A systematic literature review of the Prisoner's Dilemma and an analysis of the corresponding co author network.

Nikoleta E. Glynatsi

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1 Introduction

The emergence of cooperation is a topic of continuing and public interest for the social [24, 31], biological [32] and ecological sciences [33, 40, 50, 77]. Cooperation is essential for evolution but according to Darwin's theory it is not always easy to achieve. The game called the prisoner's dilemma offers a theoretical framework for studying the emergence of altruistic behaviour.

1.1 The Prisoner's Dilemma

The prisoner's dilemma a two player no-cooperative game [27] where the decisions of the players are made simultaneously and independently. Both players can choose between cooperation (**C**) or defection (**D**).

The fitness of each player is influenced by its own behaviour, and the behaviour of the opponent. If both players choose to cooperate, both do better than if both defect. However, a player has the temptation to deviate. If a player was to defect while the other cooperates, the defector receives more than if both had cooperated. The reward for mutual cooperation is R, for a mutual defection they receive P, and for cooperation-defection, the cooperator receives S where the defector receives T. Thus, the game's payoffs are given by,

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} \tag{1}$$

where $T > R > P > S$ and $2R > T + S$ are the conditions for a dilemma to exist. Due to rational behaviour and the knowledge that an individual is tempted to defect the game's equilibrium lies at a mutual defection and both players receive a payoff of P. Thus, the dominant strategy for the prisoner's dilemma is **D**.

However, when the game is studied in a manner where prior outcomes matter, the defecting choice is no longer necessarily the dominant choice. The repeated form of the game is called the iterated prisoner's dilemma and now two players play the game repeatedly. Interest was sparked on the iterated prisoner dilemma by R. Axelrod and his book [18] "The Evolution of Cooperation".

In his book Axelrod reports on a series of computer tournaments he organised of a finite turns games of the iterated prisoner's dilemma. Participants had to choose between **C** and **D** again and again while having memory of their previous encounters. Academics from several fields were invited design computer strategies to compete in the tournament. The pioneer work of Axelrod showed that greedy strategies did very poorly in the long run while more altruistic strategies did better.

"The Evolution of Cooperation" is considered a milestone in the field but it is not the only one. On the contrary, the prisoner's dilemma has attracted much attention ever since the game's origins. This is shown in Figure 1, which illustrates the number of publications on the prisoner's dilemma per year from the following sources:

- arXiv;
- PLOS;
- IEEE;
- Nature;
- Springer.

The choice of sources is due to the fact that they have an open access Api, the process of collecting the data (including criteria for inclusion of papers) and the analysis will be described more comprehensively in Section 3.

Each point of Figure 1 marks the starting year of a time period. Each of these time periods is reviewed and presented in 2, as an extensive literature review. This paper is the review of this type, in such detail since the origins to date.

Furthermore, in Section 3 a comprehensive data set of literature regarding the prisoner's dilemma will be presented and analysed. This allow us to review the amount of published academic articles as well as measure and explore the collaborations within the field.

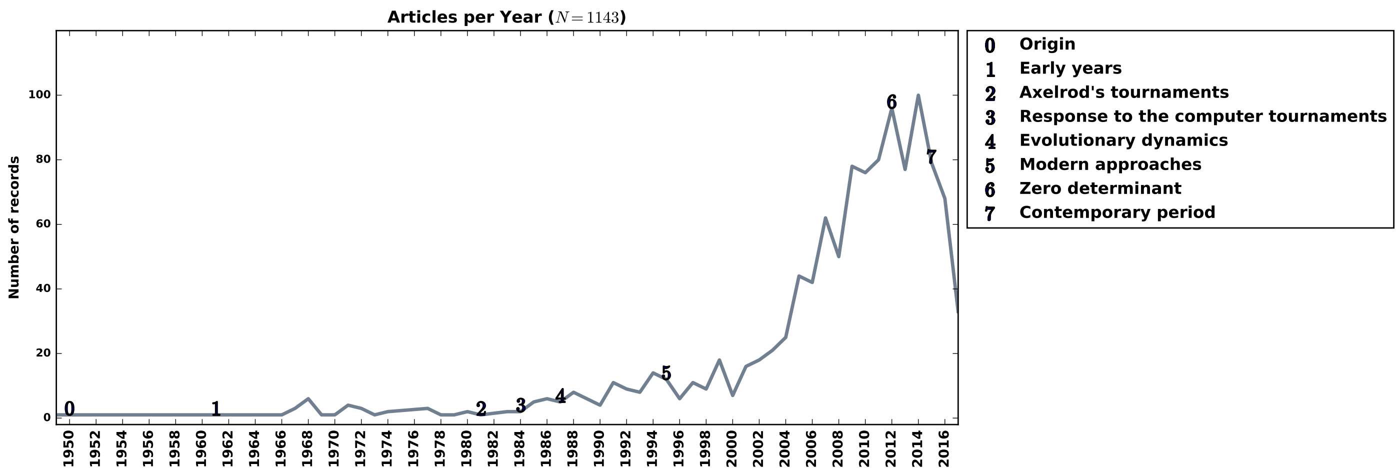

Figure 1: A timeline of the prisoner's dilemma research.

2 Timeline

2.1 Origin and Primal research (1961-1972)

The origin of the prisoner's dilemma goes back to the 1950s in early experiments conducted in RAND [27] to test the applicability of games described in [75]. Although in [27] the two player game was introduced the name behind the game was given later the same year. According to [71], A. W. Tucker (the PhD supervisor of J. Nash), in an attempt to delivery the game with a story during a talk used prisoners as players and the game has been known as the prisoner's dilemma ever since [71].

The study of the prisoner's dilemma has attracted people from various fields across the years. An early figure within the field is Prof A. Rapoport, a mathematical psychologist, whose work focused on peacekeeping. In his early work [63] Rapoport conducted experiments using humans to simulate a play of the prisoner's dilemma. Experimental groups were not been used only by Rapoport but it was a common mean of studying the game [25, 29, 46, 47, 67] and are still being used today.

Those experiments explored the conditions under which altruist behaviour emerges in human societies. Conditions such as, the gender [25, 46, 47] of individuals, the representation of the game [25], the distance between players [67], the initial effects [70] and whether the experimenter was biased [29].

Even though, several of these experiments were held and continuous research on the topic was undergoing game theorists were still in disagreement about the best way to play the game [63]. Inspired by the work of Rapoport and intrigued by the very same question the political scientist R. Axelrod took upon himself to identify the dominant strategy of the prisoners dilemma.

The main difference of Axelrod's approach was that machines were going to be used instead of humans. The issues with using humans, according to Axelrod [16], was the fact that humans can act very randomly even though the aim of the game is clear to them. Thus, Axelrod was the first researcher, to the author's knowledge, to perform a computer tournament of the iterated prisoner's dilemma. The work of Axelrod is considered one of the greatest milestones within the field. The tournaments and their results are discussed in the next sections.

2.2 Axelrod's Tournaments (1981-1984)

This section serves as a follow up from the earlier years of the topic and as an introduction to the modern ways of studying the prisoner's dilemma. It is dedicated to the computer tournaments of R. Axelrod from 1981 to 1984.

The first computer tournament was performed in 1980 [12]. Several scientists were invited to submit their strategies, written in the programming languages Fortran or Basic. There was a total number 13 submissions made by the following researchers,

1. T Nicolaus Tideman and Paula Chieruzz;
2. Rudy Nydegger;
3. Bernard Grofman;
4. Martin Shubik;
5. Stein and Anatol Rapoport;
6. James W Friedman;
7. Morton Davis;
8. Jim Graaskamp;
9. Leslie Downing;
10. Scott Feld;
11. Johann Joss;
12. Gordon Tullock;
13. Name not given.

Each competed in a 200 turn match against all 13 opponents, itself and a player that played randomly. This type of tournament is referred to as a round robin and corresponds to a complete graph from a topological point of view. The tournament was repeated 5 times to reduce variation in the results. Each participant knew the exact length of the matches and had access to the full history of each match. Furthermore, Axelrod performed an preliminary tournament and the results were known to the participants. The payoff values used for 1 where $R = 3, P = 1, T = 5$ and $S = 0$. These values are commonly used in the literature and unless specified will be the values used in the rest of the work described here.

The winner of the tournament was determined by the total average score and not by the number of matches won. The strategy that was announced the winner was submitted by Rapoport and was called **Tit For Tat**. Tit for Tat, is a strategy that always cooperates on the first round and then mimics the opponent's previous move.

Examples of Tit for Tat interacting for 8 turns with deterministic opponents are given by Tables 1, 2, 3. The opponents are, **Cooperator** a strategy that always cooperates, **Defector** an opponent that always defects and **Altenator** a player who alternates between cooperating and defecting.

The results of the first tournament were filled with surprises. Tit for Tat the simplest strategy of all had won and had managed to defeat even entrants that tried to improve on Tit for Tat after the preliminary tournament results. Axelrod justified the success of the strategy saying that the strategy was 'nice' and 'forgiving'.

The top eight ranked strategies were strategies that did no defect on the first round, thus they were described as 'nice' strategies. Compared to the rest of the "nice" strategies, Tit for Tat had also another property. That property was 'forgiveness'. Tit for Tat punished it's opponent for a defection but just once and then it would try to cooperate again. These two properties were described to be the secret of success in a prisoner's dilemma tournament.

In order to further test the robustness of the results Axelrod performed a second tournament [13] later in 1980. This time a total of 63 participants submitted strategies for the second tournament, their names were the following,

Turns	Tit for Tat	Cooperator
1	C	C
2	C	C
3	C	C
4	C	C
5	C	C
6	C	C
7	C	C
8	C	C

Table 1: Tit for Tat example match of 8 turns against Cooperator

Turns	Tit for Tat	Defector
1	C	D
2	D	D
3	D	D
4	D	D
5	D	D
6	D	D
7	D	D
8	D	D

Table 2: Tit for Tat example match of 8 turns against Defector

Turns	Tit for Tat	Altenator
1	C	C
2	C	D
3	D	C
4	C	D
5	D	C
6	C	D
7	D	C
8	C	D

Table 3: Tit for Tat example match of 8 turns against Altenator

1. Gail Grisell;
2. Harold Rabbie;
3. James W Friedman;
4. Abraham Getzler;
5. Roger Hotz;
6. George Lefevre;
7. Nelson Weiderman;
8. Tom Almy;
9. Robert Adams;
10. Herb Weiner;
11. Otto Borufsen;
12. R D Anderson;
13. William Adams;
14. Michael F McGurrin;
15. Graham J Eatherley;
16. Richard Hufford;
17. George Hufford;
18. Rob Cave;
19. Rik Smoody;
20. John Willaim Colbert;
21. David A Smith;
22. Henry Nussbacher;
23. William H Robertson;
24. Steve Newman;
25. Stanley F Quayle;
26. Rudy Nydegger;
27. Glen Rowsam;
28. Leslie Downing;
29. Jim Graaskamp and Ken Katzen;
30. Danny C Champion;
31. Howard R Hollander;
32. George Duisman;
33. Brian Yamachi;
34. Mark F Batell;
35. Ray Mikkelson;
36. Craig Feathers;
37. Fransois Leyvraz;
38. Johann Joss;
39. Robert Pebly;
40. James E Hall;
41. Edward C White Jr;
42. George Zimmerman;
43. Edward Friedland;
44. X Edward Friedland;
45. Paul D Harrington;
46. David Gladstein;
47. Scott Feld;
48. Fred Mauk;
49. Dennis Ambuehl and Kevin Hickey;
50. Robyn M Dawes and Mark Batell;
51. Martyn Jones;
52. Robert A Leyland;
53. Paul E Black;
54. T Nicolaus Tideman and Paula Chieruzz;
55. Robert B Falk and James M Langsted;
56. Bernard Grofman;
57. E E H Schurmann;
58. Scott Appold;
59. Gene Snodgrass;
60. John Maynard Smith;
61. Jonathan Pinkley;
62. Anatol Rapoport.

All the participants knew the results of the previous tournament. The rules were similar to those of the first tournament with only one exception; the number of turns was not specified instead a fixed probability (refereed to as 'shadow of the future' [17]) of the game ending on the next move was used. The fixed probability was chosen to be 0.0036 so that the expected median length of a match would be 200 turns. The topology was of a round robin and each pair of players was matched 5 times. The length of the matches was determined once by drawing a random sample. Each of the five matches had a length of 63, 77, 151 and 308.

The results of the tournament once again came as a surprise. Tit for Tat was considered to be one of the simplest submissions in the second tournament and won the second tournament as well. Tit for Tat provided proof that reciprocity behaviour can allow cooperation to emerge in the iterated prisoner's dilemma game. In [12] the main conclusions indicating strong performance was:

- that it start of by cooperating
- it would forgive it's opponent after a defection
- after opponents identified that they were playing Tit for Tat choose to cooperate for the rest of the game.

Another successful strategy from Axelrod's tournament that can been seen in literature to date is **Grudger**, originally submitted by James W. Friedman. Grudger is a strategy that will cooperate as long as the opponent does not defect.

The name Grudger was give to the strategy in [44]. Though the strategy goes by many names in the literature such as, Spite [21], Grim Trigger [20] and Grim [74].

As for the rest of the strategies, though a full explanation of all 13 submitted strategies is given in [12] the same does not hold for all 63 strategies of the second tournament [13]. The author mainly focuses on the high ranked participants and several details for the rest strategies are left unknown.

The source code of the 63 strategies be found on Axelrod's personal website [1]. The source code was written by Axelrod and several other contributors. The strategies written in Basic were translated to Fortran before the tournament. The source code includes the code only for the strategies and not for creating and performing the tournament.Figure 2 serves as an example of the source code giving the code for the winning strategy Tit for Tat. Unfortunately, the source code of the first 13 strategies is not available, as stated in Axelrod's personal website [1].

```
    FUNCTION K92R(J,M,K,L,R, JA)
C BY ANATOL RAPOPORT
C TYPED BY AX 3/27/79 (SAME AS ROUND ONE TIT FOR TAT)
c replaced by actual code, Ax 7/27/93
c  T=0
c   K92R=ITFTR(J,M,K,L,T,R)
      k92r=0
      k92r = j
c test 7/30
c   write(6,77) j, k92r
c77   format(' test k92r. j,k92r: ', 2i3)
      RETURN
      END
```

Figure 2: Source code for Tit for Tat in Fortran. Provided by [1].

So far it has been discussed how the performance of the strategy has been tested through tournaments against other strategies. A question remains: is the overall success of a strategy based only on it's performance in a round robin tournament or should it be checked through other ways as well?

Following his initial tournaments Axelrod performed an 'ecological' tournament in 1981 [18]. Axelrod argued that some strategies are so unsuccessful that there are very likely to be dropped in the future, while other more successful strategies would continue in later interactions. Influenced by evolutionary biology Axelrod introduced a way of capturing this behaviour, which included running a series of tournaments where more successful strategies would occupy a larger part of the environment and the less successful strategies would become less often. This is known as an ecological tournament.

The simulation of the process, as described in [18], is straightforward. Consider matrix 2 which provides the expected payoff when two individuals of different type interact. Starting with proportions of each type in a given generation the proportions of these strategies in the next generation is the only measure needed to be calculated. This is achieved by calculating the weighted average of the scores of a given strategy with all other players, where the weights are the numbers of the other strategies which exist in the current generation. The numbers of a given strategy in the next generation is then taken to be proportional to the product of its numbers in the current generation and its score in the current generation.

$$\begin{pmatrix} (R=3, R=3) & (S=0, T=5) \\ (T=5, S=0) & (P=1, P=1) \end{pmatrix} \tag{2}$$

Note the ecological tournament does not offer any evolutionary perspective. There is no possibility of a new strategy to be introduced, there is no mutation probability to drive the evolution. The ecological is a framework that provides the distributions of given types over time when interacting with the population.

The set of strategies from Axelrod's second tournament was used to perform the ecological tournament. Several interesting insights were reported,

- The lowest ranking 11 strategies had fallen to half their initial size by the 5 generation;
- The middle-ranking entries managed to hold their initial size;
- By the 500^{th} generation the only strategies that were larger than their initial size have been the top 11 ranked strategies;
- These formed 96% of the population at that time;
- The rule which ranked fifth in the tournament, submitted by William Adams, grew to three times its original size in the population and then began to sink after generation 100;
- The rule which ranked eighth, submitted by Paul D. Harrington, and was the only non nice rule in the top 15, grew to four times its original size but began to shrink after generation 150 to reach only a third of its original size by the 1000^{th} generation.

Overall the strategies that did rank at the top of the second tournament have also ranked top in the ecological tournament. On the same note, the strategy that was ranked at the top was again Tit for Tat. By the 1000th generation it was 14.5% of the whole population, followed by the third place rule at 13.9% and then the second place rue at 13.1%, Tit for Tat was growing at .05% per generation which was a faster rate than any other strategy. All these ar captured in Figure 3.

The ability of strategies to be favoured under natural selection and their ability to withstand invasion from other strategies soon became a measure of performance; refereed to as the stability of a strategy.

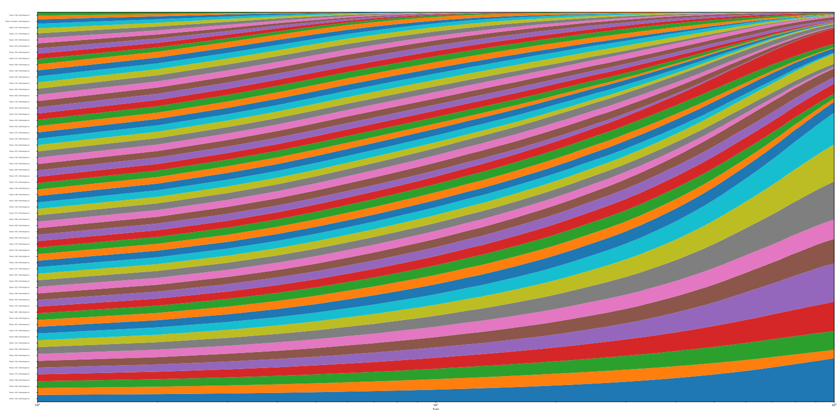

Figure 3: System evolving over time based on natural selection using [5], strategies set from Axelord's second tournament.

A much more general approach was discussed in [14]; the evolutionary approach. Imagine a population made up of individuals where everyone follows the same strategy, B and a single individual adopts a mutant strategy A. Strategy A is said to invade strategy B if,

$$V(A \mid B) > V(B \mid B) \tag{3}$$

where $V(B \mid B)$ is the expected payoff received by B against itself.

Since the strategy B is an population that interacts only with itself, the concept of invasion is equivalent to a single mutant being able to outperform the average population. This leads to the concept of the evolutionary approach. Thus for a strategy to be **evolutionary stable** it must be able to resists any invasion. There are several applications in biology for the interpretation of this approach, for example the survival of the fitness in wildlife.

Due the large number of possible strategies in the prisoner's dilemma identifying all the stable strategies was a difficulty task at the time. Axelrod focused the work of [14] in three questions,

1. Under what conditions was Tit for Tat evolutionary stable?

2. What were the necessary and sufficient conditions for any strategy to be evolutionary stable?
3. Finally, in an environment where all followed a strategy of unconditional defection, can cooperation emerge?

A series of theorems were presented which showed, that Tit for Tat is evolutionary stable y if and only if it is invadable neither by Defector nor Alternator. This is true only if the game is likely to last long enough for the retaliation to counteract the temptation to defect, according to Axelrod. Secondly, Defectors can withstand invasion by any strategy, as long as the players using other strategies come one at a time. But if they come in clusters (even in rather small clusters), the strategy could be invaded. As for the characteristics of stable strategies, Axelrod provided a series of theorems.

2.2.1 Response to the computer tournaments (1984-1993)

The pioneering work of computer tournaments and the results on the reciprocal behaviour of the prisoner's dilemma spread the knowledge of the game not only worldwide but also across different scientific principles. The study of cooperation became of critical interest once again. This section focuses on the immediate research that was carried out after the initial computer tournaments.

Ecological studies that made use of Axelrod's results include the works of [33, 50, 77], more specifically how the successful strategy Tit for Tat can be applied in nature and wildlife.

In [50] the behaviour of fish when confronting a potential predator was studied. Conflicts can arise within pairs of fish in these circumstances. Two experiments were held using a system of mirrors where sticklebacks would be accompanied by a cooperating companion or a defecting one. In both cases the hypothesis that the fish would behave according to Tit for Tat and that cooperation would evolve were supported. The works of [33, 77] looked at food sharing between vampire bats and explained behaviour based on famous at that time tournament strategies.

Axelrod's tournaments assumed that each player has perfect information of the opponent's actions. In real life situations this is not always the case. Interactions often suffer from measures of uncertainty. In the original tournaments there was no possibility of misunderstanding.

In 1985, P. Molander tested the robustness of Tit for Tat in an uncertain environment by introducing noise [53]. Noise is a probability that that one's move will be flipped. Molander findings stated that if two strategies playing Tit for Tat meet in a noisy match the average payoff that a strategy will receive will be the same as that of a Random player (with probability 0.5 of cooperating).

Further work on the performance of Tit for Tat in uncertain environments was conducted, described in [38, 33, 54]. These works focused, similar to Molander's, focused on how the strategy suffers against itself the most. In a noise environment, where a random defection can occur, the two strategies would end up in an unwanted circle of defection-cooperation. In a non noisy environment the strategies would have cooperated until the final interaction.

In [38] a similar tournament to that of Axelrod's was performed but this time noise was used. J. Bendor invited academics to submit strategies to participate in the tournament. A total of thirteen strategies were used including already existed strategies such as Tit for Tat and **Tit for Two Tats** [17]. Tit for Two Tats is a variant of the classic strategy that defects only when the opponent has defected twice in a row. The findings of the tournaments suggested that a more forgiving strategy is needed in a noisy environment. The winner of this tournament was a strategy called **Nice and Forgiving**.

The work of [54] aimed to also investigate stochastic effects. Using an evolutionary setting of a heterogeneous population where noise is taken into account, the space of reactive strategies was explored. Though a small fraction of Tit for Tat players have been essential for the emergence of cooperation, more generous strategies took over the population. This reactive strategy was is known as **Generous Tit for Tat** and can be presented as $(0, \frac{2}{3})$.

Reactive strategies are a subset of memory one strategies introduced in 1989 [55]. Reactive strategies are denoted by the probabilities to cooperate after an opponent's **C** or **D** respectively. Thus, a reactive strategy only considers the previous turn of the opponent. Memory one strategies, are a set of strategies that consider the entire last turn of the game to decide on a next move.

Memory one strategies were also introduced by M. Nowak in 1990 [56]. Depending on the simultaneous moves of two players the states of the game, when only the previous round is considered, a state where both cooperated, both defected or either of them defected. These states are represented as CC, CD, DC, DD. A memory one strategy can be written as the probability of cooperating after each of these states. Thus as a vector of four probabilities p where $p = (p_1, p_2, p_3, p_4) \in \mathbb{R}^4_{[0,1]}$. Reactive strategies are just a constrained version of memory one strategies where $p_1 = p_3$ and $p_2 = p_1$.

The above formulation offered a new framework of studying strategies. Consider that two memory one strategies are in a game of the prisoner's dilemma. Their interaction can be written as the following markov chain,

$$M = \begin{bmatrix} p_1q_1 & p_1(-q_1+1) & q_1(-p_1+1) & (-p_1+1)(-q_1+1) \\ p_2q_3 & p_2(-q_3+1) & q_3(-p_2+1) & (-p_2+1)(-q_3+1) \\ p_3q_2 & p_3(-q_2+1) & q_2(-p_3+1) & (-p_3+1)(-q_2+1) \\ p_4q_4 & p_4(-q_4+1) & q_4(-p_4+1) & (-p_4+1)(-q_4+1) \end{bmatrix} \tag{4}$$

where the opponent is denoted as $q = (q_1, q_2, q_3, q_4) \in \mathbb{R}^4_{[0,1]}$. The expected state that two opponents will end up can be estimated by calculating the steady states of the markov chain.

Nowak, as described, studied the reactive but also the memory one strategies space and introduced several other strategies, among them the most popular was **Pavlov**. Pavlov is a strategy with the tolerance of Generous Tit for Tat but also the capability of resisting and invading an all-out cooperators population. The strategy is based on the fundamental behavioural mechanism win-stay, lose-shift. It starts off with a cooperation and then repeats it's previous move only if it was awarder with a payoff of R or T. Otherwise it shifts it's last move.

A number of researchers searched for new strategies. Such strategies have been, **Handshake** [65] and **Gradual** [21]. Presented in 1989 and 1997 respectively.Handshake was developed using an evolutionary tournament, where Gradual performance was tested in both a round-robin tournament and ecological simulation.

Handshake is a strategy that starts with cooperation, defection. If the opponent plays in a similar way then it will cooperate forever, otherwise it will defect forever. Gradual starts off by cooperating, then after the first defection of the other player, it defects one time and cooperates twice. After the second defection of the opponent, it defects two times and cooperates twice. After the n^{th} defection it reacts with n consecutive defections and then two cooperations.

Another measure of uncertainty is that of mis perception. Though noise will flip a player's action it will be recorded correctly in the history. Mis perception is the probability that the opponent's current move is flipped before being recorded [37].

2.3 Evolutionary Dynamics (1987-1999)

Determining the evolutionary stability of strategies for the iterated prisoner's dilemma as we discussed is not an easy task. Methods can be use to deal with the difficulty. In [23] the author restricted the possible strategies that could be adopted to a relatively narrow set and resulted that no pure strategy is evolutionary stable, including Tit for Tat. Arguing with the results presented in [14]. The list of strategies used included strategies such as Defector and **Suspicious Tit for Tat**, a strategy that plays Tit for Tat but starts by defecting.

The results were questioned by [48], stating that much was still no fully explored and more research had to be put into the results. Farrel and Ware in 1989 [26] extended the result to include finite mixture of pure and mixtures of Tit For n Tats as well. On the same year the work of [22] looking again at a narrow set of strategies extended their results to noisy environments.

Evolutionary dynamics have been highly useful in the research of the prisoner's dilemma. In [15], an evolutionary process, called the genetic algorithm, was used to discover effective strategies. The author introduced lookup tables as a mean of representing a strategy in a gene format. A lookup table is a set of deterministic responses based on the opponents m last moves; [15] considered $m = 3$.

An extension to the natural selection was introduced in the 1992 [58], recommending a different type of topology. A population of two deterministic strategies, Defector and Cooperator, were placed on a a two dimensional square array where the individuals could interact only with the immediate neighbours. The number of immediate neighbours could be either, fourth, six or eight. As shown in Figure 4. The authors claimed that the essential results remain true of all topologies; the results also hold whether self interactions are taken into account.

Thus each cell of the lattice is occupied by a Cooperator or a Defector. At each generation step each cell owner interacts with its immediate neighbours. The score of each player is calculated as the sum of all the scores the player achieved at each generation. At the start of the next generation, each lattice cell is occupied by the player with the highest score among the previous owner and the immediate neighbours. This topology is refereed to as spatial topology.

Nowak studied the population dynamics as a function of the temptation payoff. It was shown that for different values of the temptation payoff, cooperators and defectors could persist together.

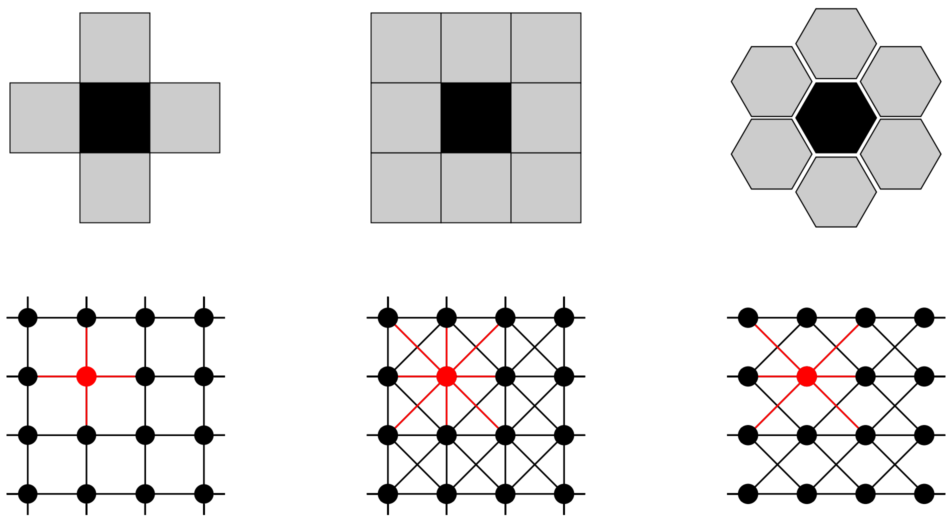

Figure 4: Spatial neighbourhoods

This work dealt with dealt with symmetric spatial lattices in two dimensions, deterministic winning and discrete time. The authors in later work [57], that the results remain valid in more realistic situations. Such as situations where the spatial distributions of cells are random in two or three dimensions, and where winning is partly probabilistic.

2.4 Modern approaches (1995-2015)

A number of aspects discussed in the previous sections such as round robin tournaments, evolutionary tournaments, training of strategies and noise environments soon became standard means of studying the iterated prisoner's dilemma. In this section we review a number of computer tournament that used these methods and introduced a number of findings that made an impact in the literature.

Initially in 1995 a combination of tournament studies, ecological simulations and theoretical analysis was used in [78] to demonstrate approaches to copy with noise. These included generosity, contrition and win-stay, lose-shift by respectively using the strategies Generous Tit for Tat, **Contrite Tit for Tat** and Pavlov. A fourth strategy was also analysed **Generous Pavlov**. A strategy that acts like Palvov but cooperates 10% of the time when it would defect otherwise.

- Adaptive Tit for Tat [73].

Less generous variants also made an appearance [36]. **Anti Tit for Tat**, is a strategy that plays the opposite of the opponents previous move. Another limitation of the strategy was discussed in [68]. Tit for Tat was proven to hit a loop between cooperation and defection. **Omega Tit For Tat** was introduced and was a strategy capable of avoiding such problem [68].

In 2011 the authors of [43] performed their own tournament where several interesting strategies made an appearance.

- **Periodic player CCD**, plays **C**, **C**, **D** periodically. Note that variations of a period player also make appearance in the article but will not be listed here.
- **Prober**, starts with the pattern **D**, **C**, **C** and then defects if the opponent has cooperated in the second and third move; otherwise, it play as Tit for Tat.
- **Reverse Pavlov**, a strategy that does the reverse of Pavlov.

In earlier work the same author introduced a strategy called **APavlov**, which stands for adaptive Pavlov [42]. The strategy attempts to classify the opponent as one of the following strategies, All Cooperator, All Defector, Pavlov, Random or **PavlovD**. PavlovD, is just Pavlov but it starts the game with a **D**. Once Adaptive Pavlov has classified the opponent plays to maximize it's payoff.

Evolutionary dynamics and optimization methods are used with different representation methods in order to discover new optimized strategies. Include lookup tables [15, 45], artificial neural networks [35, 41] and finite state machines [52, 66].

Strategies based on finite state machines are described by the number of states. The strategy selects the next action in each round based on the current state and the opponent's last move, transitioning to a new state each time. Figure 5, illustrates the finite state representation of Tit For Tat.

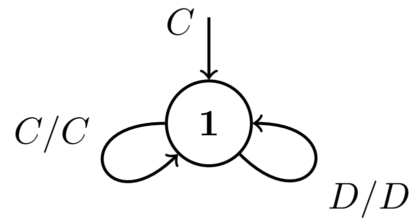

Figure 5: Finite state machine representation of Tit for Tat.

In [11] the author presented two new strategies that have been trained using a finite state machine representation. They are called, **Fortress3** and **Fortress4**. Figure 6 illustrates their diagrammatic representation where the transition arrows are labelled O/P where O is the opponent's last action and P is the player's response.

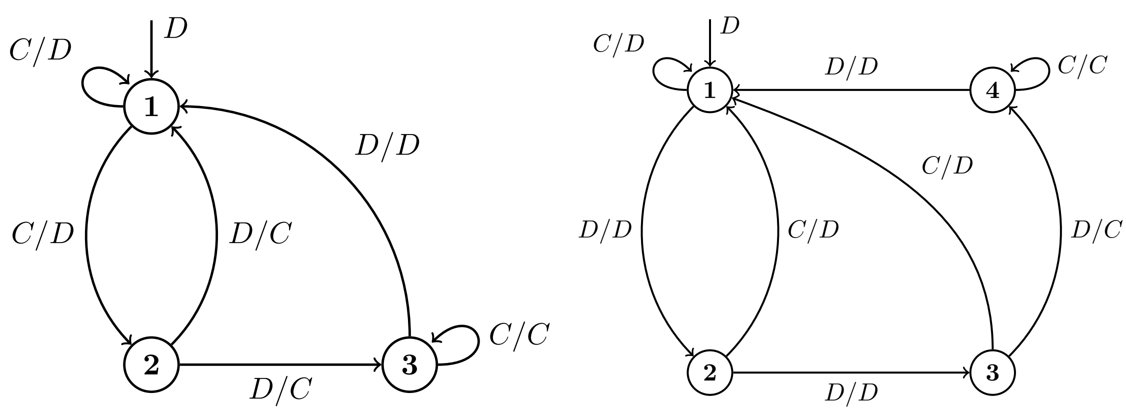

Figure 6: Representations of Fortress 3 and Fortress 4. Note that the strategy's first move, enters state 1, is defection for both strategies.

Optimisation methods will return a spectrum of strategies. In order to distinguish the strategies and assuring that they are indeed different [6] introduced a method called fingerprinting.

The method of fingerprinting is a technique for generating a functional signature for a strategy [7]. This is achieved by computing the score of a strategy against a spectrum of opponents. The basic method is to play the strategy against a probe strategy with varying noise parameters. In [6] Tit for Tat is used as the probe strategy. Fingerprint functions can then be compared to allow for easier identification of similar strategies. In Figure 7 an example of Pavlov's fingerprint is given. Fingerprinting has been studied in depth in [7, 8, 9, 10].

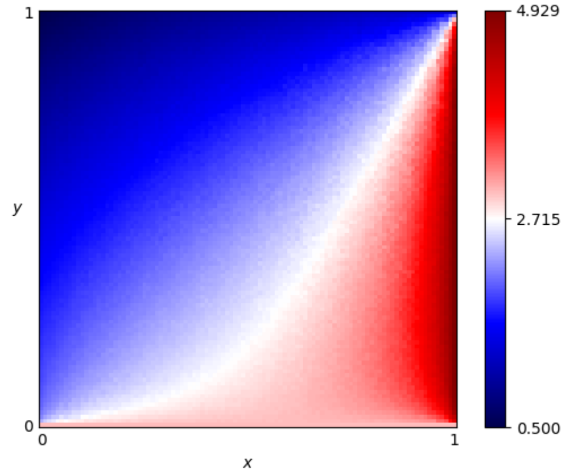

Figure 7: Pavlov fingerprinting with Tit for Tat used as the probe strategy. Figure was generated using [5].

Due the nature of the research several pieces of software are starting to appear, this includes a library called PRISON [4]. PRISON is written in the programming language Java and it has been used by it's authors in several publications. The project includes a good number of strategies from the literature but unfortunately the last update of the project dates back in 2004.

2.5 Zero determinant (2012 - 2015)

Following Section 2.4, this section is a review of an important set of strategies, the zero determinant.

In [62], a new set of memory one strategies were introduced, called **zero determinant (ZD)** strategies. The ZD strategies, manage to force a linear relationship between the score of the strategy and the opponent. Press and Dyson, prove their concept of the ZD strategies and claim that a ZD strategy can outperform any given opponent.

The ZD strategies have attracted a lot of attention. It was stated that "Press and Dyson have fundamentally changed the viewpoint on the Prisoner's Dilemma" [69]. In [69], a new tournament was performed including ZD strategies and a new set of ZD strategies the **Generous ZD**. Even so, ZD and memory one strategies have also received criticism. In [41], the 'memory of a strategy does not matter' statement was questioned. A set of more complex strategies, strategies that take in account the entire history set of the game, were trained and proven to be more stable than ZD strategies.

2.6 Contemporary period (2015 - 2017)

Following a discussion on research of short memory strategies this section reviews recent work done in complex strategies. As well as a discussion of new software and how modern approaches allows us to now revisit several pieces of work produced in the past.

Modern approaches of artificial neural networks and machine learning are now used in the field. A number of strategies based on artificial neural networks are introduced by [34]. Artificial neural networks provide a mapping function to an action based on a selection of features computed from the history of play.

These strategies are refereed to as **EvovlvedANN** strategies and are based on a pre-trained neural network with the following features,

- Opponent's first move is C
- Opponent's first move is D
- Opponent's second move is C
- Opponent's second move is D
- Player's previous move is C
- Player's previous move is D
- Player's second previous move is C
- Player's second previous move is D
- Opponent's previous move is C
- Opponent's previous move is D
- Opponent's second previous move is C
- Opponent's second previous move is D
- Total opponent cooperations
- Total opponent defections
- Total player cooperations
- Total player defections
- Round number

A representation of **EvovlvedANN 5** is given in Figure 8. The inputs of the neural network are the 17 features as listed above. Number 5 reefers to the size of the hidden layer.

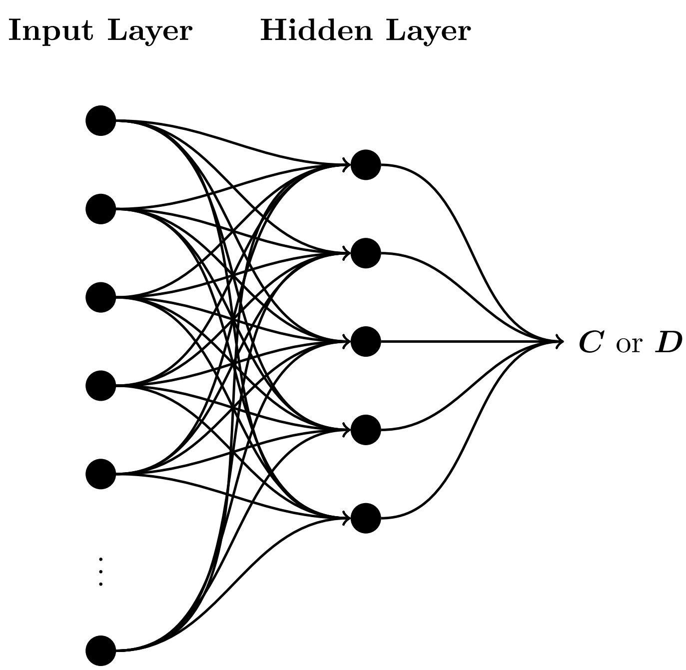

Figure 8: Neural network representation of EvovlvedANN 5.

In [34], these representing methods are refereed to as archetypes. Finite state machines and artificial neural networks are included in the work but also new archetypes are introduced, such as hidden Markov models. A variant of a finite state machine that use probabilistic transitions based on the prior round of play to other states and cooperate or defect with various probabilities at each state. Finite state machines and hidden Markov models based strategies are characterized by the number of states. Similarly, artificial neural networks based players are characterized by the size of the hidden layer and number of input features.

Additionally a variant of a look up table is also presented called the lookerup archetype. The lookerup archetype responses based on the opponent's first n_1 moves, the opponent's last m_1 moves, and the players last m_2 moves. Taking into account the initial move of the opponent can give many insights. For it is the only move a strategy is truly itself without being affected by the other player. As a reminder, Axelrod in his work highlighted the importance of the initial move and believed that it was one of the secrets of success of the strategy Tit for Tat.

Finally, a new archetype called the Gambler is also introduced, which is a stochastic variant of the lookerup archetype.

Archetypes are used with evolutionary algorithms to train set of new strategies. The evolutionary algorithm used in both [15, 30] is called genetic algorithm. Other algorithms including particle swarm optimization have been used in research of the most dominant strategy [28].

In [34] the approach in used to introduce as stated by the authors the best performing strategies for the iterated prisoner's dilemma. These strategies will be refereed as **Evolved** strategies. Several successful new strategies are,

- **EvolvedLookerUp2_2_2** a looker up strategy trained with a genetic algorithm; EvolvedLookerUp2_2_2 responses based on the opponent's 2 first and last moves and the player's 2 last moves. Thus $n_1 = 2, m_1 = 2$ and $m_2 = 2$.
- **Evolved HMM 5** a 5 states hidden markov model trained with a genetic algorithm;
- **Evolved FSM 16** a 16 state machine trained with a genetic algorithm;
- Finally **PSO Gambler 2 2 2** a looker up strategy trained with a particle swarm algorithm, where $n_1 = 2, m_1 = 2$ and $m_2 = 2$.

Though several papers have claimed before to have discovered the dominant strategies for the game the work of [34] is promising. This is due the fact that the introduced strategies have been trained using different types of evolutionary algorithms in a pool of 176 well known strategies for the literature. Including all the strategies that have been discussed in this section.

This was made possible due an open source library, called the Axelrod project [5]. The project is written in the programming language Python, it is accessible and open source. To date the list of strategies implemented within the library exceed the 200. The project has been used in several publications including [34] and a paper describing it and it's capabilities was published in 2016 [39]. The source code for Tit for Tat as implement within the library is shown in Figure 9. Furthermore, performing a tournament with a selection of strategies is possible in five lines of code, shown in Figure 10.

```python
def strategy(self, opponent: Player) -> Action:
    """This is the actual strategy"""
    # First move
    if not self.history:
        return C
    # React to the opponent's last move
    if opponent.history[-1] == D:
        return D
    return C
```

Figure 9: Source code for Tit for Tat in Python as implemented in Axelrod Python library [5]

.

```python
>>> import axelrod as axl
>>> players = (axl.Cooperator(), axl.Defector(), axl.TitForTat(), axl.Grudger())
>>> tournament = axl.Tournament(players)
>>> results = tournament.play()
>>> results.ranked_names
['Defector', 'Tit For Tat', 'Grudger', 'Cooperator']
```

Figure 10: Performing a computer tournament using [5].

Software has a crucial role in research. Well written and maintained software allows the reproducibility of prior work and can accelerate findings within the field. The field of the iterated prisoner's dilemma has suffered the consequences of poor research software. As stated above the source code of the initial computer tournament is not retrievable. Several of the strategies that competed in the tournament are not given a full explanation of how the decided on their next move. In terms of best practice and reproducibility the Axelrod library is the lead software in the field.

Other recent projects include [2, 3], both are education platforms and not research tools. In [2], several concepts such as the iterated game, computer tournaments and evolutionary dynamics are introduced through a user interface game. Project [3] offers a big collection of strategies and allows the user to try several match and tournaments configurations. Such as noise.

In [64], the authors claim that they have managed to re-run the first tournament that Axelrod performed. They tried to push his work further by altering aspects such as, the format of the tournament, the objective and the population. One of the authors claimed to have been a contributor to the first tournaments, which would explain how it was managed to reproduce the tournament.

2.6.1 Biological Applications

- [72] uses evolutionary game theory to study the spread of virus.
- [32] a shout for his work, using tit for tat to study cells.

3 Analysis

This section follows a circumstantial review of the prisoner's dilemma timeline conducted by the authors. It focuses on the analysis of the prisoner's dilemma field using a large dataset of prisoner's dilemma articles' metadata. The number of articles and connections of authors that work on the game are explored. Moreover, the collaborations within the field are examined and compared to other fields of game theory.

3.1 Data Collection

Academic articles are accessible through scholarly databases and collections of academic journals. Several databases and collections today offer access through an open Api. An Api is an application protocol interface that allows users to talk directly to the database and skipping the user interface side of a journal. Interacting with an Api has two phases:

- requesting;
- receiving;

The requesting phase includes composing a url with the requesting message. The head of the url includes the address of the Api and the tail the search argument, such as the word 'prisoner' to exists within the title. The address of the Api and the syntax of the search arguments differ from journal to journal. Thus different journals can generate completely different requesting urls.

The second phase is that of the receiving. This includes receiving a number of raw metadata of articles that satisfied the requesting message. The raw metadata are commonly received in an xml format. Similarly the number of features and the structure of the xml file differs from journal to journal.

Data collection is a crucial proceeder. We wanted to include a large number of articles from various journals for the analysis to be objective. Moreover, we wanted the collection to be fast. For these reasons an open source library was developed for the purpose of this work. The library is called Arcas and though the package itself will not be analysed here the source code is available online(`https://github.com/Nikoleta-v3/Arcas`).

Arcas serves as a translator between us and various Apis. More specifically it works in coordinate with five different journals. Arcas collects data based on a series of arguments. In the work described here, a series of keywords were given. Each keyword individually was checked weather it existed within the title or the abstract of an article. Only if this check was satisfied an article would be collected. A list of the keywords that were used in this are shown in Table 4.

1. arXiv;
2. PLOS;
3. IEEE;
4. Nature;
5. Springer.

	Keywords
1	prisoner's dilemma
2	prisoners dilemma
3	prisoners evolution
4	prisoner game theory
5	R Axelrod
6	memory one strategy
7	tit-for-tat
8	tit for tata
9	zero determinant strategies

Table 4: Keywords used in searching for articles.

Each entry retrieved by Arcas is consisted of various features which are listed on Table 5. In the following sections only a number of these features are considered. These are listed on Table 6. The search has managed to retrieve a number of different articles. The data set that was created will be analysed in the following sections.

	Result name	Explanation
1	Abstract	The abstract of the article.
2	Author	A single entity of an author from the list of authors of the respective article.
3	Date	Year of publication.
4	Journal	Journal of publication.
5	Key	A generated key containing an authors name and publication year (ex. Glynatsi2017).
6	Keyword	A single entity of a keyword assigned to the article by the given journal.
7	Labels	A single entity of labels assigned to the article manual by us.
8	Pages	Pages of publication.
9	Provenance	Scholarly database for where the article was collected.
10	Score	Score given to article by the given journal.
11	Title	Title of article.
12	Unique key	A unique key.

Table 5: Metadata for each entry/article.

	Result name	Explanation
1	Abstract	The abstract of the article.
2	Author	A single entity of an author from the list of authors of the respective article.
3	Date	Year of publication.
4	Journal	Journal of publication.
5	Provenance	Scholarly database for where the article was collected.
6	Title	Title of article.

Table 6: Structure of data set. Contained results.

3.2 Preliminary Analysis

The data set explored in this work is consisted of 1150 articles. Although only 1145 are unique titles. This is because a total of 5 articles have been collected from more than just one Api. All 5 duplicates are from the pre print server arXiv and will be dropped for the analysis, thus hereupon we consider 1145 unique article entries. The full data set has been archived and is available online.

Though Arcas was used to automatically collect data, in progress of writing the literature review of Section 2 the authors have discussed a number of articles that were not retrieved by Arcas. This is only because these articles were not published by the 5 journals accessible by the software. Even so, the meta data of those article have been manually included, more specifically 41 entries have been manually added to the data set. The overall provenance of the data is given by Table 7.

More specifically, a total number of 470 articles have been collected from arXiv, 312 from Springer and 241 from IEEE. A smaller number of entries were contributed by Nature and PLOS.

Provenance	Number of articles
IEEE	241
Manual	41
Nature	23
PLOS	63
Springer	312
arXiv	470

Table 7: Keywords used in searching for articles.

The eldest article was published in 1944 and the most recent one in 2017. Note that the last time data were collected was on December 2017. The provenance of articles can also be viewed over the years. This is illustrated in Figure 11. This allow us to view the significance of each journal's contribution to the field over the years as well as when the prisoner's dilemma fitted the scope of each journal.

Springer and IEEE are the two journals that have been publishing papers on the topic for the longest time. Even so, it can been seen that IEEE contributed more than 10 articles per year after 2002. Similarly, arXiv contributes with a large number of papers with an increasing trend after 2000. Both arXiv and IEEE are associated with computer engineering and allied disciplines. As a reminder from Section 2, the 'Modern Area' started near the 2000s where now the study of the prisoner's dilemma is associated with computer science. PLOS journal was launched on 2000 and it can been seen that accepted it's first publication on the topic on 2006. Finally Nature has a few publications over the years. Nature began it's publications after the work of Axelord in 1980s and some of it's first publications include the work of Nowak done in the 1990's.

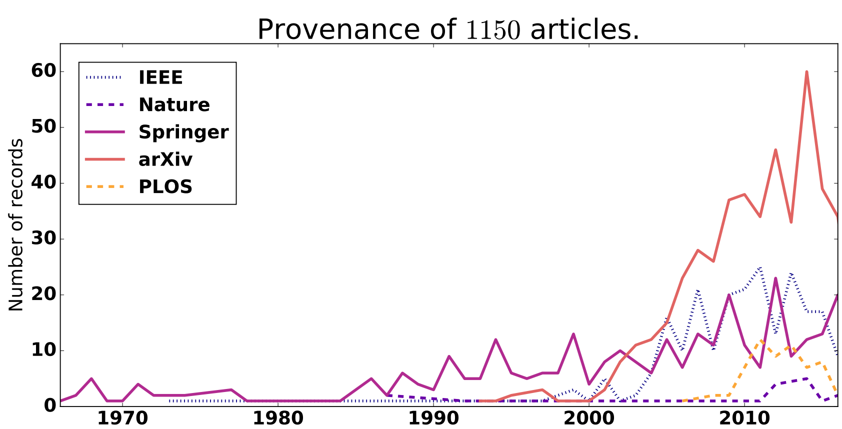

Figure 11: Provenance of articles.

The number and years of publications that have been made on the prisoner's dilemma have been discussed in this section. In the following section we explore the people that have written about the game and their connections to each other.

3.3 Authors Analysis

In this section we present and analyse the authorship in the research field of the prisoner's dilemma through a network analysis. A network is presented where each unique author is represented as a node of the network and a co-authorship is represented as an edge. Over the 1145 articles of the data set the total number of unique authors is 2101 .

Several different methods are being used from different journals for writing an author's name. Consequently, several different entries of the same author existed within the data set. In order to address the problem the Levenshtein Distance [51] was used. The Levenshtein Distance highlighted the entries with a small difference between them. A manual check was performed to assure that both entries were in did the same author and one of them was replaced by the other.

Once the network was constructed several network methods were used to perform an analysis. The connectivity of the network and the trend of co-authorship within the research field were explored. The number of authors per papers and fields can vary a lot. Some fields tend to be more collaborative than others. Thus, we use two other research topics and compare the results with those of the prisoner's dilemma authorship network. Finally, a number of sub graphs are examined more closely.

3.3.1 Co-authorship network for the prisoner's dilemma

The authors are represented using an undirected network, shown in Figure 12. The network has sets of vertices V and edges E. The 2101 vertices represent each of the unique authors from the data. The vertices are connected with an edge if and only if two authors have written together. Weights have been applied to both the vertices and the edges.

Vertices' weights corresponds to the number of publications an author has. The ten authors with the highest number of publications are given by Table 8. Authors such as M. Nowak and D. Ashlock, that have previously been discussed in Section 2, appear on the list. Though other authors such as R. Axelrod do not. On the other hand, several people whose work was not discussed in the previous section appear here, for example Matjaz Perc with a total of 34 publications. The network once the vertices' weight has been applied is shown in Figure 13.

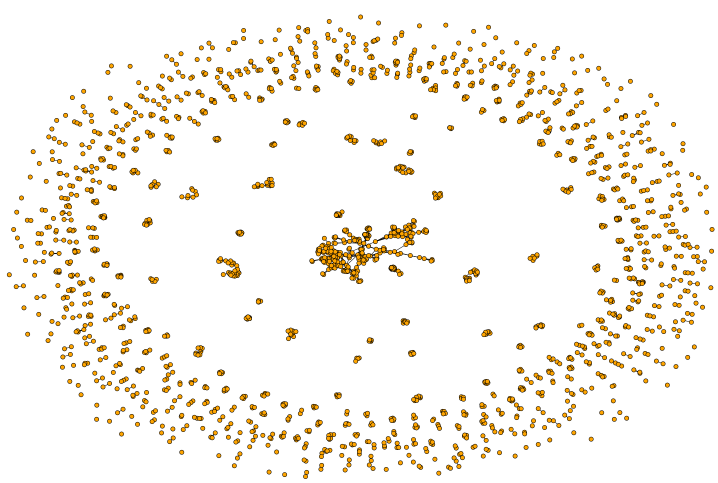

Figure 12: Co-authorship network.

Furthermore, the edge weight corresponds to the number of times the authors wrote together. From Figures 12, 13 it can bee seen that overall the co-authors network is disjoint. Several researchers on the outer circle seem to have written alone or have a single connection to another researcher. On the other hand, in the inner circle of the network some connectivity of the vertices does seem to exist, with a single large cluster located in the middle of the network.

The authors that cover the outer space, that seem to be less collaborative, are not authors that have single contributions

	Number of publications
Matjaz Perc	34
Attila Szolnoki	29
Daniel Ashlock	21
Gyorgy Szabo	19
Long Wang	16
Angel Snchez	16
Zhen Wang	13
Hisao Ishibuchi	13
Yamir Moreno	13
Zhi-Xi Wu	11

Table 8: Top 10 most published authors.

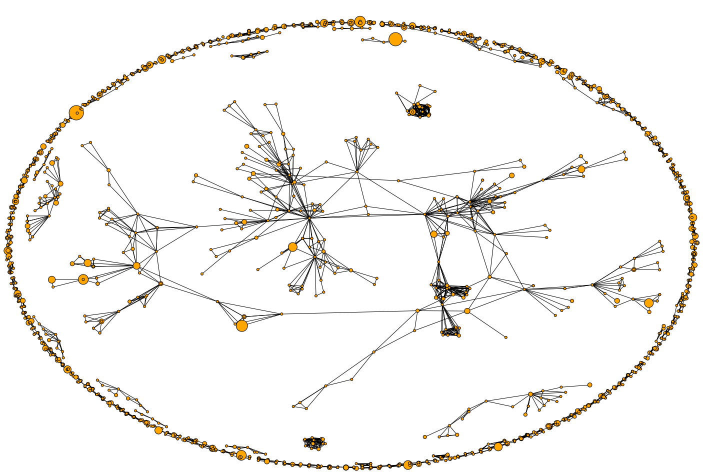

Figure 13: Co-authorship network with vertices' weight.

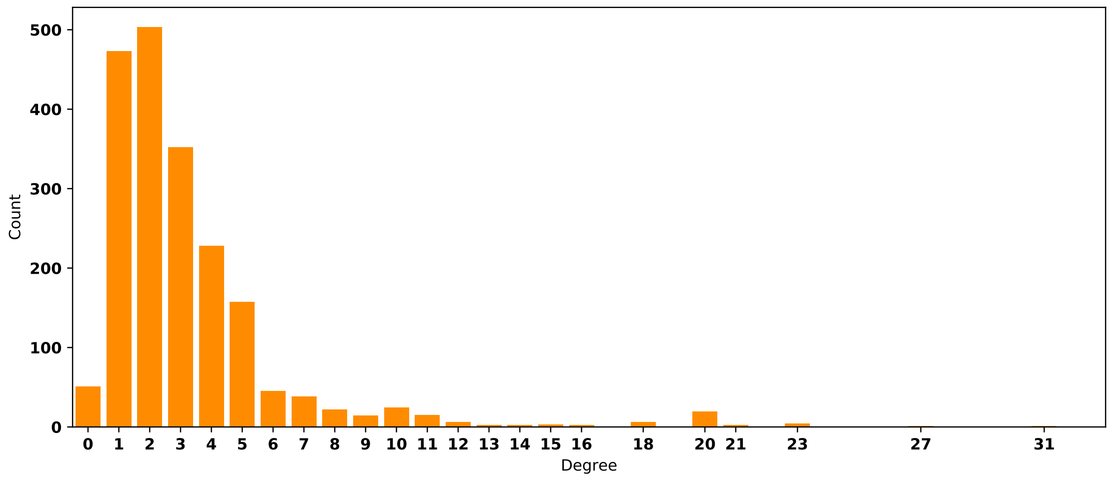

Figure 14: Degree Histogram. Co-authors network of prisoner's dilemma data.

to the field. On the other hand authors that have repeatedly published on the topic are located in both the outer and inner circles of the network.

More insights can be gained by reviewing several network measures such as the density, the average degree and centrality measures [19]. The first measure to be considered is the density of the network. The density of a network is given by, $\frac{2E}{V(V-1)}$ and is calculated to be 0.00163. The density is defined as the ratio of the number of edges and the number of possible edges. Thus a small value as 0.00163 indicates that the network as a low connectivity. Another measure closely related to the density is the average degree. The average degree is given by $\frac{2E}{V}$ and has been calculated to be 3.22. The histogram of the the degree distribution is given by Figure14. The most frequent degree appears to be that of 2 degrees.

Furthermore we compute node centrality measures. Indicators of centrality identify the most important vertices of the network. There are several centrality measures, for the purpose of this work the following are considered:

- weighted betweenness centrality;
- closeness centrality;
- weighted page rank;

The results of the different centrality measures are given by Table 9. Though several differences between the ranks of the three measures are spotted, some names appear to be on the most central vertices for all three measures. These are authors:

- Matjaz Perc;
- Yamir Moreno;
- Arne Traulsen;
- Martin A. Nowak;
- Angel Snchez;
- Krishnendu Chatterjee.

In agreement of all measures the above are the most central authors of our network. More specifically, the most central author is Matjaz Perc.

Though several insights were gained from studying the authorship within the field we do not know how these findings compare to either fields. For example, an average degree of 3.22 means that the field is more collaborative than other or not? These questions are delved into in the following section.

	Author name	Betweeness
0	Matjaz Perc	0.009332
1	Yamir Moreno	0.007424
2	Arne Traulsen	0.004272
3	Luo-Luo Jiang	0.004174
4	Martin A. Nowak	0.003881
5	Cheng-Yi Xia	0.003820
6	V. Latora	0.003455
7	Angel Snchez	0.002996
8	Krishnendu Chatterjee	0.002610
9	Dirk Helbing	0.002385

	Author name	Closeness
0	Matjaz Perc	0.044428
1	Attila Szolnoki	0.043561
2	Angel Snchez	0.038910
3	Long Wang	0.037959
4	Daniel Ashlock	0.037600
5	Yamir Moreno	0.037442
6	Zhen Wang	0.037015
7	Gyorgy Szabo	0.036824
8	Krishnendu Chatterjee	0.036301
9	Martin A. Nowak	0.035935

	Author name	Page rank
0	Matjaz Perc	0.004072
1	Attila Szolnoki	0.003041
2	Angel Snchez	0.002611
3	Long Wang	0.002581
4	Daniel Ashlock	0.002509
5	Yamir Moreno	0.002264
6	Zhen Wang	0.002183
7	Gyorgy Szabo	0.002070
8	Krishnendu Chatterjee	0.001836
9	Martin A. Nowak	0.001824

Table 9: Central authors based on different centrality measures.

3.3.2 Comparison with other topics authorship

In order to achieve objective insights regarding authorship in the field of the prisoner's dilemma we perform a comparison in this section. We compare how well connected the network of authors for the prisoner's dilemma is compared to other topics. The data collected are on two different topics within game theory, these are:

- price of anarchy and
- auction games.

The open source project Arcas was used again, this time using the following keywords respectively for each topic,

- key: price of anarchy;
- key: auction game theory.

A summary of the two data set is given by Table 10. A total of 296 unique articles have been collected on price of anarchy. The earliest entry being in 2003 and a total of 668 unique authors have written about the topic. In comparison, a total of 2103 articles are examined for auction games. Auction games are a well studied topic for several years with the earliest entry going back to 1974. Finally, 3860 different names are examined. The frequency of the prisoner's dilemma, for both articles and authors, lies between the frequencies of these two topics. In Figure 15 a timeplot for each topic is displayed and is exhibited that both topics have an increasing trend over the years. Though price of anarchy is clearly a new topic compared to auction games.

	Price of anarchy	Auction games
Unique articles	296	2103
Unique authors	668	3860
Min publication year	2003	1974
Max publication year	2017	2017

Table 10: Data sets summary.

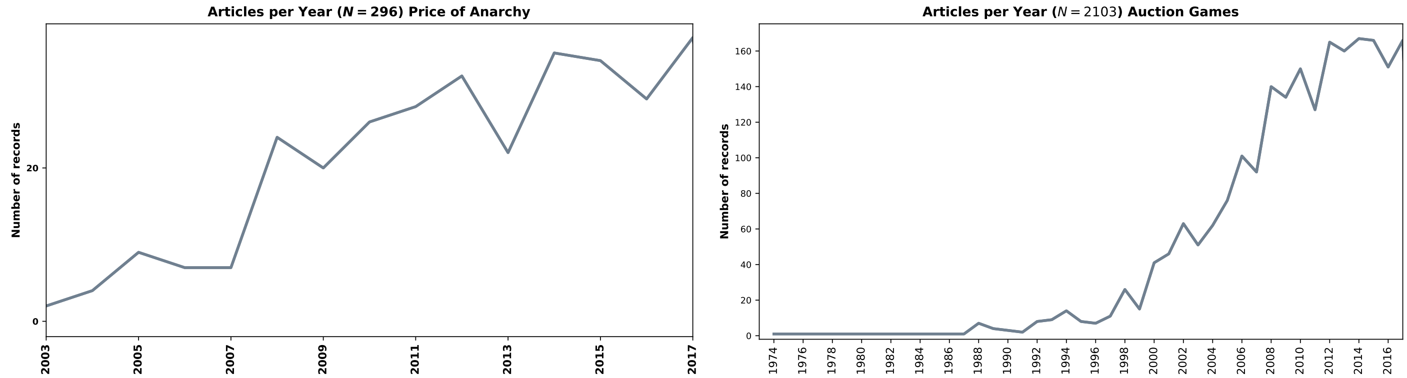

Figure 15: Timeplots.

The immediate step is to compare the networks we have constructed for all three fields. The aim of the comparison is to gain insight as to how collaborative the prisoner's dilemma authors are compared to other fields. Network measures discussed in this comparison are given by Table 11. The smallest network is that of the authorship describing the price of anarchy, followed by the prisoner's dilemma and the largest one is that of the auction games. The average degree (calculated as $\frac{2E}{V}$), is for all three networks is close to 3 and the density of all networks is very small, less than 0.005.

	Prisoner's Dilemma	Price of anarchy	Auction games
$\mid V \mid$	1970	637	3968
$\mid E \mid$	3179	865	5713
Density	0.00163	0.00427	0.00072
Average degree	3.22	2.71	2.87

Table 11: Descriptive measures of networks.

To further examined the differences or the similarities of these networks we consider the degree distributions. The normalised distributions of all three networks are given in Figure 16. They have been normalised such that the frequencies sum to one. The distributions appear to be similar but to validate the hypothesis a statistical test will be used. None of the distributions is normally distributed thus the non parametric test Kruskal-Wallis is used [49]. Kruskal-Wallis allow us to compare the medians of two or more distributions. The test returns a p-value of 0.29. Thus, there is no significant difference in the degree distributions of the three networks.

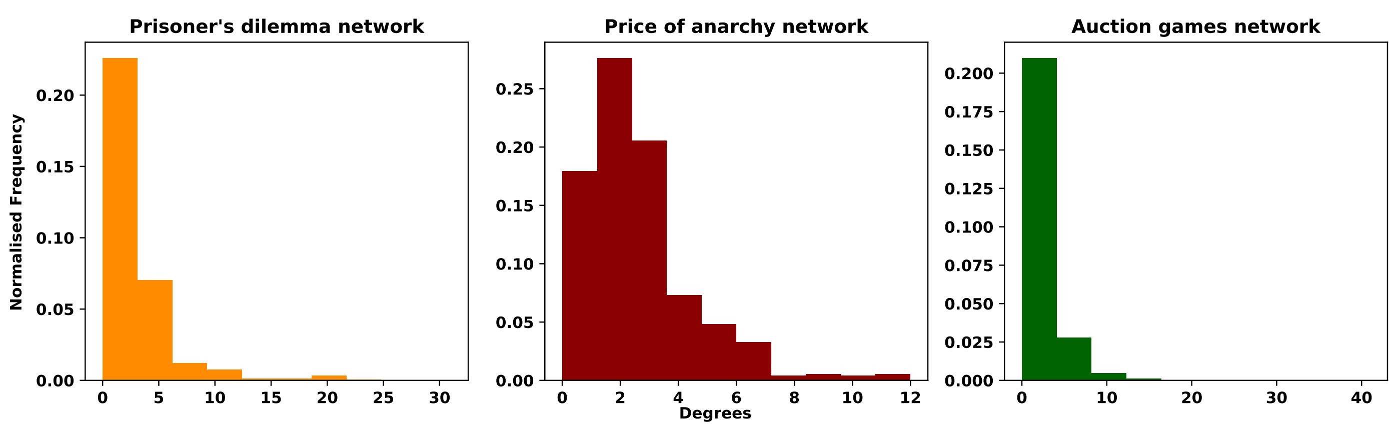

Figure 16: Degree distributions for all three networks.

An interesting insight can be gained by examining the evolution of both the average and the median degree over time. Though when the entire data sets are considered the average degree of all networks is close to 3, when observing the previous year this does not seem to be the case. The evolution of the degrees over the publication years of each topic have been very different. This can be observed by Figures 17 and 18. Auction games have had an increasing trend in co-authorships over time. The prisoner's dilemma study, though it does have an increasing trend has two sudden jumps. A decrease occurred in 1960 and a sudden increase in 1970. Finally, the price of anarchy had a big increase in authorship in 2004.

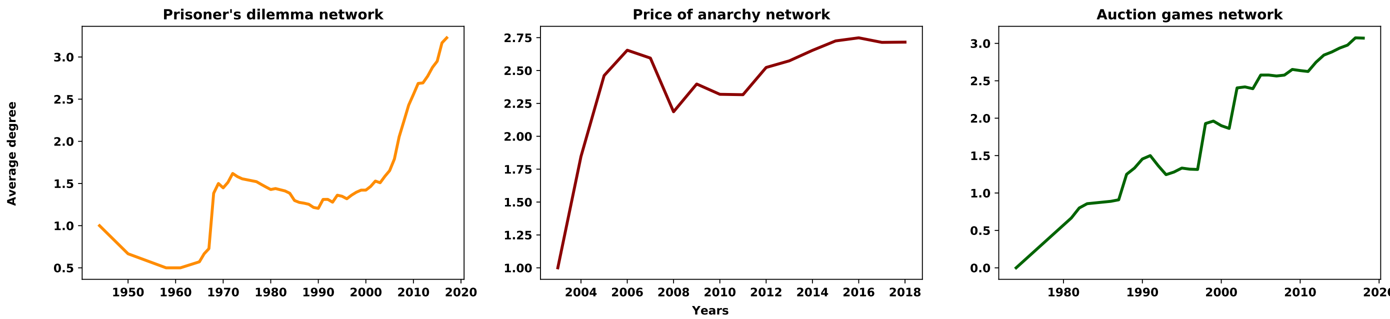

Figure 17: Average degree over time for all three networks.

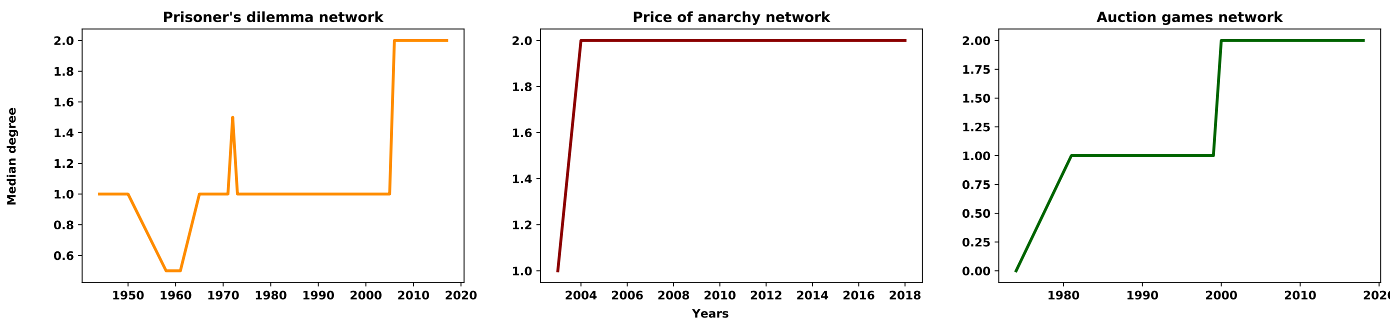

Figure 18: Median degree over time for all three networks.

Other measures that could highlight similarities in the connectivity of the graphs are centrality measures. A summary of the maximum, mean and median centrality value based on three centrality measures for all three networks is given by Table 12. The individual histograms are illustrated in Figures 19, 20 and 21. A normality test has been performed for each of the 9 distributions and with a significance level of 0.05 none of the distributions is normally distributed. Thus, a Krustal-Wallis test will be performed to test the mean difference between the topics for each centrality measure.

	Prisoner's Dilemma	Price of anarchy	Auction games
Max weighted betweenness	0.0093	0.0024	0.0011
Mean weighted betweenness	4.537e-05	0.0	0.0
Median weighted betweenness	0.0	3.4677e-05	4.5096e-06
Max closeness	0.0443	0.0333	0.0209
Mean closeness	0.0051	0.0047	0.0007
Median closeness	0.0015	0.0063	0.0017
Max weighted page rank	0.0040	0.0063	0.0012
Mean weighted page rank	0.0005	0.0015	0.0002
Median weighted page rank	0.0005	0.0015	0.0002

Table 12: Summary of centrality measures.

The results of the Kruskal-Wallis tests are shown in Table 13. It can be seen that for the closeness centrality the null hypothesis holds. Thus there is not a significant difference of the medians. On the other hand, for both the weighted betweenness and weighted page rank the null hypothesis is being rejected. Thus there seems to exist a significant difference

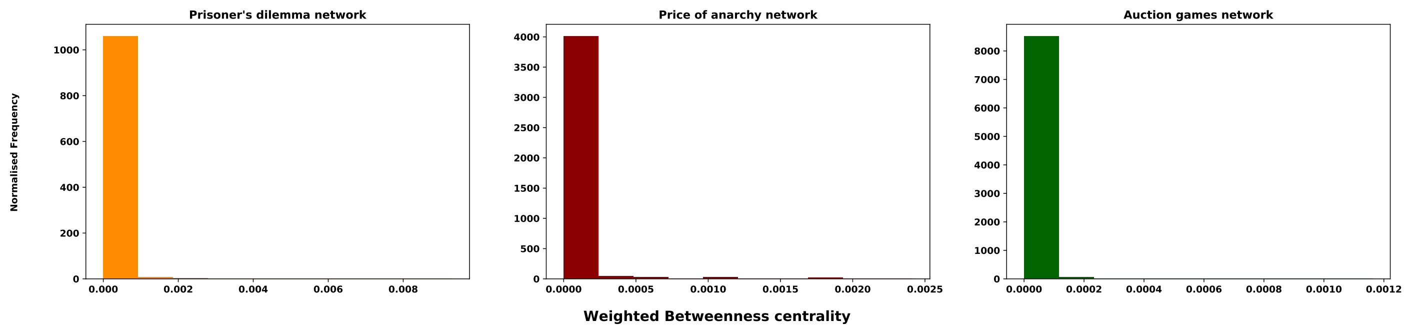

Figure 19: Between centrality histogram for all three networks.

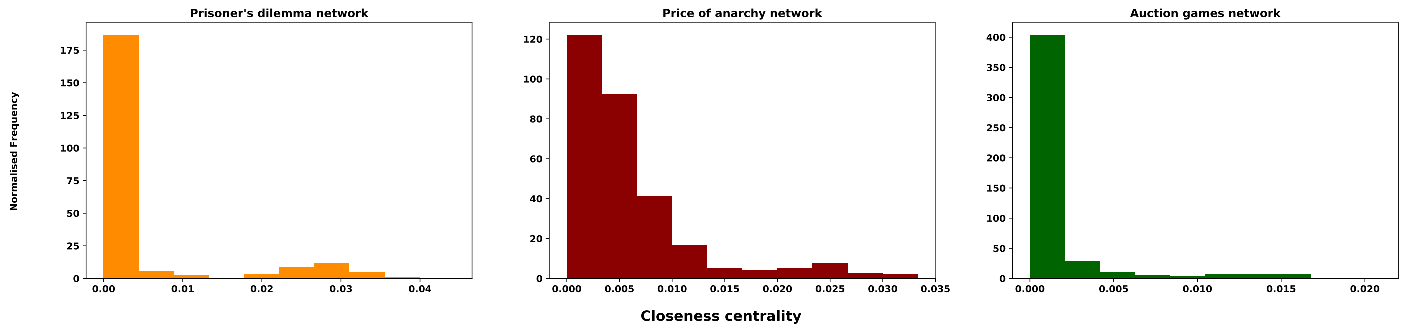

Figure 20: Closeness centrality histogram for all three networks.

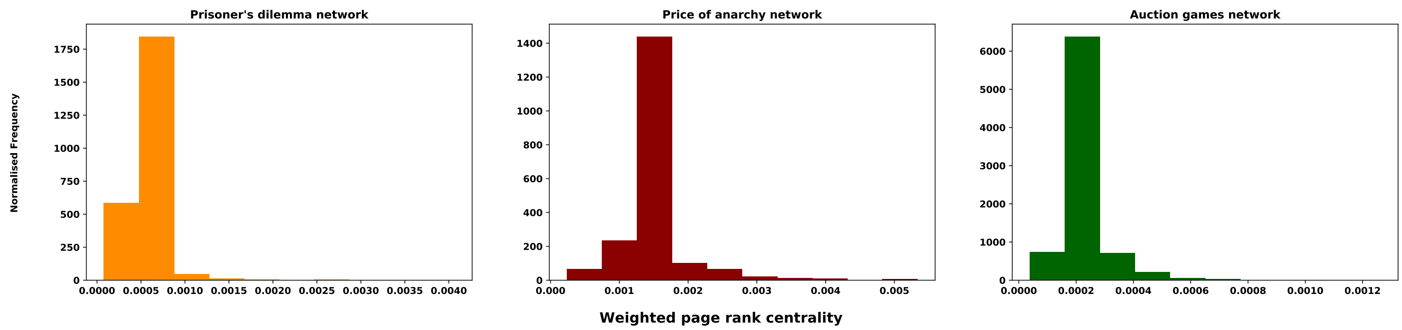

Figure 21: Page rank centrality histogram for all three networks.

of the medians for these two measures. A post-hoc test could be carried to further explore which medians are significantly different.

	$p-$value
Weighted betweenness	0.0
Closeness	0.799
Weighted page rank	0.0

Table 13: Kruskal-Wallis tests results.

In this section the amount of collaboration within the field was examined. Furthermore, to be objective this was compared to other topics within game theory. In the section to follow we focus on the exploration of the co-authorships within the field of the prisoner's dilemma. The collaboration of some well known authors alongside authors that appear to be central in our network are explored.

3.3.3 Sub graphs analysis

In this section we explore and gain insights as to who well known authors within our field tend to collaborate and write with. This is achieved by studying the cliques of the network. Cliques are subsets of vertices, all adjacent to each other. A total of 779 cliques exists within the network and the max node number of a clique is 21. The clique with the 21 authors is from [39]. Though a big number of cliques exists in this section we will only examine those of authors that have been discussed several times within this work.

The first clique is that of R. Axelrod. The subgraph, shown in Figure 22, consists of 4 vertices, 3 edges and the edge weights are all equal to 1. Thus for this subgraph we examined the betweenness, closeness and page rank centrality (all without weights). The central node is Axelrod based on all centrality measures. The results are shown in Table 14.

Furthermore, the density of the subgraph is equal to 0.5 and the average degree in 1.5. All authors are connected to Axelrod with a step of one. That is because all these authors have written only with Axelrod in [17, 18, 78].

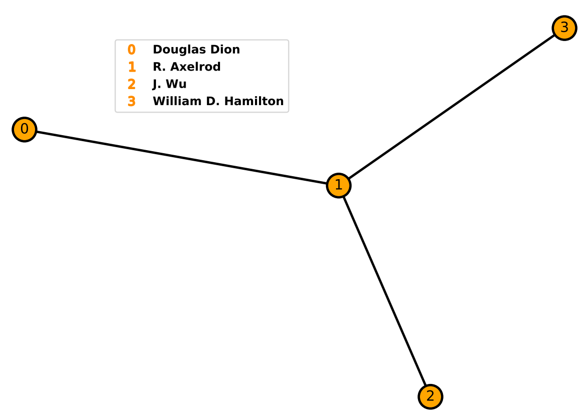

Figure 22: R. Axelrod's clique.

The next clique considered is that of D. Ashlock. The subgraph which contains Ashlock is larger than Axelrod's one. It has a total of 24 vertices and 38 edges. The density is equal to 0.137 and the average degree is 3.16. Moreover, based on

Author name	Betweenness	Closeness	Page rank
R. Axelrod	1.00	0.6	0.1734
J. WU	0.00	1.0	0.4797
William D. Hamilton	0.00	0.6	0.1734
D. Douglas	0.00	0.6	0.1734

Table 14: Centrality measures for Axelrod's clique.

all three centrality measures Ashlock appears to be the most central node of this subgraph. Table 15 present the 10 most central authors.

An interesting insight can be gained by observing the length of the shortest paths for any author to reach Ashlock. The maximum shortest path is 4. Thus, anyone can reach Ashlock at most with 4 steps.

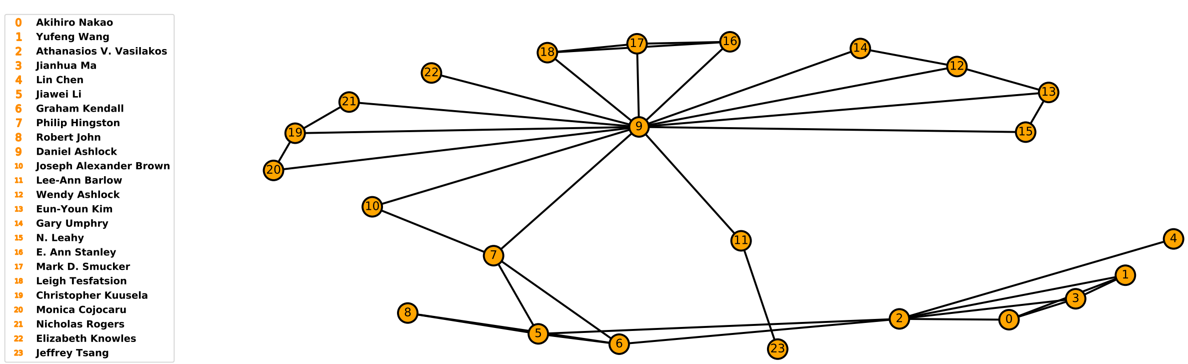

Figure 23: Subgraph that contains D. Ashlock.

R_1	Author name	Betweenness	R_2	Author name	Closeness	R_3	Author name	Page rank
0	Daniel Ashlock	0.7806	0	Daniel Ashlock	0.5476	0	Daniel Ashlock	0.1713
1	Philip Hingston	0.4743	1	Philip Hingston	0.4893	1	Athanasios V. Vasilakos	0.0743
2	Athanasios V. Vasilakos	0.3122	2	Joseph Alexander Brown	0.4181	2	Jiawei Li	0.0492
3	Jiawei Li	0.1996	3	Jiawei Li	0.4107	3	Graham Kendall	0.0492
4	Graham Kendall	0.1996	4	Graham Kendall	0.4107	4	Philip Hingston	0.0490
5	Lee-Ann Barlow	0.0869	5	Lee-Ann Barlow	0.3709	5	Christopher Kuusela	0.0405
6	Wendy Ashlock	0.0019	6	Wendy Ashlock	0.3709	6	Wendy Ashlock	0.0398
7	Eun-Youn Kim	0.0019	7	Eun-Youn Kim	0.3709	7	Eun-Youn Kim	0.0398
8	Christopher Kuusela	0.0019	8	E. Ann Stanley	0.3709	8	Yufeng Wang	0.0387
9	Akihiro Nakao	0.0000	9	Mark D. Smucker	0.3709	9	Jianhua Ma	0.0387

Table 15: Central authors based on different centrality measures of the subgraph which includes author D. Ashlock.

The final two authors considered in this section are M. Nowak and the most central author of the network M. Perc. It appreas that these two authors are in same subgraph, Figure 24. This is the largest subgraph within the network with a total of 266 vertices and 736 edges. Similarly, the centrality of the subgraph was calculated using three different measures and the results are displayed in Table 16. The density is equal to 0.02088 and the average degree is 5.533.

The subgraph does not only include these two authors but also the rest of the most central authors that we indentified in Section 3.3.1. These are,

- Yamir Moreno;

- Arne Traulsen;
- Angel Snchez;
- Krishnendu Chatterjee.

Thus this subgraph is the most connected component of the intrire network. Similarly, the average steps of reaching each author is calculated. M. Perc is, on average, three steps away from each node where M. Nowak is 4.

The short path that connects the two authors is also retrieved and is the following: M. Nowak - A. Traulsen- Y. Moreno - M. Perc. Thus the authors that connect them are two of the most central authors.

The answer as to why these people are connected can be seen if we consider the topic the have focused their research on. The common subject to all these authors works is evolunationary game theory, spatial tournaments and social applications.

M. Perc is a signicant author within the topic with some of his most well received and most citated works being [59, 60, 61, 76]. Perc is well connected author which is proven from the connectivity of his subgraph, the centrality measures and can be verified by his publications.

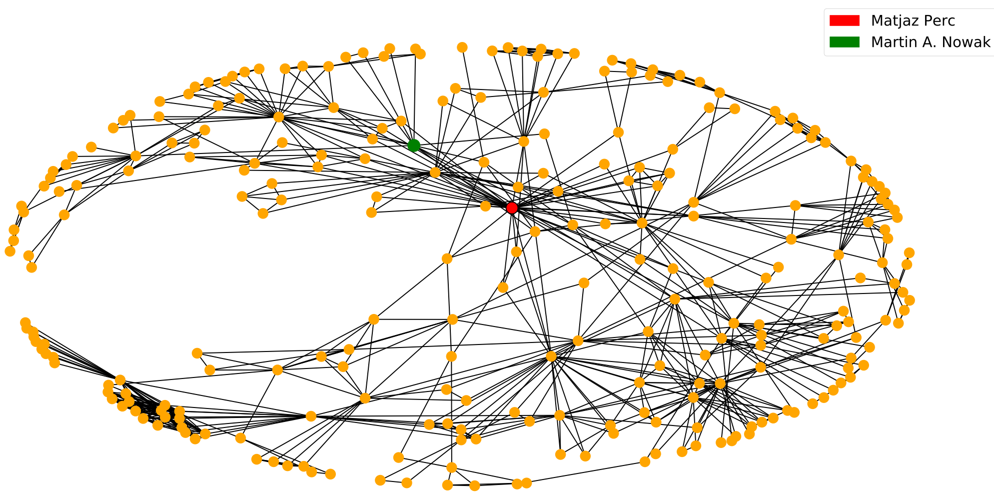

Figure 24: Subgraph that contains M. Nowak and M. Perc.

3.4 Conclusions from the analysis

In this section of the paper it was described how insights on the research of the field can be gained by performing a data analysis. Initially the method that was used to collect various metadata of articles was briefly presented.

Secondly, the number of articles and their provenance over the years was reviewed. The number of articles appears to be gradually increasing after 2000. That could be because most of the journals used in the data collection are either recent or focused on computing/engineering sciences that have been used in the field mainly after 2000.

One of the features retrieved by the collection is the authors name. Exploring the connections of these authors has been the main focus of the section as it provides several interesting findings.The aims of the network analysis have been,

R_1	Author name	Betweenness	R_2	Author name	Closeness	R_3	Author name	Page rank
0	Matjaz Perc	0.5844	0	Matjaz Perc	0.3296	0	Matjaz Perc	0.0205
1	Yamir Moreno	0.4851	1	Yamir Moreno	0.3231	1	Angel Snchez	0.0167
2	Luo-Luo Jiang	0.2385	2	Cheng-Yi Xia	0.2886	2	Long Wang	0.0156
3	Arne Traulsen	0.2164	3	Sandro Meloni	0.2816	3	Zhen Wang	0.0150
4	Martin A. Nowak	0.2116	4	Alberto Aleta	0.2789	4	Attila Szolnoki	0.0134
5	V. Latora	0.1897	5	Arne Traulsen	0.2777	5	Krishnendu Chatterjee	0.0127
6	Krishnendu Chatterjee	0.1441	6	Angel Snchez	0.2746	6	Yamir Moreno	0.0121
7	Angel Snchez	0.1403	7	Luo-Luo Jiang	0.2731	7	Gyorgy Szabo	0.0112
8	Han-Xin Yang	0.1138	8	Manfred Milinski	0.2693	8	Dirk Helbing	0.0109
9	Zhihai Rong	0.1068	9	Jos A. Cuesta	0.2666	9	Martin A. Nowak	0.0108

Table 16: Central authors based on different centrality measures of the subgraph which includes author M. Nowak and M. Perc.

- to research how collaborative the field of the prisoner's dilemma is;
- indentify the most crucial authors of the network;
- explore several specific collaborations.

Researching the collaborative behaviour of the the field is achieved by examining several network measures. The network was proven to be rather disjoint with a very low density adn an average degree of 3. The median degree of authors has been two, thus most people in the area are maninly writing with two co-authors.

In order to have an objective insights as to how collaborative the field is two other topics were considered as separate networks. The topics have been the price of anarchy and auction games. The results have shown that the collaboration behaviour is very similar to all three topics. The average degreres of all networks was close to 3 and all had a very small density. Furthermore, there was not a significant difference between the degree distributions of the topics.

However, the historical progress of the authorship is it significant different between the fields. Auction games have been attracting more authors and collaborations as time progresses. The price of anarchy is a new field that attracted a large number of authors and parternshipd after 2004. The prisoner's dilemma seems to be a more complicated, having a sudden decrease in 1960 and a sudden increase in 1970. Those this could be due the bias that this analysis suffers, which is that only 5 journals are considered. More insights can be brought in the analysis by performing further tests.

A surprise has been that the main author that was highlighted several times within the analysis has been M. Perc. An author whose work was not delved into in the literature section. A number of authors such as Perc, Moreno and Traulsen would have not come to our attention if this analysis was not held. Thought their work seems promising and may be of interest to people that focus on evolutionary game theory, spatial games and social stuff. Thus advantages of such analysis is that authors that have not previously be considered can stand out.

Another powerful results emerges by reviewing several subgraphs of authors. The connections to an author of interest can offer new opportunities and guidelines as to whose work to follow next. Nevertheless, only a small number of the subgraphs were presented here. The reader is encouraged to explore the extra material of this work and investigate the connection of their chosen author.

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