A systematic literature review of the Prisoner's Dilemma and an analysis of the corresponding co author network.

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1 Introduction

The emergence of cooperation is a topic of continuing and public interest for the social [24, 33], biological [34] and ecological sciences [36, 45, 59, 86]. Cooperation is essential for evolution but according to Darwin's theory it is not always easy to achieve. The game called the prisoner's dilemma offers a theoretical framework for studying the emergence of altruistic behaviour.

1.1 The Prisoner's Dilemma

The prisoner's dilemma is a two player non-cooperative game [28] where the decisions of the players are made simultaneously and independently. Both players can choose between cooperation (**C**) or defection (**D**).

The fitness of each player is influenced by its own behaviour, and the behaviour of the opponent. If both players choose to cooperate, both do better than if both defect. However, a player has the temptation to deviate. If a player was to defect while the other cooperates, the defector receives more than if both had cooperated. The reward for mutual cooperation is R, for a mutual defection they receive P, and for cooperation-defection, the cooperator receives S where the defector receives T. Thus, the game's payoffs are given by,

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} \tag{1}$$

where $T > R > P > S$ and $2R > T + S$ are the conditions for a dilemma to exist. Due to rational behaviour and the knowledge that an individual is tempted to defect the game's equilibrium lies at a mutual defection and both players receive a payoff of P. Thus, the dominant strategy for the prisoner's dilemma is **D**.

However, when the game is studied in a manner where prior outcomes matter, the defecting choice is no longer necessarily the dominant choice. The repeated form of the game is called the iterated prisoner's dilemma and now two players play the game repeatedly. Interest was sparked on the iterated prisoner dilemma by R. Axelrod and his book [18] "The Evolution of Cooperation".

In his book Axelrod reports on a series of computer tournaments he organised of a finite turns games of the iterated prisoner's dilemma. Participants had to choose between **C** and **D** again and again while having memory of their previous encounters. Academics from several fields were invited design computer strategies to compete in the tournament. The pioneer work of Axelrod showed that greedy strategies did very poorly in the long run while more altruistic strategies did better.

"The Evolution of Cooperation" is considered a milestone in the field but it is not the only one. On the contrary, the prisoner's dilemma has attracted much attention ever since the game's origins. This is shown in Figure 1, which illustrates the number of publications on the prisoner's dilemma per year from the following sources:

- arXiv;
- PLOS;
- IEEE;
- Nature;
- Springer.

The choice of sources is due to the fact that they have an open access Api, the process of collecting the data (including criteria for inclusion of papers) and the analysis will be described more comprehensively in Section 3.

Each point of Figure 1 marks the starting year of a time period. Each of these time periods is reviewed and presented in 2, as an extensive literature review. This paper is the review of this type, in such detail since the origins to date.

Furthermore, in Section 3 a comprehensive data set of literature regarding the prisoner's dilemma will be presented and analysed. This allow us to review the amount of published academic articles as well as measure and explore the collaborations within the field.

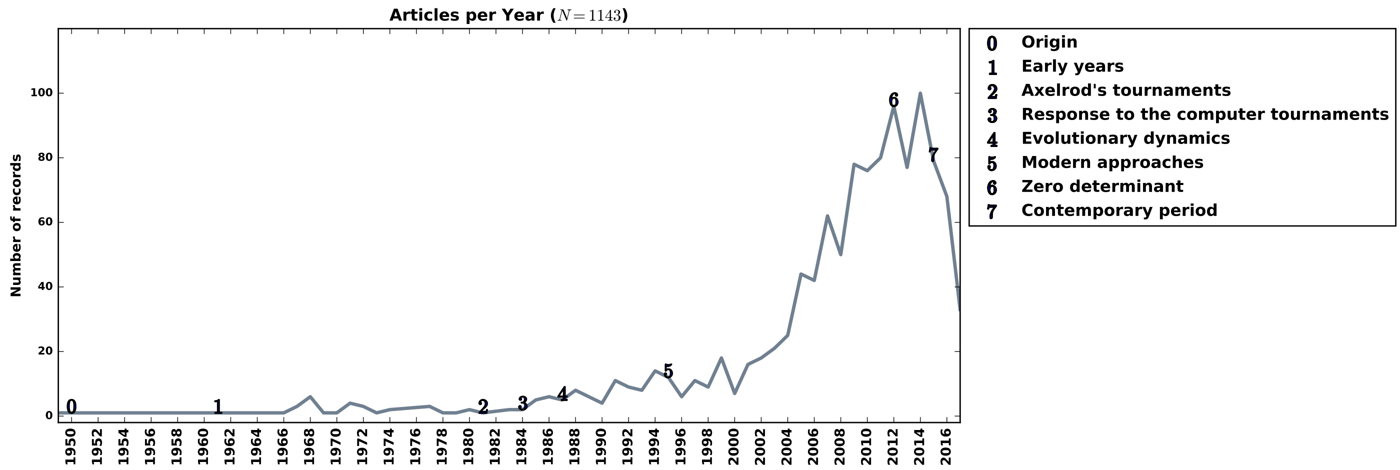


Figure 1: A timeline of the prisoner's dilemma research.

2 Timeline

2.1 Origin and Primal research (1961-1972)

The origin of the prisoner's dilemma goes back to the 1950s in early experiments conducted in RAND [28] to test the applicability of games described in [85]. Although in [28] the two player game was introduced the name behind the game was given later the same year. According to [81], A. W. Tucker (the PhD supervisor of J. Nash), in an attempt to delivery the game with a story during a talk used prisoners as players and the game has been known as the prisoner's dilemma ever since [81].

The study of the prisoner's dilemma has attracted people from various fields across the years. An early figure within the field is Prof A. Rapoport, a mathematical psychologist, whose work focused on peacekeeping. In his early work [72] Rapoport conducted experiments using humans to simulate a play of the prisoner's dilemma. Experimental groups were not been used only by Rapoport but it was a common mean of studying the game [26, 30, 52, 53, 77] and are still being used today.

Those experiments explored the conditions under which altruist behaviour emerges in human societies. Conditions such as, the gender [26, 52, 53] of individuals, the representation of the game [26], the distance between players [77], the initial effects [80] and whether the experimenter was biased [30].

Even though, several of these experiments were held and continuous research on the topic was undergoing game theorists were still in disagreement about the best way to play the game [72]. Inspired by the work of Rapoport and intrigued by the very same question the political scientist R. Axelrod took upon himself to identify the dominant strategy of the prisoners dilemma.

The main difference of Axelrod's approach was that machines were going to be used instead of humans. The issues with using humans, according to Axelrod [16], was the fact that humans can act very randomly even though the aim of the game is clear to them. Thus, Axelrod was the first researcher, to the author's knowledge, to perform a computer tournament of the iterated prisoner's dilemma. The work of Axelrod is considered one of the greatest milestones within the field. The tournaments and their results are discussed in the next sections.

2.2 Axelrod's Tournaments (1981-1984)

This section serves as a follow up from the earlier years of the topic and as an introduction to the modern ways of studying the prisoner's dilemma. It is dedicated to the computer tournaments of R. Axelrod from 1981 to 1984.

The first computer tournament was performed in 1980 [12]. Several scientists were invited to submit their strategies, written in the programming languages Fortran or Basic. There was a total number 13 submissions made by the following researchers,

1. T Nicolaus Tideman and Paula Chieruzz;
2. Rudy Nydegger;
3. Bernard Grofman;
4. Martin Shubik;
5. Stein and Anatol Rapoport;
6. James W Friedman;
7. Morton Davis;
8. Jim Graaskamp;
9. Leslie Downing;
10. Scott Feld;
11. Johann Joss;
12. Gordon Tullock;
13. Name not given.

Each competed in a 200 turn match against all 13 opponents, itself and a player that played randomly. This type of tournament is referred to as a round robin and corresponds to a complete graph from a topological point of view. The tournament was repeated 5 times to reduce variation in the results. Each participant knew the exact length of the matches and had access to the full history of each match. Furthermore, Axelrod performed an preliminary tournament and the results were known to the participants. The payoff values used for 1 where $R = 3, P = 1, T = 5$ and $S = 0$. These values are commonly used in the literature and unless specified will be the values used in the rest of the work described here.

The winner of the tournament was determined by the total average score and not by the number of matches won. The strategy that was announced the winner was submitted by Rapoport and was called **Tit For Tat**. Tit for Tat, is a strategy that always cooperates on the first round and then mimics the opponent's previous move.

Examples of Tit for Tat interacting for 8 turns with deterministic opponents are given by Tables 1, 2, 3. The opponents are, **Cooperator** a strategy that always cooperates, **Defector** an opponent that always defects and **Altenator** a player who alternates between cooperating and defecting.

The results of the first tournament were filled with surprises. Tit for Tat the simplest strategy of all had won and had managed to defeat even entrants that tried to improve on Tit for Tat after the preliminary tournament results. Axelrod justified the success of the strategy saying that the strategy was 'nice' and 'forgiving'.

The top eight ranked strategies were strategies that did no defect on the first round, thus they were described as 'nice' strategies. Compared to the rest of the "nice" strategies, Tit for Tat had also another property. That property was 'forgiveness'. Tit for Tat punished it's opponent for a defection but just once and then it would try to cooperate again. These two properties were described to be the secret of success in a prisoner's dilemma tournament.

In order to further test the robustness of the results Axelrod performed a second tournament [13] later in 1980. This time a total of 63 participants submitted strategies for the second tournament, their names were the following,

Turns	Tit for Tat	Cooperator
1	C	C
2	C	C
3	C	C
4	C	C
5	C	C
6	C	C
7	C	C
8	C	C

Table 1: Tit for Tat example match of 8 turns against Cooperator

Turns	Tit for Tat	Defector
1	C	D
2	D	D
3	D	D
4	D	D
5	D	D
6	D	D
7	D	D
8	D	D

Table 2: Tit for Tat example match of 8 turns against Defector

Turns	Tit for Tat	Altenator
1	C	C
2	C	D
3	D	C
4	C	D
5	D	C
6	C	D
7	D	C
8	C	D

Table 3: Tit for Tat example match of 8 turns against Altenator

1. Gail Grisell;
2. Harold Rabbie;
3. James W Friedman;
4. Abraham Getzler;
5. Roger Hotz;
6. George Lefevre;
7. Nelson Weiderman;
8. Tom Almy;
9. Robert Adams;
10. Herb Weiner;
11. Otto Borufsen;
12. R D Anderson;
13. William Adams;
14. Michael F McGurrin;
15. Graham J Eatherley;
16. Richard Hufford;
17. George Hufford;
18. Rob Cave;
19. Rik Smoody;
20. John Willaim Colbert;
21. David A Smith;
22. Henry Nussbacher;
23. William H Robertson;
24. Steve Newman;
25. Stanley F Quayle;
26. Rudy Nydegger;
27. Glen Rowsam;
28. Leslie Downing;
29. Jim Graaskamp and Ken Katzen;
30. Danny C Champion;
31. Howard R Hollander;
32. George Duisman;
33. Brian Yamachi;
34. Mark F Batell;
35. Ray Mikkelson;
36. Craig Feathers;
37. Fransois Leyvraz;
38. Johann Joss;
39. Robert Pebly;
40. James E Hall;
41. Edward C White Jr;
42. George Zimmerman;
43. Edward Friedland;
44. X Edward Friedland;
45. Paul D Harrington;
46. David Gladstein;
47. Scott Feld;
48. Fred Mauk;
49. Dennis Ambuehl and Kevin Hickey;
50. Robyn M Dawes and Mark Batell;
51. Martyn Jones;
52. Robert A Leyland;
53. Paul E Black;
54. T Nicolaus Tideman and Paula Chieruzz;
55. Robert B Falk and James M Langsted;
56. Bernard Grofman;
57. E E H Schurmann;
58. Scott Appold;
59. Gene Snodgrass;
60. John Maynard Smith;
61. Jonathan Pinkley;
62. Anatol Rapoport.

All the participants knew the results of the previous tournament. The rules were similar to those of the first tournament with only one exception; the number of turns was not specified instead a fixed probability (refereed to as 'shadow of the future' [17]) of the game ending on the next move was used. The fixed probability was chosen to be 0.0036 so that the expected median length of a match would be 200 turns. The topology was of a round robin and each pair of players was matched 5 times. The length of the matches was determined once by drawing a random sample. Each of the five matches had a length of 63, 77, 151 and 308.

The results of the tournament once again came as a surprise. Tit for Tat was considered to be one of the simplest submissions in the second tournament and won the second tournament as well. Tit for Tat provided proof that reciprocity behaviour can allow cooperation to emerge in the iterated prisoner's dilemma game. In [12] the main conclusions indicating strong performance was:

- that it start of by cooperating
- it would forgive it's opponent after a defection
- after opponents identified that they were playing Tit for Tat choose to cooperate for the rest of the game.

Another successful strategy from Axelrod's tournament that can been seen in literature to date is **Grudger**, originally submitted by James W. Friedman. Grudger is a strategy that will cooperate as long as the opponent does not defect.

The name Grudger was give to the strategy in [50]. Though the strategy goes by many names in the literature such as, Spite [20], Grim Trigger [19] and Grim [84].

As for the rest of the strategies, though a full explanation of all 13 submitted strategies is given in [12] the same does not hold for all 63 strategies of the second tournament [13]. The author mainly focuses on the high ranked participants and several details for the rest strategies are left unknown.

The source code of the 63 strategies be found on Axelrod's personal website [1]. The source code was written by Axelrod and several other contributors. The strategies written in Basic were translated to Fortran before the tournament. The source code includes the code only for the strategies and not for creating and performing the tournament.Figure 2 serves as an example of the source code giving the code for the winning strategy Tit for Tat. Unfortunately, the source code of the first 13 strategies is not available, as stated in Axelrod's personal website [1].

```
    FUNCTION K92R(J,M,K,L,R, JA)
C BY ANATOL RAPOPORT
C TYPED BY AX 3/27/79 (SAME AS ROUND ONE TIT FOR TAT)
c replaced by actual code, Ax 7/27/93
c  T=0
c   K92R=ITFTR(J,M,K,L,T,R)
      k92r=0
      k92r = j
c test 7/30
c   write(6,77) j, k92r
c77   format(' test k92r. j,k92r: ', 2i3)
      RETURN
      END
```

Figure 2: Source code for Tit for Tat in Fortran. Provided by [1].

So far it has been discussed how the performance of the strategy has been tested through tournaments against other strategies. A question remains: is the overall success of a strategy based only on it's performance in a round robin tournament or should it be checked through other ways as well?

Following his initial tournaments Axelrod performed an 'ecological' tournament in 1981 [18]. Axelrod argued that some strategies are so unsuccessful that there are very likely to be dropped in the future, while other more successful strategies would continue in later interactions. Influenced by evolutionary biology Axelrod introduced a way of capturing this behaviour, which included running a series of tournaments where more successful strategies would occupy a larger part of the environment and the less successful strategies would become less often. This is known as an ecological tournament.

The simulation of the process, as described in [18], is straightforward. Consider matrix 2 which provides the expected payoff when two individuals of different type interact. Starting with proportions of each type in a given generation the proportions of these strategies in the next generation is the only measure needed to be calculated. This is achieved by calculating the weighted average of the scores of a given strategy with all other players, where the weights are the numbers of the other strategies which exist in the current generation. The numbers of a given strategy in the next generation is then taken to be proportional to the product of its numbers in the current generation and its score in the current generation.

$$\begin{pmatrix} (R=3, R=3) & (S=0, T=5) \\ (T=5, S=0) & (P=1, P=1) \end{pmatrix} \tag{2}$$

Note the ecological tournament does not offer any evolutionary perspective. There is no possibility of a new strategy to be introduced, there is no mutation probability to drive the evolution. The ecological is a framework that provides the distributions of given types over time when interacting with the population.

The set of strategies from Axelrod's second tournament was used to perform the ecological tournament. Several interesting insights were reported,

- The lowest ranking 11 strategies had fallen to half their initial size by the 5 generation;
- The middle-ranking entries managed to hold their initial size;
- By the 500^{th} generation the only strategies that were larger than their initial size have been the top 11 ranked strategies;
- These formed 96% of the population at that time;
- The rule which ranked fifth in the tournament, submitted by William Adams, grew to three times its original size in the population and then began to sink after generation 100;
- The rule which ranked eighth, submitted by Paul D. Harrington, and was the only non nice rule in the top 15, grew to four times its original size but began to shrink after generation 150 to reach only a third of its original size by the 1000^{th} generation.

Overall the strategies that did rank at the top of the second tournament have also ranked top in the ecological tournament. On the same note, the strategy that was ranked at the top was again Tit for Tat. By the 1000th generation it was 14.5% of the whole population, followed by the third place rule at 13.9% and then the second place rue at 13.1%, Tit for Tat was growing at .05% per generation which was a faster rate than any other strategy. All these ar captured in Figure 3.

The ability of strategies to be favoured under natural selection and their ability to withstand invasion from other strategies soon became a measure of performance; refereed to as the stability of a strategy.

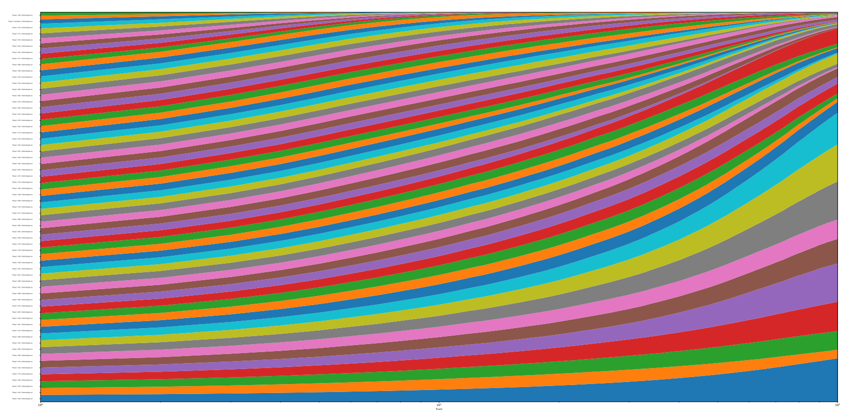


Figure 3: System evolving over time based on natural selection using [5], strategies set from Axelord's second tournament.

A much more general approach was discussed in [14]; the evolutionary approach. Imagine a population made up of individuals where everyone follows the same strategy, B and a single individual adopts a mutant strategy A. Strategy A is said to invade strategy B if,

$$V(A \mid B) > V(B \mid B) \tag{3}$$

where $V(B \mid B)$ is the expected payoff received by B against itself.

Since the strategy B is an population that interacts only with itself, the concept of invasion is equivalent to a single mutant being able to outperform the average population. This leads to the concept of the evolutionary approach. Thus for a strategy to be **evolutionary stable** it must be able to resists any invasion. There are several applications in biology for the interpretation of this approach, for example the survival of the fitness in wildlife.

Due the large number of possible strategies in the prisoner's dilemma identifying all the stable strategies was a difficulty task at the time. Axelrod focused the work of [14] in three questions,

1. Under what conditions was Tit for Tat evolutionary stable?

2. What were the necessary and sufficient conditions for any strategy to be evolutionary stable?
3. Finally, in an environment where all followed a strategy of unconditional defection, can cooperation emerge?

A series of theorems were presented which showed, that Tit for Tat is evolutionary stable y if and only if it is invadable neither by Defector nor Alternator. This is true only if the game is likely to last long enough for the retaliation to counteract the temptation to defect, according to Axelrod. Secondly, Defectors can withstand invasion by any strategy, as long as the players using other strategies come one at a time. But if they come in clusters (even in rather small clusters), the strategy could be invaded. As for the characteristics of stable strategies, Axelrod provided a series of theorems.

2.2.1 Response to the computer tournaments (1984-1993)

The pioneering work of computer tournaments and the results on the reciprocal behaviour of the prisoner's dilemma spread the knowledge of the game not only worldwide but also across different scientific principles. The study of cooperation became of critical interest once again. This section focuses on the immediate research that was carried out after the initial computer tournaments.

Ecological studies that made use of Axelrod's results include the works of [36, 59, 86], more specifically how the successful strategy Tit for Tat can be applied in nature and wildlife.

In [59] the behaviour of fish when confronting a potential predator was studied. Conflicts can arise within pairs of fish in these circumstances. Two experiments were held using a system of mirrors where sticklebacks would be accompanied by a cooperating companion or a defecting one. In both cases the hypothesis that the fish would behave according to Tit for Tat and that cooperation would evolve were supported. The works of [36, 86] looked at food sharing between vampire bats and explained behaviour based on famous at that time tournament strategies.

Axelrod's tournaments assumed that each player has perfect information of the opponent's actions. In real life situations this is not always the case. Interactions often suffer from measures of uncertainty. In the original tournaments there was no possibility of misunderstanding.

In 1985, P. Molander tested the robustness of Tit for Tat in an uncertain environment by introducing noise [62]. Noise is a probability that that one's move will be flipped. Molander findings stated that if two strategies playing Tit for Tat meet in a noisy match the average payoff that a strategy will receive will be the same as that of a Random player (with probability 0.5 of cooperating).

Further work on the performance of Tit for Tat in uncertain environments was conducted, described in [43, 36, 65]. These works focused, similar to Molander's, focused on how the strategy suffers against itself the most. In a noise environment, where a random defection can occur, the two strategies would end up in an unwanted circle of defection-cooperation. In a non noisy environment the strategies would have cooperated until the final interaction.

In [43] a similar tournament to that of Axelrod's was performed but this time noise was used. J. Bendor invited academics to submit strategies to participate in the tournament. A total of thirteen strategies were used including already existed strategies such as Tit for Tat and **Tit for Two Tats** [17]. Tit for Two Tats is a variant of the classic strategy that defects only when the opponent has defected twice in a row. The findings of the tournaments suggested that a more forgiving strategy is needed in a noisy environment. The winner of this tournament was a strategy called **Nice and Forgiving**.

The work of [65] aimed to also investigate stochastic effects. Using an evolutionary setting of a heterogeneous population where noise is taken into account, the space of reactive strategies was explored. Though a small fraction of Tit for Tat players have been essential for the emergence of cooperation, more generous strategies took over the population. This reactive strategy was is known as **Generous Tit for Tat** and can be presented as $(0, \frac{2}{}3)$.

Reactive strategies are a subset of memory one strategies introduced in 1989 [66]. Reactive strategies are denoted by the probabilities to cooperate after an opponent's **C** or **D** respectively. Thus, a reactive strategy only considers the previous turn of the opponent. Memory one strategies, are a set of strategies that consider the entire last turn of the game to decide on a next move.

Memory one strategies were also introduced by M. Nowak in 1990 [67]. Depending on the simultaneous moves of two players the states of the game, when only the previous round is considered, a state where both cooperated, both defected or either of them defected. These states are represented as CC, CD, DC, DD. A memory one strategy can be written as the probability of cooperating after each of these states. Thus as a vector of four probabilities p where $p = (p_1, p_2, p_3, p_4) \in \mathbb{R}^4_{[0,1]}$. Reactive strategies are just a constrained version of memory one strategies where $p_1 = p_3$ and $p_2 = p_1$.

The above formulation offered a new framework of studying strategies. Consider that two memory one strategies are in a game of the prisoner's dilemma. Their interaction can be written as the following markov chain,

$$M = \begin{bmatrix} p_1q_1 & p_1(-q_1+1) & q_1(-p_1+1) & (-p_1+1)(-q_1+1) \\ p_2q_3 & p_2(-q_3+1) & q_3(-p_2+1) & (-p_2+1)(-q_3+1) \\ p_3q_2 & p_3(-q_2+1) & q_2(-p_3+1) & (-p_3+1)(-q_2+1) \\ p_4q_4 & p_4(-q_4+1) & q_4(-p_4+1) & (-p_4+1)(-q_4+1) \end{bmatrix} \tag{4}$$

where the opponent is denoted as $q = (q_1, q_2, q_3, q_4) \in \mathbb{R}^4_{[0,1]}$. The expected state that two opponents will end up can be estimated by calculating the steady states of the markov chain.

Nowak, as described, studied the reactive but also the memory one strategies space and introduced several other strategies, among them the most popular was **Pavlov**. Pavlov is a strategy with the tolerance of Generous Tit for Tat but also the capability of resisting and invading an all-out cooperators population. The strategy is based on the fundamental behavioural mechanism win-stay, lose-shift. It starts off with a cooperation and then repeats it's previous move only if it was awarder with a payoff of R or T. Otherwise it shifts it's last move.

A number of researchers searched for new strategies. Such strategies have been, **Handshake** [74] and **Gradual** [20]. Presented in 1989 and 1997 respectively.Handshake was developed using an evolutionary tournament, where Gradual performance was tested in both a round-robin tournament and ecological simulation.

Handshake is a strategy that starts with cooperation, defection. If the opponent plays in a similar way then it will cooperate forever, otherwise it will defect forever. Gradual starts off by cooperating, then after the first defection of the other player, it defects one time and cooperates twice. After the second defection of the opponent, it defects two times and cooperates twice. After the n^{th} defection it reacts with n consecutive defections and then two cooperations.

Another measure of uncertainty is that of mis perception. Though noise will flip a player's action it will be recorded correctly in the history. Mis perception is the probability that the opponent's current move is flipped before being recorded [41].

2.3 Evolutionary Dynamics (1987-1999)

Determining the evolutionary stability of strategies for the iterated prisoner's dilemma as we discussed is not an easy task. Methods can be use to deal with the difficulty. In [22] the author restricted the possible strategies that could be adopted to a relatively narrow set and resulted that no pure strategy is evolutionary stable, including Tit for Tat. Arguing with the results presented in [14]. The list of strategies used included strategies such as Defector and **Suspicious Tit for Tat**, a strategy that plays Tit for Tat but starts by defecting.

The results were questioned by [54], stating that much was still no fully explored and more research had to be put into the results. Farrel and Ware in 1989 [27] extended the result to include finite mixture of pure and mixtures of Tit For n Tats as well. On the same year the work of [21] looking again at a narrow set of strategies extended their results to noisy environments.

Evolutionary dynamics have been highly useful in the research of the prisoner's dilemma. In [15], an evolutionary process, called the genetic algorithm, was used to discover effective strategies. The author introduced lookup tables as a mean of representing a strategy in a gene format. A lookup table is a set of deterministic responses based on the opponents m last moves; [15] considered $m = 3$.

An extension to the natural selection was introduced in the 1992 [69], recommending a different type of topology. A population of two deterministic strategies, Defector and Cooperator, were placed on a a two dimensional square array where the individuals could interact only with the immediate neighbours. The number of immediate neighbours could be either, fourth, six or eight. As shown in Figure 4. The authors claimed that the essential results remain true of all topologies; the results also hold whether self interactions are taken into account.

Thus each cell of the lattice is occupied by a Cooperator or a Defector. At each generation step each cell owner interacts with its immediate neighbours. The score of each player is calculated as the sum of all the scores the player achieved at each generation. At the start of the next generation, each lattice cell is occupied by the player with the highest score among the previous owner and the immediate neighbours. This topology is refereed to as spatial topology.

Nowak studied the population dynamics as a function of the temptation payoff. It was shown that for different values of the temptation payoff, cooperators and defectors could persist together.

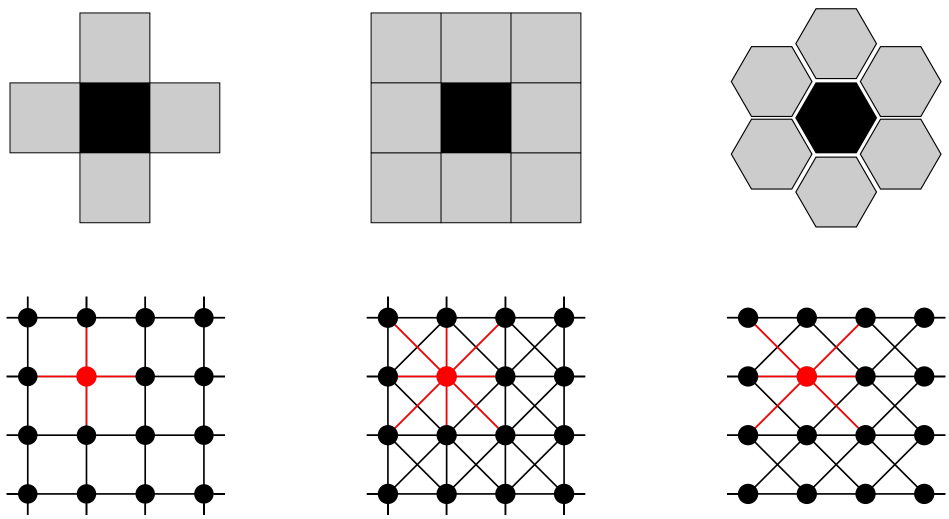


Figure 4: Spatial neighbourhoods

This work dealt with dealt with symmetric spatial lattices in two dimensions, deterministic winning and discrete time. The authors in later work [68], that the results remain valid in more realistic situations. Such as situations where the spatial distributions of cells are random in two or three dimensions, and where winning is partly probabilistic.

2.4 Modern approaches (1995-2015)

A number of aspects discussed in the previous sections such as round robin tournaments, evolutionary tournaments, training of strategies and noise environments soon became standard means of studying the iterated prisoner's dilemma. In this section we review a number of computer tournament that used these methods and introduced a number of findings that made an impact in the literature.

Initially in 1995 a combination of tournament studies, ecological simulations and theoretical analysis was used in [87] to demonstrate approaches to copy with noise. These included generosity, contrition and win-stay, lose-shift by respectively using the strategies Generous Tit for Tat, **Contrite Tit for Tat** and Pavlov. A fourth strategy was also analysed **Generous Pavlov**. A strategy that acts like Palvov but cooperates 10% of the time when it would defect otherwise.

- Adaptive Tit for Tat [83].

Less generous variants also made an appearance [40]. **Anti Tit for Tat**, is a strategy that plays the opposite of the opponents previous move. Another limitation of the strategy was discussed in [78]. Tit for Tat was proven to hit a loop between cooperation and defection. **Omega Tit For Tat** was introduced and was a strategy capable of avoiding such problem [78].

In 2011 the authors of [49] performed their own tournament where several interesting strategies made an appearance.

- **Periodic player CCD**, plays **C**, **C**, **D** periodically. Note that variations of a period player also make appearance in the article but will not be listed here.
- **Prober**, starts with the pattern **D**, **C**, **C** and then defects if the opponent has cooperated in the second and third move; otherwise, it play as Tit for Tat.
- **Reverse Pavlov**, a strategy that does the reverse of Pavlov.

In earlier work the same author introduced a strategy called **APavlov**, which stands for adaptive Pavlov [48]. The strategy attempts to classify the opponent as one of the following strategies, All Cooperator, All Defector, Pavlov, Random or **PavlovD**. PavlovD, is just Pavlov but it starts the game with a **D**. Once Adaptive Pavlov has classified the opponent plays to maximize it's payoff.

Evolutionary dynamics and optimization methods are used with different representation methods in order to discover new optimized strategies. Include lookup tables [15, 51], artificial neural networks [39, 47] and finite state machines [61, 76].

Strategies based on finite state machines are described by the number of states. The strategy selects the next action in each round based on the current state and the opponent's last move, transitioning to a new state each time. Figure 5, illustrates the finite state representation of Tit For Tat.

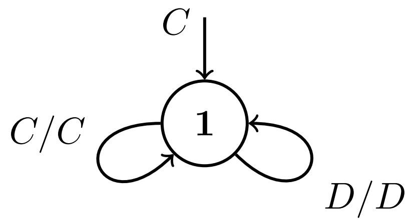


Figure 5: Finite state machine representation of Tit for Tat.

In [11] the author presented two new strategies that have been trained using a finite state machine representation. They are called, **Fortress3** and **Fortress4**. Figure 6 illustrates their diagrammatic representation where the transition arrows are labelled O/P where O is the opponent's last action and P is the player's response.

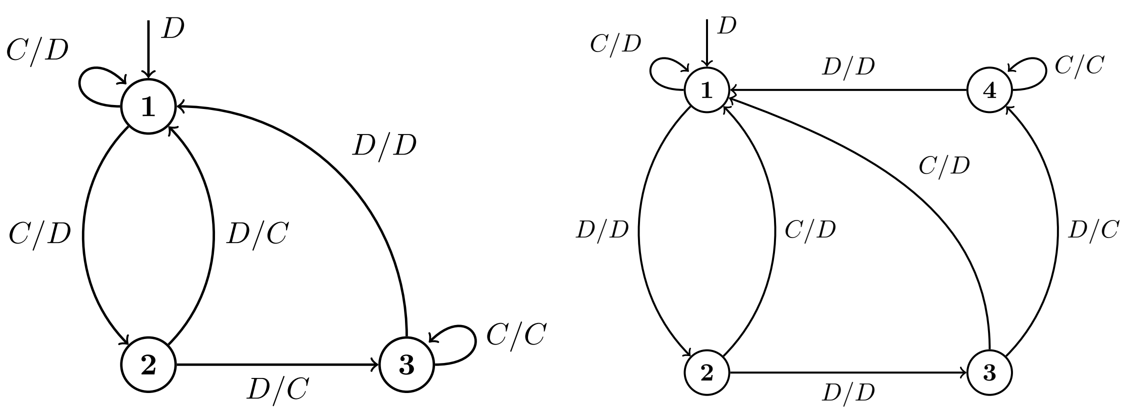


Figure 6: Representations of Fortress 3 and Fortress 4. Note that the strategy's first move, enters state 1, is defection for both strategies.

Optimisation methods will return a spectrum of strategies. In order to distinguish the strategies and assuring that they are indeed different [6] introduced a method called fingerprinting.

The method of fingerprinting is a technique for generating a functional signature for a strategy [7]. This is achieved by computing the score of a strategy against a spectrum of opponents. The basic method is to play the strategy against a probe strategy with varying noise parameters. In [6] Tit for Tat is used as the probe strategy. Fingerprint functions can then be compared to allow for easier identification of similar strategies. In Figure 7 an example of Pavlov's fingerprint is given. Fingerprinting has been studied in depth in [7, 8, 9, 10].

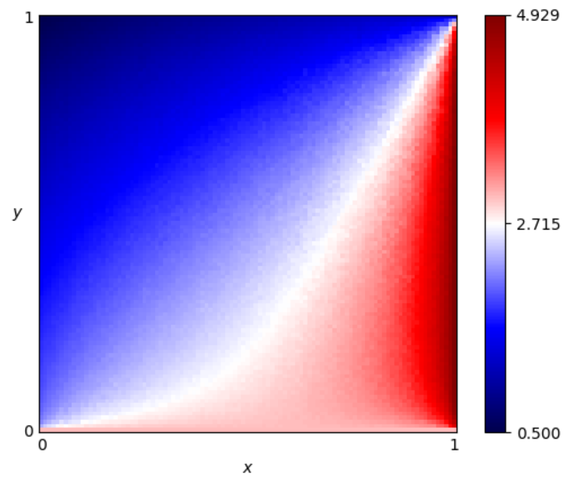


Figure 7: Pavlov fingerprinting with Tit for Tat used as the probe strategy. Figure was generated using [5].

Due the nature of the research several pieces of software are starting to appear, this includes a library called PRISON [4]. PRISON is written in the programming language Java and it has been used by it's authors in several publications. The project includes a good number of strategies from the literature but unfortunately the last update of the project dates back in 2004.

2.5 Zero determinant (2012 - 2015)

Following Section 2.4, this section is a review of an important set of strategies, the zero determinant.

In [71], a new set of memory one strategies were introduced, called **zero determinant (ZD)** strategies. The ZD strategies, manage to force a linear relationship between the score of the strategy and the opponent. Press and Dyson, prove their concept of the ZD strategies and claim that a ZD strategy can outperform any given opponent.

The ZD strategies have attracted a lot of attention. It was stated that "Press and Dyson have fundamentally changed the viewpoint on the Prisoner's Dilemma" [79]. In [79], a new tournament was performed including ZD strategies and a new set of ZD strategies the **Generous ZD**. Even so, ZD and memory one strategies have also received criticism. In [47], the 'memory of a strategy does not matter' statement was questioned. A set of more complex strategies, strategies that take in account the entire history set of the game, were trained and proven to be more stable than ZD strategies.

2.6 Contemporary period (2015 - 2017)

Following a discussion on research of short memory strategies this section reviews recent work done in complex strategies. As well as a discussion of new software and how modern approaches allows us to now revisit several pieces of work produced in the past.

Modern approaches of artificial neural networks and machine learning are now used in the field. A number of strategies based on artificial neural networks are introduced by [38]. Artificial neural networks provide a mapping function to an action based on a selection of features computed from the history of play.

These strategies are refereed to as **EvovlvedANN** strategies and are based on a pre-trained neural network with the following features,

- Opponent's first move is C
- Opponent's first move is D
- Opponent's second move is C
- Opponent's second move is D
- Player's previous move is C
- Player's previous move is D
- Player's second previous move is C
- Player's second previous move is D
- Opponent's previous move is C
- Opponent's previous move is D
- Opponent's second previous move is C
- Opponent's second previous move is D
- Total opponent cooperations
- Total opponent defections
- Total player cooperations
- Total player defections
- Round number

A representation of **EvovlvedANN 5** is given in Figure 8. The inputs of the neural network are the 17 features as listed above. Number 5 reefers to the size of the hidden layer.

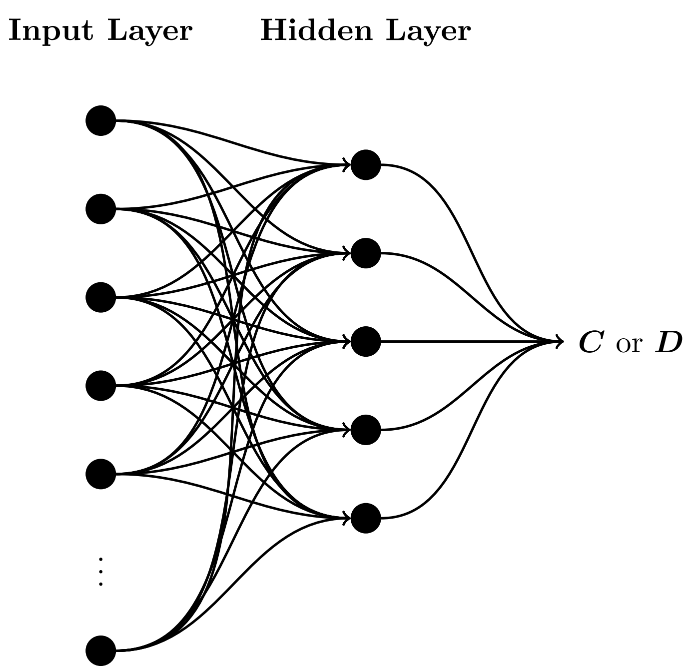


Figure 8: Neural network representation of EvovlvedANN 5.

In [38], these representing methods are refereed to as archetypes. Finite state machines and artificial neural networks are included in the work but also new archetypes are introduced, such as hidden Markov models. A variant of a finite state machine that use probabilistic transitions based on the prior round of play to other states and cooperate or defect with various probabilities at each state. Finite state machines and hidden Markov models based strategies are characterized by the number of states. Similarly, artificial neural networks based players are characterized by the size of the hidden layer and number of input features.

Additionally a variant of a look up table is also presented called the lookerup archetype. The lookerup archetype responses based on the opponent's first n_1 moves, the opponent's last m_1 moves, and the players last m_2 moves. Taking into account the initial move of the opponent can give many insights. For it is the only move a strategy is truly itself without being affected by the other player. As a reminder, Axelrod in his work highlighted the importance of the initial move and believed that it was one of the secrets of success of the strategy Tit for Tat.

Finally, a new archetype called the Gambler is also introduced, which is a stochastic variant of the lookerup archetype.

Archetypes are used with evolutionary algorithms to train set of new strategies. The evolutionary algorithm used in both [15, 32] is called genetic algorithm. Other algorithms including particle swarm optimization have been used in research of the most dominant strategy [29].

In [38] the approach in used to introduce as stated by the authors the best performing strategies for the iterated prisoner's dilemma. These strategies will be refereed as **Evolved** strategies. Several successful new strategies are,

- **EvolvedLookerUp2_2_2** a looker up strategy trained with a genetic algorithm; EvolvedLookerUp2_2_2 responses based on the opponent's 2 first and last moves and the player's 2 last moves. Thus $n_1 = 2, m_1 = 2$ and $m_2 = 2$.
- **Evolved HMM 5** a 5 states hidden markov model trained with a genetic algorithm;
- **Evolved FSM 16** a 16 state machine trained with a genetic algorithm;
- Finally **PSO Gambler 2 2 2** a looker up strategy trained with a particle swarm algorithm, where $n_1 = 2, m_1 = 2$ and $m_2 = 2$.

Though several papers have claimed before to have discovered the dominant strategies for the game the work of [38] is promising. This is due the fact that the introduced strategies have been trained using different types of evolutionary algorithms in a pool of 176 well known strategies for the literature. Including all the strategies that have been discussed in this section.

This was made possible due an open source library, called the Axelrod project [5]. The project is written in the programming language Python, it is accessible and open source. To date the list of strategies implemented within the library exceed the 200. The project has been used in several publications including [38] and a paper describing it and it's capabilities was published in 2016 [44]. The source code for Tit for Tat as implement within the library is shown in Figure 9. Furthermore, performing a tournament with a selection of strategies is possible in five lines of code, shown in Figure 10.

```python
def strategy(self, opponent: Player) -> Action:
    """This is the actual strategy"""
    # First move
    if not self.history:
        return C
    # React to the opponent's last move
    if opponent.history[-1] == D:
        return D
    return C
```

Figure 9: Source code for Tit for Tat in Python as implemented in Axelrod Python library [5]

.

```python
>>> import axelrod as axl
>>> players = (axl.Cooperator(), axl.Defector(), axl.TitForTat(), axl.Grudger())
>>> tournament = axl.Tournament(players)
>>> results = tournament.play()
>>> results.ranked_names
['Defector', 'Tit For Tat', 'Grudger', 'Cooperator']
```

Figure 10: Performing a computer tournament using [5].

Software has a crucial role in research. Well written and maintained software allows the reproducibility of prior work and can accelerate findings within the field. The field of the iterated prisoner's dilemma has suffered the consequences of poor research software. As stated above the source code of the initial computer tournament is not retrievable. Several of the strategies that competed in the tournament are not given a full explanation of how the decided on their next move. In terms of best practice and reproducibility the Axelrod library is the lead software in the field.

Other recent projects include [2, 3], both are education platforms and not research tools. In [2], several concepts such as the iterated game, computer tournaments and evolutionary dynamics are introduced through a user interface game. Project [3] offers a big collection of strategies and allows the user to try several match and tournaments configurations. Such as noise.

In [73], the authors claim that they have managed to re-run the first tournament that Axelrod performed. They tried to push his work further by altering aspects such as, the format of the tournament, the objective and the population. One of the authors claimed to have been a contributor to the first tournaments, which would explain how it was managed to reproduce the tournament.

2.6.1 Biological Applications

- [82] uses evolutionary game theory to study the spread of virus.
- [34] a shout for his work, using tit for tat to study cells.

3 Data Analysis

In this section we will focus on the analysis of the study of the prisoner's dilemma using a large dataset of articles. This data set will be used to ascertain the level of collaborative nature of the field and identify influencer. This will be done relative to:

- Other sub fields of game theory: auction games [58] and the price of anarchy [75].
- A temporal analysis.

3.1 Data Collection

Before analysing our data in this subsection we will describe the data collection process.

Academic articles are accessible through scholarly databases and collections of academic journals. Several databases and collections today offer access through an open application protocol interface (API). An API allows users to query directly a journal's database and skip the user interface side of the journal. Interacting with an API has two phases:

1. requesting;
2. receiving;

The request phase includes composing a url with the request. Figure 11 presents an example of a request message. The first part of the request is the address of the API we are querying. In this example the address corresponds to the API of arXiv. The second part of the request contains the search arguments. In our example we are requesting for a single article that the word 'prisoners dilemma' exists within it's title. The format of the request message is different from API to API.

The receive phase includes receiving a number of raw metadata of articles that satisfied the request message. The raw metadata are commonly received in a xml or a json format [70]. Similarly to the request message, the structure of the received data differs from journal to journal.

The data collection is crucial to this study. To ensure that this study can be reproduced all code used to query the different APIs has been packaged and is available online [63]. The software could be used for any type of projects similar to the one described here, documentation for it is available at: `http://arcas.readthedocs.io/en/latest/`.

```
http://export.arxiv.org/api/query?search_query=abs:prisoner's dilemma&max_results=1
```

Figure 11: A request message for the arXiv API.

Project [63] allow us to collect articles from a list of APIS by specifying just a single keyword. The following sources were used to collect data of this analysis:

1. PLOS [31];
2. Nature [35];
3. IEEE [42];
4. arXiv [55];
5. Springer [57].

These are four prominent journals in the field, as well as the arXiv [55] pre print server. In the case of an article being both in a journal and the arXiv, only the journal version was considered.

For each article [63] collects a list of the features, shown in Table 4. Note that the plain text of the article is not collected, just the metadata. The data is archived and available at. In this work only the features of Table 5 are used.

	Result name	Explanation
1	Abstract	The abstract of the article.
2	Author	A single entity of an author from the list of authors of the respective article.
3	Date	Year of publication.
4	Journal	Journal of publication.
5	Key	A generated key containing an authors name and publication year (ex. Glynatsi2017).
6	Keyword	A single entity of a keyword assigned to the article by the given journal.
7	Labels	A single entity of labels assigned to the article manual by us.
8	Pages	Pages of publication.
9	Provenance	Scholarly database for where the article was collected.
10	Score	Score given to article by the given journal.
11	Title	Title of article.
12	Unique key	A unique key.

Table 4: Metadata for each entry/article.

	Result name	Explanation
1	Abstract	The abstract of the article.
2	Author	A single entity of an author from the list of authors of the respective article.
3	Date	Year of publication.
4	Journal	Journal of publication.
5	Provenance	Scholarly database for where the article was collected.
6	Title	Title of article.

Table 5: Structure of data set. Contained results.

A series of keywords were used to identify relevant articles. Articles for which any of these keywords existed within the title or the abstract are included in the analysis. A list of the keywords that were used for the main data set are shown in Table 6.

Similarly for collecting data on auction games and the price of anarchy the following keywords were used:

	Keywords
1	prisoner's dilemma
2	prisoners dilemma
3	tit-for-tat
4	tit for tat
5	zero determinant strategies

Table 6: Keywords used in searching for articles.

- key: auction game theory;
- key: price of anarchy.

For both of these topics only a single keyword has been used. In comparison 5 different keywords were used to search of articles on the prisoner's dilemma. This is because the main focus of this analysis is the prisoner's dilemma. The aim has been been to collect as many articles as possible for the sources, so that the results of this work would be accurate.

Even so, the amount of articles collected from the key such as 'tit for tat' and 'zero determinant' had a small contribution to the size of the data set.

3.2 Preliminary Analysis

A total of three data sets are explored in this work. A summary of each data is presented in this section. The three data sets are:

- The main data set which contains articles on the prisoner's dilemma.
- A secondary data set which contains article on auction games.
- A secondary data set which contains articles on the price of anarchy.

3.2.0.1 The prisoner's dilemma data set

The main data set and the main focus of this analysis. The data set [] consists of 1150 articles, where 1145 have unique titles. This is because a total of 5 articles have been collected from both a journal and arXiv. All duplicates from arXiv are dropped, thus hereupon we consider 1145 unique article entries.

There are a number of 41 articles that have not been collected from the aforementioned APIs. These articles were manually added to the dataset throughout the writing of Section 2. A more detailed summary of the articles' provenance is given by Table 7.

The larger number of articles were collected from arXiv, Springer and IEEE. Both Nature and PLOS have a small contribution to the size of the data set. The oldest article was published in 1944 and the most recent one in 2017. Note that the latest data collection was on December 2017.

Moreover we calculate the average publication over time. This is done for the overall data set and for each journal individually. This is denotes as,

$$\mu_P = \frac{N_A}{N_Y},$$

	Provenance	Total articles
1	arXiv	470
2	Springer	312
3	IEEE	241
4	PLOS	63
5	Nature	23
6	Manual	41

Table 7: Articles' provenance for [].

where N_A is the total number of articles and N_Y is the years of publication. The years of publication is calculated as the range between 2017 and the first published article within the data.

Table 8 summarises these averages. Overall an average of 21 articles are published per year on the topic. The most significant contribution to this appears to be from arXiv with 8 articles per year, followed by Springer with 5 articles per year.

	Av. publication
Overall	21.167
IEEE	4.463
Nature	0.426
PLOS	1.167
Springer	5.741
arXiv	8.611

Table 8: Average publication for [].

Though the average publication can have insights of the publications of the fields it is still a constant number. Our data are changing are over time. More specifically the data are non stationary. Time series data are said to be stationary if their statistical properties such as mean and variance remain constant over time. An Augmented Dickey-Fuller test is used to verify that the data are non stationary. The results show that the data is non stationary: the p value is greater than 0.005.

Calculating the rolling average we can see that our data have an increasing trend. This is presented in Figure 12. The rolling average of each time point is calculated as the average of the points on either side of it. The number of neighbourhood points is specified as window size and here we have chosen a window size of 5.

The rolling average indicates that the time series has an increasing trend. Even so there seems to be a small decrease by the last time point. In order to offer some insights as to what expected of the field in the next years we conduct a forecast of the next time periods. An Autoregressive Integrated Moving Average Model (ARIMA (1, 1, 1)) has been used.

Though details of the ARIMA models are not going to be given here the models parameters had to be fitted [23]. This was done using the Akaike Information Criterion value. ARIMA also has been chosen because it can handle non stationary data. The results of the forecast of the next following 10 years are given by Table.

thought the time series have indicated a slight decrease we can see that the model forecasts an increase over the next years.

3.2.0.2 Auction games and the price of anarchy data sets

The secondary data sets are described in this subsection. More specifically, a summary of both data sets, auction games [] and price of anarchy [], is given by Table 10.

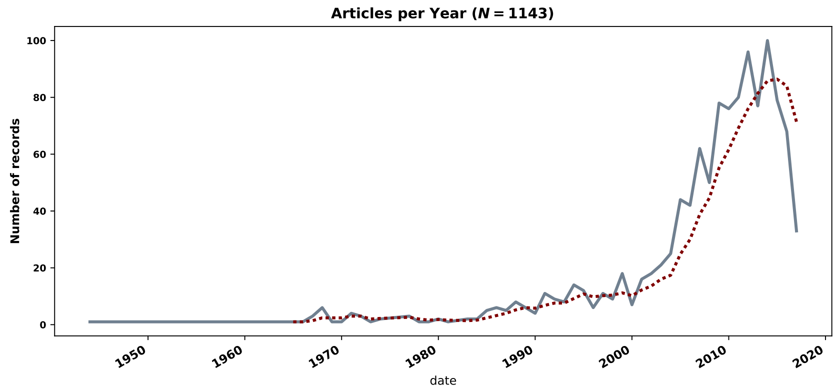


Figure 12: Time series of main data set and the rolling average.

	Forecast
1	27.301817
2	24.500512
3	26.490962
4	26.208756
5	27.004436
6	27.288891
7	27.815812
8	28.227735
9	28.694200
10	29.134797

Table 9: Forecasting the number of publications over the next 10 years.

A total of 2103 articles with 3860 unique authors are examined for auction games. Auction games is well studied topic with the earliest entry going back to 1974. In comparison, 296 unique articles have been collected on price of anarchy. The earliest entry being in 2003 and a total of 668 unique authors have written about the topic.

In Figure 13 a time plot for each topic is displayed and is exhibited that both topics have had an increasing trend over the years. Though price of anarchy is clearly a new topic compared to auction games.

The frequency of the prisoner's dilemma, for both articles and authors, lies between the frequencies of these two topics.

	Price of anarchy	Auction games
Unique articles	296	2103
Unique authors	668	3860
Min publication year	2003	1974
Max publication year	2017	2017

Table 10: Data sets [] and [] summary.

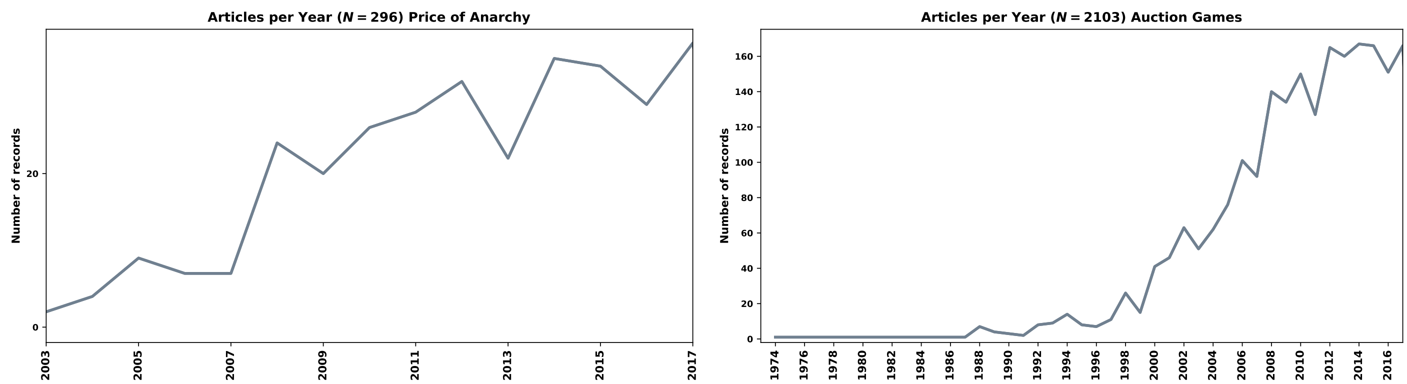


Figure 13: Time plots of [] and [].

The provenance of the articles is given by Table 11. Almost 1500 article for auction games have been collected from Springer, that is more than three times the articles that have been collected from other sources. PLOS and Nature have a minor contribution and PLOS and Nature had no articles on the price of anarchy.

Provenance	**Total articles** (auction games)	**Total articles** (price of anarchy)
Springer	1429	78
arXiv	436	108
IEEE	301	131
PLOS	15	-
Nature	1	-

Table 11: Articles' provenance for [] and [].

The overall average publication for [] and [] are 59 and 20 articles respectively. It appears that auction games publication is largely different for both the prisoner's dilemma and the price of anarchy. These two topics have the same average publication. Note that the significance of each journal differs from topic to topic. Though this analysis will not focus on individual sources from hereupon.

Provenance	**Av. publication** (auction games)	**Av. publication** (price of anarchy)
Overall	58.973	19.812
Springer	38.622	4.875
arXiv	11.784	6.750
IEEE	8.135	8.188
PLOS	0.405	-
Nature	0.027	-

Table 12: Average publication for auction games and the price of anarchy.

3.2.0.3 Temporal analysis

For comparison reasons in the following subsections the analysis will also be held relative to a temporal analysis. The main data set [] is divided into time period according to the subsections of Section 2. The respective measures of unique titles and unique authors for each period is given by Table 13.

	Unique articles	Unique authors
period 1	21	38
period 2	5	6
period 3	64	70
period 4	121	169
period 5	926	1730
period 6	453	1008
period 7	180	466

Table 13: Periods and their respective measures.

In this section we have described the three data sets that we are going to use in the following sections in order to identify collaborative behaviour and influence. Two data sets of different topics are used for comparison reasons. The frequency of articles and authors differs within the three data sets which is ideal. Furthermore the journals effect also differ but not much will be done at that level form now onwards.

Finally a temporal analysis of the data sets [] will also assists us in obtaining more insights. The period have also be presented here. In the following section we explore the authors of the papers and their connections to one another.

3.3 Co authorship Analysis

Broadly speaking, most academic research is undertaken in the form of collaborative effort. As discussed in [46], it is rationale that two or more people can do better as a group than individually. Academic collaborations have many different forms. Researchers might have immediate collaborated and written together. Others might have collaborated through an common co author.

Collaboration in groups has a long tradition in experimental sciences and it has be proven to be productive according to [25]. Even so, the number of collaborations can be very different between research fields and measuring collaboration is not always an easy task. Moreover, we are not interested only in collaborative behaviour but also in the power of influence. For example academics can influence us through workshops, talks or by collaborating we people in our environment. Such as other researchers in our department or our supervisors.

Several studies tend to consider academic citations as a measure for these things. As discussed in [64], depending on citations can often be misleading. This is because:

- The true number of citations can not be known. Citations can be missed due data entry errors or typos in a journal.
- Academics are influenced by many more papers than they actually cite.
- Several citations are superficial.

We are in agreement with the above reasons and here we suggest that new way of measuring collaboration and influence must be introduced. We suggest that this should be done by looking the co authorship network. A co authorship network, is a network where academics that have written and published together are connected. Moreover, people that have not written together but have with the same co author can reach each other.

We must properly define what collaboration and influence means to us in order to measure them using the co authorship network.

An academic field will never be fully connected, instead there will be various academic groups of people who are connected to each other. By collaborative behaviour we mean the average connections an author can have in their academic group. Moreover, how strongly close they are in that group of theirs.

Additionally, influence is defined as how much authors connect their groups. In essence, how many connections are made possible because of an author. A strong influence can have many advantages for a field. Authors can have a wider perspective for the undergoing research in their field, they can compare results and be more accurate. Thus they authors that do not necessarily have influence but can have a big gain from it. We are interested in both types, people that influence and people that gain from it.

In the following section we give a rigorous definition of the co authorship network. moreover we introduced several network measures that we will be using such as:

- Number of connected components.
- Clustering coefficient.
- Degrees distributions.
- Centrality measures.

3.3.1 Constructing a co authorship network

To construct a co authorship network we need to consider all the unique authors of a data set. The issue with retrieving the unique authors is that authors names can be written in different ways in different sources. For example consider the author of this paper:

- Nikoleta Glynatsi
- Nikoleta E. Glynatsi
- Nikoleta Evdokia Glynatsi

Consequently, several different entries of the same author existed within the data set. Thus we wanted to figure out when two author names were the same in real life. Though identifying if two string correspond to the same author is human possible the data sets consited of more than 1000 authors, thus we wanted to automate the procedure.

This was done by using the Levenshtein Distance [60]. The Levenshtein Distance is a metric for measuring the difference between two sequences. It is based upon the number of actions one has to take to transform one string into the other. These actions include:

1. Insertion;

2. Deletion;
3. Substitution of a single character.

Let us consider an example where we are trying to calculate the distance between the two strings ‘Wang’ and ‘Yang’. To compute the distance in a non-recursive way, we use a matrix D containing the distances between all the prefixes of the two strings. The first row and column are indexed by empty strings. The rest of the rows and columns are index by the prefixes of the two strings.

The matrix is filled from left to right. The first row is filled as follows:

1. To go from an empty string to an empty string zero actions are needed. Thus the $D_{\mathrm{e,e}}$ is 0.
2. To go from an empty string to ‘W’, or the other way around, 1 action is needed. Thus the $D_{\mathrm{e,W}}$ is 1.
3. For every new letter we have to take another action (+1).

Similarly, this is done for the first columns. For rest of the elements we follow a similar approach, but this time the previous distances are also taken in account. For example, $D_{\mathrm{Y,W}}$. For the letter ‘Y’ to go to ‘W’ a single action is required. Note that now 1 is added to the minimum distance between of $D_{\mathrm{e,e}}, D_{\mathrm{e,W}}$ and $D_{\mathrm{e,Y}}$.

Similarly we fill the rest of matrix. The last value computed, bottom right, is the Levenshtein Distance of the two strings. In our example it is calculated to be 1.

$$D = \begin{array}{c} \\ \scriptstyle e \\ \scriptstyle Y \\ \scriptstyle A \\ \scriptstyle N \\ \scriptstyle G \end{array} \begin{array}{c} \begin{array}{ccccc} \scriptstyle e & \scriptstyle W & \scriptstyle A & \scriptstyle N & \scriptstyle G \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 & 4 \\ 2 & 2 & 1 & 2 & 3 \\ 3 & 3 & 2 & 1 & 2 \\ 4 & 4 & 3 & 2 & 1 \end{bmatrix} \end{array}$$

In this work we calculate the ratio of two string matching for all possible pairs of authors in the data sets. The matching ratio is calculated as,

$$(1 - \frac{\mathrm{lev}}{m}) \times 100,$$

where lev is the distance and m is the length of the longest of the two words. If the ratio of a pair was between 85 and 99 both entries were highlighted. The highlighted entries were manually checked to assure that there were indeed the same author and then one of them was replaced by the other.

For example all entries with author name written as example 1 were replaced by 2.

1. Y. Moreno
2. Yamir Moreno

The manual check is performed because not all highlighted entries are indeed the same. For example:

1. Zhen Yang and
2. Zhen Wang

are two different authors. Once the name entries have been cleaned the co authorship networks can be defined. The definition of a co authorship network is given by:

Definition 3.1. Co authorship network. A co authorship network is an undirected network G of vertices V and edges E where vertices representing each unique author and an edge connects two authors if and only if those authors have written together. No weight has been applied to the edges nor the nodes.

Three such networks are studied here. These are:

- G_1 a prisoner's dilemma network, where $V(G_1) = 2101$ and $E(G_1) = 3174$.
- G_2 an auction games network, where $V(G_2) = 3676$ and $E(G_2) = 5643$.
- G_3 an auction games network, where $V(G_3) = 637$ and $E(G_3) = 865$.

The respective illustrations of G_1, G_2 and G_3 are given by Figures 14 and 15.

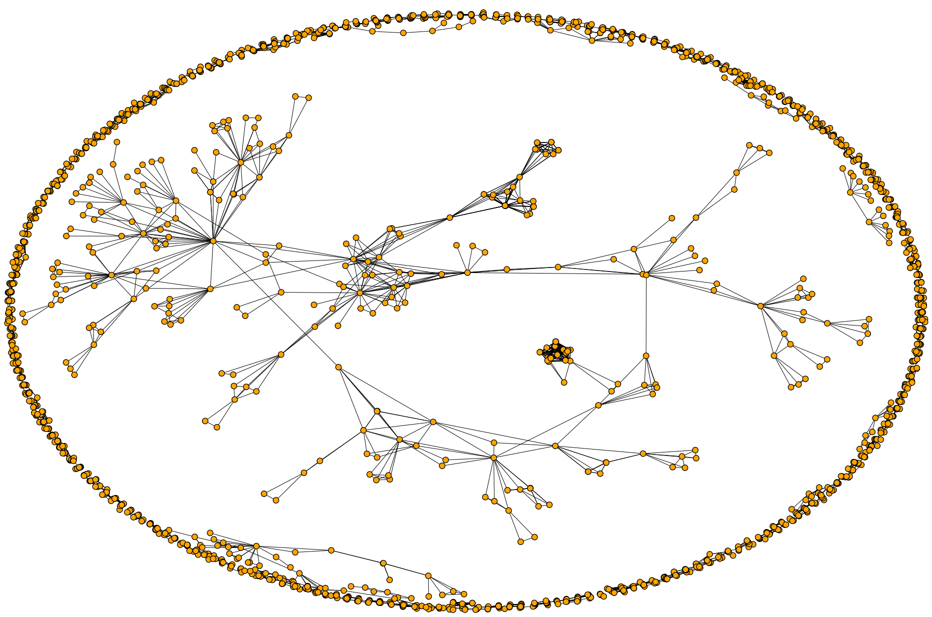
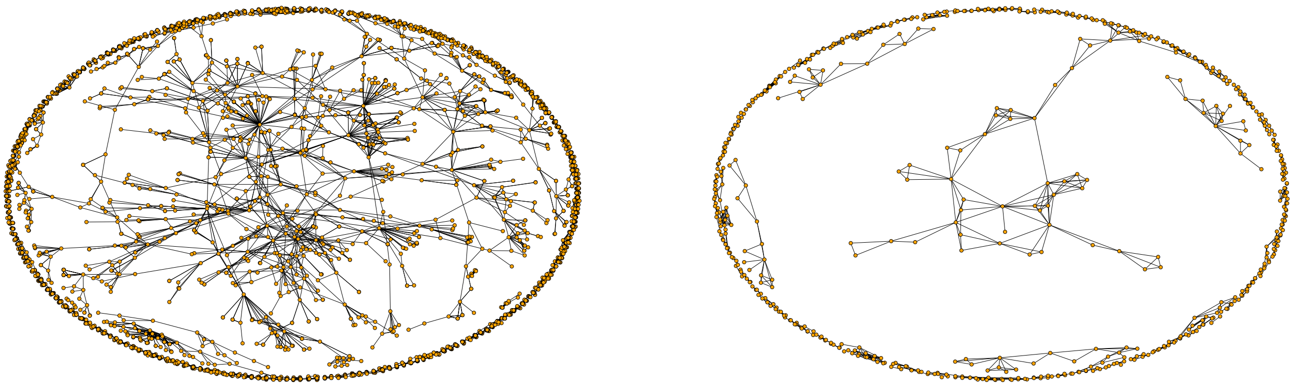


Figure 14: Co authorship network for prisoner's dilemma.

3.3.2 Collaborative behaviour

In this section we ascertain the level of collaborative nature of the field. The collaborative behaviour is measured as the connections authors can have within their groups. Moreover, how strongly connected these groups are. Several connectivity measures will be used to explain such behaviour, in this section these are introduced through various examples.

3.3.2.1 The measures

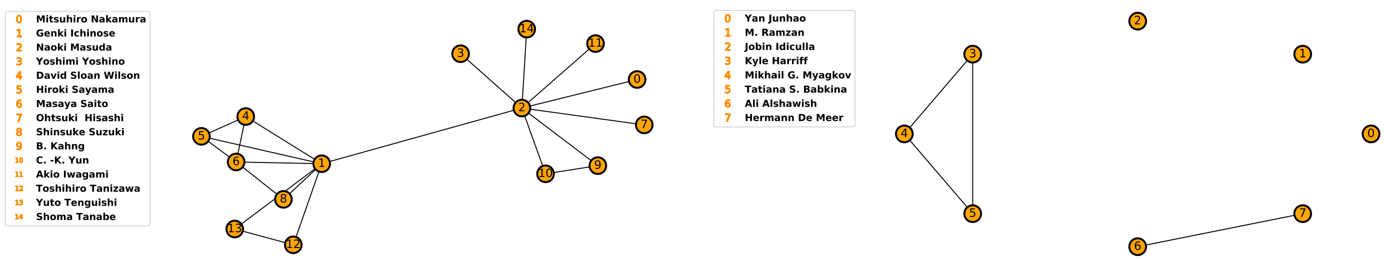


(a) Co authorship network for auction games.

(b) Co authorship network for the price of anarchy.

Figure 15: Co authorship network for other topics.

The first measure introduced is the number of connected components. A connected component of an undirected graph is a maximal set of nodes such that each pair of nodes is connected by a path. Two examples are illustrated in Figure 16. These are two different sub graphs of G_1 with a number of connected components of 1 and 5.



(a) A sub graph of G_1 with 1 connected component.

(b) A sub graph of G_1 with 5 connected component.

Figure 16: Connected components examples.

Note that a vertex with no incident edges is itself a connected component. Connected components show us how disjoint the network is. In essence the number of groups in the field.

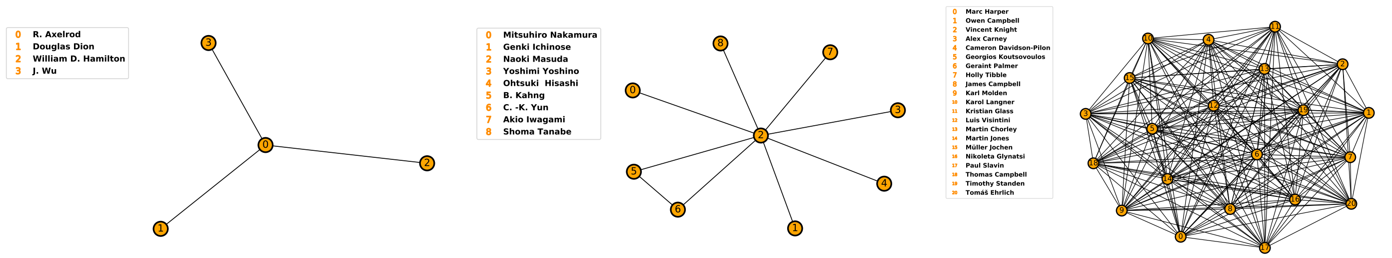
The second measure considered is the degree. The degree of a node express the number of connections a node has. We will consider the degree distribution of a network. It will allow us to understand the mean connection that authors have in the network's groups.

The final measure is the clustering coefficient. Clustering coefficient is a measure of the degree to which nodes in a graph tend to cluster together. There are two versions of the measure. The local and the global coefficients. The local coefficient is the clustering coefficient of a single vertex. It is calculated as,

$$C_u = \frac{2 \times L_u}{(k_u(k_u - 1)},$$

where k_u is the degree of vertex u and L_u is the number of edges between k_u neighbours of vertex u.

The global coefficient, $\bar{C}$, is calculated by averaging all the local coefficients of the graph. The values of the measure can range between 0 and 1. Figure 17 illustrates several sub graphs with different $\bar{C}$ values.



(a) A sub graph of G_1 with a clustering coefficient of 0.

(b) A sub graph of G_1 with a clustering coefficient of 0.23.

(c) A sub graph of G_1 with a clustering coefficient of 1.

Figure 17: Clustering coefficients examples.

A clustering coefficient of 1 indicates that that a sub graph is a complete graph. On the contrary, a coefficient of 0 indicates that authors write only with just a single co author. This case is not ideal in a co authorship group. A high clustering coefficient indicates that people within groups have several connections. There are not there due to a single publication with a random researcher but on the contrary that group is said to be collaborative.

3.3.2.2 Comparison with other topics co authorship

All connectivity measures are calculated for G_1, G_2 and G_3. This is done using the open source package [37].

We are aware that all three of the networks are disjoint. This is also verified by the number of connected components. More specifically there are 529 , 797 and 162 for graphs G_1, G_2 and G_3 respectively. Though the number of connected components for G_1 is between G_2 and G_3 we believe that this could be due the size of the data sets.

An interesting insights can be given by looking at the number of the isolated authors, thus the authors that have written by themselves. These are 51 , 63 , 4 . The price of anarchy has only a total of 4 authors that have not written with anyone else, indicating high collaboration. Graphs G_1 and G_2 appear to have a similar number of isolated authors.

Now that the number of groups for each topic have been calcutated we can still be very ojective. That is because the these could be affected by the size of the data sets. Thus to further examine the collaborative behaviour we consider the degree distributions. The normalised distributions of all three networks are shown in Figure 18. They have been normalised such that the frequencies sum to one. The distributions appear to be similar but to validate the hypothesis a statistical test is used.

None of the distributions is normally distributed thus the non parametric test Kruskal-Wallis is used [56]. Kruskal-Wallis allow us to compare the medians of two or more distributions. The test returns a p-value of 0.29. Thus, there is no significant difference in the degree distributions of the three networks.

Finally we calculate $\bar{C}$ for all three networks.

The price of anarchy appears to have the largest clustering coefficient. The price of anarchy did not only not have authors writting on their own but authors is groups appear to be clustered together. The prisoners dilemma and auction games have a similar clustering coefficient.

3.3.2.3 Temporal comparison

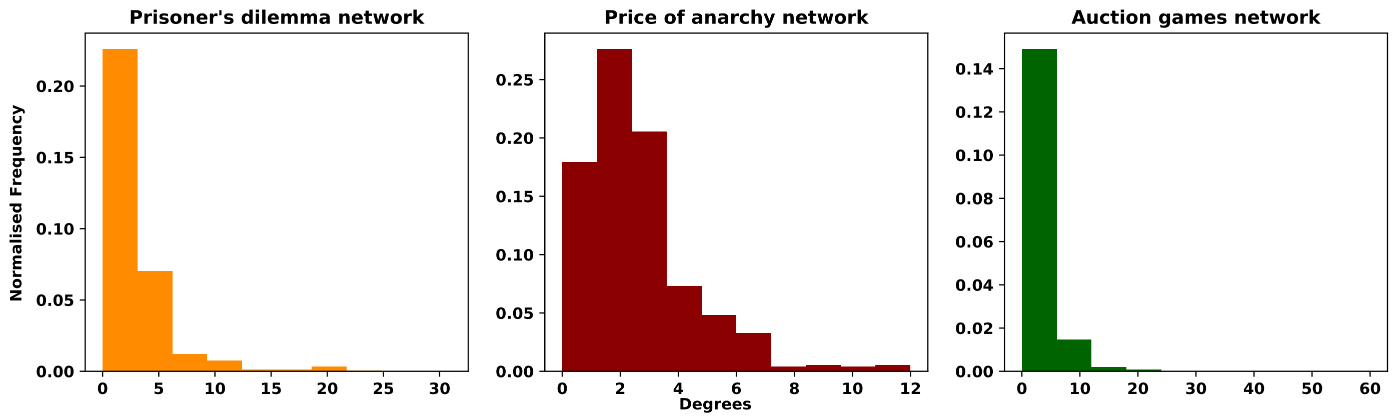


Figure 18: Degree distributions for all three networks.

	$\bar{C}$
G_1	0.68
G_2	0.677
G_3	0.712

Table 14: Clustering coefficient for all three networks.

The collaborative behaviour of G_1 is also studied over time. This is achieved by calculating the number of connected components, the degrees distribution and the clustering coefficients for each time period. Table 17 summarises the results.

The number of connected components indicate that after period 3 to 7 the number of author is increasing. Period 2 is a very poor period of publication in our sources. For periods 3 to 7 it is shown in Figure 19 that the degrees are stabilising over 3.Thus in the study of the prisoners dilemma according to our data, papers with 3 authors seems to be favoured.

Furthermore, the clustering coefficients are also increasing over these periods. This analysis indicates that not only more people are were attracted to the the topic over the years but also that the field was becoming more collaborative.

	Connected components	**Clustering coefficient**
period 1	15	0.5
period 2	5	0.0
period 3	49	0.14
period 4	96	0.3
period 5	534	0.64
period 6	281	0.74
period 7	134	0.76

Table 15: Collaborative behaviour measures over time periods.

3.3.3 Influence

Network centrality is used in network theory to study which nodes of a graph are the most important. There are several centrality measures used to explain different behaviours of the nodes. Centrality will be used here to explain influence. The two centrality which are used are:

- Closeness centrality C_C.

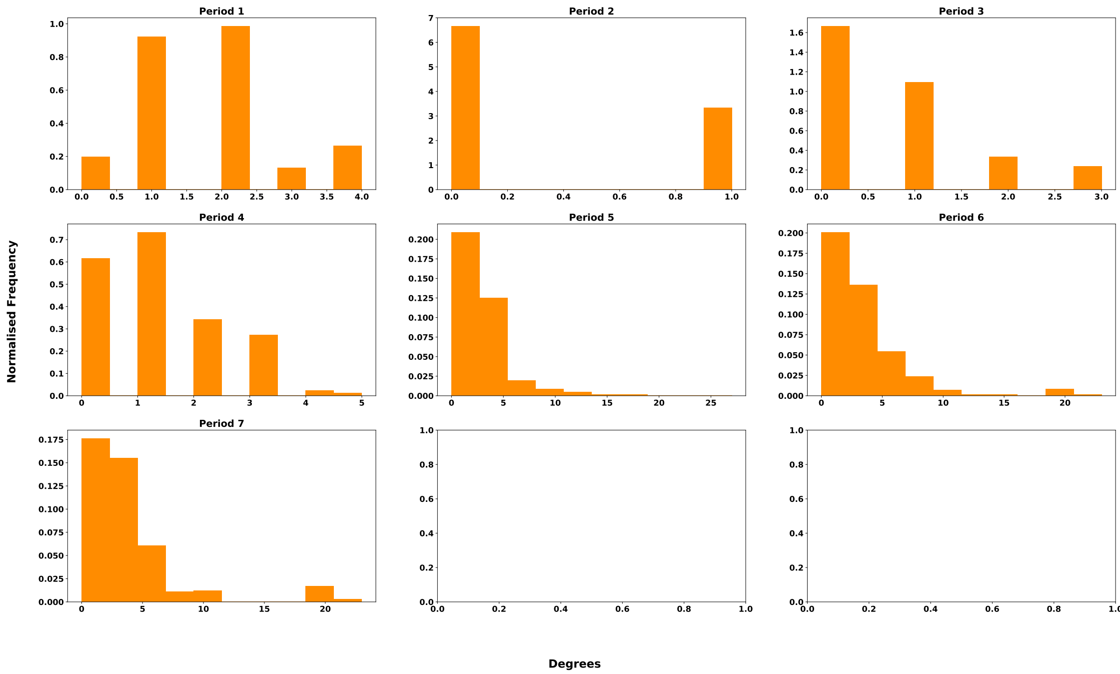


Figure 19: Degrees distribution over time.

- Betweenness centrality C_B.

3.3.3.1 Measures

Both network measures are explained with an example. The definitions for both centralities are given by Definition 3.2 and 3.3.

Definition 3.2. Closeness. Closenesse centrality of a node u is the reciprocal of the average shortest path distance to u over all $n-1$ reachable nodes. It is denoted as,

$$C_C(u) = \frac{n-1}{\Sigma_{v=1}^{n-1} d(v,u)}$$

where $d(v,u)$ is the shortest-path distance between v and u, and n is the number of nodes that can reach u. The mornalised centrality is C_C normalised by the number of nodes in the connected part of the graph.

Definition 3.3. Betweenness. Betweenness centrality of a node u is the sum of the fraction of all-pairs shortest paths that pass through u. It is denoted as,

$$C_B(u) = \Sigma_{s,t \in V} \frac{\sigma(s,t|u)}{\sigma(s,t)}$$

where V is the set of nodes, $\sigma(s,t)$ is the number of shortest (s,t)-paths, and $\sigma(s,t|v)$ is the number of those paths passing through some node u other than s,t. If $s=t$, $\sigma(s,t)=1$, and if $u \in s,t, \sigma(s,t|u)=0$. Normalised C_B is normalized by $\frac{2}{((n-1)(n-2))}$

Closeness is a measure that shows how well a node connects other nodes. Equivalently, how well an author is connected to other authors and make them collaborate. This is called influence. On the other hand betweenness is about how connected a node is, thus how much influence an author can gain from their environment.

As an example consider a sub graph of G_1 which is illustrated in Figure 20. Note that nodes 1, 2 and 3 are connected to three authors. Thus we expect their betweenness centrality to be the same. However, this is not true for closeness centrality. Node 3 is the connecting link between at least 4 people. Thus node 3 is the person in the sub graph that influences most authors. Node 3 also gains influence due ot its rule in the team, but node 2 achieves the same without connecting people as much as node 3.

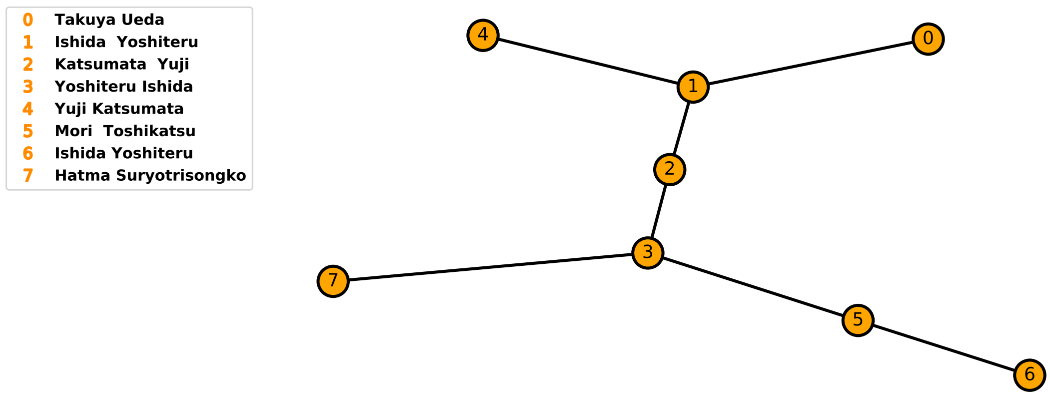


Figure 20: A sub graph of G_1.

The centrlity for all three networks are calculated using [37]. Table 16 summarises the most important authors of network G_1 based on the two centralities. The people that influence the field the most are Matjaz Perc, Yamir Moreno, Luo-Luo Jiang, Arne Traulsen and Martin A. Nowak. Their work have been discussed in Section 2. Though Matjaz Perc and Yamir Moreno appear to both influence and gain from the networks influence, it does not hold for the rest of the three authors.

	Author name	Betweeness		Author name	Closeness
1	Matjaz Perc	0.010584	1	Matjaz Perc	0.044428
2	Yamir Moreno	0.008786	2	Yamir Moreno	0.043561
3	Luo-Luo Jiang	0.004319	3	Cheng-Yi Xia	0.038910
4	Arne Traulsen	0.003920	4	Sandro Meloni	0.037959
5	Martin A. Nowak	0.003832	5	Alberto Aleta	0.037600

Table 16: Top 5 ranked authors of G_1 based on different centrality measures.

3.3.3.2 Comparison with other topics co authorship

Influence for a given network is calculated easily. However, we want to assert the power of the influence of G_1 by comparing the results to those of G_2 and G_3. Using the distribution of the centralities will we statistically tests whether a difference does exist between them.

For C_C all distributions are not normally distributed so we will use the non parametric test Kruskal Wallis. The test returns a p value of 0.0 thus it can be stated with 95% confidence of that the distributions are statistically different. These are plotted as violin plots in Figure 22.

Note that the centrality values range between 0 and 1. For all three graphs C_C have very low values. Both G_1 and G_2 appear to have a similar distributions. Most authors have have a coefficient of zero and a few authors by the tails appear to have a larger coefficient. This means that for the two topics, the highest frequency of authors have a very small influence in their field. Though there are people with a high influence they are only a few.

The coefficients of G_3 are different from the other topics. Overall it can be seen that authors' centrality is more spread through the different values.

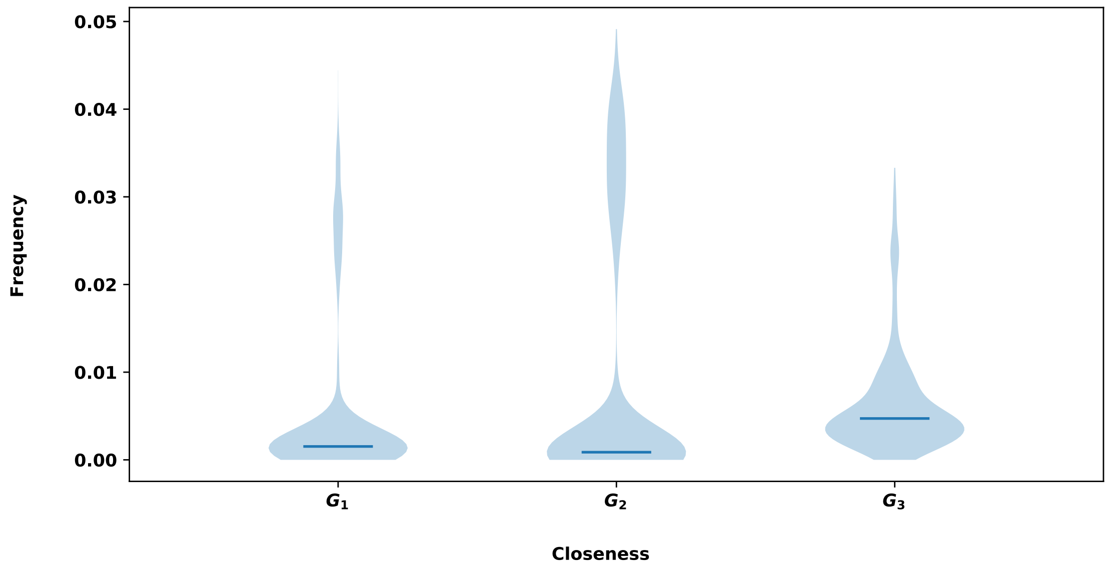


Figure 21: Closeness distributions for all three networks.

The betweenness centrality of the networks is compared in a similar manner. All three distribution are not normally distributed thus Kruskal Wallis test is used again the assert whether they medians are the same. The test returns a p value of thus the medians are statistically not the same.

Though the are different Figure 22 that all coefficients are clustered around the value of zero. Non of the topics authors gain influence from their networks.

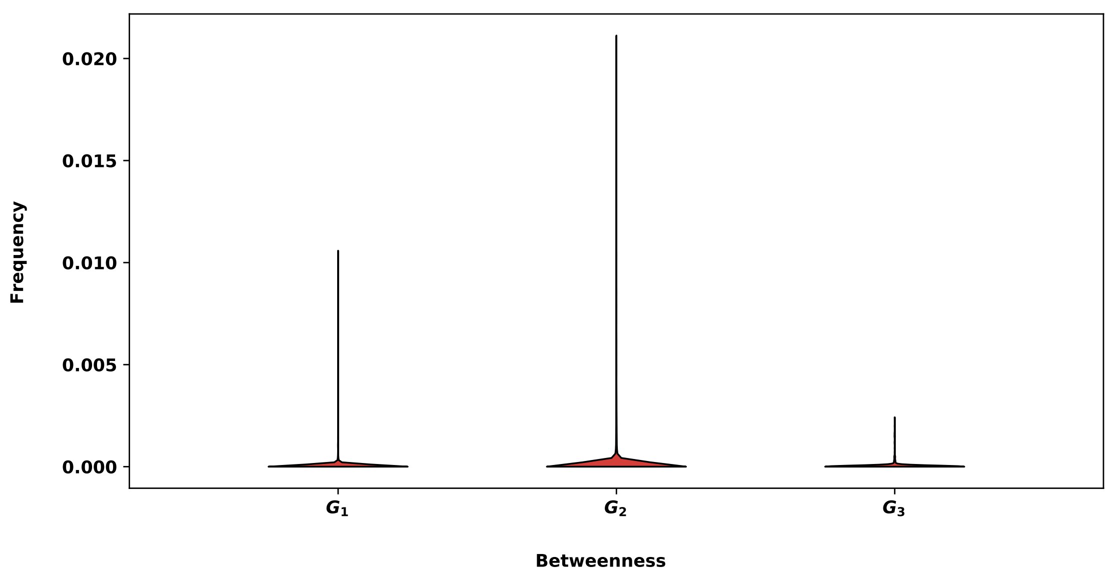


Figure 22: Betweenness distributions for all three networks.

3.3.3.3 Temporal comparison

The two centralities are also studied over time. Table 17 higlights the authors with the most influnce at each time period. Equivalently, Table 18 highlights the authors that gained more from the networks' influence at each time period.

For periods 1, 3, 4, 5 and 6 the authors with the highest closeness centrality are the authors with the highest betweenness centrality as well. Several of these authors have been shown to also have a strong influence to the entire network as well.

For period 7 though March Harper is the person that influences most authors, Chen Yi Xia is the person that gains the most. For period 2 we have a similar case. Robert Axelrod appears to be the most influenced author but not gaining as much from it.

Note that the centrality values are decreasing over the time periods. This could be an effect due the size of the network which is increasing. In smaller networks authors have more motivation to collaborate with the few people on their fields.

	Author name	Closeness
1	Svenn Lindskold	0.108108
2	R. Axelrod	0.200000
3	Martin A. Nowak	0.043478
4	Lee A. Dugatkin	0.029762
5	Matjaz Perc	0.043443
6	Yamir Moreno	0.039311
7	Marc Harper	0.049462

Table 17: Authors with the most influnce at each time period.

	Author name	Betweeness
1	Svenn Lindskold	0.006006
2	A. W. Tucker	0.000000
3	Martin A. Nowak	0.001279
4	Lee A. Dugatkin	0.000499
5	Matjaz Perc	0.010689
6	Yamir Moreno	0.005468
7	Cheng-Yi Xia	0.001242

Table 18: Authors that gained more from the networks influence at each time period.

3.3.4 Conclusion

In this section we have conducted an investigation of the literature based on a data analysis. More specifically, this was mainly done using network theory.

Initially, we gave a summary on the data collection. An open source project which was developed for the purpose of this work was used [63]. The project takes advantage of the API system several academic journals offer today. The procedure, the sources as well as the keywords used in the process of collection have been clearly specified making the process reproducible.

Three data sets have been composed for three different topics of game theory. These are:

- The prisoner's dilemma. The main focus of this paper.
- Auction games. A sub field of game theory used for comparison reasons.
- The price of anarchy. A sub field of game theory used for comparison reasons.

We conducted a brief preliminary analysis on these data sets. Mainly to understand the sizes, provenance and trends of each topic. The main data set was also partitioned into time periods such that an temporal comparison could be conducted.

The main focus of the analysis has been to explain collaborative behaviour and influence. Both terms have been defined and we have explained how network measures of the co authorship network were used to quantify them. The co authorship network is a network representing all the unique authors of a topic. An edge exists within two authors if they have written together. Co authorship was decided to be used as be believe other measures, such as citations, perform less well.

The findings of this analysis are presented in two parts. The comparison of the collaborative behaviour and influence of the prisoner's dilemma field based on other topics and how these measures change for the field over time.

All three networks have been disjointed with a large number of connected components. The collaborative behaviour was based on the nature of these connected components. The median connection of an author has been the same for all three networks. However, the price of anarchy had a smaller number of authors that prefer to write on their own and based on the clustering coefficient the collaboration of the field authors appears to be stronger. The collaboration for both the prisoner's dilemma and auction games it's similar. Note though that the price of anarchy is a new topic with less authors. It makes more sense for a few people that work on the topic to be more collaborative which each other.

Similarly influence was studied using two centrality measures. For G_1 and G_2 we conclude that there are only a few authors that have power of influence on the network. For research this is not ideal. It could mean that the research is only driven from the work of specific people. It could also indicate a hostile environment for new authors. In comparison, G_3 has several authors that have different influence on their neighbours. A wider spread of influence could indicate a nice flow of knowledge across the field from different people. This could help the growth of a field and accelerate findings. The influence that authors gain from the respective networks was also explored. The results argued that the gain was very low for all three networks.

Collaborations and influence of the field have changed over time. As argued by [25] over time the amount of research groups was increasing for the topic. However, as stated earlier, as the number of researchers increases the amount of influence and collaboration decreases. This is because researchers can not have full knowledge of their entire network and the more people the network has the more people we actually do not know. The results from the temporal analysis imply that this is true for the field of the prisoner's dilemma.

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