A systematic literature review of the Prisoner's Dilemma; collaboration and influence.

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Abstract

The prisoner's dilemma is a well known game used since the 1950's as a framework for studying the emergence of cooperation; a topic of continuing interest for mathematical, social, biological and ecological sciences. The iterated version of the game attracted attention in the 1980's after the publication of the "The Evolution of Cooperation" and has been a topic of pioneering research ever since. In this work we aim to provide a detailed literature review of the field. This is achieved by partitioning the timeline into six different sections. Furthermore, a comprehensive data set of literature is analysed using network theoretic approaches in order to explore the influence and the collaborative behaviour of the field itself.

1 Introduction

To add a sentence about selfness and selfishness. There is a simple way of representing these behaviours/concepts. This is to use a particular two player non-cooperative game called the prisoner's dilemma, originally described in [32].

Each player has two choices, to either be selfness and cooperate or to act in a selfish manner and choose to defect. Each decision is made simultaneously and independently. The fitness of each player is influenced by its own behaviour, and the behaviour of the opponent. Both players do better if they choose to cooperate than if both choose to defect. However, a player has the temptation to deviate as that player will receive a higher payoff than that of a mutual cooperation.

A player's payoffs are generally represented by (1). Both players receive a reward for mutual cooperation, R, and a payoff P for mutual defection. A player that defects while the other cooperates receives a payoff of T, whereas the cooperator receives S. The dilemma exists due to constrains (2) and (3).

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} \tag{1}$$

$$T > R > P > S \tag{2}$$

$$2R > T + S \tag{3}$$

Another common representation of the payoff matrix is given by (1), where b is the benefit of the altruistic behaviour and c is the cost and constrains (2) - (3) need to hold.

$$\begin{pmatrix} b - c & c \\ b & 0 \end{pmatrix} \tag{4}$$

Constrains (2) - (3) and rational behaviour guarantee that it never benefits a player to cooperate. However, when the game is studied in a manner where prior outcome matters, defecting is no longer necessarily the dominant choice.

The repeated form of the game is called the iterated prisoner's dilemma and theoretical works have shown that cooperation can emerge once players interact repeatedly. Arguably, the most important of these works has been R. Axelrod's "The Evolution of Cooperation" [20]. In his book Axelrod reports on a series of computer tournaments he organised of a finite turns games of the iterated prisoner's dilemma. Participants had to choose between cooperation and defection again and again while having memory of their previous encounters. Academics from several fields were invited to design computer strategies to compete. The pioneering work of Axelrod showed that greedy strategies did very poorly in the long run whereas altruistic strategies did better.

"The Evolution of Cooperation" is considered a milestone in the field but it is not the only one. On the contrary, the prisoner's dilemma has attracted much attention ever since the game's origins. In Section 3 a comprehensive data set of literature regarding the prisoner's dilemma collected from the the following sources

- arXiv;
- PLOS;
- IEEE;
- Nature;
- Springer.

will be presented and analysed. This allow us to review the amount of published academic articles as well as measure and explore the collaborations within the field.

2 Timeline

In this section we review a large amount of literature regarding the prisoner's dilemma. We start from the year the game was formulated all the way to today.

2.1 Origins of the Prisoner's Dilemma

The origin of the prisoner's dilemma goes back to the 1950s in early experiments conducted at the RAND [32] to test the applicability of games described in [84]. The game received it's name later the same year. According to [81], A. W. Tucker (the PhD supervisor of J. Nash [57]), in an attempt to delivery the game with a story during a talk he described the players as prisoners and the game has been known as the prisoner's dilemma ever since.

The early experiments were very constrained. The only source of date was through groups of humans and those came with disadvantages. Firstly, humans can behave randomly and secondly both the size and the background of the individuals were different from experiment to experiment.

An early figure within the field of game theory is Prof A. Rapoport, a mathematical psychologist, whose work focused on how to promote international and national cooperation. Rapoport sought to conceptualize strategies that could promote international cooperation. In his teaching and research he used the prisoner's dilemma. In [68] Rapoport conducted experiments using human subjects to simulate rounds of the iterated prisoner's dilemma. He sought out to understand the conditions under which altruist behaviour emerged. This was a common area of research. Conditions explored were the gender [31, 48, 49] of individuals, the representation of the game [31], the distance between players [72], the initial effects [80] and whether the experimenter was biased [33].

In the next section we will introduce the pioneer computer tournaments of R. Axelrod, that largely replaced human subjects in the study of the iterated prisoner's dilemma.

2.2 Axelrod's Tournaments and intelligent design of strategies

As discussed in Section 2.1, before the 1980s a great deal of research was done in the field. However, as described in [18], the political scientist R. Axelrod believed that there was no clear answer to the question of how to avoid conflict, or even how an individual should play the game. Combining his interest in artificial intelligence and political sciences Axelrod created a framework for exploring these questions using computer tournaments. Axelrod asked a number of researchers to design their own strategies and submit them in the form of computer code. In essence, the competitors were asked to design a strategy, to set a number of rules, with the purpose of wining an iterated prisoner's dilemma tournament; an intelligent design of strategies.

As described in [69], "Axelrod's new approach has been extremely successful and immensely influential in casting light on the conflict between an individual and the collective rationality reflected in the choices of a population whose members are unknown and its size unspecified, thereby opening a new avenue of research." Several researchers responded immediately to Axelrods tournaments and the study of cooperation became of critical interest once again. In a collaboration [19] with a colleague, Douglas Dion, they summarized a number of works inspired from the "Evolution of Cooperation".

Several tournaments have been performed since Axelrod's and a number of strategies of intelligent design have made an appearance ever since. These were created mainly to surpass limitations of Axelrod's tournaments or to argue with his results. In 2012, [39] wrote a review on iterated prisoner's dilemma strategies and competitions that had occurred. In this section we will cover Axelrod's original tournaments as well as research that introduced new strategies of intelligent design. The strategies discussed in this section are strategies were constructed by an intelligent cause and not an undirected process. Such strategies will be covered in a following section.

The first reported computer tournament took place in 1980 [14]. Several scientists were invited to submit their strategies, written in the programming languages Fortran or Basic. There was a total of 13 submissions made by the following researchers,

1. T Nicolaus Tideman and Paula Chieruzz;
2. Rudy Nydegger;
3. Bernard Grofman;
4. Martin Shubik;
5. Stein and Anatol Rapoport;
6. James W Friedman;
7. Morton Davis;
8. Jim Graaskamp;
9. Leslie Downing;
10. Scott Feld;
11. Johann Joss;
12. Gordon Tullock;

and a 13^{th} who remained anonymous.

Each competed in a 200 turn match against all 12 opponents, itself and a player that played randomly (called Random). This type of tournament is referred to as a round robin. The tournament was run only once. Each participant knew the exact length of the matches and had access to the full history of each match. Furthermore, Axelrod performed a preliminary tournament and the results were known to the participants. The payoff values used for equation (1) where $R = 3, P = 1, T = 5$ and $S = 0$. These values are commonly used in the literature and unless specified will be the values used in the rest of the work described here.

The winner of the tournament was determined by the total average score and not by the number of matches won. The strategy that was announced the winner was submitted by Rapoport and was called **Tit For Tat**. Tit for Tat, is a strategy that always cooperates on the first round and then mimics the opponent's previous move. The success of Tit for Tat came as a surprise. It was not only the simplest submitted strategy but it had also won the tournament even though it could never do better than any player it was interacting with.

In order to further test the results Axelrod performed a second tournament later in 1980 [15]. The results of the first tournament had been publicised and the second tournament received much more attention, with 62 entries made by the following people,

1. Gail Grisell;
2. Harold Rabbie;
3. James W Friedman;
4. Abraham Getzler;
5. Roger Hotz;
6. George Lefevre;
7. Nelson Weiderman;
8. Tom Almy;
9. Robert Adams;
10. Herb Weiner;
11. Otto Borufsen;
12. R D Anderson;
13. William Adams;
14. Michael F McGurrin;
15. Graham J Eatherley;
16. Richard Hufford;
17. George Hufford;
18. Rob Cave;
19. Rik Smoody;
20. John Willaim Colbert;
21. David A Smith;
22. Henry Nussbacher;
23. William H Robertson;
24. Steve Newman;
25. Stanley F Quayle;
26. Rudy Nydegger;
27. Glen Rowsam;
28. Leslie Downing;
29. Jim Graaskamp and Ken Katzen;
30. Danny C Champion;
31. Howard R Hollander;
32. George Duisman;
33. Brian Yamachi;
34. Mark F Batell;
35. Ray Mikkelson;
36. Craig Feathers;
37. Fransois Leyvraz;
38. Johann Joss;
39. Robert Pebly;
40. James E Hall;
41. Edward C White Jr;
42. George Zimmerman;
43. Edward Friedland;
44. X Edward Friedland;
45. Paul D Harrington;
46. David Gladstein;
47. Scott Feld;
48. Fred Mauk;
49. Dennis Ambuehl and Kevin Hickey;
50. Robyn M Dawes and Mark Batell;
51. Martyn Jones;
52. Robert A Leyland;
53. Paul E Black;
54. T Nicolaus Tideman and Paula Chieruzz;
55. Robert B Falk and James M Langsted;
56. Bernard Grofman;
57. E E H Schurmann;
58. Scott Appold;
59. Gene Snodgrass;
60. John Maynard Smith;
61. Jonathan Pinkley;
62. Anatol Rapoport.

The new participants knew the results of the previous tournament. The rules were similar with only a few alternations. The tournament was repeated 5 times and the length of each match was not known to the participants. Axelrod intended to use a fixed probability (refereed to as 'shadow of the future' [19]) of the game ending on the next move. However, 5 different length matches were selected for each match 63, 77, 151, 308 and 401, such that the average length would be around 200 turns.

Several entries of the second tournament tended to be variants of Tit for Tat, such as **Tit for Two Tats** submitted by John Maynard Smith. Tit for Two Tats defects only when the opponent has defected twice in a row. Another well known entry was **Grudger**. Grudger was originally submitted by James W. Friedman. Grudger is a strategy that will cooperate as long as the opponent does not defect. The name Grudger was give to the strategy in [46]. Though the strategy goes by many names in the literature such as, Spite [23], Grim Trigger [21] and Grim [83].

Despite the size of this tournament, none of the variants and new strategies managed to outperform the simple designed strategy. The winner was once again Tit for Tat. The conclusions made from the first two tournaments were that the strong performance of the strategy was due to:

- The strategy would start of by cooperating.
- It would forgive it's opponent after a defection.

- It would always be provoked by a defection no matter the history.
- As soon as the opponents identified that they were playing Tit for Tat, they would choose to cooperate for the rest of the game.

However, the success of Tit for Tat was not unquestionable. Several papers showed that stochastic uncertainty severely undercut the effectiveness of reciprocating strategies. Though such stochastic uncertainty are unlikely to occur in a computer tournament, but have to be expected in real life situations [53].

In [55] it was proved that in an environment where **noise** is introduced two strategies playing Tit for Tat receive the same average payoff as two Random players. Noise is a probability that a player's move will be flipped. In 1986, [28] ran a computer tournament with a 10 percent chance of noise and Tit for Tat finished sixth out of 21 strategies. Bendor in [37] performed tournament similar to Axelrod's with noise and a probability of 0.0067 of ending in the next turn. His results demonstrated the poor performance of Tit for Tat was again and the highest ranked strategies were more generous ones. His top ranked strategy was called **Nice and Forgiving**.

Hammerstein [71], pointed out another weakness of Tit for Tat that if by mistake, one of two Tit for Tat players makes a wrong move, this locks the two opponents into a hopeless sequence of alternating defections and cooperations. To overcome this error [73] introduced their strategy **OmegaTFT**. They also altered their strategy so that it had the ability to to recognize and exploit the Random strategy in a way that after an opponent strategy crosses a certain randomness threshold they conclude that the opponent is a Random strategy and change the behaviour to act as a Defector, a strategy that always choose to defect.

A second type of noise is misperception, where a player's action is made correctly but it's recorder incorrectly by the opponent. In 1986, [78] introduced a strategy called **Contrite Tit for Tat** that was more successful than Tit for Tat in such environments. The difference between the strategies was that Contrite Tit for Tat was not so fast to retaliate a defection. This hints, alongside more generous versions of Tit for Tat as discussed above, that the counter attack to stochastic uncertainties is a strategy's readiness to defect after a defection.

Another protagonist in the literature and better perform strategy than Tit for Tat is a strategy called **Pavlov**. Though the name Pavlov was given to the strategy in 1993 by Nowak [62] the strategy had been around since 1965 known as Simpleton [68]. The strategy is based on the fundamental behavioural mechanism win-stay, lose-shift. It starts off with a cooperation and then repeats it's previous move only if it was awarder with a payoff of R or T. Pavlov is heavily studied in the literature and Similarly to Tit for Tat it's used in tournaments perform until today and has had many variants trying to build upon it's success. **PavlovD**, just a Pavlov which starts the game with a defection and **Adaptive Pavlov**. Adaptive Pavlov tried to classify it's opponents as as one of the following strategies, Cooperator, Defector, Pavlov, Random or Pavlov and chooses a strategy that maximise it's payoff against the now 'known' opponent. Cooperator and Defector are deterministic strategies that conditionally cooperate and defect respectively.

Another limitation of [16] was that the interactions have been between pairs of players, as argued by [38]. In several applications, however, interactions involve more than two players. This can be modelled using the corresponding n-player prisoner's dilemma (NPD), in which players make a choice (cooperate or defect) which they play with all other players. In evolutionary settings [38] found that if individuals play a "hard" Tit For Tat, meaning that they will cooperate until one player defects, and the number of individuals playing hard Tir Fot Tat passes a certain threshold, then hard Tit For Tat can dominate a population of Defectors. But this threshold rises as the number of individuals in the society increases.

Several researchers, and this will discussed in later sections as well, argued with Axelrod's result on simplicity. The advantages of complexity were shown by [23] in 1997 where they introduced another well known strategy **Gradual**. Gradual starts off by cooperating, then after the first defection of the other player, it defects one time and cooperates twice. After the second defection of the opponent, it defects two times and cooperates twice. After the n^{th} defection it reacts with n consecutive defections and then two cooperations. In a tournament of 12 strategies [23], Gradual had managed to outperform strategies such as Tit for Tat and Pavlov. Gradual was surpassed by yet another complex and intelligent designed strategy **Adaptive Tit for Tat**. The authors of [82] conducted the exact same tournament as [23] with now 13 strategies and their strategy ranked first.

Another interesting research on teams of strategies [26, 27, 70]. These strategies are of intelligent design and they have been programmed with a recognition mechanism by default. Once the strategies recognise one another, one would act as leader and the other as a follower. The follower then plays as a Cooperator, cooperates unconditionally and the leader would play as a Defector gaining the hghest achievable score. Followers would behave as Defector towards other strategies to defect their score and help the leader. In [70], a team for the University of Southampton used teams and recognition patterns and managed to win the 2004 Anniversary IPD Tournament.

2.3 Evolutionary Dynamics

Following the original tournaments Axelrod wanted to further test the robustness of his conclusions. In the later sections of [15], he discusses about an ecological tournament he performed using the 62 strategy of the second tournament and in [16] he also studied the evolutionary stability of the strategy. His results alongside side a collaboration with the biologist William Donald Hamilton on biological applications won the pair the Newcomb-Cleveland prize of the American Association for the Advancement of Science.

In general, the complex nature of iterated prisoner's dilemma strategies makes their evolutionary stability more complex to study. Though 'when does cooperation emergences in situations where individuals a more likely to be selected if they are doing better' still remains open several researchers have provided results over the years. This has been done in both unstructured and structured populations. Structure populations has attracted a lot of attention in the community as it can be used to explain how cooperation can emerge in social interactions where we only interact with a selected few. Szabo in [79] conducted a comprehensive review on evolutionary dynamics on structured populations, work on the iterated prisoner's dilemma is referenced. In this section we will review several works on the evolutionary dynamics of the game in both structured and unstructured.

The ecological approach is a simulation of theoretical future rounds of the game where strategies that do better are more likely to be included in future rounds than others. The simulation of the process, as described in [15], is straightforward. Let us consider an example. Let the four strategies Tit for Tat, Tit for Two Tat, Cooperator and Defector compete in an ecological tournament. The expected payoff matrix, when these four strategies interact, is give by,

$$\begin{bmatrix} 3.0, & 3.0, & 3.0, & 0.99 \\ 3.0, & 3.0, & 3.0, & 0.99 \\ 3.0, & 3.0, & 3.0, & 0.0 \\ 1.02, & 1.039, & 5.0, & 1.0 \end{bmatrix}$$

Starting with proportions of each type in a given generation, their proportions for the next generation needs to be calculated. This is achieved by calculating the weighted average of the scores of a given strategy with all other players.

- The weights are the numbers of the other strategies which exist in the current generation.
- The numbers of a given strategy in the next generation is then taken to be proportional to the product of its numbers in the current generation and its score in the current generation.

The process is then repeated for a given number of future tournaments. Figure 1 illustrates a simulation of our hypothetical ecological tournament, as shown strategies that cooperate quickly kill off the Defector. A similar result was presented by Axelrod. In his ecological tournament cooperative strategies started to take over the population over time. On the other hand exploitative strategies started to die off as weaker strategies were becoming extinct. In other words they were dying because there was fewer and fewer prey for them to exploit.

In 1981, Axelrod also studied the prisoner's dilemma in an evolutionary context based on the evolutionary approaches of John Maynard Smith [52, 74, 75]. Smith is a well know evolutionary biologist as well as an attendant of Axelord's second tournament. John Maynard Smith alongside George Price are considered fundamental figures of evolutionary game theory. In [52] they introduced the definition of an evolutionary stable strategy (ESS).

Imagine a population made up of individuals where everyone follows the same strategy B and a single individual adopts a mutant strategy A. Strategy A is said to invade strategy B if the payoff of A against B is greater than the expected payoff received by B against itself. Since strategy B is in a population that interacts only with itself, the concept of invasion is

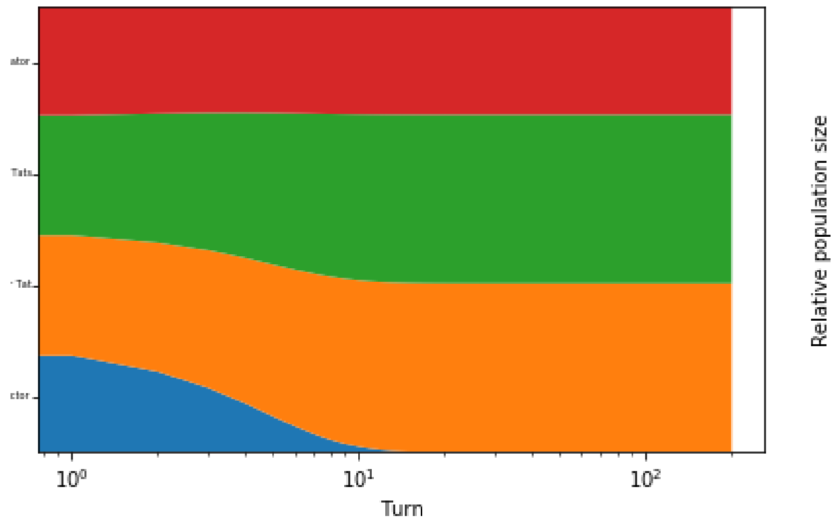

Figure 1: Results on an ecological tournament with Tit for Tat, Tit for Two Tats, Cooperator and Defector.

equivalent to a single mutant being able to outperform the average population. Thus for a strategy to be ESS it must be able to resists any invasion.

The work described in [16], studied the evolutionary stability of Tit for Tat and although the strategy was likely to take over the population, its stability was conditional on the importance of the future of the game. This is represented by a discounting factor denoted as w. Axelrod showed that if w was sufficiently large, Tit for Tat could resist invasion by any other strategy. Moreover, he showed how a small cluster of Tit for Tat players could invade a extortionate environment.

In [25] Boyd and Lorderbaum show that if w, the importance of the future of the game, is large enough then no deterministic strategy is ESS because it can always be invaded by any pair of other strategies. This was also independently proven by [67]. Furthermore, Boyd Boyd showed that introducing noise into the evolutionary IPD allows such strategies to be evolutionary stable and that for given combination of strategies cooperation can start without a cluster of cooperative strategies. Arguing with this remark with Axelrod's results.

Then, in [60], Nowak and Sigmund studied the dynamics of the evolutionary iterated prisoner's dilemma with a spectrum of stochastic strategies that only remember their opponent's last move, not their own. They found that there can be multiple fixed points that there can be an evolutionary stable coexistence among multiple such strategies.

Another dynamic approach that was considered in 1992 is that of using very simplistic strategies but in more complex topological structures. An extension to the natural selection where who meets whom is not random anymore. In [63], a population of two deterministic strategies, Defector and Cooperator, were placed onto a two dimensional square array where the individuals could interact only with the immediate neighbours. The number of immediate neighbours could be either, fourth, six or eight. As shown in Figure 2, where each node represents a player and the edges denote whether two players will interact.

Thus each cell of the lattice is occupied by a Cooperator or a Defector.

- At each generation step each cell owner interacts with its immediate neighbours.
- The score of each player is calculated as the sum of all the scores the player achieved at each generation.
- At the start of the next generation, each lattice cell is occupied by the player with the highest score among the previous owner and the immediate neighbours.

This topology is refereed to as spatial topology. The population dynamics of these experiments were studied as a function of the temptation (T) payoff. More specifically the following payoff matrix was used, which is equivalent to equation (1):

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix} \tag{5}$$

where ($b > 1$).

It was shown that for different values of the temptation payoff b, this purely deterministic spatial version could generate chaotically change patterns in which cooperators and defectors could persist together in a mix population. Though it was known that in unstructured populations natural selection would favour defection, [59] provided evidence that in structured populations the results can be wildly different. The authors claimed that the essential results remain true of all topologies; the results also hold whether self interactions are taken into account.

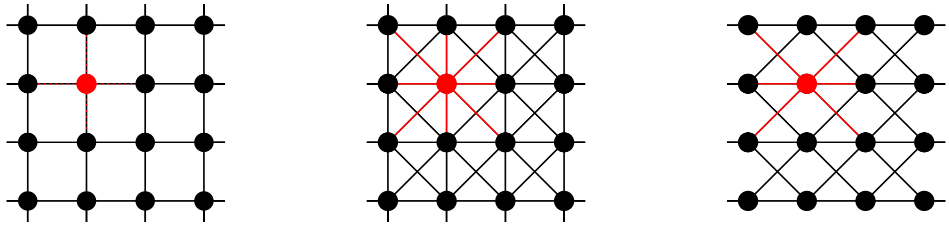

Figure 2: Spatial neighbourhoods

Later in 2003, the authors [50] decided to consider small world networks instead of regular graphs. More specifically they used the well known Watts and Strogatz [85] graphs. The experiment starts with each node having $k/2$ nearest neighbours on each side . Then a proportion ρ of the total edges is rewired by removing $\rho kn/2$ edges and creating $\rho kn/2$ new edges each of whose initial vertex is the initial vertex of a removed edge and its terminal vertex is randomly chosen so that the generated graph does not have multiple edges or once removed edges.

The authors argued that by applying these experimental rules we re closer to capturing real life behaviour where we constantly change who we interact with. The results of their work had been that:

- For small b, it is not so tempting for players to exploit cooperators. Thus cooperators converge regardless the value of ρ.
- The number of cooperators highly depends on ρ roughly for for higher values of the temptation payoff b.
- Lastly once temptation if very strong, even cooperators happen to form tight clusters, they cannot survive once they face defectors. Finally, the cooperators eventually extinguish.

In 2006, Ohtsuki studied regular k degree graphs and introduced a "rule" regarding when is cooperation favoured in spatial tournaments. Ohtsuki used the following game matrix,

where b represents the altruistic act and c represents the cost. Ohtsuki etc all proved that natural selection favours cooperation if the ration of b/c exceeds the average number of neighbours. Their result, however, holds for weak selection. That means that the fitness of an individual is only proportional to their payoff.

Later the same year, Ohtsuki and Nowak [65], managed to approximate the dynamics of a population on graphs using an approximation of the replicator equations. The replicator equation was introduced as the first closed form differential equation to describe the dynamics of natural selected populations.

Another topic worth mentioning is that of coevolution on graphs. In [88] studied graphs were a probability of rewiring ones connections was in place, however, the rewire could be with any given node in the graphs and not just with imitate neighbours. Perc etc all showed that "making of new friends" may be an important activity for the successful evolution of cooperation, but also that partners must be selected carefully and one should keep their number limited.

2.4 Structured strategies and training

There is limitation to all of these dynamic treatments. That is their inability to develop new strategies. A way of overcoming is to use a genetic approach. An evolutionary process called the genetic algorithm was used to discover effective strategies in [17]. A genetic algorithm is a search heuristic that is inspired by the theory of natural selection. In a population of candidate solutions, the fittest individuals are selected for reproduction. They produce offsprings that will replace the weakest members of the population as long as they do better than them.

In order to use a genetic algorithm [17] needed to represent strategies in a format such that the algorithm could optimise. Axelord considered deterministic strategies that took into account the last 3 turns of the game. For each turn there are 4 possible outcomes (CC, CD, DC, DD), thus for 3 turns there are a total of $4 \times 4 \times 4 = 64$ possible combinations. Axelord therefore used a list of 64 C's and D's to represent different strategies. We refer to this representation structure as a lookup table, which is a set of deterministic responses based on the opponents m last moves; for example [17] considered $m = 3$. In later section we discuss some more recent work that has been done using lookup tables.

The pioneering work of the computer tournaments and the results of the reciprocal behaviour in the prisoner's dilemma spread the knowledge of the game worldwide and across disciplines. Several researchers responded immediately to Axelrod's tournaments and the study of cooperation became of critical interest once again. This section focuses on the research that was carried out after the initial computer tournaments, over the time period between 1984 and 1993.

The work of [51] pointed out that it is important 'to take more account of intrinsic stochasticities'. This suggested considering stochastic strategies and [60] studied such strategies. An iterated prisoner's dilemma strategy was represented by using three parameters (y, p_1, p_2), where y is the probability to cooperate in the first move, and p_1 and p_2 the conditional probabilities to cooperate, given that the opponent's last move was a cooperation or a defection. These are a very specific set of strategies that only remember their opponent's last move, not their own and they are called reactive strategies. Using the above notation a strategy can now be defined by a triple. For example,

- Defector: (0, 0, 0)
- Cooperator: (1, 1, 1)
- Tit for Tat: (1, 1, 0)

This framework was used in [60] to study game dynamical aspects of the iterated prisoner's dilemma, the results of which will be presented in the next section which is dedicated to such research. Another outcome of the framework came the next year. In 1990, [61] gave a formal definition of a memory one strategy. Memory one strategies consider the entire history of the previous turn to make a decision (thus reactive strategies are a subset of memory one).

If only a single turn of the game is taken into account and depending on the simultaneous moves of two players there are only four possible states that players could possibly be in. These are CC, CD, DC and DD. A memory one strategy is denoted by the probabilities of cooperating after each of these states, $p = (p_1, p_2, p_3, p_4) \in \mathbb{R}^4_{[0,1]}$. A match between two memory one players p and q can be modelled as a stochastic process, where the players move from state to state. More specifically, it can be modelled by the use of a Markov chain [34], which is described by a matrix M.

$$M = \begin{bmatrix} p_1q_1 & p_1(-q_1+1) & q_1(-p_1+1) & (-p_1+1)(-q_1+1) \\ p_2q_3 & p_2(-q_3+1) & q_3(-p_2+1) & (-p_2+1)(-q_3+1) \\ p_3q_2 & p_3(-q_2+1) & q_2(-p_3+1) & (-p_3+1)(-q_2+1) \\ p_4q_4 & p_4(-q_4+1) & q_4(-p_4+1) & (-p_4+1)(-q_4+1) \end{bmatrix} \tag{6}$$

The players are assumed to move from each state until the system reaches a state steady, let the steady states vector be denoted as $\bar{v}$. The utility of a player can be given by multiplying the steady states of M by the payoffs of equation (1). Thus [61] offered a mathematical framework to calculate the utility of two players without actually simulating the game. The payoff of a player p can be obtained by,

$$s_p = \bar{v} \times \begin{pmatrix} R \\ S \\ T \\ P \end{pmatrix}$$

In 1992 reactive strategies were used to investigate which strategies would manage to take over the population and would be ESS in an environment with noise. The results demonstrated that though a small fraction of Tit for Tat players have been essential for the emergence of cooperation, more generous strategies took over the population. More specifically the re-active strategy known as **Generous Tit for Tat** which is give by the triplet $(1, 0, \frac{2}{3})$. Generous Tit for Tat was not the only strategy to outperform Tit for Tat in a noisy environment, same conclusions were made by [35, 37].

In this section we will cover several research projects published between 1993 and 2017. The research reviewed here focuses on computer tournaments and serves as an introduction to various strategies that have made an impact in the literature.

An interesting approach of capturing promising strategies for the game was written in 1996 by [54]. Strategies represented by finite automata were learning to update their choices through a genetic algorithm. The specific type of finite automata that were used were Moore machines [56]. Finite state machine consist of a set of internal states. One of these states is the initial state of the machine. A machine also consists of transitions arrows associated with the states. Each arrow is labelled with A/R where A is the opponent's last action and R is the player's response. For example let us consider a graphical representation of the famous Tit for Tat given by a finite machine, Figure 3.

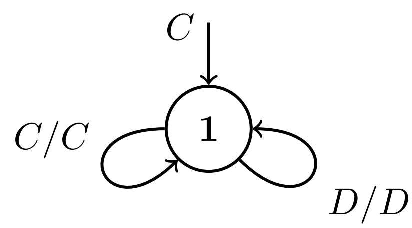

Figure 3: Finite state machine representation of Tit for Tat.

Miller used the genetic algorithm to train finite state machines in environments with noise. His results showed that even a small difference in noise (from 1% to 3%) significantly changed the characteristics of the evolving strategies. Three machines described in his paper are the following:

- **Punish Twice**: A strategy that punishes defection with 2 defections.
- **Punish Once for Two Tats**: A strategy which will defect only if the opponent has defected twice in a row.
- **Punish Twice and Wait**: A variant of Punish Twice which will answer defection with 2 defections and will cooperate if an only if the opponent cooperated.

Similar to Miller's work, in [12] the author presented two new strategies that have been trained using a finite state machine representation. These strategies are called, **Fortress3** and **Fortress4**. Figure 4 illustrates their diagrammatic representation.

During the experiments that introduced Fortress3 and Fortress4 a large diversity of strategies were found. Differentiating between strategies represented as a finite machine is not an easy task. It is not obvious looking at a finite state diagram how a machine will behave, and many different machines can represent the same strategy. In order to distinguish the strategies and assuring that they are indeed different [7] introduced a method called fingerprinting. The method of fingerprinting is a technique for generating a functional signature for a strategy [8]. This is achieved by computing the score of a strategy against a spectrum of opponents. The basic method is to play the strategy against a probe strategy with varying noise parameters. In [7] Tit for Tat is used as the probe strategy. Fingerprint functions can then be compared to allow for easier identification of similar strategies. In Figure 5 an example of Pavlov's fingerprint is given. Fingerprinting has been studied in depth in [8, 9, 10, 11].

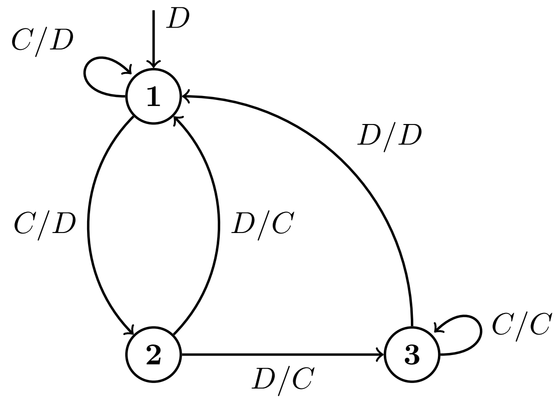

Figure 4: Representations of Fortress 3 and Fortress 4. Note that the strategy's first move, enters state 1, is defection for both strategies.

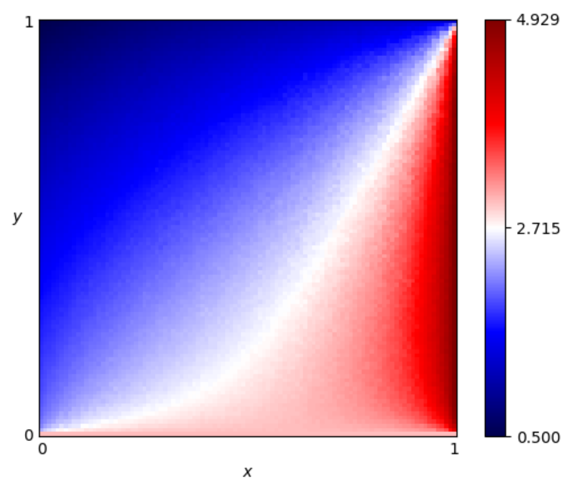

Figure 5: Pavlov fingerprinting with Tit for Tat used as the probe strategy. Figure was generated using [5].

In 2012 Press and Dyson [66] studied the iterated prisoner's dilemma and presented a new set of strategies called **zero determinant (ZD)**. The ZD strategies are memory one strategies that manage to force a linear relationship between their score and that of the opponent. In Section 2.4 it was described how the payoffs of two players could be retrieved by formulating their interactions using a Markov chain. Let us denote the payoffs of players p and q as:

$$s_p = vS_p$$
$$s_q = vS_q$$

where v is a vector of the steady states of matrix M and S_p, S_q are the equivalent payoff values of the players for each state CC, CD, DC, DD. Using linear algebra, Press and Dyson showed that the dot product of the stationary distribution of v with any vector f can be expressed as a 4×4 determinant. In which one column is f, one column is entirely under the control of player p and another column is entirely under the control of player q.

This meant that either p or q could independently force the dot product of v with some other chosen vector f to be zero by choosing their strategy so as to make the column they control be proportional to f. In particular, by $f = \alpha S_p + \beta S_q + \gamma$, any player can force a given linear relation to hold between the long-run scores of both players.

Press and Dyson's results suggested that the best strategies were selfish ones that led to extortion, not cooperation. Arguing with Axelrod's reports. All the more, their work stated that in the iterated prisoner's dilemma, memory is not advantageous.

The ZD strategies have attracted a lot of attention. It was stated that "Press and Dyson have fundamentally changed the viewpoint on the Prisoner's Dilemma" [76]. In [76], they ran a variant of Axelrod's tournament with 19 strategies to test the effectiveness of ZD strategies. While conducting their tournament they have implement several strategies discussed by [66] and revealed a set of generous ZDs the **Generous ZD**.

In [44], the 'memory of a strategy does not matter' statement was questioned. A set of more complex strategies, strategies that take in account the entire history set of the game, were trained and proven to be more stable than ZD strategies. Complex strategies were also studied by [36, 42]. This was done using an open source package, called the Axelrod project [5] which launched on 2015.

The project is written in the programming language Python, it is accessible and open source. To date the list of strategies implemented within the library exceed the 200. The project has been used in several publications including [36] and a paper describing it and it's capabilities was published in 2016 [41].

The two paper using the Axelrod project [36, 42] present several powerful strategies created using reinforcement learning techniques. Reinforcement learning refers to a collection of algorithms that trains a model by exploring a space of actions and evaluating consequences of those actions. In these papers the authors used genetic algorithms and particle swarm optimisation algorithms [77]. A number of strategy representations, referred as archetypes, were used to train strategies. These included, lookerup tables, finite state machines, artificial neural networks [87] and hidden Markov models [30].

Hidden Markov models, are a variant of a finite state machine that use probabilistic transitions based on the prior round of play to other states and cooperate or defect with various probabilities at each state. Finite state machines and hidden Markov models based strategies are characterized by the number of states. Similarly, artificial neural networks based players are characterized by the size of the hidden layer and number of input features.

Additionally a variant of a look up table is also presented called the lookerup archetype. The lookerup archetype responses based on the opponent's first n_1 moves, the opponent's last m_1 moves, and the players last m_2 moves. Taking into account the initial move of the opponent can give many insights. For it is the only move a strategy is truly itself without being affected by the other player. As a reminder, Axelrod in his work highlighted the importance of the initial move and believed that it was one of the secrets of success of the strategy Tit for Tat. Finally, a new archetype called the Gambler is also introduced, which is a stochastic variant of the lookerup archetype.

The training of these archetypes was done in two following settings:

- A Moran process, which is an evolutionary model of invasion and resistance across time during which high performing individuals are more likely to be replicated.

- A round robin tournament.

The result of [42] show that the trained strategies evolve an ability to recognise themselves by using a handshake. This characteristic of the strategies was an important one because in a Moran process this recognition mechanism allowed these strategies to resist invasion. In [36], they performed a a standard tournament with 200 turns but also a noisy tournament. For the standard tournament the newly introduced trained strategies outperform the designed ones. In the case of noise there is one particular strategy that has not seen much attention in the literature called “Desired Belief Strategy” [13]. These experiments are, to the authors knowledge, the biggest ones done in the field in terms of different strategies.

2.5 Contemporary period

In recent years the study of the iterated prisoners’ dilemma is still active and papers are still being published. New strategies, new variants of the game and new applications are being introduced every year. In this section we will briefly review some articles that have been published between 2017 and 2018.

The iterated prisoner’s dilemma serves as a model in a wide range of applications. For example in [40] cancer cells and how they can resist treatment has been modelled using evolutionary approaches described in Section **??**. Furthermore, in [29] they explore whether learning in social situations can be driven by rewards.

A lot of work has been done on evolutionary dynamics on structured populations. This is mainly because the applications they can offer in real life prolem, such as social interactions. In [47] the authors consider the evolutionary spatial prisoner dilemma with memory one strategies and their results indicate that a Pavlov like behaviour is stable and dominant. But how does cooperation evolve in structure situations and more specifically in situations where the number of neighbours can vary? In [6] the authors tried to shed some light to this question. However this was done for weak selection. Their results argue that, by considering the coalescence times of random walks of any given graph they can approximate if cooperation will emerge.

Several variations to the actual game are still being introduced. Ohtsuki in [64], studies the NPD, briefly introduced in Section 2.4. Ohtsuki propose a model called coordinated cooperation. It’s an NPD game which starts with is a negotiation before an actual game is played. Each individual can flexibly change their decision, either to cooperation or to defection, according to the number of those who show the intention of cooperation/defection. This NPD model was introduced in order to provide one explanation of why people tend to take into account others’ decisions even when doing so gives them no payoff consequences at all.

2.6 Software

However, it is important to mention that the only source code available is the code for the 62 strategies of the second tournament, found on Axelrod’s personal website [1].

Though several of this tournament discussed so far were generated using computer code not all of the source code was made available by the authors. Several open projects were created and published through the year. The first one discussed in this work, excluding the code for Axelrod’s strategies, is PRISON [4]. PRISON is written in the programming language Java and preliminary version was launched on 1998. It was used by it’s authors in several publications, such as [23] which introduced Gradual and [22]. The project includes a good number of strategies from the literature but unfortunately the last update of the project dates back in 2004.

Other recent software projects include [2, 3], both are education platforms and not research tools. In [2], several concepts such as the iterated game, computer tournaments and evolutionary dynamics are introduced through a user interface game. Project [3] offers a big collection of strategies and allows the user to try several matches and tournament configurations.

Finally, as described in Section **??**, the open source package Axelrod. Axelrod package is a software written following best practice approaches and contains the larger to date data set of strategies. The strategy list of the project has been cited by publications [58] and the package has not been used only by the contributors for academic research but from several such as: [43].

2.7 Conclusion

3 Analysing a large corpus of articles

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