469	0.333
1976 - 1977	68	37	23	33.8	41	6	1.088	41	0.885318	0.294
1977 - 1978	70	38	23	32.9	42	6	1.086	42	0.890582	0.286
1978 - 1979	73	42	23	31.5	42	6	1.151	42	0.893424	0.292
1979 - 1980	77	45	25	32.5	44	6	1.169	44	0.899753	0.307
1980 - 1981	80	50	26	32.5	45	6	1.250	45	0.8928	0.318
1981 - 1982	84	56	26	31.0	46	6	1.333	46	0.903061	0.350
1982 - 1983	87	57	27	31.0	48	6	1.310	48	0.906125	0.338
1983 - 1984	94	58	32	34.0	54	6	1.234	54	0.909037	0.313
1984 - 1985	95	58	33	34.7	55	6	1.221	55	0.909037	0.309
1985 - 1986	104	59	40	38.5	63	6	1.135	63	0.911807	0.283
1986 - 1987	116	61	48	41.4	73	6	1.052	73	0.916958	0.253
1987 - 1988	121	65	48	39.7	75	6	1.074	75	0.924497	0.268
1988 - 1989	134	76	47	35.1	80	6	1.134	80	0.937673	0.272
1989 - 1990	145	82	49	33.8	86	6	1.131	86	0.944676	0.272
1990 - 1991	158	88	53	33.5	94	6	1.114	94	0.950413	0.268
1991 - 1992	169	91	59	34.9	102	6	1.077	102	0.953025	0.251
1992 - 1993	186	104	62	33.3	110	6	1.118	110	0.95932	0.266
1993 - 1994	220	134	72	32.7	127	6	1.218	127	0.965471	0.317
1994 - 1995	239	144	74	31.0	137	6	1.205	137	0.969329	0.304
1995 - 1996	257	163	77	30.0	145	6	1.268	145	0.970831	0.318
1996 - 1997	279	178	81	29.0	156	6	1.276	156	0.974309	0.336
1997 - 1998	311	215	65	20.9	160	6	1.383	160	0.979773	0.354
1998 - 1999	329	239	58	17.6	162	6	1.453	162	0.981741	0.376
1999 - 2000	373	273	67	18.0	183	6	1.464	183	0.983778	0.387
2000 - 2001	400	320	54	13.5	184	7	1.600	184	0.983066	0.410
2001 - 2002	450	366	61	13.6	206	7	1.627	206	0.984547	0.418
2002 - 2003	509	414	58	11.4	229	7	1.627	229	0.987083	0.421
2003 - 2004	580	489	58	10.0	253	10	1.686	253	0.988052	0.429
2004 - 2005	679	599	57	8.4	284	19	1.764	284	0.98891	0.463
2005 - 2006	854	806	66	7.7	342	21	1.888	342	0.990724	0.496
2006 - 2007	1056	1117	76	7.2	402	24	2.116	402	0.989663	0.527
2007 - 2008	1255	1460	85	6.8	454	32	2.327	455	0.989734	0.549
2008 - 2009	1462	1759	104	7.1	520	56	2.406	521	0.987517	0.550
2009 - 2010	1700	2301	114	6.7	581	99	2.707	584	0.979084	0.571
2010 - 2011	2040	2954	121	5.9	665	121	2.896	668	0.980451	0.603
2011 - 2012	2422	3676	126	5.2	756	210	3.036	759	0.978972	0.629
2012 - 2013	2807	4398	138	4.9	843	330	3.134	849	0.977892	0.639
2013 - 2014	3199	5044	148	4.6	942	406	3.153	950	0.974719	0.651
2014 - 2015	3798	6221	159	4.2	1064	514	3.276	1074	0.976286	0.668
2015 - 2016	4472	8344	169	3.8	1184	614	3.732	1197	0.975044	0.690
2016 - 2017	4925	9235	173	3.5	1274	703	3.750	1288	0.975975	0.700
2017 - 2018	5385	10379	176	3.3	1356	815	3.855	1369	0.977065	0.708

Table 6: Collaborativeness metrics for cumulative graphs, $G \subseteq G_1$.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
1954 - 1950	1	0	1	100.0	1	1	0.000	-	-	0.000
1954 - 1955	1	0	1	100.0	1	1	0.000	-	-	0.000
1955 - 1956	1	0	1	100.0	1	1	0.000	-	-	0.000
1956 - 1957	1	0	1	100.0	1	1	0.000	-	-	0.000
1957 - 1958	1	0	1	100.0	1	1	0.000	-	-	0.000
1958 - 1959	1	0	1	100.0	1	1	0.000	-	-	0.000
1959 - 1961	1	0	1	100.0	1	1	0.000	-	-	0.000
1961 - 1962	1	0	1	100.0	1	1	0.000	-	-	0.000
1962 - 1964	1	0	1	100.0	1	1	0.000	-	-	0.000
1964 - 1965	1	0	1	100.0	1	1	0.000	-	-	0.000
1965 - 1966	2	1	0	0.0	1	2	1.000	1	0	0.000
1966 - 1967	2	1	0	0.0	1	2	1.000	1	0	0.000
1967 - 1968	5	8	0	0.0	1	5	3.200	1	0	0.867
1968 - 1969	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1969 - 1970	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1970 - 1971	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1971 - 1972	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1972 - 1973	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1973 - 1974	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1974 - 1975	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1975 - 1976	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1976 - 1977	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1977 - 1978	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1978 - 1979	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1979 - 1980	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1980 - 1981	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1981 - 1982	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1982 - 1983	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1983 - 1984	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1984 - 1985	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1985 - 1986	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1986 - 1987	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1987 - 1988	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1988 - 1989	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1989 - 1990	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1990 - 1991	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1991 - 1992	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1992 - 1993	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1993 - 1994	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1994 - 1995	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1995 - 1996	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1996 - 1997	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1997 - 1998	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1998 - 1999	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1999 - 2000	6	9	0	0.0	1	6	3.000	2	0.166667	0.900
2000 - 2001	7	21	0	0.0	1	7	6.000	1	0	1.000
2001 - 2002	7	21	0	0.0	1	7	6.000	1	0	1.000
2002 - 2003	7	21	0	0.0	1	7	6.000	1	0	1.000
2003 - 2004	10	13	0	0.0	1	10	2.600	2	0.37574	0.553
2004 - 2005	19	28	0	0.0	1	19	2.947	3	0.544005	0.730
2005 - 2006	21	32	0	0.0	1	21	3.048	4	0.543945	0.713
2006 - 2007	24	36	0	0.0	1	24	3.000	5	0.563272	0.678
2007 - 2008	32	59	0	0.0	1	32	3.688	3	0.627837	0.732
2008 - 2009	56	102	0	0.0	1	56	3.643	4	0.716792	0.699
2009 - 2010	99	238	0	0.0	1	99	4.808	7	0.781539	0.734
2010 - 2011	121	288	0	0.0	1	121	4.760	9	0.771858	0.713
2011 - 2012	210	610	0	0.0	1	210	5.810	13	0.780145	0.747
2012 - 2013	330	908	0	0.0	1	330	5.503	17	0.817439	0.753
2013 - 2014	406	1125	0	0.0	1	406	5.542	21	0.818674	0.749
2014 - 2015	514	1390	0	0.0	1	514	5.409	20	0.827712	0.757
2015 - 2016	614	1682	0	0.0	1	614	5.479	29	0.830544	0.765
2016 - 2017	703	1925	0	0.0	1	703	5.477	31	0.835941	0.774
2017 - 2018	815	2300	0	0.0	1	815	5.644	30	0.855152	0.775

Table 7: Collaborativeness metrics for cumulative graphs' main clusters, $G \subseteq \bar{G}_1$.

- In the price of anarchy cumulative graphs a sharp increase since the beginning of the field can be observed for all metrics. There are not many data points due to the recent development of the field, however these steep trends could be an indication that game theoretic and potentially all scientific research has over time been more collaborative. This could be due to logistic and technical solutions.
- The high values of modularity throughout time is not true only for the network reported in [76] but also for all three networks of this field. Indicating that authors tend to create communities and write only with people from their communities and not others.

The cumulative collaborative metrics have also been calculated for each main cluster, given in Table 7. Similarly, the results do not appear to change over the main cluster.

The next results discussed here are on centrality measures. As a reminder, two centrality measures are reported here, these are the closeness centrality and the betweenness centrality. Closeness centrality is a measure of how easy is for an author to contact others, and consequently affect them; influence them. Thus closeness centrality here is a measure of influence. Betweenness centrality is a measure of how many paths pass through a specific node, thus the amount of information this person has access to. Betweenness centrality is used here as a measure of how much an author gain from the field. All centrality measure can have values ranging from $[0, 1]$.

For G_1 the most central author based on closeness and betweenness are given by Tables 13 and 14 respectively. The betweenness centrality of the most central authors in G_1 are rather low with the highest ranked author being Matjaz Perc with a between centrality of 0.008, Table 14. A publication of Perc's work has been briefly discussed in Section, and the centrality measure suggest that the network is influenced by him. He is connected to a total of 58 nodes and he has published to all five of the different sources considered in the study. Though he also gains from his position in the network, the gain is minor. An author who is not in the top influencers but does indeed gain from his position in the network is Martin Nowak, who was extensively discussed in Section 2.

	Name	Closeness
1	Matjaz Perc	0.048447
2	Yamir Moreno	0.044840
3	Zhen Wang	0.044005
4	Long Wang	0.043770
5	Attila Szolnoki	0.043338
6	Luo-Luo Jiang	0.042148
7	Arne Traulsen	0.041790
8	Valerio Capraro	0.041257
9	Cheng-Yi Xia	0.040791
10	Angel Sanchez	0.040562

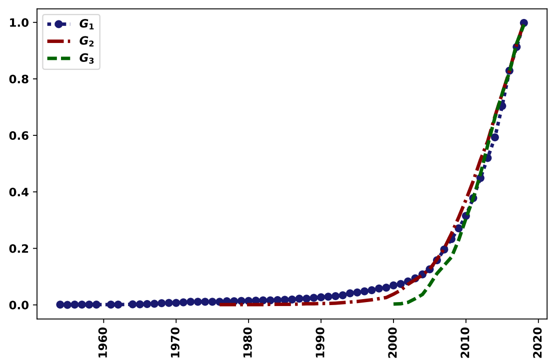
Figure 13: Ten most influenced authors in G_1.

	Name	Betweeness
1	Matjaz Perc	0.008331
2	Zhen Wang	0.006356
3	Yamir Moreno	0.004806
4	Long Wang	0.003538
5	Martin Nowak	0.003230
6	Valerio Capraro	0.002739
7	Arne Traulsen	0.002479
8	Angel Sanchez	0.002319
9	Jianye Hao	0.002188
10	Franz Weissing	0.002186

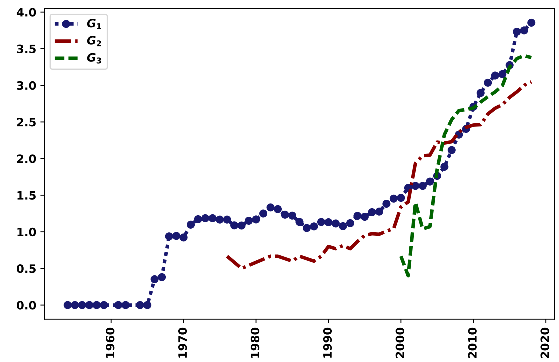
Figure 14: Authors that gain the most influence in G_1.

From Tables 13 and 14 it can be seen that authors in G_1 are more likely to affect their field instead of gaining from it. This can be better explored by considering the distributions of the centralities and by comparing them to other fields. The distributions for both centralities are plotted in Figures 15a and 16a, and in Figures 15b and 16b for their respective main clusters.

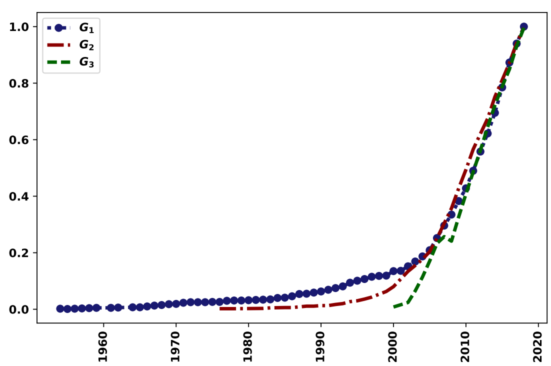
Regarding gaining from your network. An author is more likely to gain more from the influence of the field if they were authors in auction games or the price of anarchy. Though if it were an author in the main cluster it would make to statistical difference in which field they were to published. Overall, all the betweenness values are rather small and the distributions skewed to the left. This could imply that in all three networks, authors do not gain much from the influence of their fields.



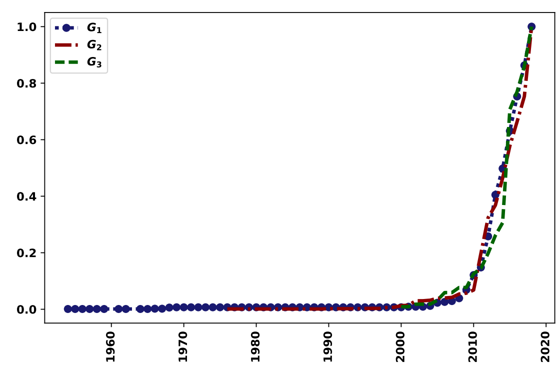
(a) % Nodes.



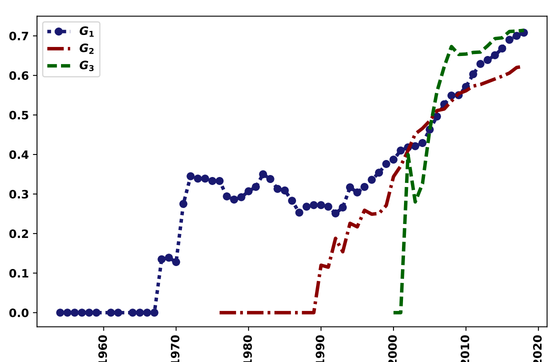
(b) Average Degree.



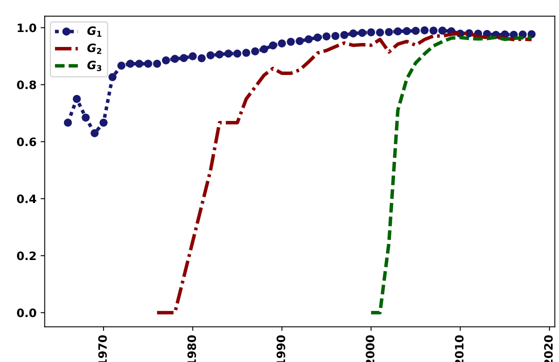
(c) % Connected components.



(d) % Size of largest connected component.

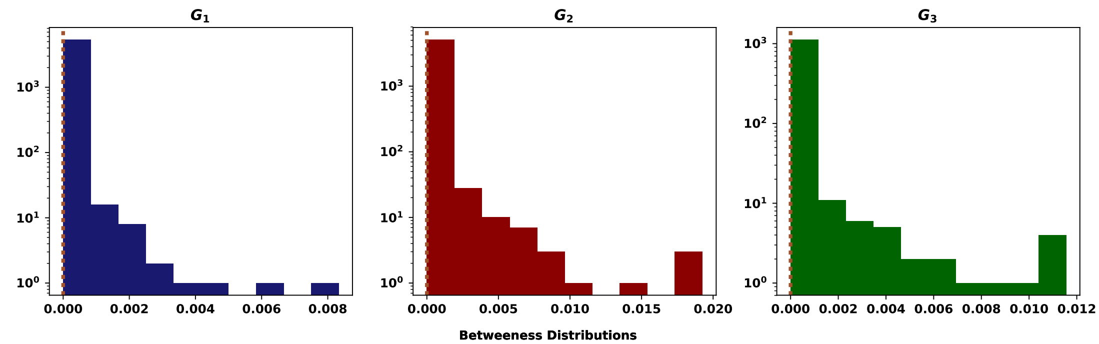


(e) Clustering coefficient.

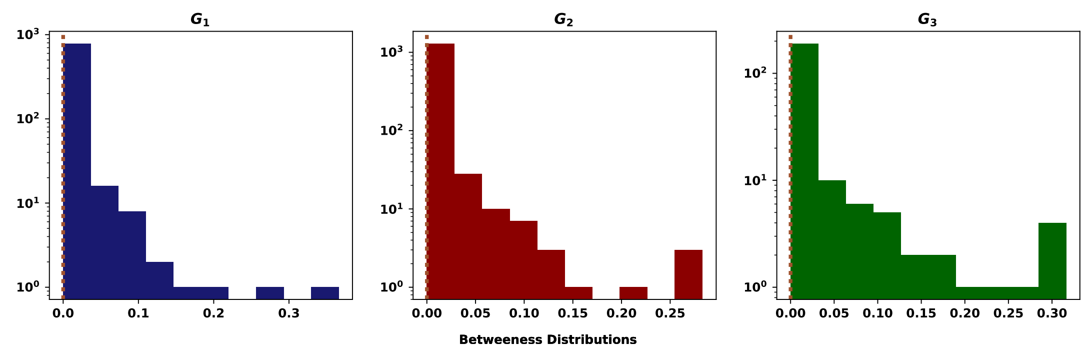


(f) Modularity.

Figure 12: Collaborative metrics over time for cumulative networks for G_1, G_2 and G_3.



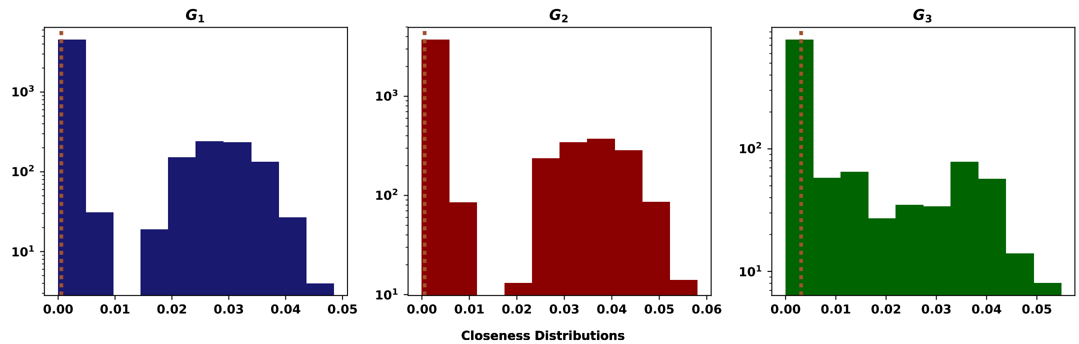
(a) Betweenness centrality distributions G_1, G_2, G_3. The descriptive statistics for each of the distribution are for G_1: mean= 0.000019, median= 0.0, std= 0.000207. For G_2: mean= 0.000086, median= 0.0, std= 0.000693 and for G_3: mean= 0.000151, median= 0.0, std= 0.000931. None of the three distributions is normally distributed and there is significant difference between the means (these have been tested using appropriate statistical difference). According to a Mann Whitney both G_2 and G_3 medians are significant larger than that of G_1 however there is not statistical difference between those two medians.



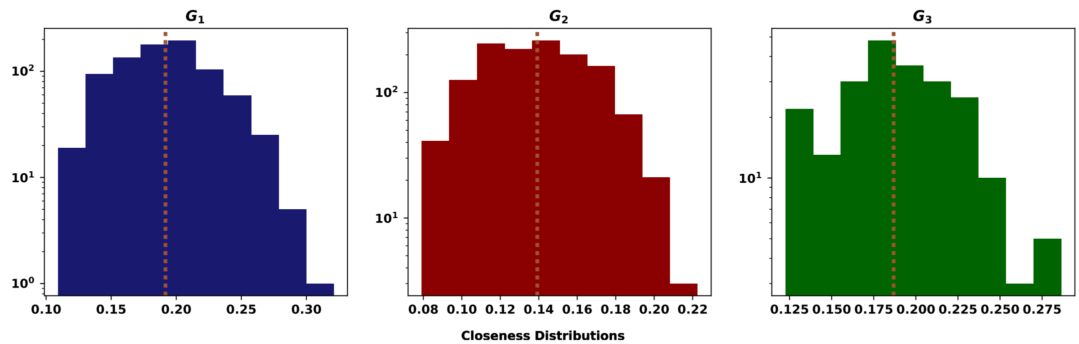
(b) Betweenness centrality distributions for G_1, G_2, G_3 respective main clusters.The descriptive statistics for each of the distribution are for G_1: mean= 0.0054, median= 0.0, std= 0.022. For G_2: mean= 0.0048, median= 0.0, std= 0.019 and for G_3: mean= 0.02, median= 0.0, std= 0.055. None of the three distributions is normally distributed. There is no statistical difference between the medians of G_1 and G_3. There is however, statistical difference between the median of G_2. These have been tested using a Kruskal Wallis test.

Figure 15

In relation to influencing your field. An author is most likely to influence their field if they write for the price of anarchy and authors that publish on auctions game are more likely to influence compared to authors in the IPD. Though if an author was to be placed in the main cluster of the respective field they would chose to be in either G_1 or G_3. In conclusion, authors regarding both influence metrics that have been defined here are more likely to gain more if they were to published on either topics of auction games or the price of anarchy. Though the value of gaining is actual small, you are more likely to influence your field more in another field compared to that of the PD.



(a) Closeness centrality distributions G_1, G_2, G_3. The descriptive statistics for each of the distribution are for G_1: mean= 0.0050, median= 0.00056, std= 0.010. For G_2: mean= 0.000086, median= 0.00058, std= 0.000693 and for G_3: mean= 0.000151, median= 0, std= 0.000931. None of the three distributions is normally distributed and the median of G_3 is statistically larger than that of G_2, which is larger than that of G_1.



(b) Closeness centrality distributions for G_1, G_2, G_3 respective main clusters. The descriptive statistics for each of the distribution are for G_1: mean= 0.19, median= 0.19, std= 0.035. For G_2: mean= 0.14, median= 0.14, std= 0.026 and for G_3: mean= 0.19, median= 0.19, std= 0.035. None of the three distributions is normally distributed. All medians are statistically different. The medians of G_1 and G_3 are greater than that of G_2.

Figure 16

4 Conclusion

This manuscript presented a coherent literature review on the Iterated Prisoner's Dilemma. The opening sections focused on the research trends of the field. Each trend and it's milestones were covered, followed by a presentation on research and educational software that has implemented for the Iterated Prisoner's Dilemma. The later sections presented a meta analysis of publications with the aim of examining the collaborativeness of the authors and their influence in the research of the game.

The research trends covered in this manuscript included the early experiments using human subject research, the investigation of cooperative behaviour and the search of dominant strategies for the game. Human subject research had limitations and thought it is used by several researchers to date, they have been gradually replaced by the computer tournaments introduced by Axelrod in 1980s. The search of strategies includes strategies that have been manually designed by an intelligent design and strategies that have been found by training processes of structures such as finite state automata and neural networks. Moreover, cooperative behaviour and it's emergence under natural section was discussed in Section 2.3. The results of several milestones have been summarised in this review. These included the emerge of cooperation in the PD in structured populations, the success and deficiency of the infamous Tit For Tat, and the training of complex strategies that evolved a handshake mechanism to combat invasion.

The meta analysis which was covered in the second part of this paper, explored the publications in field, the authors collaborative behaviour and influence. The publications from five different journals excited the 3000 and predicted a continuous growth to the number of publications in the following years. The authors of these papers were used to create a co authorship network which was studied and showed that compered to two other prominent sub fields of game theory the Iterated Prisoner's Dilemma field is more collaborative.

5 Acknowledgements

A variety of software have been used in this work:

- The Axelrod library for IPD simulations [6].
- The Matplotlib library for visualisation [62].
- The Numpy library for data manipulation [126].
- The Networkx [52] package for analysing networks.
- Gephi [26] open source package for visualising networks.
- The louvain library for calculating the networks modularity.

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