A systematic literature review of the Prisoner's Dilemma; collaboration and influence.

Nikoleta E. Glynatsi, Vincent A. Knight

1 Timeline

2 Analysing a large corpus of articles

The focus of Section 1 has been the academic publications on the topic of the iterated prisoner's dilemma. Whilst in Section 1 several publications of specific interest were covered and the literature was manually partitioned in different sections, in the second part of this paper the publications are analysed using a large dataset of articles. In Section 2.1 some background research on bibliometrics is covered. The data collection process is covered in Section 2.2 and a preliminary analysis of the data is conducted in Section 2.3. In Section 2.4, the methodology which will be used to analyse the author relationships is presented. In summary, graph theoretical methods will be used to ascertain the level of collaborative nature of the field and identify influence. This type of analysis has been carried out in [12]. The novelty here is to consider approaches not considered in [12] and new origins of publications. A further comparison of the results are made, relative to two other sub fields of game theory: auction games [13] and the price of anarchy [19] and a temporal analysis. Finally in Section 2.5, the results of the analysis are presented.

2.1 Background

As discussed in [22], bibliometrics or the statistical analysis of published works (originally described by [17]) have been used to support historical assumptions about the development of fields [18], identify connections between scientific growth and policy changes [5], to develop a quantitative understanding of author order [20], and investigate the collaborative structure of an interdisciplinary field [12]. Most academic research is undertaken in the form of collaborative effort and as [10] points out, it is rationale that two or more people have the potential to do better as a group than individually. Collaboration in groups has a long tradition in experimental sciences and it has be proven to be productive according to [6]. The number of collaborations can be very different between research fields and understanding how collaborative a field is, is not always an easy task. Several studies tend to consider academic citations as a measure for these things. A blog post published in Nature [15] argues that depending on citations can often be misleading because the true number of citations can not be known. Citations can be missed due to data entry errors, academics are influenced by many more papers than they actually cite and several of the citations are superficial.

A more recent approach to measure collaborative behaviour is to use the co authorship network, as described in [12]. Using this approach has many advantages as several graph theoretic measures can be used as proxies to explain authors relationship. In [12], they analyse the development of the field "evolution of cooperation" using this approach. The topic "evolution of cooperation" is a multidisciplinary field which also includes a large number of publications on the prisoner's dilemma. This paper builds upon the work done by [12] and extends their methodology. Though in [12], they considered a data set from a single source, Web of Science, the data set described here has been collected from five different sources. Moreover, the collaborative results of the analysis are compared to those of two different sub fields. Co authorship networks have also been used in [22] for classifying topics of an interdisciplinary field. This was done using centrality measures, which will be covered below, here centrality measures are used in order to understand the influence an author can have and can receive by being part of an academic group. Furthermore, in [1] they study the relationship between research impact and five classes of diversity: ethnicity, discipline, gender, affiliation, and academic age. These characteristics of the authors described here are not being captured. This is considered to be a limitation to this work which will be will explored in future work.

2.2 Data Collection

Academic articles are accessible through scholarly databases. Several databases and collections today offer access through an open application protocol interface (API). An API allows users to query directly a journal's database and bypass the user interface side of the journal. Interacting with an API has two phases: requesting and receiving. The request phase includes composing a url with the details of the request. For example, `http://export.arxiv.org/api/query?search_query=abs:prisoner'sdilemma&max_results=1` represents a request message. The first part of the request is the address of the API we are querying. In this example the address corresponds to the API of arXiv. The second part of the request contains the search arguments. In this example it is requested that the word 'prisoners dilemma' exists within the article's title. The format of the request message is different from API to API. The receive phase includes receiving a number of raw metadata of articles that satisfies the request message. The raw metadata are commonly received in extensive markup language (xml) or Javascript object notation (json) formats [16]. Similarly to the request message, the structure of the received data differs from journal to journal.

The data collection is crucial to this study. To ensure that this study can be reproduced all code used to query the different APIs has been packaged as a Python library and is available online [14]. The software could be used for any type of projects similar to the one described here, documentation for it is available at: `http://arcas.readthedocs.io/en/latest/`. Project [14] allow users to collect articles from a list of APIs by specifying just a single keyword. Four prominent journals in the field and a pre print server were used as sources to collect data for this analysis: PLOS, Nature, IEEE, Springer and arXiv.

A series of search terms were used to identify relevant articles:

- "prisoner's dilemma",
- "prisoners dilemma",
- "prisoner dilemma",
- "prisoners evolution",
- "prisoner game theory"

and articles for which any of these terms existed within the title, the abstract or the text are included in the analysis. More specifically, 23% of article considered here were included because any of the above terms existed within the abstract, 50% within the main text and 27% within the title. As will be described in Section 2.3, two other game theoretic sub fields were also considered in this work, auction games and the price of anarchy. For collecting data on these sub fields the search terms used were "auction game theory" and "price of anarchy". The three data sets are archived and available at. Note that the latest data collection was perform on November 2018.

2.3 Preliminary Analysis

A summary of each of the three data sets used is presented in this section. The three data sets are:

- The main data set which contains articles on the prisoner's dilemma.
- A data set which contains article on auction games.
- A data set which contains articles on the price of anarchy.

The main data set is archived at [ref]. It consists of 3089 articles with unique titles. In case of duplicates the preprint version of an article (collected from arXiv) was dropped. Of these 3089 article, 89 have not been collected from the aforementioned APIs. These articles were of specific interest and manually added to the dataset throughout the writing of Section 1. A similar approach was used in [12] where a number of articles of interest where manually added to the data set. A more detailed summary of the articles' provenance is given by Table 1. Only 3% of the data set consists of articles that were manually added and 33% of the articles were collected from arXiv.

provenance	# of Articles	Percentage
Manual	89	2.88
IEEE	295	9.55
PLOS	482	15.60
Springer	572	18.52
Nature	673	21.79
arXiv	1056	34.19

Table 1: Articles' provenance for the main data set.

The average number of publications was calculated for the entire dataset and for each provenance. The average number of publications is denoted as, $\mu_P = \frac{N_A}{N_Y}$, where N_A is the total number of articles and N_Y is the years of publication. The years of publication is calculated as the range between the collection date and the first published article, for each provenance, within the data. These averages are summarised in Table 2. Overall an average of 49 articles are published per year on the topic. The most significant contribution to this appears to be from arXiv with 16 articles per year, followed by Nature with 10 and Springer with 9.

	Av. Yearly publication
IEEE	5.0
PLOS	8.0
Springer	9.0
Nature	11.0
arXiv	16.0
Overall	49.0

Table 2: Average publication for main data set.

Though the average publication offers insights about the publications of the fields, it remains a constant number. The data handled here is a time series between 1950, when the game was introduced, and 2018 (Figure 1). Two observations can be made from Figure 1.

1. A steady increase to the number of publications since the 1980s and the introduction of computer tournaments.
2. A decrease in 2017-2018. This is due to our data set being incomplete. Articles that have been written in 2017-2018 have either not being published or have are not retrievable by the APIs yet.

These observations can be confirmed by studying the time series. Using [8], an exponential distribution is fitted to the data from 1980-2016. The exponential fitting proves that since 1980 there has been an increase in the number of publications till 2016 (Figure 2). The fitted model can also be used to project the behaviour of the field for the next 5 years. The forecasted periods are plotted in Figure 3 and their exact values are given by Table 3. The time series has indicated a slight decrease however we can see that the model forecasts that the number of publications will keep increasing, thus indicating that the field of the iterated prisoner's dilemma still attracts academic attention.

To allow for a comparative analysis two sub fields of game theory have been chosen for this work; auction games and the price of anarchy.

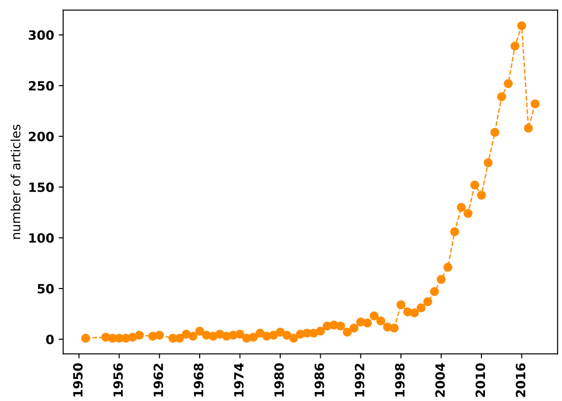


Figure 1: Line plot; # of articles published on the PD 1950-2019.

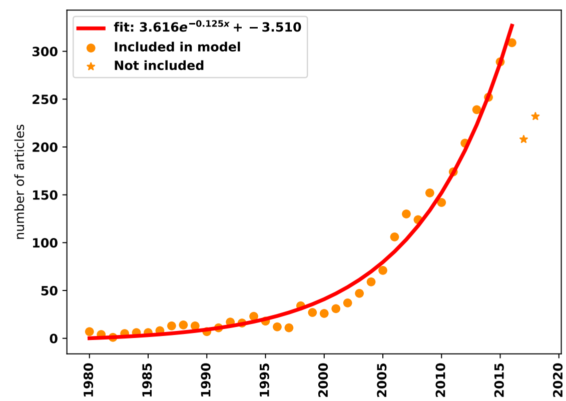


Figure 2: Scatter plot; # of articles published on the PD 1980-2019.

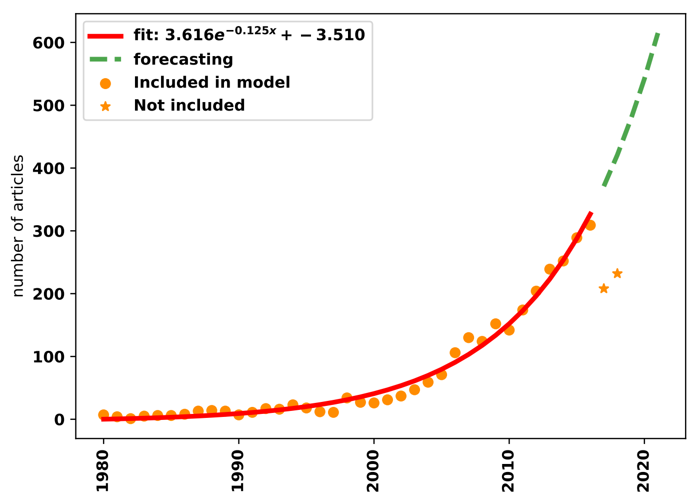


Figure 3: Forecast for 2017-2022.

	Forecast
2017	371.0
2018	421.0
2019	478.0
2020	542.0
2021	615.0

Table 3: Forecasting the number of publications over the next 10 years.

- Auction theory is a branch of game theory which researches the properties of auction markets. Game theory is used for years to study auctions and the behaviour of bidders [21]. The earliest entry in our data set [ref] goes back to 1974 (Figure 4). Note that no articles have been added manually for auction games.
- Price of Anarchy is a concept in game theory which measures how the efficiency of a system degrades due to selfish behaviour of it's agents. There is a variety of such measures however the price of anarchy has attracted a lot of attention since it's informal introduction in 1999 by [9]. Note that [9] has been manually added to the date set and it's the first entry (Figure 5).

A summary of both data sets collected on both topics, in comparison to that of [ref], is given by Table 4.

	Num. Articles	Num. Authors	Manual (%)	PLOS (%)	Nature (%)	Springer (%)	IEEE (%)	arXiv (%)	Av. Yearly Publication
Prisoner's Dilemma	3089	5811	2.88	15.6	21.79	18.52	9.55	34.19	49.0
Auction Games	3444	5362	-	-	5.89	37.63	7.46	51.36	93.0
Price of Anarchy	747	1315	0.13	1.74	24.63	38.02	30.66	8.84	39.0

Table 4: Measures of all three data sets.

The iterated prisoner's dilemma and auction theory are very well studied topics that have been publicising for decades. A large number of articles have been collected for both topics, 3089 and 34444 respectively. Though, auction games have a larger number of articles, the iterated prisoner's dilemma has almost 300 more authors.

Auction games have an overall average yearly publication of 93 articles per year compared to the prisoner's dilemma with 49 per year. 50% of articles for [ref] have been collected from the pre print server arXiv and no articles have been published in PLOS.

Compared to these two topics the price of anarchy is a fairly recent one. Only a total of 747 articles have been collected, however it has a large number of 1229 authors. On average each paper had had at least two authors. It has an overall average publication of 39 articles and the biggest contribution has been made to Springer.

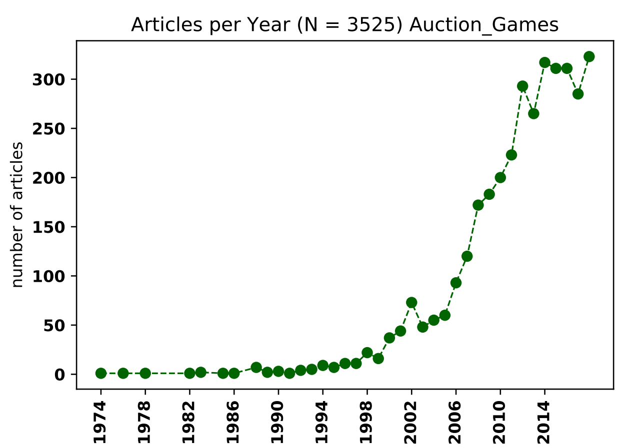


Figure 4: Line plot; # articles published on auction games 1974-2018.

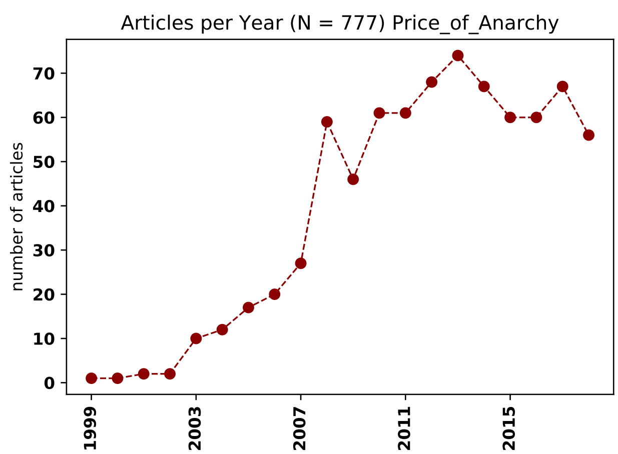


Figure 5: Line plot; # articles published on the price of anarchy.

2.4 Methodology

The relationship between the authors within a field will be modelled as a graph G with a set V_G of nodes and E_G of edges. The set V_G represents the authors and an edge connects two authors if and only if those authors have written together. The co authorship network is constructed using the main data set described in Section 2.3 and the open source package [7]. The prisoner's dilemma network is denoted as G_1 where the number of unique authors $|V(G_1)|$ is 5394 and $|E(G_1)| =$ 10397 . Note that the names of all authors names were formatted as their last name and first initial (i.e. Martin A.

Nowak to Martin Nowak). This was done to avoid errors such as Martin A. Nowak and Martin Nowak, being treated as a different person.

Collaborativeness, will be analysed using measures such as, isolated nodes, connected components, clustering coefficient, communities, modularity and average degree. These measures show the number of connections authors can have and how strongly connected these people are. The number of isolated nodes is the number of nodes that are not connected to another node, thus the number of authors that have published alone. The average degree denotes the average number of neighbours for each nodes, i.e. the average number of collaborations between the authors.

A connected component is a maximal set of nodes such that each pair of nodes is connected by a path. The number of connected components as well as the size of the largest connected component in the network are reported. The size of largest connected component represents the scale of the central cluster of the entire network, as it will discussed in the analysis section. Clustering coefficient and modularity and are also calculated. The clustering coefficient, defined as 3 times the number of triangle on the graph divided by the number of connected triples of nodes, is a local measure of the degree to which nodes in a graph tend to cluster together in a clique. It is precisely the probability that the collaborators of an author also write together.

In comparison, modularity is a global measure designed to measure the strength of division of a network into communities. The number of communities will be reported using the Clauset-Newman-Moore method [4]. Also the modularity index is calculated using the Louvain method described in [3]. The value of the modularity index can vary between $[-1, 1]$, a high value of modularity corresponds to a structure where have dense connections between the nodes within communities but sparse connections between nodes in different communities. That means that authors in the network are mainly connected co-authors that they all have written together, and not to several different collaborators.

Networks are commonly dominated by one person who controls information flow and people that receive a great amount of information due to their position. Two further points are aimed to be explored in this work, (1) which people control the flow; as in which people influence the field the most and (2) which are the authors that gain the most from the influence of the field. To measure these concepts graph theoretic metrics, more specifically centrality measures are going to be used. Centrality measures are often used to understand different aspects fo social networks [11]. In order to achieve that two centrality measures that have been chosen are closeness and betweenness centrality.

1. In networks some nodes have a short distance to other nodes and consequently are able to spread information on the network very effectively. A representative of this idea is **closeness centrality**, where a person is seen as centrally involved in the network if it requires only few intermediaries to contact others and thus is structurally relatively independent. Here, this is defined as influence. Authors with a high value of closeness centrality, are the authors that spread scientific knowledge easier on the network and they have high influence.
2. Another centrality measure is the **betweenness centrality**, where the determination of an author's centrality is based on the quotient of the number of all shortest paths between nodes in the network that include the regarded node and the number of all shortest paths in the network. In betweenness centrality the position of the node matters. Nodes with a higher value of betweenness centrality are located in positions that a lot of information pass through them, this is defined as the gain from the influence, thus these authors gain the most from their networks.

In the next section all the metrics discussed here are calculated for the data sets in order to provide insights into the field.

2.5 Analysis of co authorship network

As mentioned previously, G_1 denotes the co authorship network of the iterated prisoner's dilemma. It's graphical representation is given by Figure 6a. It is evident that the network is disjoint, which is only natural as many authors write academic articles on their own. More specifically, a total of 176 authors, have had single author publications, which corresponds to the 3.3 (%) of authors in G_1.

There are a total of 1356 connected components and the largest one has a size of 815 nodes. The largest connected component is shown in Figure 6b and is going to be refereed to as the main cluster of the network. There are total of 1369 communities in G_1. The network has a clustering coefficient of 0.708 , thus authors are 70% likely to write with a collaborator's co author and the degree distribution, Figure 7, shows that the average degree is ≈ 4. Thus authors are on average connected to 4 other authors, however there are authors with far more connections, the largest one being 58.

In [12] the collaborative metrics for the "evolution of cooperation" co authorship network were reported. Though their network is of smaller size (number of nodes $3670 < 5394$), the collaborative metrics are fairly similar between the two graphs (clustering coeff. $0.632 \approx 0.708$ and modularity $0.950 \approx 0.977$), indicating that for the same multidisciplinary field the same remarks can be made from a different co authors network. But how does these compare to other fields and more specifically to other fields of game theory? The representation of the two graphs,

- G_2 for auction games and
- G_3 for the price of anarchy,

are given by Figures 6c and 6e and their respective clusters in Figures 6d and 6f. As stated before G_3 is the smallest network of all three, this is also clearly seen from it's graphical representation. The G_2 network appears to be very similar to G_1, however it's main cluster is larger in size.

A summary of the collaborative metrics for all three co authorship networks is given by Table 5 and shown in Figure 7 are the degree distributions of all three networks. In G_1 and G_2 there are cases of high degree (> 20) but this could be an affect of the size of the data, networks and subsequently the size of the main clusters.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
Prisoner's Dilemma	5394	10397	176	3.3	1356	815	3.855	1369	0.977	0.708
Auction Games	5165	7861	256	5.0	1272	1348	3.044	1295	0.957	0.622
Price of Anarchy	1155	1953	4	0.3	245	222	3.382	253	0.965	0.712

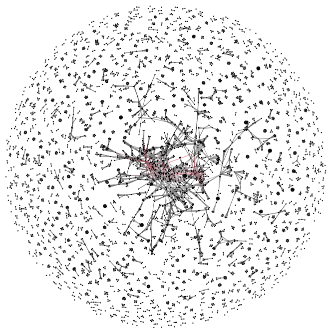
Table 5: Network metrics for G_1, G_2, G_3.

Regarding the three sub fields of game theory and using Table 5 and Figure 7 the following remarks can be made:

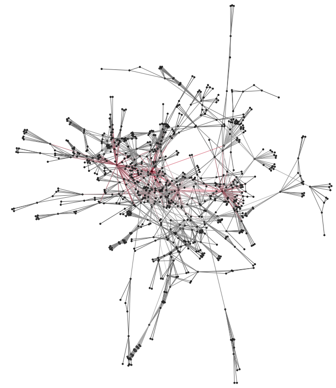
- All three networks have similar values of modularity index, and they are all very high (Table 5), indicating that the networks are partitioned in many communities. Note that the number of communities is very much similar to the number of connected components. This is all to expected. Due to the nature of our network, the number of connected components and the number of communities are very close. Most connected components represent a single publication written by all the authors in the component (corresponding to a fully connected graph), and due to that density they are also a community on their own.
- Comparing to another well studied topic, auction games, the field of the iterated prisoner's dilemma appear to be more collaborative. Due to the value of the average degree, authors in G_1 are known to have on average almost one more collaboration than G_2. A slightly lower cluster coefficient ($.622 < .702$) of auction games indicate that is less likely for authors in G_2 to collaborate with a collaborators co author.
- Regarding the price of anarchy, the measures indicate that the field is not as mature as the other two sub fields. There are no isolated authors, which is more of an indication of the time the field has been active. As a more recent field there had been better communication tools that enable more collaborations between researches. The average degree as well as the clustering coefficient (clustering coeff.$= 0.713$) of G_3 is comparable to those of the iterated prisoner's dilemma.

These results can be extend to the main clusters of each network, Table 6. The metrics's values are fairly similar and the size of G_2's main cluster does not appear to gave any significant effect; all the same conclusions are made. Compared to auction games the iterated prisoner's dilemma is a more collaborative field, and it is fairly similar to the price of anarchy. However, our analysis suggests that the price of anarchy is still a maturing field.

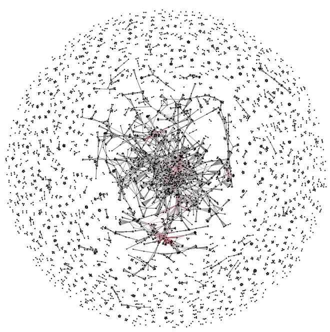
The change of the networks over time is also studied by constructing the network cumulatively with a year interval. A total of 64 sub graphs over 64 periods, starting in 1950, were created and all the collaborative metrics for each sub graph have been calculated. Note that years 1952 and 1953 have no publications in our data set. The metrics of each network for



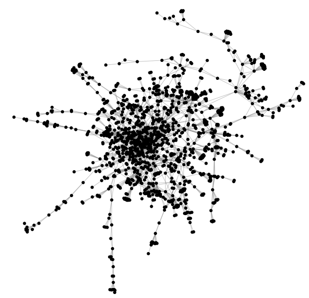
(a) G_1 network.



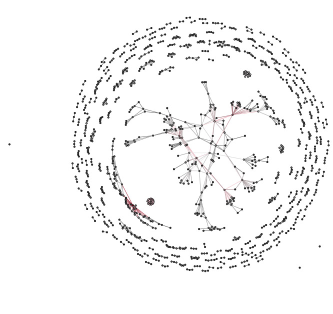
(b) G_1 larger connected component.



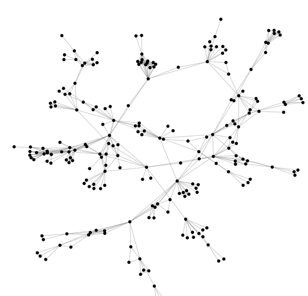
(c) G_2 network.



(d) G_2 larger connected component.



(e) G_3 network.



(f) G_3 larger connected component.

Figure 6: Graphical representations of G_1, G_2, G_3 and their respective main clusters.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
Prisoner's Dilemma	815	2300	0	0.0	1	815	5.644	29	0.855	0.775
Auction Games	1348	3158	0	0.0	1	1348	4.685	26	0.856	0.699
Price of Anarchy	222	521	0	0.0	1	222	4.694	12	0.817	0.711

Table 6: Network metrics for G_1, G_2, G_3.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
1954 - 1950	3	0	3	100.0	3	1	0.000	-	-	0.000
1954 - 1955	2	0	2	100.0	2	1	0.000	-	-	0.000
1955 - 1956	3	0	3	100.0	3	1	0.000	-	-	0.000
1956 - 1957	4	0	4	100.0	4	1	0.000	-	-	0.000
1957 - 1958	6	0	6	100.0	6	1	0.000	-	-	0.000
1958 - 1959	7	0	7	100.0	7	1	0.000	-	-	0.000
1959 - 1961	7	0	7	100.0	7	1	0.000	-	-	0.000
1961 - 1962	8	0	8	100.0	8	1	0.000	-	-	0.000
1962 - 1964	9	0	9	100.0	9	1	0.000	-	-	0.000
1964 - 1965	10	0	10	100.0	10	1	0.000	-	-	0.000
1965 - 1966	17	3	11	64.7	14	2	0.353	14	0.666667	0.000
1966 - 1967	21	4	13	61.9	17	2	0.381	17	0.75	0.000
1967 - 1968	32	15	13	40.6	21	5	0.938	21	0.684444	0.135
1968 - 1969	36	17	16	44.4	24	6	0.944	24	0.629758	0.139
1969 - 1970	39	18	17	43.6	26	6	0.923	26	0.666667	0.128
1970 - 1971	51	28	18	35.3	31	6	1.098	31	0.826531	0.275
1971 - 1972	58	34	19	32.8	34	6	1.172	34	0.866782	0.345
1972 - 1973	59	35	18	30.5	34	6	1.186	34	0.873469	0.339
1973 - 1974	59	35	18	30.5	34	6	1.186	34	0.873469	0.339
1974 - 1975	60	35	19	31.7	35	6	1.167	35	0.873469	0.333
1975 - 1976	60	35	19	31.7	35	6	1.167	35	0.873469	0.333
1976 - 1977	68	37	23	33.8	41	6	1.088	41	0.885318	0.294
1977 - 1978	70	38	23	32.9	42	6	1.086	42	0.890582	0.286
1978 - 1979	73	42	23	31.5	42	6	1.151	42	0.893424	0.292
1979 - 1980	77	45	25	32.5	44	6	1.169	44	0.899753	0.307
1980 - 1981	80	50	26	32.5	45	6	1.250	45	0.8928	0.318
1981 - 1982	84	56	26	31.0	46	6	1.333	46	0.903061	0.350
1982 - 1983	87	57	27	31.0	48	6	1.310	48	0.906125	0.338
1983 - 1984	94	58	32	34.0	54	6	1.234	54	0.909037	0.313
1984 - 1985	95	58	33	34.7	55	6	1.221	55	0.909037	0.309
1985 - 1986	104	59	40	38.5	63	6	1.135	63	0.911807	0.283
1986 - 1987	116	61	48	41.4	73	6	1.052	73	0.916958	0.253
1987 - 1988	121	65	48	39.7	75	6	1.074	75	0.924497	0.268
1988 - 1989	134	76	47	35.1	80	6	1.134	80	0.937673	0.272
1989 - 1990	145	82	49	33.8	86	6	1.131	86	0.944676	0.272
1990 - 1991	158	88	53	33.5	94	6	1.114	94	0.950413	0.268
1991 - 1992	169	91	59	34.9	102	6	1.077	102	0.953025	0.251
1992 - 1993	186	104	62	33.3	110	6	1.118	110	0.95932	0.266
1993 - 1994	220	134	72	32.7	127	6	1.218	127	0.965471	0.317
1994 - 1995	239	144	74	31.0	137	6	1.205	137	0.969329	0.304
1995 - 1996	257	163	77	30.0	145	6	1.268	145	0.970831	0.318
1996 - 1997	279	178	81	29.0	156	6	1.276	156	0.974309	0.336
1997 - 1998	311	215	65	20.9	160	6	1.383	160	0.979773	0.354
1998 - 1999	329	239	58	17.6	162	6	1.453	162	0.981741	0.376
1999 - 2000	373	273	67	18.0	183	6	1.464	183	0.983778	0.387
2000 - 2001	400	320	54	13.5	184	7	1.600	184	0.983066	0.410
2001 - 2002	450	366	61	13.6	206	7	1.627	206	0.984547	0.418
2002 - 2003	509	414	58	11.4	229	7	1.627	229	0.987083	0.421
2003 - 2004	580	489	58	10.0	253	10	1.686	253	0.988052	0.429
2004 - 2005	679	599	57	8.4	284	19	1.764	284	0.98891	0.463
2005 - 2006	854	806	66	7.7	342	21	1.888	342	0.990724	0.496
2006 - 2007	1056	1117	76	7.2	402	24	2.116	402	0.989663	0.527
2007 - 2008	1255	1460	85	6.8	454	32	2.327	455	0.989753	0.549
2008 - 2009	1462	1759	104	7.1	520	56	2.406	521	0.987517	0.550
2009 - 2010	1700	2301	114	6.7	581	99	2.707	584	0.979084	0.571
2010 - 2011	2040	2954	121	5.9	665	121	2.896	668	0.980396	0.603
2011 - 2012	2422	3676	126	5.2	756	210	3.036	759	0.979127	0.629
2012 - 2013	2807	4398	138	4.9	843	330	3.134	849	0.977978	0.639
2013 - 2014	3199	5044	148	4.6	942	406	3.153	951	0.973882	0.651
2014 - 2015	3798	6221	159	4.2	1064	514	3.276	1074	0.975594	0.668
2015 - 2016	4472	8344	169	3.8	1184	614	3.732	1197	0.9754	0.690
2016 - 2017	4925	9235	173	3.5	1274	703	3.750	1293	0.975251	0.700
2017 - 2018	5385	10379	176	3.3	1356	815	3.855	1369	0.97706	0.708

Table 7: Collaborativeness metrics for cumulative graphs.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
1954 - 1950	1	0	1	100.0	1	1	0.000	-	-	0.000
1954 - 1955	1	0	1	100.0	1	1	0.000	-	-	0.000
1955 - 1956	1	0	1	100.0	1	1	0.000	-	-	0.000
1956 - 1957	1	0	1	100.0	1	1	0.000	-	-	0.000
1957 - 1958	1	0	1	100.0	1	1	0.000	-	-	0.000
1958 - 1959	1	0	1	100.0	1	1	0.000	-	-	0.000
1959 - 1961	1	0	1	100.0	1	1	0.000	-	-	0.000
1961 - 1962	1	0	1	100.0	1	1	0.000	-	-	0.000
1962 - 1964	1	0	1	100.0	1	1	0.000	-	-	0.000
1964 - 1965	1	0	1	100.0	1	1	0.000	-	-	0.000
1965 - 1966	2	1	0	0.0	1	2	1.000	1	0	0.000
1966 - 1967	2	1	0	0.0	1	2	1.000	1	0	0.000
1967 - 1968	5	8	0	0.0	1	5	3.200	1	0	0.867
1968 - 1969	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1969 - 1970	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1970 - 1971	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1971 - 1972	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1972 - 1973	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1973 - 1974	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1974 - 1975	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1975 - 1976	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1976 - 1977	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1977 - 1978	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1978 - 1979	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1979 - 1980	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1980 - 1981	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1981 - 1982	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1982 - 1983	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1983 - 1984	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1984 - 1985	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1985 - 1986	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1986 - 1987	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1987 - 1988	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1988 - 1989	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1989 - 1990	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1990 - 1991	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1991 - 1992	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1992 - 1993	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1993 - 1994	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1994 - 1995	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1995 - 1996	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1996 - 1997	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1997 - 1998	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1998 - 1999	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1999 - 2000	6	10	0	0.0	1	6	3.333	2	0.02	0.833
2000 - 2001	7	21	0	0.0	1	7	6.000	1	0	1.000
2001 - 2002	7	21	0	0.0	1	7	6.000	1	0	1.000
2002 - 2003	7	21	0	0.0	1	7	6.000	1	0	1.000
2003 - 2004	10	13	0	0.0	1	10	2.600	2	0.37574	0.553
2004 - 2005	19	28	0	0.0	1	19	2.947	3	0.524235	0.730
2005 - 2006	21	32	0	0.0	1	21	3.048	4	0.530273	0.713
2006 - 2007	24	36	0	0.0	1	24	3.000	5	0.563272	0.678
2007 - 2008	32	59	0	0.0	1	32	3.688	4	0.627837	0.732
2008 - 2009	56	102	0	0.0	1	56	3.643	4	0.713331	0.699
2009 - 2010	99	238	0	0.0	1	99	4.808	8	0.781601	0.734
2010 - 2011	121	288	0	0.0	1	121	4.760	9	0.776054	0.713
2011 - 2012	210	610	0	0.0	1	210	5.810	12	0.779433	0.747
2012 - 2013	330	908	0	0.0	1	330	5.503	17	0.813721	0.753
2013 - 2014	406	1125	0	0.0	1	406	5.542	18	0.814914	0.749
2014 - 2015	514	1390	0	0.0	1	514	5.409	21	0.828125	0.757
2015 - 2016	614	1682	0	0.0	1	614	5.479	25	0.83153	0.765
2016 - 2017	703	1925	0	0.0	1	703	5.477	25	0.839089	0.774
2017 - 2018	815	2300	0	0.0	1	815	5.644	29	0.854812	0.775

Table 8: Collaborativeness metrics for cumulative graphs' main clusters.

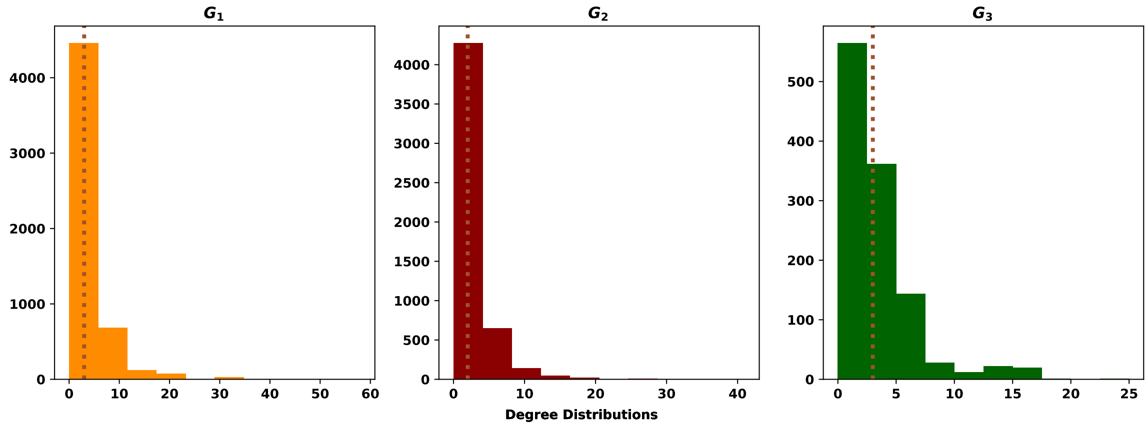


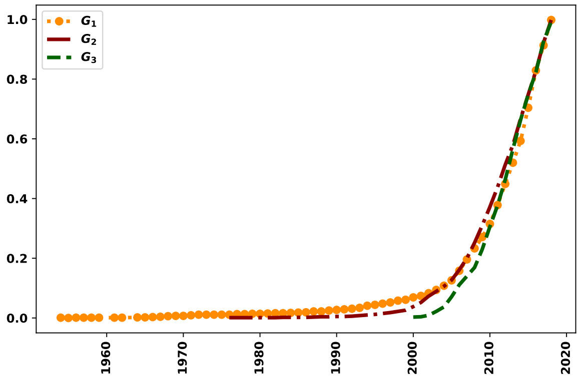
Figure 7: Degree distribution for networks G_1, G_2 and G_3. The descriptive statistics for each of the distribution are for G_1: mean = 3.85, median = 3, std = 4.25. For G_2: mean = 3.04, median = 2, std = 3.01 and for G_3: mean = 3.38, median = 3, std = 2.90. It is clear that these distributions are not normally distributed, which has also been verified using a statistical test. Moreover, the statistical difference of the medians has been tested using a Kruskal Wallis test. The medians of G_1 and G_3 are not significantly different, however they are are significantly large than that of G_2.

each period are given by Table 7 and for each main cluster by Table 8. Similar to the results of [12], it can been observed that the network G_1 grows over time and that the network always had a high value of modularity.

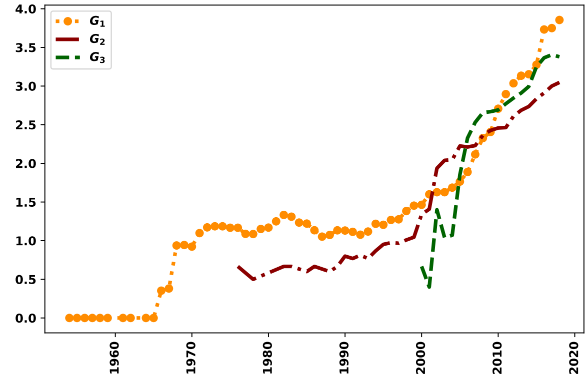
To better assess the change over time for each metric they have been plotted in Figure 8. The number of nodes, connected components and the size of largest component have been normalised such that we can compare the trend between the three networks.

- In Figure 8a the normalised number of nodes, which is calculated by dividing by the total number of nodes in each respective network, is shown. A steep increase in the size of all three networks is spotted soon after 2000. This could indicate that more data have been available in the sources used in this work following the year 2000. It is however, definitely not a effect of a single field, as it is true for all three sub fields considered here. The sudden increase following the year 2000, is also reported by the number of connected components and the size of the main cluster, Figures 8c, 8d. A connected components represents at least one publication which means that indeed more articles are being gathered from 2000 onwards.
- Auction games have been through out time less collaborative compared to the iterated prisoner's dilemma. The average degree (Figure 8b) and the clustering coefficient (Figure 8e) of the cumulative sub graphs have been lower than that of G_1 throughout time. The only exception is during years 2001-2008. For these year auction games appear to have had a more collaborative environment.
- In the price of anarchy cumulative graphs a sharp increase since the beginning of the field can be observed for all metrics. There are not many data points due to the recent development of the field, however these steep trends could be an indication that game theoretic and potentially all scientific research has over time been more collaborative. This could be due to logistic and techical solutions.
- The high values of modularity through out time is not true only for the network reported in [12] but also for all three networks of this field. This could indicate a limitation to the co authorship network. As the measure is likely to be skewed, as each paper is more likely a connected component on it's own.

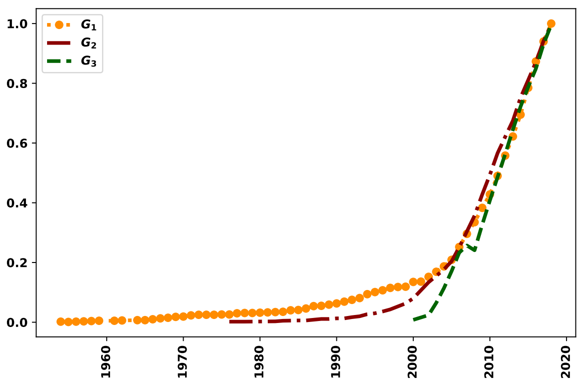
The metrics that change over time for the main clusters have also been plotted, to further gain understanding in the change of the main cluster for all three networks over the course of time. Summary the results do not appear to change over the main cluster. The networks have been always modular which could be a limitation to the co authorship network. Over time auction gae have only a small amount of period where the collaborativeness was more, and the historical data with the steep increases support our hypothesis that the anarchy is not mature yet.



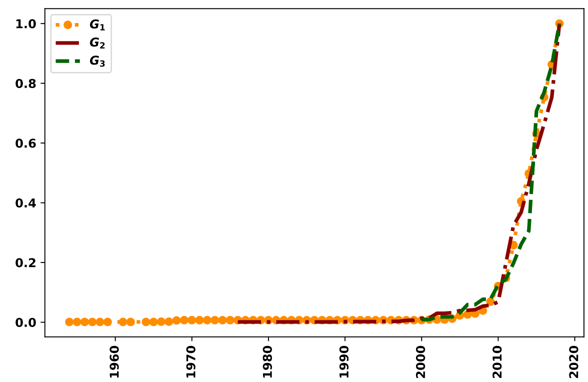
(a) % Nodes.



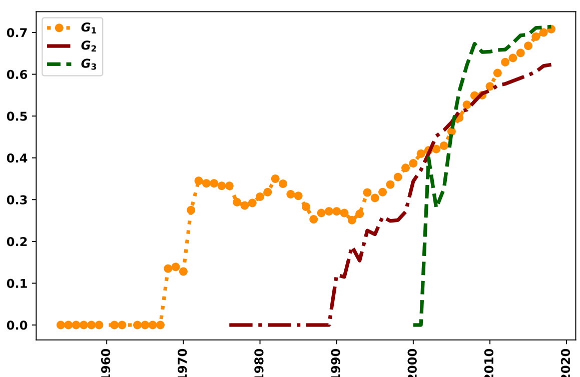
(b) Average Degree.



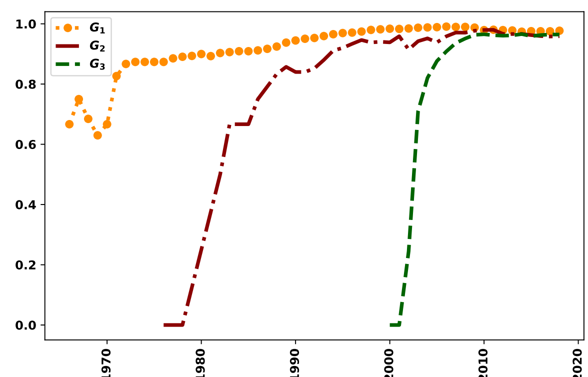
(c) % Connected components.



(d) % Size of largest connected component.



(e) Clustering coefficient.



(f) Modularity.

Figure 8: Collaborative metrics over time for cumulative networks for G_1, G_2 and G_3.

The next results discussed here are on centrality measures. As a reminder, two centrality measures are reported here, these are the closeness centrality and the betweenness centrality. Closeness centrality is a measure of how easy is for an author to contact others, and consequently affect them; influence them. Thus closeness centrality here is a measure of influence. Betweenness centrality is a measure of how many paths pass through a specific nodes, thus the amount of information this person has access to. Betweenness centrality is used here as a measure of how much an author gain from the field. All centrality measure can have values ranging from $[0, 1]$.

For G_1 the most central author based on closeness and betweenness are given by Tables 9 and 10 respectively. Centrality measures range between $[0, 1]$. The betweenness centrality of the most central authors in G_1 are rather low with the highest ranked author being Matjaz Perc with a between centrality of 0.008, Table 10. Matjaz Perc is also the first ranked author based on closeness centrality, with a centrality of 0.04. Perc's work has been briefly discussed in Section, and the centrality measure suggest that the network is very influenced by him. He is connected to a total of 58 nodes and he has published to all five of the different sources we are considering in the study. Though he also gains from his position in the network, the gain is minor. An author who is not in the top influencers but does indeed gain from his position in the network is Martin Nowak. An author that his work has been discussed in Section 1.

	Name	Closeness
1	Matjaz Perc	0.048447
2	Yamir Moreno	0.044840
3	Zhen Wang	0.044005
4	Long Wang	0.043770
5	Attila Szolnoki	0.043338
6	Luo-Luo Jiang	0.042148
7	Arne Traulsen	0.041790
8	Valerio Capraro	0.041257
9	Cheng-Yi Xia	0.040791
10	Angel Sanchez	0.040562

Figure 9: Ten most influenced authors in G_1.

	Name	Betweeness
1	Matjaz Perc	0.008331
2	Zhen Wang	0.006356
3	Yamir Moreno	0.004806
4	Long Wang	0.003538
5	Martin Nowak	0.003230
6	Valerio Capraro	0.002739
7	Arne Traulsen	0.002479
8	Angel Sanchez	0.002319
9	Jianye Hao	0.002188
10	Franz Weissing	0.002186

Figure 10: Authors that gain the most influence in G_1.

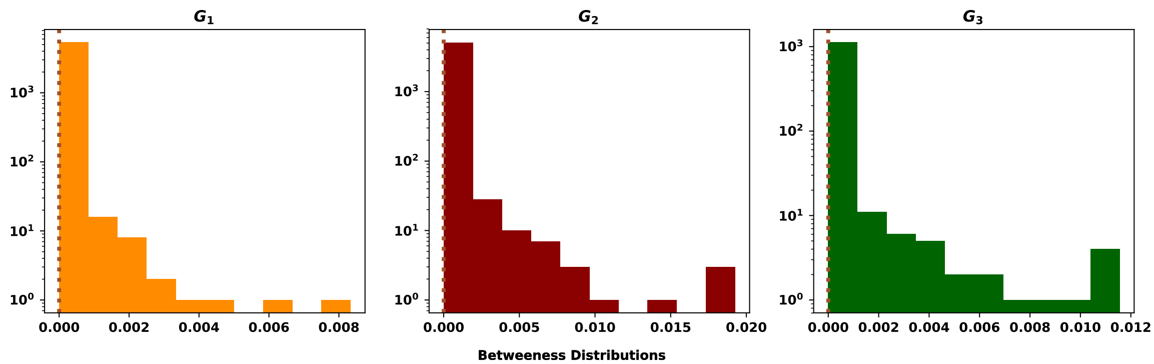
From Tables 9 and 10 it can be seen that authors in G_1 are more likely to affect their field instead of gaining from it. This can be better explored by considering the distributions of the centralities and by comparing them to other fields.

Overall, the values of closeness centrality appear to be higher than those of betweenness. These can be better explored by considering the centralities' distributions for all three networks. The distributions for both centralities are plotted in Figures 11a and 12a, and in Figures 11b and 12b for their respective main clusters.

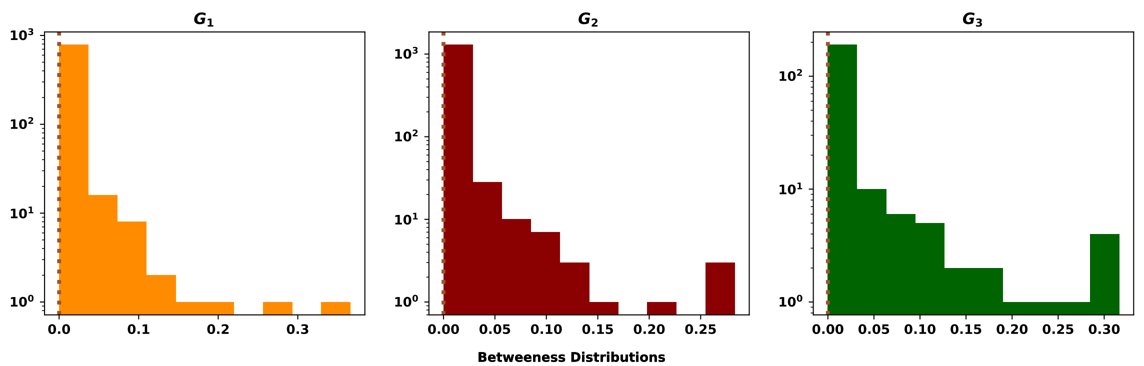
Regarding gaining from your network. For all the distributions the values are vely low and skewed to the left. That implies that in all three networks, authors do not gain much from the influence of their fields.

On the other hand, closeness distributions have more variation. The following observations are made from the distributions:

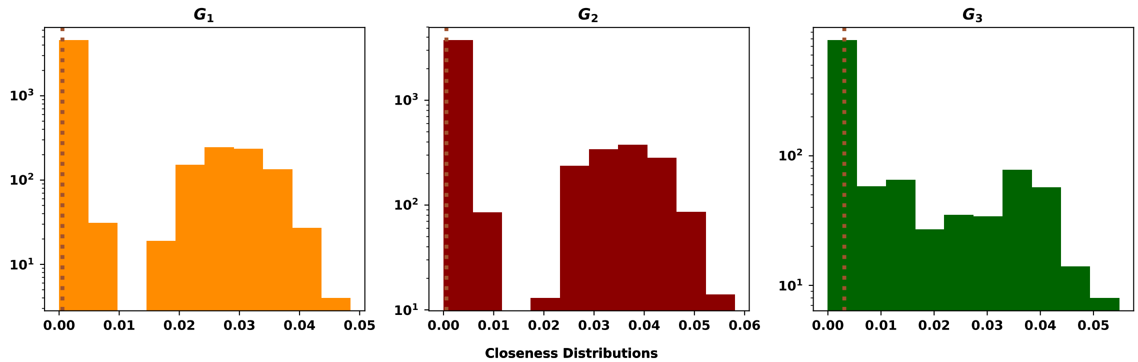
- Neither are normally distributed and there is a significant difference between the medians of all three distributions, with G_3 having a larger median.
- There are clusters from all three networks for which a number of authors have a closeness centrality greater than 0.02. The authors in these clusters were explored but not pattern was found behind their publications. The provenance and the year of publication were checked.
- The authors in these clusters, are the authors which are in the main clusters of their relative networks. Thus, the people that influence the field the most are the most central authors in the main cluster of the co authorship network of a field.
- Both G_2 and G_3 have more people influencing the field compared to G_1.



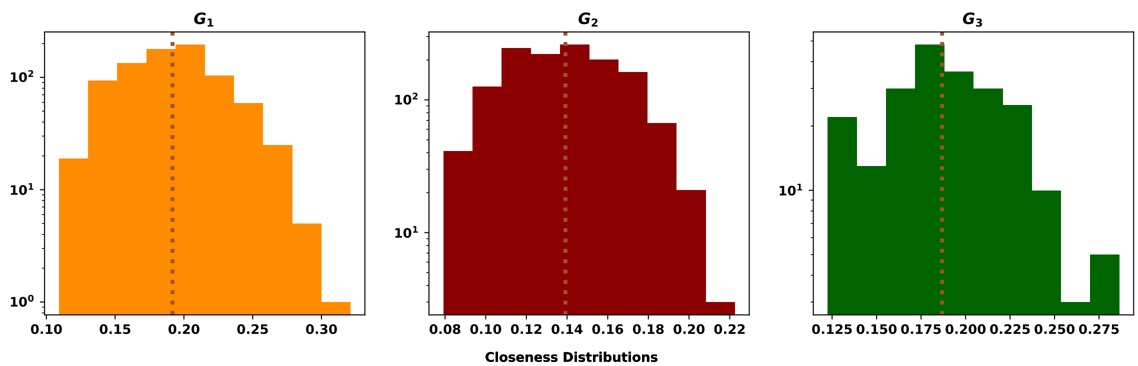
(a) Betweenness centrality distributions G_1, G_2, G_3. The descriptive statistics for each of the distribution are for G_1: mean= 0.000019, median= 0.0, std= 0.000207. For G_2: mean= 0.000086, median= 0.0, std= 0.000693 and for G_3: mean= 0.000151, median= 0.0, std= 0.000931. None of the three distributions is normally distributed. There is no statistical difference between the medians of G_1 and G_3. There is however, statistical difference between the median of G_2. These have been tested using a Kruskal Wallis test.



(b) Betweenness centrality distributions for G_1, G_2, G_3 respective main clusters.The descriptive statistics for each of the distribution are for G_1: mean= 0.0054, median= 0.0, std= 0.022. For G_2: mean= 0.0048, median= 0.0, std= 0.019 and for G_3: mean= 0.02, median= 0.0, std= 0.055. None of the three distributions is normally distributed. There is no statistical difference between the medians of G_1 and G_3. There is however, statistical difference between the median of G_2. These have been tested using a Kruskal Wallis test.



(a) Closeness centrality distributions G_1, G_2, G_3. The descriptive statistics for each of the distribution are for G_1: mean= 0.0050, median= 0.00056, std= 0.010. For G_2: mean= 0.000086, median= 0.00058, std= 0.000693 and for G_3: mean= 0.000151, median= 0, std= 0.000931. None of the three distributions is normally distributed. All medians are statistically different.



(b) Closeness centrality distributions for G_1, G_2, G_3 respective main clusters. The descriptive statistics for each of the distribution are for G_1: mean= 0.19, median= 0.19, std= 0.035. For G_2: mean= 0.14, median= 0.14, std= 0.026 and for G_3: mean= 0.19, median= 0.19, std= 0.035. None of the three distributions is normally distributed. All medians are statistically different.

2.6 Conclusion

3 Acknowledges

A variety of software libraries have been used in this work:

- Networkx [7], library for analysing networks.
- Gephi [2] open source package for visualising networks.
- louvain, library for calculating the networks modularity.

References

[1] Bedoor K AlShebli, Talal Rahwan, and Wei Lee Woon. The preeminence of ethnic diversity in scientific collaboration. *Nature communications*, 9(1):5163, 2018.

[2] Mathieu Bastian, Sebastien Heymann, and Mathieu Jacomy. Gephi: An open source software for exploring and manipulating networks, 2009.

[3] Vincent D Blondel, Jean-Loup Guillaume, Renaud Lambiotte, and Etienne Lefebvre. Fast unfolding of communities in large networks. *Journal of statistical mechanics: theory and experiment*, 2008(10):P10008, 2008.

[4] Aaron Clauset, Mark EJ Newman, and Cristopher Moore. Finding community structure in very large networks. *Physical review E*, 70(6):066111, 2004.

[5] Raymundo das Neves Machado, Benjamín Vargas-Quesada, and Jacqueline Leta. Intellectual structure in stem cell research: exploring brazilian scientific articles from 2001 to 2010. *Scientometrics*, 106(2):525–537, 2016.

[6] Henry Etzkowitz. Individual investigators and their research groups. *Minerva*, 30(1):28–50, 1992.

[7] Aric A. Hagberg, Daniel A. Schult, and Pieter J. Swart. Exploring network structure, dynamics, and function using NetworkX. In *Proceedings of the 7th Python in Science Conference (SciPy2008)*, pages 11–15, Pasadena, CA USA, August 2008.

[8] Eric Jones, Travis Oliphant, Pearu Peterson, et al. SciPy: Open source scientific tools for Python, 2001–. [Online; accessed ¡today¿].

[9] Elias Koutsoupias and Christos Papadimitriou. Worst-case equilibria. In *Proceedings of the 16th Annual Conference on Theoretical Aspects of Computer Science*, STACS'99, pages 404–413, Berlin, Heidelberg, 1999. Springer-Verlag.

[10] Svein Kyvik and Ingvild Reymert. Research collaboration in groups and networks: differences across academic fields. *Scientometrics*, 113(2):951–967, 2017.

[11] Andrea Landherr, Bettina Friedl, and Julia Heidemann. A critical review of centrality measures in social networks. *Business & Information Systems Engineering*, 2(6):371–385, Dec 2010.

[12] Peng Liu and Haoxiang Xia. Structure and evolution of co-authorship network in an interdisciplinary research field. *Scientometrics*, 103(1):101–134, Apr 2015.

[13] Flavio M Menezes and Paulo Klinger Monteiro. *An introduction to auction theory.* OUP Oxford, 2005.

[14] Nikoleta and Vince Knight. Nikoleta-v3/arcas: Arcas v 0.0.4, December 2017.

[15] Richard Van Noorden. The science thats never been cited. `https://www.nature.com/articles/d41586-017-08404-0`, 2017.

[16] Nurzhan Nurseitov, Michael Paulson, Randall Reynolds, and Clemente Izurieta. Comparison of json and xml data interchange formats: a case study. *Caine*, 2009:157–162, 2009.

[17] Alan Pritchard et al. Statistical bibliography or bibliometrics. *Journal of documentation*, 25(4):348–349, 1969.

[18] Dhruv Raina and BM Gupta. Four aspects of the institutionalization of physics research in india (1990–1950): Substantiating the claims of histortical sociology through bibliometrics. *Scientometrics*, 42(1):17–40, 1998.

[19] Tim Roughgarden. *Selfish routing and the price of anarchy*, volume 174. MIT press Cambridge, 2005.

[20] Vedran Sekara, Pierre Deville, Sebastian E. Ahnert, Albert-László Barabási, Roberta Sinatra, and Sune Lehmann. The chaperone effect in scientific publishing. *Proceedings of the National Academy of Sciences*, 115(50):12603–12607, 2018.

[21] Martin Shubik. The dollar auction game: a paradox in noncooperative behavior and escalation. *Journal of Conflict Resolution*, 15(1):109–111, 1971.

[22] Mason Youngblood and David Lahti. A bibliometric analysis of the interdisciplinary field of cultural evolution. *Palgrave Communications*, 4(1):120, 2018.