A systematic literature review of the Prisoner's Dilemma and an analysis of the corresponding co author network.

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1 Introduction

The emergence of cooperation is a topic of continuing and public interest for the social [23, 31], biological [32] and ecological sciences [33, 40, 52, 79]. Cooperation is essential for evolution but according to Darwin's theory it is not always easy to achieve. The game called the prisoner's dilemma offers a theoretical framework for studying the emergence of altruistic behaviour.

1.1 The Prisoner's Dilemma

The prisoner's dilemma is a two player non-cooperative game [26] where the decisions of the players are made simultaneously and independently. Both players can choose between cooperation (**C**) or defection (**D**).

The fitness of each player is influenced by its own behaviour, and the behaviour of the opponent. If both players choose to cooperate, both do better than if both defect. However, a player has the temptation to deviate. If a player was to defect while the other cooperates, the defector receives more than if both had cooperated. The reward for mutual cooperation is R, for a mutual defection they receive P, and for cooperation-defection, the cooperator receives S where the defector receives T. Thus, the game's payoffs are given by,

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} \tag{1}$$

where $T > R > P > S$ and $2R > T + S$ are the conditions for a dilemma to exist. Due to rational behaviour and the knowledge that an individual is tempted to defect the game's equilibrium lies at a mutual defection and both players receive a payoff of P. Thus, the dominant strategy for the prisoner's dilemma is **D**.

However, when the game is studied in a manner where prior outcomes matter, the defecting choice is no longer necessarily the dominant choice. The repeated form of the game is called the iterated prisoner's dilemma and now two players play the game repeatedly. Interest was sparked on the iterated prisoner dilemma by R. Axelrod and his book [18] "The Evolution of Cooperation".

In his book Axelrod reports on a series of computer tournaments he organised of a finite turns games of the iterated prisoner's dilemma. Participants had to choose between **C** and **D** again and again while having memory of their previous encounters. Academics from several fields were invited design computer strategies to compete in the tournament. The pioneer work of Axelrod showed that greedy strategies did very poorly in the long run while more altruistic strategies did better.

"The Evolution of Cooperation" is considered a milestone in the field but it is not the only one. On the contrary, the prisoner's dilemma has attracted much attention ever since the game's origins. This is shown in Figure 1, which illustrates the number of publications on the prisoner's dilemma per year from the following sources:

- arXiv;
- PLOS;
- IEEE;
- Nature;
- Springer.

The choice of sources is due to the fact that they have an open access Api, the process of collecting the data (including criteria for inclusion of papers) and the analysis will be described more comprehensively in Section 3.

Each point of Figure 1 marks the starting year of a time period. Each of these time periods is reviewed and presented in 2, as an extensive literature review. This paper is the review of this type, in such detail since the origins to date.

Furthermore, in Section 3 a comprehensive data set of literature regarding the prisoner's dilemma will be presented and analysed. This allow us to review the amount of published academic articles as well as measure and explore the collaborations within the field.

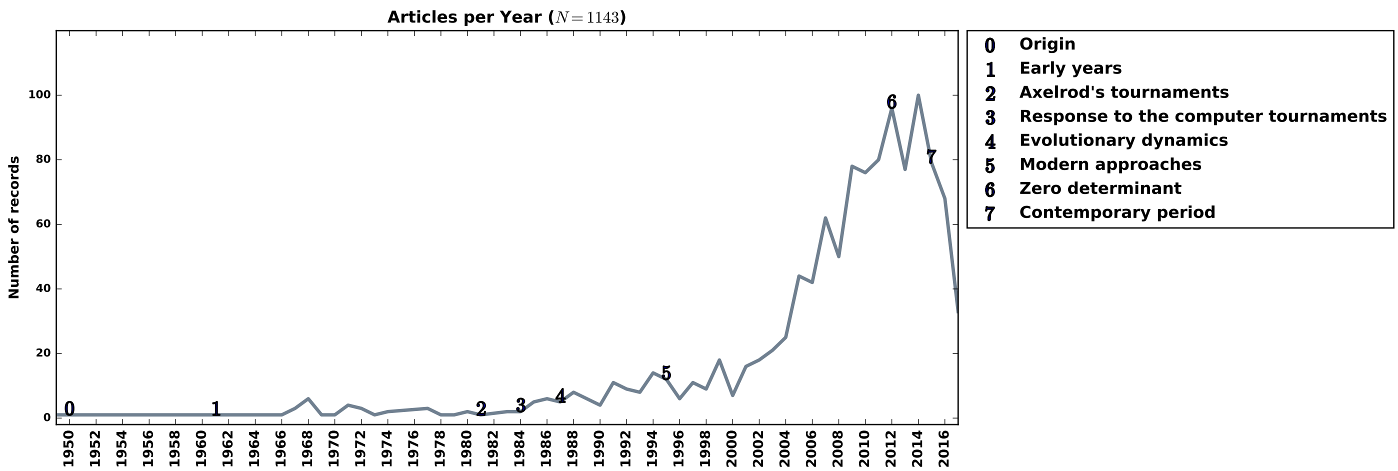


Figure 1: A timeline of the prisoner's dilemma research.

2 Timeline

2.1 Origin and Primal research (1961-1972)

The origin of the prisoner's dilemma goes back to the 1950s in early experiments conducted in RAND [26] to test the applicability of games described in [78]. Although in [26] the two player game was introduced the name behind the game was given later the same year. According to [74], A. W. Tucker (the PhD supervisor of J. Nash), in an attempt to delivery the game with a story during a talk used prisoners as players and the game has been known as the prisoner's dilemma ever since [74].

The study of the prisoner's dilemma has attracted people from various fields across the years. An early figure within the field is Prof A. Rapoport, a mathematical psychologist, whose work focused on peacekeeping. In his early work [65] Rapoport conducted experiments using humans to simulate a play of the prisoner's dilemma. Experimental groups were not been used only by Rapoport but it was a common mean of studying the game [24, 28, 46, 47, 70] and are still being used today.

Those experiments explored the conditions under which altruist behaviour emerges in human societies. Conditions such as, the gender [24, 46, 47] of individuals, the representation of the game [24], the distance between players [70], the initial effects [73] and whether the experimenter was biased [28].

Even though, several of these experiments were held and continuous research on the topic was undergoing game theorists were still in disagreement about the best way to play the game [65]. Inspired by the work of Rapoport and intrigued by the very same question the political scientist R. Axelrod took upon himself to identify the dominant strategy of the prisoners dilemma.

The main difference of Axelrod's approach was that machines were going to be used instead of humans. The issues with using humans, according to Axelrod [16], was the fact that humans can act very randomly even though the aim of the game is clear to them. Thus, Axelrod was the first researcher, to the author's knowledge, to perform a computer tournament of the iterated prisoner's dilemma. The work of Axelrod is considered one of the greatest milestones within the field. The tournaments and their results are discussed in the next sections.

2.2 Axelrod's Tournaments (1981-1984)

This section serves as a follow up from the earlier years of the topic and as an introduction to the modern ways of studying the prisoner's dilemma. It is dedicated to the computer tournaments of R. Axelrod from 1981 to 1984.

The first computer tournament was performed in 1980 [12]. Several scientists were invited to submit their strategies, written in the programming languages Fortran or Basic. There was a total number 13 submissions made by the following researchers,

1. T Nicolaus Tideman and Paula Chieruzz;
2. Rudy Nydegger;
3. Bernard Grofman;
4. Martin Shubik;
5. Stein and Anatol Rapoport;
6. James W Friedman;
7. Morton Davis;
8. Jim Graaskamp;
9. Leslie Downing;
10. Scott Feld;
11. Johann Joss;
12. Gordon Tullock;
13. Name not given.

Each competed in a 200 turn match against all 13 opponents, itself and a player that played randomly. This type of tournament is referred to as a round robin and corresponds to a complete graph from a topological point of view. The tournament was repeated 5 times to reduce variation in the results. Each participant knew the exact length of the matches and had access to the full history of each match. Furthermore, Axelrod performed an preliminary tournament and the results were known to the participants. The payoff values used for 1 where $R = 3, P = 1, T = 5$ and $S = 0$. These values are commonly used in the literature and unless specified will be the values used in the rest of the work described here.

The winner of the tournament was determined by the total average score and not by the number of matches won. The strategy that was announced the winner was submitted by Rapoport and was called **Tit For Tat**. Tit for Tat, is a strategy that always cooperates on the first round and then mimics the opponent's previous move.

Examples of Tit for Tat interacting for 8 turns with deterministic opponents are given by Tables 1, 2, 3. The opponents are, **Cooperator** a strategy that always cooperates, **Defector** an opponent that always defects and **Altenator** a player who alternates between cooperating and defecting.

The results of the first tournament were filled with surprises. Tit for Tat the simplest strategy of all had won and had managed to defeat even entrants that tried to improve on Tit for Tat after the preliminary tournament results. Axelrod justified the success of the strategy saying that the strategy was 'nice' and 'forgiving'.

The top eight ranked strategies were strategies that did no defect on the first round, thus they were described as 'nice' strategies. Compared to the rest of the "nice" strategies, Tit for Tat had also another property. That property was 'forgiveness'. Tit for Tat punished it's opponent for a defection but just once and then it would try to cooperate again. These two properties were described to be the secret of success in a prisoner's dilemma tournament.

In order to further test the robustness of the results Axelrod performed a second tournament [13] later in 1980. This time a total of 63 participants submitted strategies for the second tournament, their names were the following,

Turns	Tit for Tat	Cooperator
1	C	C
2	C	C
3	C	C
4	C	C
5	C	C
6	C	C
7	C	C
8	C	C

Table 1: Tit for Tat example match of 8 turns against Cooperator

Turns	Tit for Tat	Defector
1	C	D
2	D	D
3	D	D
4	D	D
5	D	D
6	D	D
7	D	D
8	D	D

Table 2: Tit for Tat example match of 8 turns against Defector

Turns	Tit for Tat	Altenator
1	C	C
2	C	D
3	D	C
4	C	D
5	D	C
6	C	D
7	D	C
8	C	D

Table 3: Tit for Tat example match of 8 turns against Altenator

1. Gail Grisell;
2. Harold Rabbie;
3. James W Friedman;
4. Abraham Getzler;
5. Roger Hotz;
6. George Lefevre;
7. Nelson Weiderman;
8. Tom Almy;
9. Robert Adams;
10. Herb Weiner;
11. Otto Borufsen;
12. R D Anderson;
13. William Adams;
14. Michael F McGurrin;
15. Graham J Eatherley;
16. Richard Hufford;
17. George Hufford;
18. Rob Cave;
19. Rik Smoody;
20. John Willaim Colbert;
21. David A Smith;
22. Henry Nussbacher;
23. William H Robertson;
24. Steve Newman;
25. Stanley F Quayle;
26. Rudy Nydegger;
27. Glen Rowsam;
28. Leslie Downing;
29. Jim Graaskamp and Ken Katzen;
30. Danny C Champion;
31. Howard R Hollander;
32. George Duisman;
33. Brian Yamachi;
34. Mark F Batell;
35. Ray Mikkelson;
36. Craig Feathers;
37. Fransois Leyvraz;
38. Johann Joss;
39. Robert Pebly;
40. James E Hall;
41. Edward C White Jr;
42. George Zimmerman;
43. Edward Friedland;
44. X Edward Friedland;
45. Paul D Harrington;
46. David Gladstein;
47. Scott Feld;
48. Fred Mauk;
49. Dennis Ambuehl and Kevin Hickey;
50. Robyn M Dawes and Mark Batell;
51. Martyn Jones;
52. Robert A Leyland;
53. Paul E Black;
54. T Nicolaus Tideman and Paula Chieruzz;
55. Robert B Falk and James M Langsted;
56. Bernard Grofman;
57. E E H Schurmann;
58. Scott Appold;
59. Gene Snodgrass;
60. John Maynard Smith;
61. Jonathan Pinkley;
62. Anatol Rapoport.

All the participants knew the results of the previous tournament. The rules were similar to those of the first tournament with only one exception; the number of turns was not specified instead a fixed probability (refereed to as 'shadow of the future' [17]) of the game ending on the next move was used. The fixed probability was chosen to be 0.0036 so that the expected median length of a match would be 200 turns. The topology was of a round robin and each pair of players was matched 5 times. The length of the matches was determined once by drawing a random sample. Each of the five matches had a length of 63, 77, 151 and 308.

The results of the tournament once again came as a surprise. Tit for Tat was considered to be one of the simplest submissions in the second tournament and won the second tournament as well. Tit for Tat provided proof that reciprocity behaviour can allow cooperation to emerge in the iterated prisoner's dilemma game. In [12] the main conclusions indicating strong performance was:

- that it start of by cooperating
- it would forgive it's opponent after a defection
- after opponents identified that they were playing Tit for Tat choose to cooperate for the rest of the game.

Another successful strategy from Axelrod's tournament that can been seen in literature to date is **Grudger**, originally submitted by James W. Friedman. Grudger is a strategy that will cooperate as long as the opponent does not defect.

The name Grudger was give to the strategy in [44]. Though the strategy goes by many names in the literature such as, Spite [20], Grim Trigger [19] and Grim [77].

As for the rest of the strategies, though a full explanation of all 13 submitted strategies is given in [12] the same does not hold for all 63 strategies of the second tournament [13]. The author mainly focuses on the high ranked participants and several details for the rest strategies are left unknown.

The source code of the 63 strategies be found on Axelrod's personal website [1]. The source code was written by Axelrod and several other contributors. The strategies written in Basic were translated to Fortran before the tournament. The source code includes the code only for the strategies and not for creating and performing the tournament.Figure 2 serves as an example of the source code giving the code for the winning strategy Tit for Tat. Unfortunately, the source code of the first 13 strategies is not available, as stated in Axelrod's personal website [1].

```
    FUNCTION K92R(J,M,K,L,R, JA)
C BY ANATOL RAPOPORT
C TYPED BY AX 3/27/79 (SAME AS ROUND ONE TIT FOR TAT)
c replaced by actual code, Ax 7/27/93
c  T=0
c   K92R=ITFTR(J,M,K,L,T,R)
      k92r=0
      k92r = j
c test 7/30
c   write(6,77) j, k92r
c77   format(' test k92r. j,k92r: ', 2i3)
      RETURN
      END
```

Figure 2: Source code for Tit for Tat in Fortran. Provided by [1].

So far it has been discussed how the performance of the strategy has been tested through tournaments against other strategies. A question remains: is the overall success of a strategy based only on it's performance in a round robin tournament or should it be checked through other ways as well?

Following his initial tournaments Axelrod performed an 'ecological' tournament in 1981 [18]. Axelrod argued that some strategies are so unsuccessful that there are very likely to be dropped in the future, while other more successful strategies would continue in later interactions. Influenced by evolutionary biology Axelrod introduced a way of capturing this behaviour, which included running a series of tournaments where more successful strategies would occupy a larger part of the environment and the less successful strategies would become less often. This is known as an ecological tournament.

The simulation of the process, as described in [18], is straightforward. Consider matrix 2 which provides the expected payoff when two individuals of different type interact. Starting with proportions of each type in a given generation the proportions of these strategies in the next generation is the only measure needed to be calculated. This is achieved by calculating the weighted average of the scores of a given strategy with all other players, where the weights are the numbers of the other strategies which exist in the current generation. The numbers of a given strategy in the next generation is then taken to be proportional to the product of its numbers in the current generation and its score in the current generation.

$$\begin{pmatrix} (R=3,R=3) & (S=0,T=5) \\ (T=5,S=0) & (P=1,P=1) \end{pmatrix} \tag{2}$$

Note the ecological tournament does not offer any evolutionary perspective. There is no possibility of a new strategy to be introduced, there is no mutation probability to drive the evolution. The ecological is a framework that provides the distributions of given types over time when interacting with the population.

The set of strategies from Axelrod's second tournament was used to perform the ecological tournament. Several interesting insights were reported,

- The lowest ranking 11 strategies had fallen to half their initial size by the 5 generation;
- The middle-ranking entries managed to hold their initial size;
- By the 500^{th} generation the only strategies that were larger than their initial size have been the top 11 ranked strategies;
- These formed 96% of the population at that time;
- The rule which ranked fifth in the tournament, submitted by William Adams, grew to three times its original size in the population and then began to sink after generation 100;
- The rule which ranked eighth, submitted by Paul D. Harrington, and was the only non nice rule in the top 15, grew to four times its original size but began to shrink after generation 150 to reach only a third of its original size by the 1000^{th} generation.

Overall the strategies that did rank at the top of the second tournament have also ranked top in the ecological tournament. On the same note, the strategy that was ranked at the top was again Tit for Tat. By the 1000th generation it was 14.5% of the whole population, followed by the third place rule at 13.9% and then the second place rue at 13.1%, Tit for Tat was growing at .05% per generation which was a faster rate than any other strategy. All these ar captured in Figure 3.

The ability of strategies to be favoured under natural selection and their ability to withstand invasion from other strategies soon became a measure of performance; refereed to as the stability of a strategy.

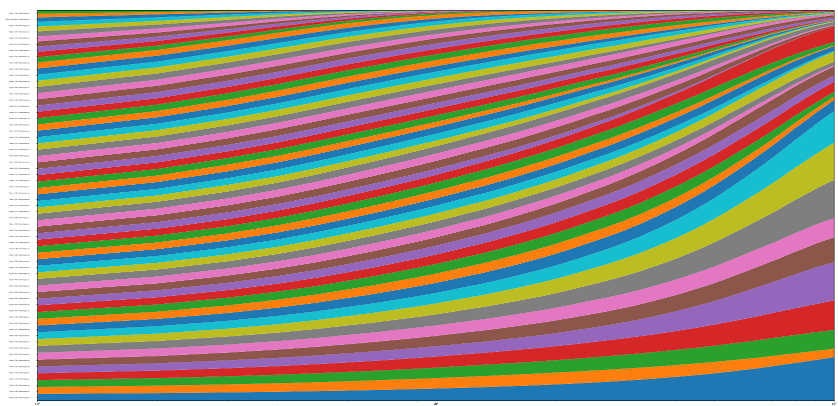


Figure 3: System evolving over time based on natural selection using [5], strategies set from Axelord's second tournament.

A much more general approach was discussed in [14]; the evolutionary approach. Imagine a population made up of individuals where everyone follows the same strategy, B and a single individual adopts a mutant strategy A. Strategy A is said to invade strategy B if,

$$V(A \mid B) > V(B \mid B) \tag{3}$$

where $V(B \mid B)$ is the expected payoff received by B against itself.

Since the strategy B is an population that interacts only with itself, the concept of invasion is equivalent to a single mutant being able to outperform the average population. This leads to the concept of the evolutionary approach. Thus for a strategy to be **evolutionary stable** it must be able to resists any invasion. There are several applications in biology for the interpretation of this approach, for example the survival of the fitness in wildlife.

Due the large number of possible strategies in the prisoner's dilemma identifying all the stable strategies was a difficulty task at the time. Axelrod focused the work of [14] in three questions,

1. Under what conditions was Tit for Tat evolutionary stable?

2. What were the necessary and sufficient conditions for any strategy to be evolutionary stable?
3. Finally, in an environment where all followed a strategy of unconditional defection, can cooperation emerge?

A series of theorems were presented which showed, that Tit for Tat is evolutionary stable y if and only if it is invadable neither by Defector nor Alternator. This is true only if the game is likely to last long enough for the retaliation to counteract the temptation to defect, according to Axelrod. Secondly, Defectors can withstand invasion by any strategy, as long as the players using other strategies come one at a time. But if they come in clusters (even in rather small clusters), the strategy could be invaded. As for the characteristics of stable strategies, Axelrod provided a series of theorems.

2.2.1 Response to the computer tournaments (1984-1993)

The pioneering work of computer tournaments and the results on the reciprocal behaviour of the prisoner's dilemma spread the knowledge of the game not only worldwide but also across different scientific principles. The study of cooperation became of critical interest once again. This section focuses on the immediate research that was carried out after the initial computer tournaments.

Ecological studies that made use of Axelrod's results include the works of [33, 52, 79], more specifically how the successful strategy Tit for Tat can be applied in nature and wildlife.

In [52] the behaviour of fish when confronting a potential predator was studied. Conflicts can arise within pairs of fish in these circumstances. Two experiments were held using a system of mirrors where sticklebacks would be accompanied by a cooperating companion or a defecting one. In both cases the hypothesis that the fish would behave according to Tit for Tat and that cooperation would evolve were supported. The works of [33, 79] looked at food sharing between vampire bats and explained behaviour based on famous at that time tournament strategies.

Axelrod's tournaments assumed that each player has perfect information of the opponent's actions. In real life situations this is not always the case. Interactions often suffer from measures of uncertainty. In the original tournaments there was no possibility of misunderstanding.

In 1985, P. Molander tested the robustness of Tit for Tat in an uncertain environment by introducing noise [55]. Noise is a probability that that one's move will be flipped. Molander findings stated that if two strategies playing Tit for Tat meet in a noisy match the average payoff that a strategy will receive will be the same as that of a Random player (with probability 0.5 of cooperating).

Further work on the performance of Tit for Tat in uncertain environments was conducted, described in [38, 33, 58]. These works focused, similar to Molander's, focused on how the strategy suffers against itself the most. In a noise environment, where a random defection can occur, the two strategies would end up in an unwanted circle of defection-cooperation. In a non noisy environment the strategies would have cooperated until the final interaction.

In [38] a similar tournament to that of Axelrod's was performed but this time noise was used. J. Bendor invited academics to submit strategies to participate in the tournament. A total of thirteen strategies were used including already existed strategies such as Tit for Tat and **Tit for Two Tats** [17]. Tit for Two Tats is a variant of the classic strategy that defects only when the opponent has defected twice in a row. The findings of the tournaments suggested that a more forgiving strategy is needed in a noisy environment. The winner of this tournament was a strategy called **Nice and Forgiving**.

The work of [58] aimed to also investigate stochastic effects. Using an evolutionary setting of a heterogeneous population where noise is taken into account, the space of reactive strategies was explored. Though a small fraction of Tit for Tat players have been essential for the emergence of cooperation, more generous strategies took over the population. This reactive strategy was is known as **Generous Tit for Tat** and can be presented as $(0, \frac{2}{}3)$.

Reactive strategies are a subset of memory one strategies introduced in 1989 [59]. Reactive strategies are denoted by the probabilities to cooperate after an opponent's **C** or **D** respectively. Thus, a reactive strategy only considers the previous turn of the opponent. Memory one strategies, are a set of strategies that consider the entire last turn of the game to decide on a next move.

Memory one strategies were also introduced by M. Nowak in 1990 [60]. Depending on the simultaneous moves of two players the states of the game, when only the previous round is considered, a state where both cooperated, both defected or either of them defected. These states are represented as CC, CD, DC, DD. A memory one strategy can be written as the probability of cooperating after each of these states. Thus as a vector of four probabilities p where $p = (p_1, p_2, p_3, p_4) \in \mathbb{R}^4_{[0,1]}$. Reactive strategies are just a constrained version of memory one strategies where $p_1 = p_3$ and $p_2 = p_1$.

The above formulation offered a new framework of studying strategies. Consider that two memory one strategies are in a game of the prisoner's dilemma. Their interaction can be written as the following markov chain,

$$M = \begin{bmatrix} p_1q_1 & p_1(-q_1+1) & q_1(-p_1+1) & (-p_1+1)(-q_1+1) \\ p_2q_3 & p_2(-q_3+1) & q_3(-p_2+1) & (-p_2+1)(-q_3+1) \\ p_3q_2 & p_3(-q_2+1) & q_2(-p_3+1) & (-p_3+1)(-q_2+1) \\ p_4q_4 & p_4(-q_4+1) & q_4(-p_4+1) & (-p_4+1)(-q_4+1) \end{bmatrix} \tag{4}$$

where the opponent is denoted as $q = (q_1, q_2, q_3, q_4) \in \mathbb{R}^4_{[0,1]}$. The expected state that two opponents will end up can be estimated by calculating the steady states of the markov chain.

Nowak, as described, studied the reactive but also the memory one strategies space and introduced several other strategies, among them the most popular was **Pavlov**. Pavlov is a strategy with the tolerance of Generous Tit for Tat but also the capability of resisting and invading an all-out cooperators population. The strategy is based on the fundamental behavioural mechanism win-stay, lose-shift. It starts off with a cooperation and then repeats it's previous move only if it was awarder with a payoff of R or T. Otherwise it shifts it's last move.

A number of researchers searched for new strategies. Such strategies have been, **Handshake** [67] and **Gradual** [20]. Presented in 1989 and 1997 respectively.Handshake was developed using an evolutionary tournament, where Gradual performance was tested in both a round-robin tournament and ecological simulation.

Handshake is a strategy that starts with cooperation, defection. If the opponent plays in a similar way then it will cooperate forever, otherwise it will defect forever. Gradual starts off by cooperating, then after the first defection of the other player, it defects one time and cooperates twice. After the second defection of the opponent, it defects two times and cooperates twice. After the n^{th} defection it reacts with n consecutive defections and then two cooperations.

Another measure of uncertainty is that of mis perception. Though noise will flip a player's action it will be recorded correctly in the history. Mis perception is the probability that the opponent's current move is flipped before being recorded [37].

2.3 Evolutionary Dynamics (1987-1999)

Determining the evolutionary stability of strategies for the iterated prisoner's dilemma as we discussed is not an easy task. Methods can be use to deal with the difficulty. In [22] the author restricted the possible strategies that could be adopted to a relatively narrow set and resulted that no pure strategy is evolutionary stable, including Tit for Tat. Arguing with the results presented in [14]. The list of strategies used included strategies such as Defector and **Suspicious Tit for Tat**, a strategy that plays Tit for Tat but starts by defecting.

The results were questioned by [48], stating that much was still no fully explored and more research had to be put into the results. Farrel and Ware in 1989 [25] extended the result to include finite mixture of pure and mixtures of Tit For n Tats as well. On the same year the work of [21] looking again at a narrow set of strategies extended their results to noisy environments.

Evolutionary dynamics have been highly useful in the research of the prisoner's dilemma. In [15], an evolutionary process, called the genetic algorithm, was used to discover effective strategies. The author introduced lookup tables as a mean of representing a strategy in a gene format. A lookup table is a set of deterministic responses based on the opponents m last moves; [15] considered $m = 3$.

An extension to the natural selection was introduced in the 1992 [62], recommending a different type of topology. A population of two deterministic strategies, Defector and Cooperator, were placed on a a two dimensional square array where the individuals could interact only with the immediate neighbours. The number of immediate neighbours could be either, fourth, six or eight. As shown in Figure 4. The authors claimed that the essential results remain true of all topologies; the results also hold whether self interactions are taken into account.

Thus each cell of the lattice is occupied by a Cooperator or a Defector. At each generation step each cell owner interacts with its immediate neighbours. The score of each player is calculated as the sum of all the scores the player achieved at each generation. At the start of the next generation, each lattice cell is occupied by the player with the highest score among the previous owner and the immediate neighbours. This topology is refereed to as spatial topology.

Nowak studied the population dynamics as a function of the temptation payoff. It was shown that for different values of the temptation payoff, cooperators and defectors could persist together.

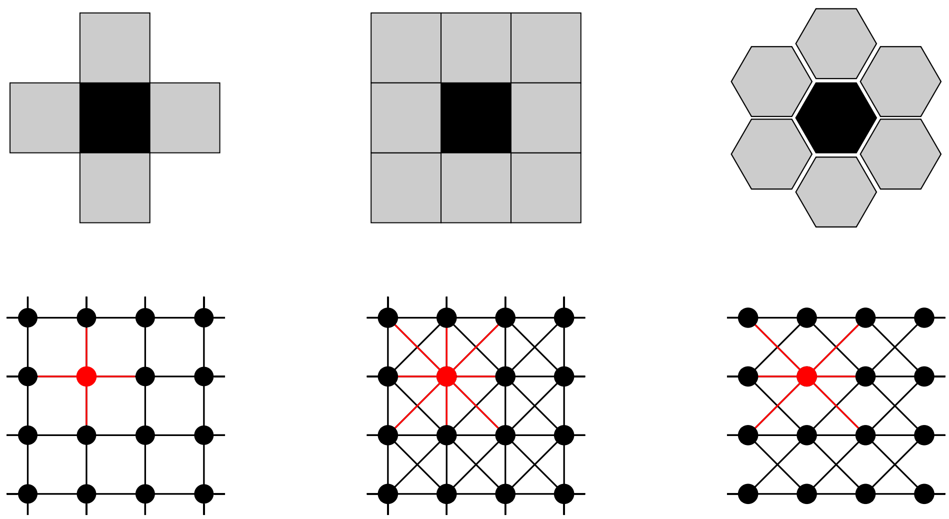


Figure 4: Spatial neighbourhoods

This work dealt with dealt with symmetric spatial lattices in two dimensions, deterministic winning and discrete time. The authors in later work [61], that the results remain valid in more realistic situations. Such as situations where the spatial distributions of cells are random in two or three dimensions, and where winning is partly probabilistic.

2.4 Modern approaches (1995-2015)

A number of aspects discussed in the previous sections such as round robin tournaments, evolutionary tournaments, training of strategies and noise environments soon became standard means of studying the iterated prisoner's dilemma. In this section we review a number of computer tournament that used these methods and introduced a number of findings that made an impact in the literature.

Initially in 1995 a combination of tournament studies, ecological simulations and theoretical analysis was used in [80] to demonstrate approaches to copy with noise. These included generosity, contrition and win-stay, lose-shift by respectively using the strategies Generous Tit for Tat, **Contrite Tit for Tat** and Pavlov. A fourth strategy was also analysed **Generous Pavlov**. A strategy that acts like Palvov but cooperates 10% of the time when it would defect otherwise.

- **Adaptive Tit for Tat** [76].

Less generous variants also made an appearance [36]. **Anti Tit for Tat**, is a strategy that plays the opposite of the opponents previous move. Another limitation of the strategy was discussed in [71]. Tit for Tat was proven to hit a loop between cooperation and defection. **Omega Tit For Tat** was introduced and was a strategy capable of avoiding such problem [71].

In 2011 the authors of [43] performed their own tournament where several interesting strategies made an appearance.

- **Periodic player CCD**, plays **C**, **C**, **D** periodically. Note that variations of a period player also make appearance in the article but will not be listed here.
- **Prober**, starts with the pattern **D**, **C**, **C** and then defects if the opponent has cooperated in the second and third move; otherwise, it play as Tit for Tat.
- **Reverse Pavlov**, a strategy that does the reverse of Pavlov.

In earlier work the same author introduced a strategy called **APavlov**, which stands for adaptive Pavlov [42]. The strategy attempts to classify the opponent as one of the following strategies, All Cooperator, All Defector, Pavlov, Random or **PavlovD**. PavlovD, is just Pavlov but it starts the game with a **D**. Once Adaptive Pavlov has classified the opponent plays to maximize it's payoff.

Evolutionary dynamics and optimization methods are used with different representation methods in order to discover new optimized strategies. Include lookup tables [15, 45], artificial neural networks [35, 41] and finite state machines [54, 69].

Strategies based on finite state machines are described by the number of states. The strategy selects the next action in each round based on the current state and the opponent's last move, transitioning to a new state each time. Figure 5, illustrates the finite state representation of Tit For Tat.

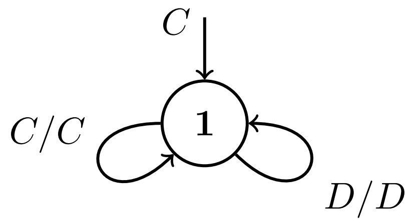


Figure 5: Finite state machine representation of Tit for Tat.

In [11] the author presented two new strategies that have been trained using a finite state machine representation. They are called, **Fortress3** and **Fortress4**. Figure 6 illustrates their diagrammatic representation where the transition arrows are labelled O/P where O is the opponent's last action and P is the player's response.

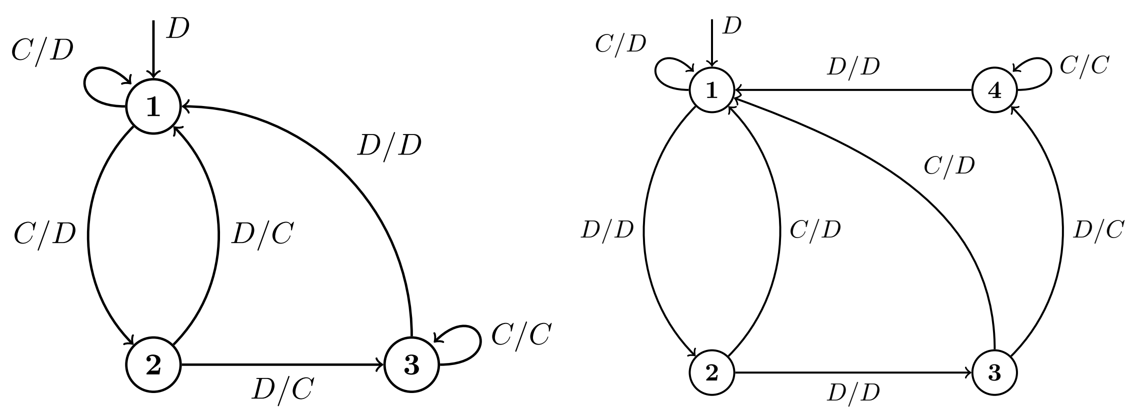


Figure 6: Representations of Fortress 3 and Fortress 4. Note that the strategy's first move, enters state 1, is defection for both strategies.

Optimisation methods will return a spectrum of strategies. In order to distinguish the strategies and assuring that they are indeed different [6] introduced a method called fingerprinting.

The method of fingerprinting is a technique for generating a functional signature for a strategy [7]. This is achieved by computing the score of a strategy against a spectrum of opponents. The basic method is to play the strategy against a probe strategy with varying noise parameters. In [6] Tit for Tat is used as the probe strategy. Fingerprint functions can then be compared to allow for easier identification of similar strategies. In Figure 7 an example of Pavlov's fingerprint is given. Fingerprinting has been studied in depth in [7, 8, 9, 10].

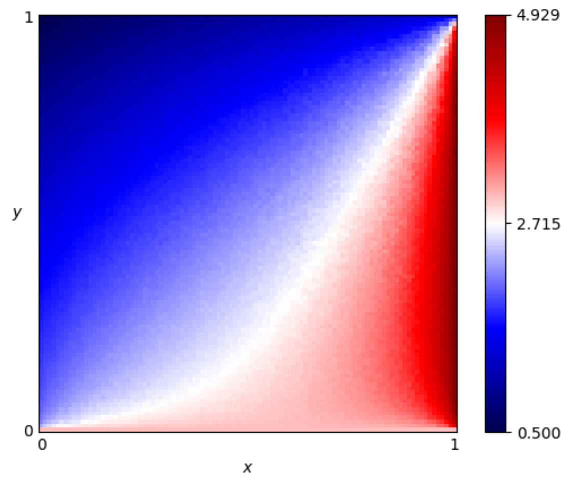


Figure 7: Pavlov fingerprinting with Tit for Tat used as the probe strategy. Figure was generated using [5].

Due the nature of the research several pieces of software are starting to appear, this includes a library called PRISON [4]. PRISON is written in the programming language Java and it has been used by it's authors in several publications. The project includes a good number of strategies from the literature but unfortunately the last update of the project dates back in 2004.

2.5 Zero determinant (2012 - 2015)

Following Section 2.4, this section is a review of an important set of strategies, the zero determinant.

In [64], a new set of memory one strategies were introduced, called **zero determinant (ZD)** strategies. The ZD strategies, manage to force a linear relationship between the score of the strategy and the opponent. Press and Dyson, prove their concept of the ZD strategies and claim that a ZD strategy can outperform any given opponent.

The ZD strategies have attracted a lot of attention. It was stated that "Press and Dyson have fundamentally changed the viewpoint on the Prisoner's Dilemma" [72]. In [72], a new tournament was performed including ZD strategies and a new set of ZD strategies the **Generous ZD**. Even so, ZD and memory one strategies have also received criticism. In [41], the 'memory of a strategy does not matter' statement was questioned. A set of more complex strategies, strategies that take in account the entire history set of the game, were trained and proven to be more stable than ZD strategies.

2.6 Contemporary period (2015 - 2017)

Following a discussion on research of short memory strategies this section reviews recent work done in complex strategies. As well as a discussion of new software and how modern approaches allows us to now revisit several pieces of work produced in the past.

Modern approaches of artificial neural networks and machine learning are now used in the field. A number of strategies based on artificial neural networks are introduced by [34]. Artificial neural networks provide a mapping function to an action based on a selection of features computed from the history of play.

These strategies are refereed to as **EvovlvedANN** strategies and are based on a pre-trained neural network with the following features,

- Opponent's first move is C
- Opponent's first move is D
- Opponent's second move is C
- Opponent's second move is D
- Player's previous move is C
- Player's previous move is D
- Player's second previous move is C
- Player's second previous move is D
- Opponent's previous move is C
- Opponent's previous move is D
- Opponent's second previous move is C
- Opponent's second previous move is D
- Total opponent cooperations
- Total opponent defections
- Total player cooperations
- Total player defections
- Round number

A representation of **EvovlvedANN 5** is given in Figure 8. The inputs of the neural network are the 17 features as listed above. Number 5 reefers to the size of the hidden layer.

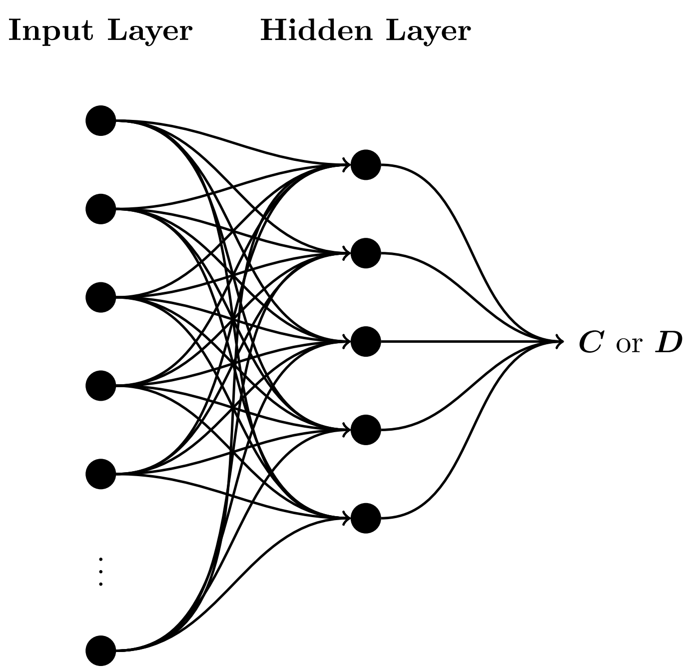


Figure 8: Neural network representation of EvovlvedANN 5.

In [34], these representing methods are refereed to as archetypes. Finite state machines and artificial neural networks are included in the work but also new archetypes are introduced, such as hidden Markov models. A variant of a finite state machine that use probabilistic transitions based on the prior round of play to other states and cooperate or defect with various probabilities at each state. Finite state machines and hidden Markov models based strategies are characterized by the number of states. Similarly, artificial neural networks based players are characterized by the size of the hidden layer and number of input features.

Additionally a variant of a look up table is also presented called the lookerup archetype. The lookerup archetype responses based on the opponent's first n_1 moves, the opponent's last m_1 moves, and the players last m_2 moves. Taking into account the initial move of the opponent can give many insights. For it is the only move a strategy is truly itself without being affected by the other player. As a reminder, Axelrod in his work highlighted the importance of the initial move and believed that it was one of the secrets of success of the strategy Tit for Tat.

Finally, a new archetype called the Gambler is also introduced, which is a stochastic variant of the lookerup archetype.

Archetypes are used with evolutionary algorithms to train set of new strategies. The evolutionary algorithm used in both [15, 30] is called genetic algorithm. Other algorithms including particle swarm optimization have been used in research of the most dominant strategy [27].

In [34] the approach in used to introduce as stated by the authors the best performing strategies for the iterated prisoner's dilemma. These strategies will be refereed as **Evolved** strategies. Several successful new strategies are,

- **EvolvedLookerUp2_2_2** a looker up strategy trained with a genetic algorithm; EvolvedLookerUp2_2_2 responses based on the opponent's 2 first and last moves and the player's 2 last moves. Thus $n_1 = 2, m_1 = 2$ and $m_2 = 2$.
- **Evolved HMM 5** a 5 states hidden markov model trained with a genetic algorithm;
- **Evolved FSM 16** a 16 state machine trained with a genetic algorithm;
- Finally **PSO Gambler 2 2 2** a looker up strategy trained with a particle swarm algorithm, where $n_1 = 2, m_1 = 2$ and $m_2 = 2$.

Though several papers have claimed before to have discovered the dominant strategies for the game the work of [34] is promising. This is due the fact that the introduced strategies have been trained using different types of evolutionary algorithms in a pool of 176 well known strategies for the literature. Including all the strategies that have been discussed in this section.

This was made possible due an open source library, called the Axelrod project [5]. The project is written in the programming language Python, it is accessible and open source. To date the list of strategies implemented within the library exceed the 200. The project has been used in several publications including [34] and a paper describing it and it's capabilities was published in 2016 [39]. The source code for Tit for Tat as implement within the library is shown in Figure 9. Furthermore, performing a tournament with a selection of strategies is possible in five lines of code, shown in Figure 10.

```python
def strategy(self, opponent: Player) -> Action:
    """This is the actual strategy"""
    # First move
    if not self.history:
        return C
    # React to the opponent's last move
    if opponent.history[-1] == D:
        return D
    return C
```

Figure 9: Source code for Tit for Tat in Python as implemented in Axelrod Python library [5]

.

```python
>>> import axelrod as axl
>>> players = (axl.Cooperator(), axl.Defector(), axl.TitForTat(), axl.Grudger())
>>> tournament = axl.Tournament(players)
>>> results = tournament.play()
>>> results.ranked_names
['Defector', 'Tit For Tat', 'Grudger', 'Cooperator']
```

Figure 10: Performing a computer tournament using [5].

Software has a crucial role in research. Well written and maintained software allows the reproducibility of prior work and can accelerate findings within the field. The field of the iterated prisoner's dilemma has suffered the consequences of poor research software. As stated above the source code of the initial computer tournament is not retrievable. Several of the strategies that competed in the tournament are not given a full explanation of how the decided on their next move. In terms of best practice and reproducibility the Axelrod library is the lead software in the field.

Other recent projects include [2, 3], both are education platforms and not research tools. In [2], several concepts such as the iterated game, computer tournaments and evolutionary dynamics are introduced through a user interface game. Project [3] offers a big collection of strategies and allows the user to try several match and tournaments configurations. Such as noise.

In [66], the authors claim that they have managed to re-run the first tournament that Axelrod performed. They tried to push his work further by altering aspects such as, the format of the tournament, the objective and the population. One of the authors claimed to have been a contributor to the first tournaments, which would explain how it was managed to reproduce the tournament.

2.6.1 Biological Applications

- [75] uses evolutionary game theory to study the spread of virus.
- [32] a shout for his work, using tit for tat to study cells.

3 Data Analysis

In this section we will focus on the analysis of the study of the prisoner's dilemma using a large dataset of articles. This data set will be used to ascertain the level of collaborative nature of the field and identify influencers. This will be done relative to:

- Other sub fields of game theory: auction games [51] and the price of anarchy [68].
- A temporal analysis.

3.1 Data Collection

Before analysing our data in this subsection we will describe the data collection process.

Academic articles are accessible through scholarly databases and collections of academic journals. Several databases and collections today offer access through an open application protocol interface (API). An API allows users to query directly a journal's database and skip the user interface side of the journal. Interacting with an API has two phases:

1. requesting;
2. receiving;

The request phase includes composing a url with the request. Figure 11 demonstrates an example request. The first part of the request is the address of the API we are querying. In this example the address is that of the arXiv API. The second part of the request contains the search arguments. In our example we are requesting for a single article that the word 'prisoners dilemma' exists within it's title. The format of the request message is different from API to API.

The receive phase includes receiving a number of raw metadata of articles that satisfied the request message. The raw metadata are commonly received in a xml or a json format [63]. Similarly to the request message, the structure of the received data differs from journal to journal.

```
http://export.arxiv.org/api/query?search_query=abs:prisoner’s dilemma&max_results=1
```

Figure 11: A request message for the arXiv API.

The data collection is crucial to this study, to ensure that this study can be reproduced all code used to query the different APIs has been packaged and is available online [56]. The software could be used for any type of project similar to the one described here, documentation for it is available at: `http://arcas.readthedocs.io/en/latest/`.

The following sources were used to collect data,

1. arXiv [49];
2. PLOS [29];
3. IEEE;
4. Nature;
5. Springer.

These are four prominent journals in the field, as well as the arXiv [49] pre print server. In the case of an article being both in a journal and the arXiv, only the journal version was considered.

For each article [56] collects a list of the features, shown in Table 4. Note that the plain text of the article is not collected, just the metadata. The data is archived and available at. In this work only the features of Table 5 are used.

	Result name	Explanation
1	Abstract	The abstract of the article.
2	Author	A single entity of an author from the list of authors of the respective article.
3	Date	Year of publication.
4	Journal	Journal of publication.
5	Key	A generated key containing an authors name and publication year (ex. Glynatsi2017).
6	Keyword	A single entity of a keyword assigned to the article by the given journal.
7	Labels	A single entity of labels assigned to the article manual by us.
8	Pages	Pages of publication.
9	Provenance	Scholarly database for where the article was collected.
10	Score	Score given to article by the given journal.
11	Title	Title of article.
12	Unique key	A unique key.

Table 4: Metadata for each entry/article.

	Result name	Explanation
1	Abstract	The abstract of the article.
2	Author	A single entity of an author from the list of authors of the respective article.
3	Date	Year of publication.
4	Journal	Journal of publication.
5	Provenance	Scholarly database for where the article was collected.
6	Title	Title of article.

Table 5: Structure of data set. Contained results.

In the work described here, a series of keywords were used to identify relevant articles. Articles for which these keywords were in the title or the abstract are included in the analysis. A list of the keywords that were used are shown in Table 6.

Similarly, for collecting data on auction games and the price of anarchy the following keywords respectively for each topic,

- key: auction game theory;
- key: price of anarchy.

	Keywords
1	prisoner's dilemma
2	prisoners dilemma
3	prisoners evolution
4	prisoner game theory
5	R Axelrod
6	memory one strategy
7	tit-for-tat
8	tit for tata
9	zero determinant strategies

Table 6: Keywords used in searching for articles.

3.2 Preliminary Analysis

A total of three data sets are explored in this work. A summary of each data is presented in this section. The three data sets are,

- the main data set, which contains articles on the prisoner's dilemma, []
- a data set containing data on auction games [] and
- a data set containing data on the price of anarchy [].

The prisoner's dilemma data set.

The main data set and the main focus of this analysis. The data set [] consists of 1150 articles, where 1145 have unique titles. This is because a total of 5 articles have been collected from both a journal and arXiv. All duplicates from arXiv are dropped, thus hereupon we consider 1145 unique article entries.

There are a number of 41 articles that have not been collected from the aforementioned APIs. These articles were manually added to the dataset throughout the writing of Section 2. A more detailed summary of the articles' provenance is given by Table 7.

	Provenance	Total articles
1	arXiv	470
2	Springer	312
3	IEEE	241
4	PLOS	63
5	Nature	23
6	Manual	41

Table 7: Articles' provenance for [].

The larger number of articles were collected from arXiv, Springer and IEEE. Both Nature and PLOS have a small contribution to the size of the data set. The oldest article was published in 1944 and the most recent one in 2017. Note that the last data collection was on December 2017.

The average publication over the year has been calculated. This is done for the overall data set and for each journal individually. The average publication is estimated as the ratio of the number of total articles and the years of publication. Thus:

$$\text{av. publication} = \frac{\text{number of articles}}{\text{years of publication}}$$

The years of publication is calculated as the range between 2017 and the first published article within the data.

Table 8 summarises these averages. Overall an average of 21 articles are published per year. The most significant contribution to this appears to be from arXiv with 8 articles per year, followed by Springer with 5 articles per year.

	Av. publications
Overall	21.167
IEEE	4.463
Nature	0.426
PLOS	1.167
Springer	5.741
arXiv	8.611

Table 8: Average publication for [].

Auction games and the price of anarchy data sets.

A summary of both data sets, auction games [] and price of anarchy [], is given by Table 9.

A total of 2103 articles are examined for auction games. Auction games are a well studied topic for several years with 3860 authors and the earliest entry going back to 1974. In comparison, 296 unique articles have been collected on price of anarchy. The earliest entry being in 2003 and a total of 668 unique authors have written about the topic.

In Figure 12 a time plot for each topic is displayed and is exhibited that both topics have an increasing trend over the years. Though price of anarchy is clearly a new topic compared to auction games.

The frequency of the prisoner's dilemma, for both articles and authors, lies between the frequencies of these two topics.

	Price of anarchy	Auction games
Unique articles	296	2103
Unique authors	668	3860
Min publication year	2003	1974
Max publication year	2017	2017

Table 9: Data sets [] and [] summary.

The provenance of the articles is given by Table 10. Almost 1500 article for auction games have been collected from Springer, that is more than three times the articles that have been collected from other sources. Where PLOS and Nature have a minor contribution and PLOS and Nature had no articles on the price of anarchy.

The overall average publication for [] and [] are 59 and 20 articles respectively. It appears that auction games publication is largely different for both the prisoner's dilemma and the price of anarchy. These two topics have the same average publication. Note that the significance of each journal differs from topic to topic. Though this analysis will not focus on individual sources from hereupon.

Temporal analysis.

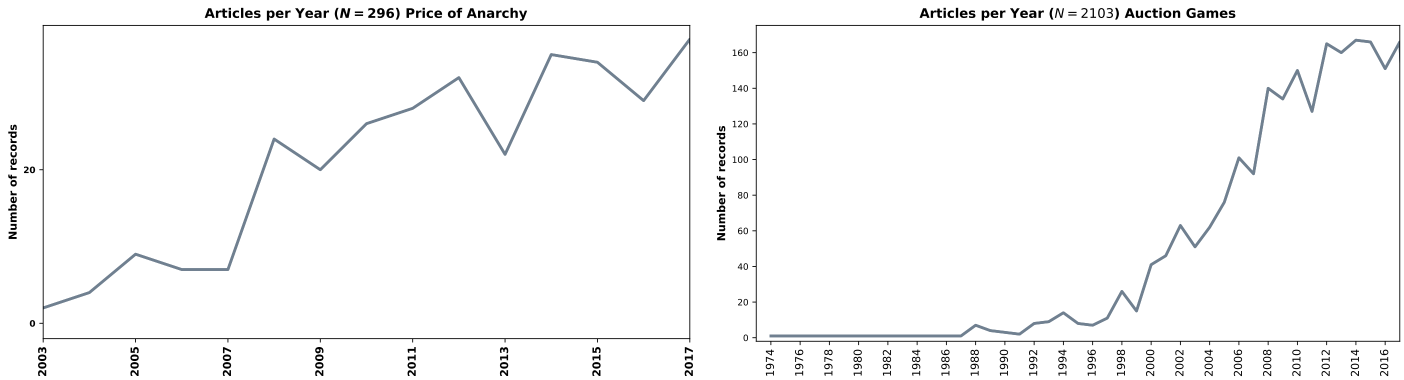


Figure 12: Time plots of [] and [].

Provenance	Total articles (auction games)	Total articles (price of anarchy)
Springer	1429	78
arXiv	436	108
IEEE	301	131
PLOS	15	-
Nature	1	-

Table 10: Articles' provenance for [] and [].

Provenance	Av. publication (auction games)	Av. publication (price of anarchy)
Overall	58.973	19.812
Springer	38.622	4.875
arXiv	11.784	6.750
IEEE	8.135	8.188
PLOS	0.405	-
Nature	0.027	-

Table 11: Average publication for auction games and the price of anarchy.

For comparison reasons in the following subsections the analysis will also be held relative to a temporal analysis. The main data set [] is divided into time period according to the subsections of Section 2. The respective measures of unique titles and unique authors for each period is also given by Table 12.

	Unique articles	**Unique authors**
period 1: (1961 - 1972)	21	38
period 2: (1981 - 1984)	5	6
period 3: (1984 - 1993)	64	70
period 4: (1987 - 1999)	121	169
period 5: (1995 - 2015)	926	1730
period 6: (2012 - 2017)	453	1008
period 7: (2015 - 2017)	180	466

Table 12: Periods and their respective measures.

In this section we have described the the three data sets that we are going to use in the following sections in order to identify collaborative behaviour and influence. Two data sets of different topics are used for comparison reasons. The frequency of articles and authors differs within the three data sets which is ideal. Furthermore the journals effect also differ but not much will be done at that level form now onwards.

Finally a temporal analysis of the data sets [] will also assists us in obtaining more insights. The period have also be presented here. In the following section we explore the authors of the papers and their connections to one another.

3.3 Co authorship Analysis

Measuring academic collaborations and influences of academic projects is not always trivial. Many academics tend to consider citations as a measure of how good a research project is. According to a blog post [57] written by Nature in 2017, depending on citation can be misleading. This is because:

- The true number of citations can not be known. Citations can be missed due data entry errors or typos in a journal.
- Academics are influenced by many more papers than they actually cite.
- Several citations are superficial.

For these reasons in this work we introduce an alternative way. Initially, we construct a network of co authorship for a given topic, then several network measures, such as,

- number of connected components,
- clustering coefficient,
- degrees and
- centrality measures

are used to explain **collaboration** and **influence** within the field. All network measures are introduced with examples in the following sections.

3.3.1 Constructing the networks

To construct a co authorship network all the unique authors of a data set are needed. The issue with retrieving the unique authors is that authors names can be written in different ways in different sources. For example consider the author of this paper:

- Nikoleta Glynatsi
- Nikoleta E. Glynatsi
- Nikoleta Evdokia Glynatsi

Consequently, several different entries of the same author existed within the data set. To address the problem the Levenshtein Distance [53] was used. The Levenshtein Distance is a metric for measuring the difference between two sequences. The Levenshtein distance between two strings is defined as the minimum number of edits needed to transform one string into the other, with the allowable edit operations:

1. insertion;
2. deletion;
3. substitution of a single character.

The Levenshtein distance of all possible pairs of authors in the data set was computed. If the distance was between 85 and 99 the entries were highlighted. The highlighted entries were manually checked to assure that there were indeed the same author and then one of them was replaced by the other.

For example all entries with author name written as example 1 were replaced by 2.

1. Y. Moreno
2. Yamir Moreno

The manual check is performed because not all highlighted entries are indeed the same. For example:

1. Zhen Yang and
2. Zhen Wang

are two individual authors. Once the names were cleaned the total number of unique authors can be estimated.

Thus, the co authorship network of the prisoner's dilemma is now defined as:

An undirected graph G of of vertices V and edges E. There are 2101 vertices representing each unique author. The vertices are joined by 3174 edges. An edge connects two authors if and only if those authors have written together. No weight has been applied to the edges or the nodes.

The network is shown in Figure 13.

The same approached is used to construct the networks for auction games and the price of anarchy. Their respective networks are illustrated in Figure 14.

Now that the networks have been defined in the following sections the collaborative trend of the field and the influence are explored.

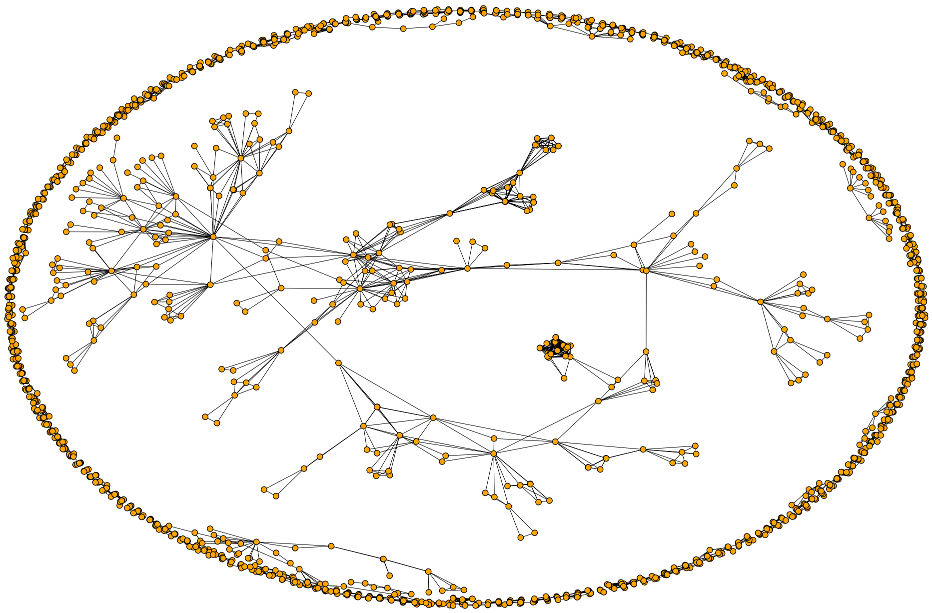
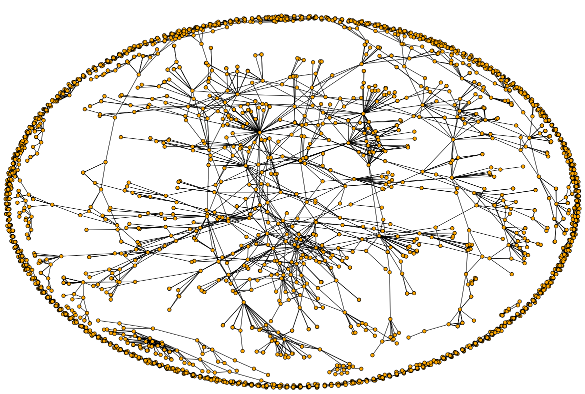
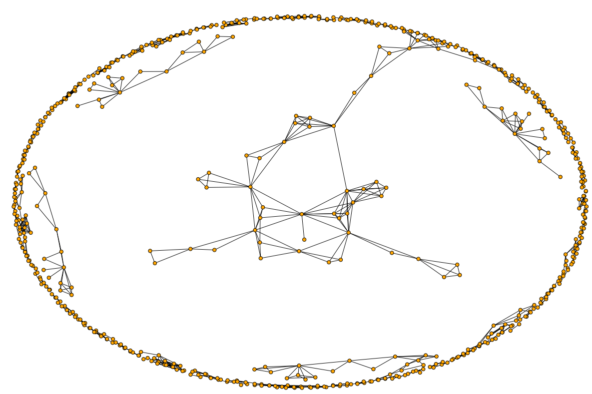


Figure 13: Co-authorship network for prisoner's dilemma.



(a) Co-authorship network for auction games.



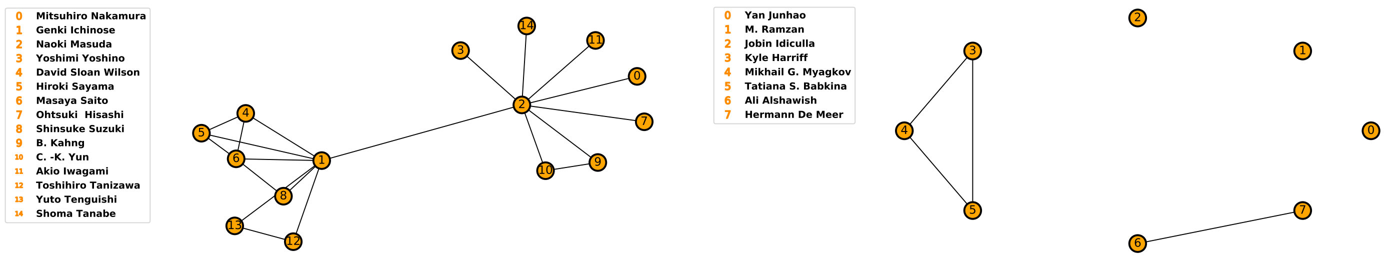
(b) Co-authorship network for the price of anarchy.

Figure 14: Co-authorship network for other topics.

3.3.2 Collaborative behaviour

In this section we ascertain the level of collaborative nature of the field using several connectivity measures such as, number of connected components, clustering coefficients and degree distribution. To explain the notion of some measures we will be using several sub graphs of G.

The first measure is the number of connected components. A connected component of an undirected graph is a sub graph in which any two vertices are connected to each other by paths. Let us consider two sub graphs of G, Z and H (shown in Figure 15) where sub graph Z is a connected graph and H is not.



(a) Sub graph Z.

(b) Sub graph H.

Figure 15: Connected components examples.

The number of connected components indicates the number of connected sub graphs within the co authorship network. Thus the number of authors teams that are connected either because their wrote with each other or with someone that wrote with someone else.

The second connectivity measure considered is the clustering coefficient. Clustering coefficient is a measure of the degree to which nodes in a graph tend to cluster together, thus a measure of cliqueness. There are two types of clustering coefficient of a graph. The local and the global measure. The local measure, measures the clustering coefficients of a specific node and is calculated as,

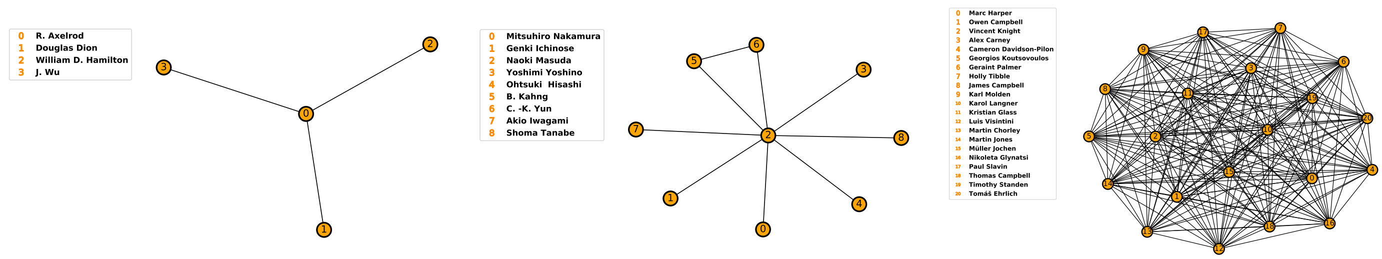
$$\frac{2 \times \text{Number of edges between neighbours}}{\text{Node degree (Node degree - 1).}}$$

The global version of the measure can be calculating by averaging all the local coefficients of the graph. The clustering coefficient ranges between values of 1 and 0. The closer to 1 indicates that the graph is getting closer to a complete graph, sub graph K, where the closer to 0 indicates being a star graph, sub graph A. Sub graph M lies within the two extreme cases and has a clustering coefficient of 0.23.

In this work having a high clustering coefficient indicates that the connectivity of the connected component is because people wrote a paper together. Thus there is no larger connection to the sub graph other than that clique of authors that worked together.

The final measure considered in this section is the degree distribution. Degree shows the number of connections a node has. The degree distribution will allow us to understand the most common degree in our networks.

Comparison with other topics authorship.



(a) Sub graph A.

(b) Sub graph M.

(c) Sub graph K.

Figure 16: Clustering coefficients examples.

	Connected components	**Clustering coefficient**
Prisoner's dilemma	529	0.68
Auction games	797	0.67
The price of anarchy	162	0.71

Table 13: Collaborative behaviour measures for auction games and the price of anarchy.

Let us compare the collaborative behaviour of the prisoners dilemma against two other topics. Table 13 summarizes the measures.

The field of the prisoner's dilemma has smaller number connected components than auction games and a higher number than the price of anarchy. This could be due the size of the data sets. The interesting result can be obtained by the clustering coefficients by considering the difference in the number of connected components.

The prisoner's dilemma has the same clustering coefficient to auction games. Based on their number of connected components we can result that authors of the prisoner's dilemma tend to collaborate only with specific authors, and avoid interactions with other sub graph compared to auction game authors. However, the field appears to be more collaborative compared to the topic of price of anarchy, which has a clustering coefficient of 0.77 with only 162 connected components.

To further examined the differences or the similarities of these networks we consider the degree distributions. The normalised distributions of all three networks are given in Figure 17. They have been normalised such that the frequencies sum to one. The distributions appear to be similar but to validate the hypothesis a statistical test is used.

None of the distributions is normally distributed thus the non parametric test Kruskal-Wallis is used [50]. Kruskal-Wallis allow us to compare the medians of two or more distributions. The test returns a p-value of 0.29. Thus, there is no significant difference in the degree distributions of the three networks.

Temporal comparison.

The collaborative behaviour of G is also studied over time. That is achieve by calculating the number of connected components, the clustering coefficients and the degrees distribution.

The number of connected connected components and the clustering coefficients are both increasing from period 2 to period 7.

Period 2 was a very poor period of publications in our sources. Period 5 has the highest number of connected components but period 7 has the highest clustering coefficient of 0.76 with only 134 connected components. A big increase of connected components is spotted from periods 4 to 5.

Furthermore, over time the degree seems to be stabilising over 2 degrees, Figure 18. Thus in the study of the prisoners dilemma according to our data, papers with 3 authors seems to be favoured.

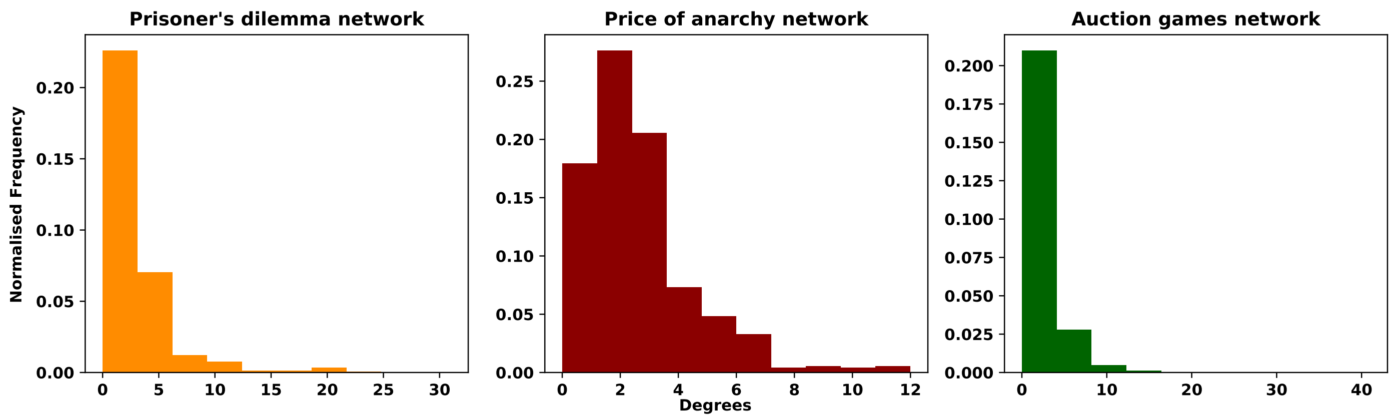


Figure 17: Degree distributions for all three networks.

	Connected components	Clustering coefficient
period 1	15	0.50
period 2	5	0.00
period 3	49	0.14
period 4	96	0.30
period 5	534	0.64
period 6	281	0.74
period 7	134	0.76

Table 14: Collaborative behaviour measures over time periods.

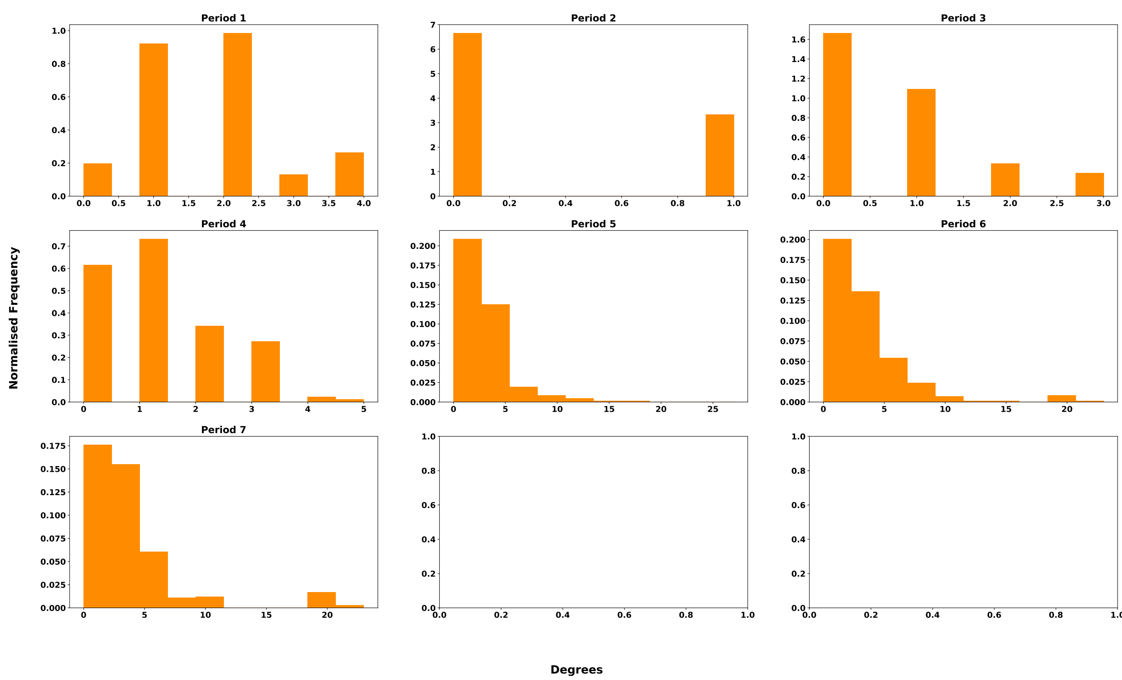


Figure 18: Degrees distribution over time.

3.3.3 Influence

Network centrality is used in network theory to study which nodes of a graph are the most important. There are several centrality measures used in different projects to explain different behaviours of the nodes. The advantage of constructing a co authorship network is that measure such that can used to measure influence in the literature of the fields.

Two centrality measures are used here:

- betweenness centrality and closeness centrality.

Betweenness will be used to explain the people in a network that gain influence and closeness the people that influence the field. Similarly both network measures are explained with exampled using sub graphs.

Closeness centrality

Closeness centrality of a node u is the reciprocal of the average shortest path distance to u over all $n-1$ reachable nodes.

$$C(u) = \frac{n-1}{\Sigma_{v=1}^{n-1} d(v,u)}$$

where $d(v,u)$ is the shortest-path distance between v and u, and n is the number of nodes that can reach u.

Betweenness centrality

Betweenness centrality of a node v is the sum of the fraction of all-pairs shortest paths that pass through v

$$C_B(v) = \Sigma_{s,t \in V} \frac{\sigma(s,t|v)}{\sigma(s,t)}$$

where V is the set of nodes, $\sigma(s,t)$ is the number of shortest (s,t)-paths, and $\sigma(s,t|v)$ is the number of those paths passing through some node v other than s, t. If $s = t$, $\sigma(s,t) = 1$, and if $v \in s, t, \sigma(s,t|v) = 0$.

These two measures can be used to explain different things. Closeness is a measure that shows how well a node conenects other nodes. In out work how well an author is connected to other uahtors and make them collaborate. We call this influence. How much an author influene their enviroment. On the other hand betweeness is about how connected a node is, how much influence an author can gain from their enviroment.

As an example consider sub graph T. Note that nodes 2, 3 and 1 are immiatly connected to three authors. thus we expect their between measure to be the same but closeness is different. Node 3 is the connecting link between at leat 4 people. Thus node 3 is the person in the subgraph that influences most authors cause it connects people work. And that is hld to be true when we calculate, C_b.

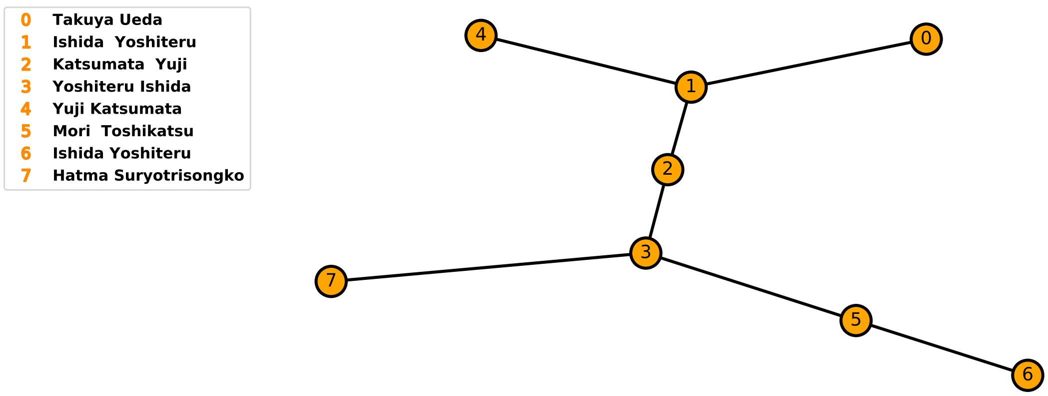
Node 3 also gains influence due ot its rule in the team, but node 2 that does not connected people as much has the same betweeness. That is because node 2 without connecting operations byt haing the right collaborations can gain from the influence of the team.

We indentify those two measures to be importnant and easily calcutalted due to an open source package. We base our further analysis to these measures in order to test how the authors affect the field.

Comparison to other fields.

Temporal comparison.

3.3.4 Conclusion



	Author name	Betweeness		Author name	Closeness
0	Matjaz Perc	0.010584	0	Matjaz Perc	0.044428
1	Yamir Moreno	0.008786	1	Yamir Moreno	0.043561
2	Luo-Luo Jiang	0.004319	2	Cheng-Yi Xia	0.038910
3	Arne Traulsen	0.003920	3	Sandro Meloni	0.037959
4	Martin A. Nowak	0.003832	4	Alberto Aleta	0.037600

Table 15: Central authors based on different centrality measures.

	Author name	Betweeness		Author name	Closeness
0	Chen Jing	0.021126	0	Michal Feldman	0.049135
1	Martin md	0.019925	1	Brendan Lucier	0.049014
2	Nick Gravin	0.014590	2	Tim Roughgarden	0.048825
3	Liu Zhe	0.013759	3	Martin md	0.048100
4	Huang He	0.012792	4	Chen Jing	0.047613

Table 16: Central authors based on different centrality measures.

	Author name	Betweeness		Author name	Closeness
0	Angelo Fanelli	0.002418	0	Ioannis Caragiannis	0.033312
1	Ioannis Caragiannis	0.001983	1	Brendan Lucier	0.032486
2	Alexander Skopalik	0.001664	2	Eva Tardos	0.030710
3	Eva Tardos	0.001596	3	Michal Feldman	0.030471
4	Michal Feldman	0.001497	4	Angelo Fanelli	0.030471

Table 17: Central authors based on different centrality measures.

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