A systematic literature review of the Prisoner's Dilemma; collaboration and influence.

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Abstract

The prisoner's dilemma is a well known game used since the 1950's as a framework for studying the emergence of cooperation; a topic of continuing interest for mathematical, social, biological and ecological sciences. The iterated version of the game attracted attention in the 1980's after the publication of the "The Evolution of Cooperation" and has been a topic of pioneering research ever since. This paper aims to provide a detailed literature review of the field. This is achieved by partitioning the timeline into six different sections. Furthermore, a comprehensive data set of literature is analysed using network theoretic approaches in order to explore the influence and the collaborative behaviour of the field itself.

1 Introduction

In everyday life people choose between being selfish or being selfless when they interact with others. The question that arises is: which behaviour is more beneficial? There is a simple way of representing these behaviours/concepts. This is to use a particular two player non-cooperative game called the prisoner's dilemma, originally described in [42].

Each player has two choices, to either be selfness and cooperate or to act in a selfish manner and choose to defect. Each decision is made simultaneously and independently. The fitness of each player is influenced by its own behaviour, and the behaviour of the opponent. Both players do better if they choose to cooperate than if both choose to defect. However, a player has the temptation to deviate as that player will receive a higher payoff than that of a mutual cooperation.

A player's payoffs are generally represented by (1). Both players receive a reward for mutual cooperation, R, and a payoff P for mutual defection. A player that defects while the other cooperates receives a payoff of T, whereas the cooperator receives S. The dilemma exists due to constrains (2) and (3).

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} \tag{1}$$

$$T > R > P > S \tag{2}$$

$$2R > T + S \tag{3}$$

Another common representation of the payoff matrix is given by (4), where b is the benefit of the altruistic behaviour and c it's its cost (constrains (2) - (3) still hold).

$$\begin{pmatrix} b-c & c \\ b & 0 \end{pmatrix} \tag{4}$$

Constrains (2) - (3) and rational behaviour guarantee that it never benefits a player to cooperate. However, when the game is studied in a manner where prior outcome matters, defecting is no longer necessarily the dominant choice.

The repeated form of the game is called the iterated prisoner's dilemma and theoretical works have shown that cooperation can emerge once players interact repeatedly. Arguably, the most important of these works has been R. Axelrod's "The Evolution of Cooperation" [24]. In his book Axelrod reports on a series of computer tournaments he organised of a finite turns games of the iterated prisoner's dilemma. Participants had to choose between cooperation and defection again and again while having memory of their previous encounters. Academics from several fields were invited to design computer strategies to compete. The pioneering work of Axelrod showed that greedy strategies did very poorly in the long run whereas altruistic strategies did better.

"The Evolution of Cooperation" is considered a milestone in the field but it is not the only one. On the contrary, the prisoner's dilemma has attracted much attention ever since the game's origins. In Section 3 a comprehensive data set of literature regarding the prisoner's dilemma, and collected from the following sources, is presented and analysed.

- arXiv [77]; a repository of electronic preprints. It consists of scientific papers in the fields of mathematics, physics, astronomy, electrical engineering, computer science, quantitative biology, statistics, and quantitative finance, which all can be accessed online.
- PLOS [5]; a library of open access journals and other scientific literature under an open content license. It launched its first journal, PLOS Biology, in October 2003 and publishes seven journals, as of October 2015.
- IEEE Xplore Digital Library (IEEE) [58]; a research database for discovery and access to journal articles, conference proceedings, technical standards, and related materials on computer science, electrical engineering and electronics, and allied fields. It contains material published mainly by the Institute of Electrical and Electronics Engineers and other partner publishers.
- Nature [46]; a British multidisciplinary scientific journal, first published on 4 November 1869. It was ranked the world's most cited scientific journal by the Science Edition of the 2010 Journal Citation Reports and is ascribed an impact factor of 40.137, making it one of the world's top academic journals.
- Springer [78]; a leading global scientific publisher of books and journals. It publishes close to 500 academic and professional society journals.

The aim of the analysis is to review the amount of published academic articles as well as to measure and explore the collaborations within the field.

2 Timeline

In this section a large amount of literature regarding the prisoner's dilemma is reviewed. The review starts from the year the game was formulated and covers publications all the way to today.

2.1 Origins of the prisoner's dilemma

The origin of the prisoner's dilemma goes back to the 1950s in early experiments conducted at RAND [42] to test the applicability of games described in [86]. The game received it's name later the same year. According to [118], A. W. Tucker (the PhD supervisor of J. Nash [85]), in an attempt to delivery the game with a story during a talk described the players as prisoners and the game has been known as the prisoner's dilemma ever since.

The early research on the prisoner's dilemma had been very constrained. The only source of experimental results was through groups of humans that simulated rounds of the games and human groups came with disadvantages. Human can behave very randomly and in several experiments both the size and the background of the individuals were different. Thus comparing results of two or more studies became tricky.

The main aim of these experiments had been to understand the conditions behind the emergence of cooperation. Conditions such as the gender [41, 73, 75] of individuals, the representation of the game [41], the distance between players [108], the initial effects [117] and whether the experimenter was biased [43] were being explored and still are today.

An early figure that sought out to understand the conditions under which altruist behaviour emerged was Prof A. Rapoport. A mathematical psychologist, whose work focused on how to promote international and national cooperation. Rapoport tried to conceptualize strategies that could promote international cooperation. In his teaching and research he used the prisoner's dilemma. Rapoport offered the field many insights, he is the creator of strategies such as Tit for Tat and Pavlov, which are going to be discussed in later parts of this paper.

Decades later the political scientist R. Axelrod introduced the pioneer computer tournaments that have largely replaced human subjects in the study of the iterated prisoner's dilemma ever since. In the next section these tournaments and several strategies that were design by researchers, such as Rapoport, are introduced.

2.2 Axelrod's tournaments and intelligent design of strategies

As discussed in Section 2.1, before 1980 a great deal of research was done in the field, however, as described in [22], the political scientist R. Axelrod believed that there was no clear answer to the question of how to avoid conflict, or even how an individual should play the game. Combining his interest in artificial intelligence and political sciences Axelrod created a framework for exploring these questions using computer tournaments.

Axelrod's tournaments made the study of cooperation of critical interest once again, academic articles were being published reproducing Axelord's work, accessing and further developing his results. As described in [102], "Axelrod's new approach has been extremely successful and immensely influential in casting light on the conflict between an individual and the collective rationality reflected in the choices of a population whose members are unknown and its size unspecified, thereby opening a new avenue of research". In a collaboration with a colleague, Douglas Dion, Axelrod in [23] summarized a number of works that were immediately inspired from the "Evolution of Cooperation" and in 2012, [61] wrote a review on iterated prisoner's dilemma strategies and big competitions that had occurred since the originals.

In essence, Axelrod asked researchers to design a strategy, set a number of rules, with the purpose of wining an iterated prisoner's dilemma tournament. These strategies were constructed by an intelligent cause and not an undirected process, and here there are refereed to as strategies of intelligent design. This section covers Axelrod's original tournaments as well as research that introduced new strategies of intelligent design.

The first reported computer tournament took place in 1980 [18]. Several scientists were invited to submit their strategies, written in the programming languages Fortran or Basic. There was a total of 13 submissions made by the following researchers,

1. T Nicolaus Tideman and Paula Chieruzz;
2. Rudy Nydegger;
3. Bernard Grofman;
4. Martin Shubik;
5. Stein and Anatol Rapoport;
6. James W Friedman;
7. Morton Davis;
8. Jim Graaskamp;
9. Leslie Downing;
10. Scott Feld;
11. Johann Joss;
12. Gordon Tullock;

and a 13^{th} who remained anonymous.

Each competed in a 200 turn match against all 12 opponents, itself and a player that played randomly (called **Random**). This type of tournament is referred to as a round robin. The tournament was run only once, each participant knew the exact length of the matches and had access to the full history of each match. Furthermore, Axelrod performed a preliminary tournament and the results were known to the participants. The payoff values used for equation (1) where $R = 3, P = 1, T = 5$ and $S = 0$. These values are commonly used in the literature and unless specified will be the values used in the rest of the work described here.

The winner of the tournament was determined by the total average score and not by the number of matches won. The strategy that was announced the winner was submitted by Rapoport and was called **Tit For Tat**. Tit for Tat, is a strategy that always cooperates on the first round and then mimics the opponent's previous move. The success of Tit for

Tat came as a surprise. It was not only the simplest submitted strategy but it had also won the tournament even though it could never do better than any player it was interacting with.

In order to further test the results Axelrod performed a second tournament later in 1980 [19]. The results of the first tournament had been publicised and the second tournament received much more attention, with 62 entries made by the following people,

1. Gail Grisell;
2. Harold Rabbie;
3. James W Friedman;
4. Abraham Getzler;
5. Roger Hotz;
6. George Lefevre;
7. Nelson Weiderman;
8. Tom Almy;
9. Robert Adams;
10. Herb Weiner;
11. Otto Borufsen;
12. R D Anderson;
13. William Adams;
14. Michael F McGurrin;
15. Graham J Eatherley;
16. Richard Hufford;
17. George Hufford;
18. Rob Cave;
19. Rik Smoody;
20. John Willaim Colbert;
21. David A Smith;
22. Henry Nussbacher;
23. William H Robertson;
24. Steve Newman;
25. Stanley F Quayle;
26. Rudy Nydegger;
27. Glen Rowsam;
28. Leslie Downing;
29. Jim Graaskamp and Ken Katzen;
30. Danny C Champion;
31. Howard R Hollander;
32. George Duisman;
33. Brian Yamachi;
34. Mark F Batell;
35. Ray Mikkelson;
36. Craig Feathers;
37. Fransois Leyvraz;
38. Johann Joss;
39. Robert Pebly;
40. James E Hall;
41. Edward C White Jr;
42. George Zimmerman;
43. Edward Friedland;
44. X Edward Friedland;
45. Paul D Harrington;
46. David Gladstein;
47. Scott Feld;
48. Fred Mauk;
49. Dennis Ambuehl and Kevin Hickey;
50. Robyn M Dawes and Mark Batell;
51. Martyn Jones;
52. Robert A Leyland;
53. Paul E Black;
54. T Nicolaus Tideman and Paula Chieruzz;
55. Robert B Falk and James M Langsted;
56. Bernard Grofman;
57. E E H Schurmann;
58. Scott Appold;
59. Gene Snodgrass;
60. John Maynard Smith;
61. Jonathan Pinkley;
62. Anatol Rapoport.

The new participants knew the results of the previous tournament. The rules were similar with only a few alternations. The tournament was repeated 5 times and the length of each match was not known to the participants. Axelrod intended to use a fixed probability (refereed to as ‘shadow of the future’ [23]) of the game ending on the next move. However, 5 different length matches were selected for each match 63, 77, 151, 308 and 401, such that the average length would be around 200 turns.

Several entries of the second tournament tended to be variants of Tit for Tat, such as **Tit for Two Tats** submitted by John Maynard Smith. Tit for Two Tats defects only when the opponent has defected twice in a row. Another well known entry was **Grudger**. Grudger was originally submitted by James W. Friedman. Grudger is a strategy that will cooperate as long as the opponent does not defect. The name Grudger was give to the strategy in [70]. Though the strategy goes by many names in the literature such as, Spite [27], Grim Trigger [25] and Grim [120].

Despite the larger size of the second tournament, none of the variants and new strategies managed to outperform the simple designed strategy. The winner was once again Tit for Tat. The conclusions made from the first two tournaments were that the strong performance of the strategy was due to:

- The strategy would start of by cooperating.
- It would forgive it's opponent after a defection.
- It would always be provoked by a defection no matter the history.
- As soon as the opponents identified that they were playing Tit for Tat, they would choose to cooperate for the rest of the game.

However, the success of Tit for Tat was not unquestionable. Several papers showed that stochastic uncertainty severely undercut the effectiveness of reciprocating strategies. Though such stochastic uncertainty are unlikely to occur in a computer tournament, but have to be expected in real life situations [80].

In [83] it was proved that in an environment where **noise** is introduced two strategies playing Tit for Tat receive the same average payoff as two Random players. Noise is a probability that a player's move will be flipped. In 1986, [37] ran a computer tournament with a 10 percent chance of noise and Tit for Tat finished sixth out of 21 strategies. Bendor in [29] performed tournament similar to Axelrod's with noise and a probability of 0.0067 of ending in the next turn. His results demonstrated the poor performance of Tit for Tat once again and showed that the highest ranked strategies were more generous ones. His top ranked strategy was called **Nice and Forgiving**. Nice and Forgiving, differs in significant ways from Tit for Tat. Initially, Nice's generosity takes the form of a benign indifference. It will continue to play cooperation as long as its rival's cooperation level exceeded 80%. Secondly, although it will retaliate if its rival's observed cooperation fell below 80, it is willing to revert to full cooperation before its partner does, so long as the partner satisfies a certain thresholds of acceptable behaviour.

Hammerstein [107], pointed out another weakness of Tit for Tat in noisy environments. If by mistake, one of two Tit for Tat players makes a wrong move, this locks the two opponents into a hopeless sequence of alternating defections and cooperations. To overcome this error [122] introduced another more generous variant of Tit for Tat, **OmegaTFT**. They also altered their strategy so that it had the ability to recognize and exploit the Random strategy in a way that after an opponent strategy crosses a certain randomness threshold they conclude that the opponent is a Random strategy and change the behaviour to act as a **Defector**, a strategy that always choose to defect.

A second type of stochastic uncertainty is misperception, where a player's action is made correctly but it's recorder incorrectly by the opponent. In 1986, [115] introduced a strategy called **Contrite Tit for Tat** that was more successful than Tit for Tat in such environments. The difference between the strategies was that Contrite Tit for Tat was not so fast to retaliate a defection. This hints, alongside more generous versions of Tit for Tat as discussed above, that the counter attack to stochastic uncertainties is a strategy's readiness to defect after a defection.

Another protagonist in the literature and better perform strategy than Tit for Tat came along in 1993. The strategy was **Pavlov** and though the name was formally given by Nowak [90] the strategy had been around since 1965 known as Simpleton [101], introduced by Rapoport himself. The strategy is based on the fundamental behavioural mechanism win-stay, lose-shift. It starts off with a cooperation and then repeats it's previous move only if it was awarder with a payoff of R or T. Pavlov is heavily studied in the literature and similarly to Tit for Tat it's used in tournaments perform until today and has had many variants trying to build upon it's success. **PavlovD**, just a Pavlov which starts the game with a defection and **Adaptive Pavlov**. Adaptive Pavlov tried to classify it's opponents as as one of the following strategies, **Cooperator**, Defector, Pavlov, Random or Pavlov and chooses a strategy that maximise it's payoff against the now 'known' opponent. Cooperator is a deterministic strategy that conditionally cooperates.

Several researchers, and this will discussed in later sections as well, argued with Axelrod's result on simplicity. The advantages of complexity were shown by [27] in 1997 where they introduced another well known strategy **Gradual**. Gradual starts off by cooperating, then after the first defection of the other player, it defects one time and cooperates twice. After the second defection of the opponent, it defects two times and cooperates twice. After the n^{th} defection it reacts with n consecutive defections and then two cooperations. In a tournament of 12 strategies [27], Gradual had managed to outperform strategies such as Tit for Tat and Pavlov. Gradual was surpassed by yet another complex and intelligent designed strategy **Adaptive Tit for Tat**. The authors of [119] conducted the exact same tournament as [27] with now 13 strategies and their strategy ranked first.

Another interesting research on intelligent design strategies is on teams [35, 36, 104]. The strategies which make the team have been programmed with a recognition mechanism by default. Once the strategies recognise one another, one would act as leader and the other as a follower. The follower then plays as a Cooperator, cooperates unconditionally and the leader would play as a Defector gaining the highest achievable score. Followers would behave as Defector towards other strategies to defect their score and help the leader. In [104], a team for the University of Southampton used teams and recognition patterns and managed to win the 2004 Anniversary Iterated Prisoner's Dilemma Tournament.

The next section focuses on a different aspect of research which is evolutionary dynamics.

2.3 Evolutionary Dynamics

Following Axelrod's tournaments it was proven that direct reciprocity can make cooperation successful is an round robin setting. A long standing question has been to understand the conditions required for the emergence and maintenance of cooperation in evolving populations. Is reciprocity the key here as well? and what other mechanisms could favour cooperation? Due to the complex nature of the iterated prisoner's dilemma strategies makes their evolutionary stability more complex to study and though the question and these still remain open questions. Even so, several insights have been published over the years and in this section several remarks that have been made on the evolutionary dynamics are discussed.

In the later sections of [19], Axelrod discusses about an ecological tournament he performed using the 62 strategy of the second tournament An ecological approach is a simulation of theoretical future rounds of the game where strategies that do better are more likely to be included in future rounds than others. The simulation of the process, as described in [19], is straightforward. Let us consider an example with four strategies Tit for Tat, Tit for Two Tat, Cooperator and Defector compete in an ecological tournament. The expected payoff matrix, when these four strategies interact, is give by,

$$\begin{bmatrix} 3.0, & 3.0, & 3.0, & 0.99 \\ 3.0, & 3.0, & 3.0, & 0.99 \\ 3.0, & 3.0, & 3.0, & 0.0 \\ 1.02, & 1.039, & 5.0, & 1.0 \end{bmatrix}$$

Starting with proportions of each type in a given generation, their proportions for the next generation needs to be calculated. This is achieved by calculating the weighted average of the scores of a given strategy with all other players.

- The weights are the numbers of the other strategies which exist in the current generation.
- The numbers of a given strategy in the next generation is then taken to be proportional to the product of its numbers in the current generation and its score in the current generation.

The process is then repeated for a given number of future tournaments. Figure 1 illustrates a simulation of our ecological tournament, as shown strategies that cooperate quickly kill off the Defector. A similar result was presented by Axelrod. In his ecological tournament cooperative strategies managed to take over the population over time. On the other hand exploitative strategies started to die off as weaker strategies were becoming extinct. In other words they were dying because there was fewer and fewer prey for them to exploit. In 1981, Axelrod also studied the prisoner's dilemma in an evolutionary context based on the evolutionary approaches of John Maynard Smith [112, 110, 111]. Smith is a well know evolutionary biologist as well as an attendant of Axelord's second tournament. John Maynard Smith alongside George Price are considered fundamental figures of evolutionary game theory. In [112] they introduced the definition of an evolutionary stable strategy (ESS).

Imagine a population made up of individuals where everyone follows the same strategy B and a single individual adopts a mutant strategy A. Strategy A is said to invade strategy B if the payoff of A against B is greater than the expected payoff received by B against itself. Since strategy B is in a population that interacts only with itself, the concept of invasion is equivalent to a single mutant being able to outperform the average population. Thus for a strategy to be ESS it must be able to resists any invasion.

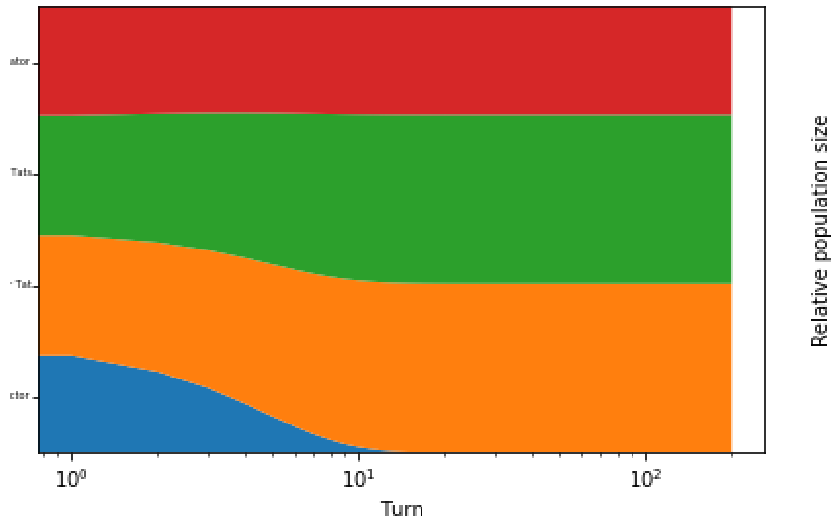


Figure 1: Results on an ecological tournament with Tit for Tat, Tit for Two Tats, Cooperator and Defector.

The work described in [20], studied the evolutionary stability of Tit for Tat and although the strategy was likely to take over the population, its stability was conditional on the importance of the future of the game. This is represented by a discounting factor denoted as w. Axelrod showed that if w was sufficiently large, Tit for Tat could resist invasion by any other strategy. Moreover, he showed how a small cluster of Tit for Tat players could invade a extortionate environment. Alongside the biologist William Donald Hamilton they wrote about the biological applications of the evolutionary dynamics of the iterated prisoner's dilemma [24] and won the Newcomb-Cleveland prize of the American Association for the Advancement of Science. Arguing with Axelrod's results. In [31] Boyd and Lorderbaum show that if w, the importance of the future of the game, is large enough then no deterministic strategy is ESS because it can always be invaded by any pair of other strategies. This was also independently proven by [99], so is reciprocity the answer to the emergence of cooperation?

Cooperation can be favoured in several other settings, in [91], Nowak and Sigmund studied the dynamics of the evolutionary iterated prisoner's dilemma with a spectrum of stochastic strategies that only remember their opponent's last move, not their own. They found that there can be multiple fixed points that there can be an evolutionary stable coexistence among multiple such strategies and in 1992, [74] explored how local interaction alone can facilitate population wide cooperation in a one shot prisoner's dilemma.

The two deterministic strategies Defector and Cooperator, were placed onto a two dimensional square array where the individuals could interact only with the immediate neighbours. The number of immediate neighbours could be either, fourth, six or eight. As shown in Figure 2, where each node represents a player and the edges denote whether two players will interact, this topology is refereed to as spatial topology. Thus each cell of the lattice is occupied by a Cooperator or a Defector.

- At each generation step each cell owner interacts with its immediate neighbours.
- The score of each player is calculated as the sum of all the scores the player achieved at each generation.
- At the start of the next generation, each lattice cell is occupied by the player with the highest score among the previous owner and the immediate neighbours.

This topology is refereed to as spatial topology. The population dynamics of these experiments were studied as a function of the temptation (T) payoff. More specifically the following payoff matrix was used, which is equivalent to equation (1):

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix} \tag{5}$$

where $(b > 1)$. Note that this representation fails constraint (2).

Local interactions proved that as long as small clusters of cooperators form and they can benefit from interactions with other cooperators and while avoiding interactions with defectors, global cooperation will continue. Cooperation was also proven to be evolutionary beneficial under conditions were the benefit to cost ratio $\frac{b}{c}$ is higher to the number of neighbours composed of unconditional cooperators and defectors [96] and [76] showed that cooperation is more likely to emerge in a small world topology. Moreover, [125] studied graphs were a probability of rewiring ones connections was in place, however, the rewire could be with any given node in the graphs and not just with imitate neighbours. Perc etc all showed that "making of new friends" may be an important activity for the successful evolution of cooperation, but also that partners must be selected carefully and one should keep their number limited.

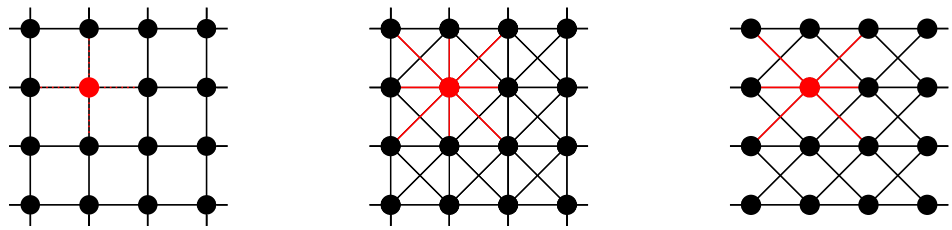


Figure 2: Spatial neighbourhoods

Other mechanisms that have studied how cooperators may interact more so that cooperation can emerge include reputation [59, 93, 116] and communication tokens [82] which are partner identification methods. Another related approach is using a tag based partner identification [32, 48, 82, 103]. Tags allow cooperative strategies to distinguish 'them' from the rest.

This section has focused on evolutionary dynamics and how different evolutionary settings can used to shed some light to how cooperation emerges in populations. In the following section evolutionary settings are used to train strategies.

2.4 Structured strategies and training

In Section 2.3 several evolutionary dynamic approaches used in the iterated prisoner's dilemma research were covered. All of these approaches have a limitation, and that is their inability to develop new strategies. In evolutionary settings strategies can learn to adapt their actions over time based upon what has been effective and what has not. In essence evolutionary reinforcement learning techniques can be used to train strategies for playing the iterated prisoner's dilemma. A strategy must be represented in a generic strategy archetype so it can be trained. Strategies that are discovered via strategy archetypes are refereed to as structured strategies. This does not include only strategies that have been trained, but any strategy that can emerge for any given structure that covers a range of iterated prisoner's dilemma strategies. They are the opposite to that of intelligent designed discussed in Section 2.2. This section covers papers that proposed new structured strategies as well as papers that explore new training algorithms are covered.

In Axelrod, [21] having realised the limitations of his evolutionary work, decided to use reinforcement learning to demonstrated how to fine tune the responses of an iterated prisoner's dilemma strategy and obtain a trained strategy. The algorithm used in [21] was the genetic algorithm [56] and the structure was that of a lookup table. Axelrod decided to considered deterministic strategies that took into account the last 3 turns of the game. For each turn there are 4 possible outcomes (CC, CD, DC, DD), thus for 3 turns there are a total of $4 \times 4 \times 4 = 64$ possible combinations. Therefore, the strategy can be defined by a series of 64 C's/D's, corresponding to each history; a lookup table. A more generic lookup definition is a deterministic strategy which take into account the last m actions.

In 1989 [91], Nowak and Sigmund proposed a structure for studying sophisticated strategies instead of deterministic ones. They studied a set of very simple strategies that remember only the previous turn, and moreover, only record the move of the opponent. They are called reactive strategies and they can be represented by using three parameters (y, p_1, p_2), where y is the probability to cooperate in the first move, and p_1 and p_2 the conditional probabilities to cooperate, given that the opponent's last move was a cooperation or a defection. Using the above notation a strategy can now be defined by a triple. For example, Defector: (0, 0, 0), Cooperator: (1, 1, 1), Tit for Tat: (1, 1, 0) and Pavlov: (0, 1, 1, 0).

This framework was used by the authors to study the evolution of a population composed of 99 reactive strategies. The strategy that managed to take over was **Generous Tit for Tat** which is given by the triplet $(1, 0, \frac{2}{3})$. In [92] the reactive strategies structure was expanded and a formal definition of a **memory one strategy** was given for the first time. Memory one strategies consider the entire history of the previous turn to make a decision (thus reactive strategies are a subset of memory one).

If only a single turn of the game is taken into account and depending on the simultaneous moves of two players there are only four possible states that players could possibly be in. These are CC, CD, DC and DD. A memory one strategy is denoted by the probabilities of cooperating after each of these states, $p = (p_1, p_2, p_3, p_4) \in \mathbb{R}^4_{[0,1]}$. A match between two memory one players p and q can be modelled as a stochastic process, where the players move from state to state. More specifically, it can be modelled by the use of a Markov chain [44], which is described by a matrix M.

$$M = \begin{bmatrix} p_1q_1 & p_1(-q_1+1) & q_1(-p_1+1) & (-p_1+1)(-q_1+1) \\ p_2q_3 & p_2(-q_3+1) & q_3(-p_2+1) & (-p_2+1)(-q_3+1) \\ p_3q_2 & p_3(-q_2+1) & q_2(-p_3+1) & (-p_3+1)(-q_2+1) \\ p_4q_4 & p_4(-q_4+1) & q_4(-p_4+1) & (-p_4+1)(-q_4+1) \end{bmatrix} \tag{6}$$

The players are assumed to move from each state until the system reaches a state steady, let the steady states vector be denoted as $\bar{v}$. The utility of a player can be given by multiplying the steady states of M by the payoffs of equation (1). Thus [92] offered a mathematical framework to calculate the utility of two players without actually simulating the game. The payoff of a player p can be obtained by,

$$s_p = \bar{v} \times \begin{pmatrix} R \\ S \\ T \\ P \end{pmatrix}$$

The family of memory one strategies have been proven rather useful in the terms of exploring strategies. The most famous work of memory one strategies is that of Press and Dyson [97]. In 2012, [97] presented a new set of strategies called **zero determinant (ZD)**. The ZD strategies are memory one strategies that manage to force a linear relationship between their score and that of the opponent. The payoffs of players p and q are denoted as:

$$s_p = vS_p$$
$$s_q = vS_q$$

where v is a vector of the steady states of matrix M and S_p, S_q are the equivalent payoff values of the players for each state CC, CD, DC, DD. Using linear algebra, Press and Dyson showed that the dot product of the stationary distribution of v with any vector f can be expressed as a 4×4 determinant. In which one column is f, one column is entirely under the control of player p and another column is entirely under the control of player q. This meant that either p or q could independently force the dot product of v with some other chosen vector f to be zero by choosing their strategy so as to make the column they control be proportional to f. In particular, by

$$f = \alpha S_p + \beta S_q + \gamma \tag{7}$$

any player can force a given linear relation to hold between the long-run scores of both players. Press and Dyson's suggested that these extortionate strategies are the dominant family of strategies and that memory does not benefit them, thus it would not benefit any strategy.

The ZD strategies have attracted a lot of attention. It was stated that "Press and Dyson have fundamentally changed the viewpoint on the Prisoner's Dilemma" [113] and as stated in [55] the American Mathematical Society's news section said that "the world of game theory is currently on fire". In [7, 54, 53, 55, 65, 69, 113] they question the effectiveness of ZD strategies. In [113], they revealed a more generous set of ZDs the **Generous ZD**, [7] showed that ZD strategies were not evolutionarily stable and in [69], the 'memory of a strategy does not matter' statement was questioned. A set of more complex strategies, strategies that take in account the entire history set of the game, were trained and proven to be more stable than ZD strategies.

Complex strategy can be explored by using different structures. In 1996, two were introduced by [50] and [81]. Harrald and Fogel [50], considered a neural network structure. Their neural network used a memory length of 3 and the actions were encoded as continuous values in $[-1, 1]$, where 1 meant complete cooperation. The input nodes represented 3 previous steps of the player and the opponent and there was a single hidden layer of N fully connected nodes and an output node that produced values from the range $[-1, 1]$. Same year [81] considered finite state automata. The specific type of finite automata that were used were Moore machines [84]. Finite state machine consist of a set of internal states. One of these states is the initial state of the machine. A machine also consists of transitions arrows associated with the states. Each arrow is labelled with A/R where A is the opponent's last action and R is the player's response. [81] used a genetic algorithm to train finite state machines in environments with noise. His results showed that even a small difference in noise (from 1% to 3%) significantly changed the characteristics of the evolving strategies. Three machines described in his paper were **Punish Twice**, **Punish Once for Two Tats** and **Punish Twice and Wait**.

Arguably the most complex strategies that have been trained are these of [49, 65]. In [49, 65] present several powerful strategies created using training. In these papers the authors used genetic algorithms and particle swarm optimisation algorithms [114]. Their selected structures included, lookerup tables, finite state machines, artificial neural networks [123] and hidden Markov models [39]. Hidden Markov models, are a variant of a finite state machine that use probabilistic transitions based on the prior round of play to other states and cooperate or defect with various probabilities at each state. Additionally a variant of a look up table is also presented called the lookerup archetype. The lookerup archetype responses based on the opponent's first n_1 moves, the opponent's last m_1 moves, and the players last m_2 moves. Taking into account the initial move of the opponent can give many insights. For it is the only move a strategy is truly itself without being affected by the other player. Finally, a new structure called the Gambler was also introduced, which is a stochastic variant of the lookerup archetype.

The structured strategies were put up against a large number of strategies in two following settings:

- A Moran process, which is an evolutionary model of invasion and resistance across time during which high performing individuals are more likely to be replicated.
- A round robin tournament.

The authors made use of an open source package called Axelrod-Python (paper describing it and it's capabilities was published in 2016 [64]). The package includes more than 200 implemented strategies, this includes most the strategies covered in this review. These experiments are, to the authors knowledge, the biggest ones done in the field in terms of different strategies. In [49], they performed a standard tournament and a noisy tournament. For the standard tournament the newly introduced trained strategies outperform the strategies outperform all the strategies of intelligent design.In the case of noise there is one particular strategy that has not seen much attention in the literature called "Desired Belief Strategy" [17].

The result of [65] show that the trained strategies evolve an ability to recognise themselves by using a handshake. This characteristic of the strategies was an important one because in a Moran process this recognition mechanism allowed these strategies to resist invasion.

Training can return a series of strategies. Differentiating between strategies is not always an easy task. It is not obvious looking at a finite state diagram how a machine will behave, and many different machines, or neural networks can represent the same strategy. For example Figure 3 shows two finite automata and both are a representation of Tit for Tat.

In order to distinguish the strategies and assuring that they are indeed different [12] introduced a method called fingerprinting. The method of fingerprinting is a technique for generating a functional signature for a strategy [13]. This is achieved by computing the score of a strategy against a spectrum of opponents. The basic method is to play the strategy

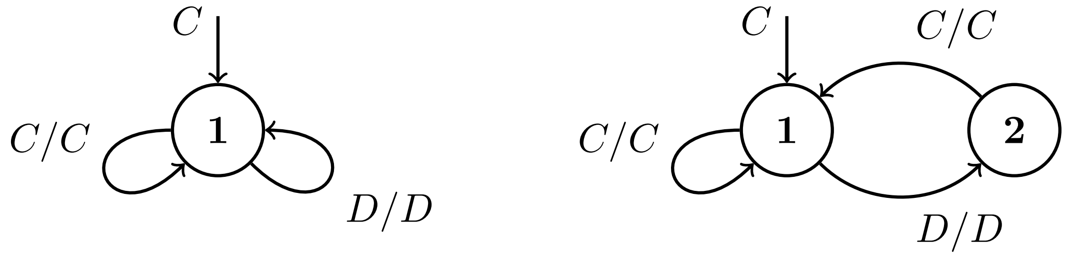


Figure 3: Finite state machine representations of Tit for Tat.

against a probe strategy with varying noise parameters. In [12] Tit for Tat is used as the probe strategy. Fingerprint functions can then be compared to allow for easier identification of similar strategies. In Figure 4 an example of Pavlov's fingerprint is given. Fingerprinting has been studied in depth in [13, 14, 15, 16].

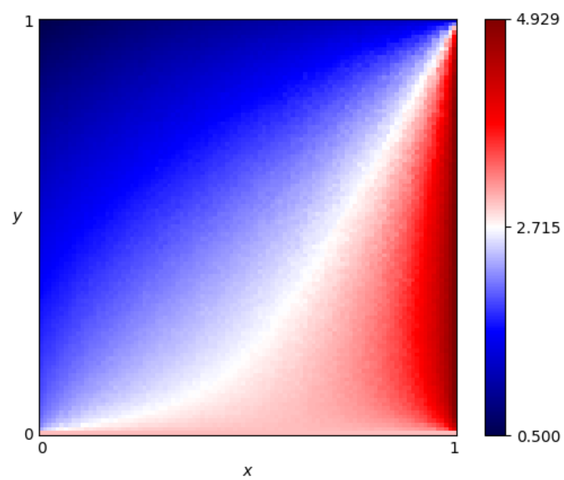


Figure 4: Pavlov fingerprinting with Tit for Tat used as the probe strategy. Figure was generated using [6].

This section covered structured strategies and training methods. In the following section software that has been developed with main aim simulating the iterated prisoner's dilemma is presented.

2.5 Software

The research of the iterated prisoner's dilemma heavily relays on software. This is to be expected as the pioneer computer tournaments have become the main mean of simulating the interactions in an iterated prisoner's dilemma game. Many academics fields suffer from the lack of source code availability and the prisoner's dilemma is not an exception. Though several of the tournaments that have been discussed so far were generated using computer code not all of the source code was made available by the authors. The code for Axelrod's original tournament is known to be lost and moreover for the second tournament the only source code available is the code for the 62 strategies (found on Axelrod's personal website [1]).

Several projects, however, are open, available and have been used as research tools or educational platforms over the years. Two research tools are briefly mentioned here [4, 6] and two educational tools [2, 3]. Both [4, 6] are open source projects used as research tools. PRISON is written in the programming language Java and preliminary version was launched on 1998. It was used by it's authors in several publications, such as [27] which introduced Gradual and [28]. The project includes a good number of strategies from the literature but unfortunately the last update of the project dates back in 2004. Axelrod-Python is a package used by several papers covered in Section 2.4. It is written in the programming language Python following best practice approaches and contains the larger to date data set of strategies, known to the author. The strategy list of the project has been cited by publications [10, 52, 87] and the package has been for several manuscripts such as [45, 121].

The ‘Game of Trust’ [2] is an on-line, graphical user interface educational platform for learning the basics of game theory, the iterated prisoner’s dilemma and the notion of strategies. It attracted a lot of attention due to being “well-presented with scribble-y hand drawn characters” [57] and “a whole heap of fun” [63]. Finally [3] is a personal project written in PHP. It’s graphical user interface platform that offers a big collection of strategies and allows the user to try several matches and tournament configurations.

2.6 Conclusion and Contemporary period

This section of the paper served as a review of publications on the prisoner’s dilemma. This review has partitioned the literature into five different sections, focusing on different aspects of the research. Section 2.1 covered the early years of research. This was when simulating turns of the game was only possible with human subject research.

Following the early years, the pioneer tournaments of Axelord were introduced in Section 2.2. Axelrod’s work offered the field an agent based game theoretic framework to study the iterated prisoner’s dilemma. In his original papers he asked researchers to design strategies to test their performance with the new framework. The winning strategy of both his tournaments was Tit for Tat. The strategy however came with limitations which were explored by other researchers and new strategies of intelligent design were introduced in order to surpass Tit for Tat with some contributions such as Pavlov and Gradual.

Soon researchers came to realise that strategies should not just do well in a tournament setting but should also be evolutionary robust. Evolutionary dynamics methods were applied to many works in the fields, and factors under which cooperation emerges were explored, as described in Section 2.3. This was not done only for unstructured populations, where all strategies in the population interacted with each other, but also in population where interactions were limited to only strategies that were close to each other. In such topologies it was proven that even in the one shot game cooperation can indeed emerge.

Evolutionary approaches can offer many insights in the study of the prisoner’s dilemma. In evolutionary settings strategies can learn to adapt and take over population by adjusting their actions; Such algorithms can be applied so that evolutionary robust strategies can emerge. Algorithms and structures used to train strategies in the literature were covered in Section 2.4. Several strategies can emerge from such processes, and to be able to differentiate between strategies fingerprinting was introduced. The research of best play and cooperation has been going on since the 1950s, and software has been developed along the way. Few have been briefly discussed in Section 2.5.

The study of the prisoner’s dilemma is still an ongoing field of pioneer and innovating research, where new variants and new structures of strategies are continuously being explored [95]. The game now serves as a model in a wide range of applications, for example in medicine and the study of cancer cells [11, 62], as well as in social situations and how they can be driven by rewards [38]. A lot of work is still being published on evolutionarily dynamics on graphs [8, 51, 72].

A large scale of articles has been covered in each of the corresponding sections of this review. This literature doe not pretend to have covered all the publications in the field. It will soon in the following section that the field has hab many publications, exceeding 3000 articles. However, several important milestones of the field have been presented here.

3 Analysing a large corpus of articles

The focus of Section 2 has been the academic publications on the topic of the iterated prisoner’s dilemma. Whilst in Section 2 several publications of specific interest were covered and the literature was manually partitioned in different sections, in the second part of this paper the publications are analysed using a large dataset of articles. In Section 3.1 some background research on bibliometrics is covered. The data collection process is covered in Section 3.2 and a preliminary analysis of the data is conducted in Section 3.3. In Section 3.4, the methodology which will be used to analyse the author relationships is presented. In summary, graph theoretical methods will be used to ascertain the level of collaborative nature of the field and identify influence. This type of analysis has been carried out in [71]. The novelty here is to consider approaches not considered in [71] and new origins of publications. A further comparison of the results are made, relative to two other sub fields of game theory: auction games [79] and the price of anarchy [105] and a temporal analysis. Finally in Section 3.5, the results of the analysis are presented.

3.1 Background

As discussed in [124], bibliometrics or the statistical analysis of published works (originally described by [98]) have been used to support historical assumptions about the development of fields [100], identify connections between scientific growth and policy changes [34], develop a quantitative understanding of author order [106], and investigate the collaborative structure of an interdisciplinary field [71]. Most academic research is undertaken in the form of collaborative effort and as [67] points out, it is rationale that two or more people have the potential to do better as a group than individually. Collaboration in groups has a long tradition in experimental sciences and it has be proven to be productive according to [40]. The number of collaborations can be very different between research fields and understanding how collaborative a field is, is not always an easy task. Several studies tend to consider academic citations as a measure for these things. A blog post published in Nature [89] argues that depending on citations can often be misleading because the true number of citations can not be known. Citations can be missed due to data entry errors, academics are influenced by many more papers than they actually cite and several of the citations are superficial.

A more recent approach to measure collaborative behaviour is to use the co authorship network, as described in [71]. Using this approach has many advantages as several graph theoretic measures can be used as proxies to explain authors relationship. In [71], they analyse the development of the field "evolution of cooperation" using this approach. The topic "evolution of cooperation" is a multidisciplinary field which also includes a large number of publications on the prisoner's dilemma. This paper builds upon the work done by [71] and extends their methodology. Though in [71], they considered a data set from a single source, Web of Science, the data set described here has been collected from five different sources. Moreover, the collaborative results of the analysis are compared to those of two different sub fields. Co authorship networks have also been used in [124] for classifying topics of an interdisciplinary field. This was done using centrality measures, which will be covered below, here centrality measures are used in order to understand the influence an author can have and can receive by being part of an academic group. Furthermore, in [9] they look at the relationship between research impact and five classes of diversity: ethnicity, discipline, gender, affiliation, and academic age. These characteristics of the authors are not being captured here. In future work these characteristics would be included in the analysis.

3.2 Data Collection

Academic articles are accessible through scholarly databases. Several databases and collections today offer access through an open application protocol interface (API). An API allows users to query directly a journal's database and bypass the user interface side of the journal. Interacting with an API has two phases: requesting and receiving. The request phase includes composing a url with the details of the request. For example, `http://export.arxiv.org/api/query?search_query=abs:prisoner'sdilemma&max_results=1` represents a request message. The first part of the request is the address of the API. In this example the address corresponds to the API of arXiv. The second part of the request contains the search arguments. In this example it is requested that the word 'prisoners dilemma' exists within the article's title. The format of the request message is different from API to API. The receive phase includes receiving a number of raw metadata of articles that satisfies the request message. The raw metadata are commonly received in extensive markup language (xml) or Javascript object notation (json) formats [94]. Similarly to the request message, the structure of the received data differs from journal to journal.

The data collection is crucial to this study. To ensure that this study can be reproduced all code used to query the different APIs has been packaged as a Python library and is available online [88]. The software could be used for any type of projects similar to the one described here, documentation for it is available at: `http://arcas.readthedocs.io/en/latest/`. Project [88] allow users to collect articles from a list of APIs by specifying just a single keyword. Four prominent journals in the field and a pre print server were used as sources to collect data for this analysis: PLOS, Nature, IEEE, Springer and arXiv.

A series of search terms were used to identify relevant articles:

- “prisoner’s dilemma”,
- “prisoners dilemma”,
- “prisoner dilemma”,
- “prisoners evolution”,
- “prisoner game theory”

and articles for which any of these terms existed within the title, the abstract or the text are included in the analysis. More specifically, 23% of article considered here were included because any of the above terms existed within the abstract, 50% within the main text and 27% within the title. As will be described in Section 3.3, two other game theoretic sub fields were also considered in this work, auction games and the price of anarchy. For collecting data on these sub fields the search terms used were “auction game theory” and “price of anarchy”. The three data sets are archived and available at. Note that the latest data collection was perform on November 2018.

3.3 Preliminary Analysis

A summary of each of the three data sets used is presented in this section. The three data sets are:

- The main data set which contains articles on the prisoner’s dilemma.
- A data set which contains article on auction games.
- A data set which contains articles on the price of anarchy.

The main data set is archived at [ref]. It consists of 3089 articles with unique titles. In case of duplicates the preprint version of an article (collected from arXiv) was dropped. Of these 3089 article, 89 have not been collected from the aforementioned APIs. These articles were of specific interest and manually added to the dataset throughout the writing of Section 2. A similar approach was used in [71] where a number of articles of interest where manually added to the data set. A more detailed summary of the articles’ provenance is given by Table 1. Only 3% of the data set consists of articles that were manually added and 33% of the articles were collected from arXiv.

provenance	# of Articles	Percentage
Manual	89	2.88
IEEE	295	9.55
PLOS	482	15.60
Springer	572	18.52
Nature	673	21.79
arXiv	1056	34.19

Table 1: Articles’ provenance for the main data set.

The average number of publications was calculated for the entire dataset and for each provenance. The average number of publications is denoted as, $\mu_P = \frac{N_A}{N_Y}$, where N_A is the total number of articles and N_Y is the years of publication. The years of publication is calculated as the range between the collection date and the first published article, for each provenance, within the data. These averages are summarised in Table 2. Overall an average of 49 articles are published per year on the topic. The most significant contribution to this appears to be from arXiv with 16 articles per year, followed by Nature with 10 and Springer with 9.

Though the average publication offers insights about the publications of the fields, it remains a constant number. The data handled here is a time series between 1950, when the game was introduced, and 2018 (Figure 5). Two observations can be made from Figure 5.

	Av. Yearly publication
IEEE	5.0
PLOS	8.0
Springer	9.0
Nature	11.0
arXiv	16.0
Overall	49.0

Table 2: Average publication for main data set.

1. There is a steady increase to the number of publications since the 1980s and the introduction of computer tournaments.
2. There is a decrease in 2017-2018. This is due to our data set being incomplete. Articles that have been written in 2017-2018 have either not being published or have are not retrievable by the APIs yet.

These observations can be confirmed by studying the time series. Using [60], an exponential distribution is fitted to the data from 1980-2016. The exponential fitting proves that since 1980 there has been an increase in the number of publications till 2016 (Figure 6). The fitted model can also be used to project the behaviour of the field for the next 5 years. The forecasted periods are plotted in Figure 7 and their exact values are given by Table 3. The time series has indicated a slight decrease however the model forecasts that the number of publications will keep increasing, thus indicating that the field of the iterated prisoner's dilemma still attracts academic attention.

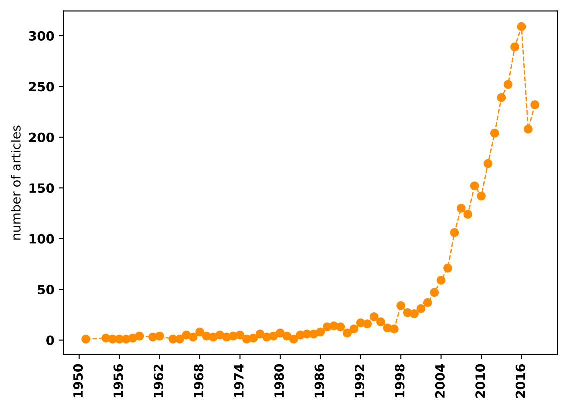


Figure 5: Line plot; # of articles published on the PD 1950-2019.

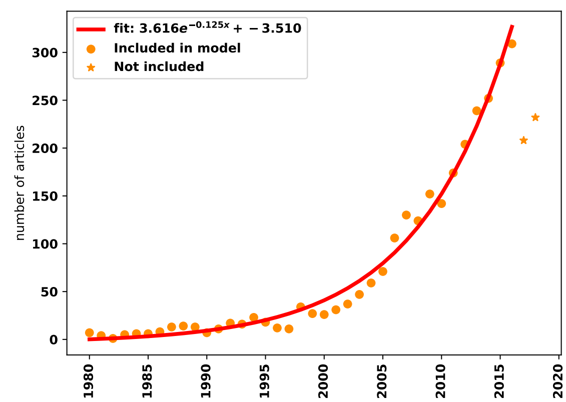


Figure 6: Scatter plot; # of articles published on the PD 1980-2019.

	Forecast
2017	371.0
2018	421.0
2019	478.0
2020	542.0
2021	615.0

Table 3: Forecasting the number of publications over the next 10 years.

To allow for a comparative analysis two sub fields of game theory have been chosen for this work; auction games and the price of anarchy.

- Auction theory is a branch of game theory which researches the properties of auction markets. Game theory is used

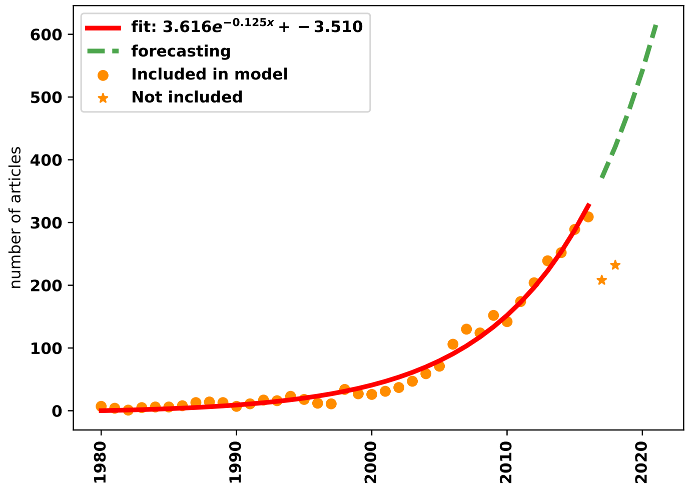


Figure 7: Forecast for 2017-2022.

for years to study auctions and the behaviour of bidders [109]. The earliest entry in our data set [ref] goes back to 1974 (Figure 8). Note that no articles have been added manually for auction games.

- Price of Anarchy is a concept in game theory which measures how the efficiency of a system degrades due to selfish behaviour of it's agents. There is a variety of such measures however the price of anarchy has attracted a lot of attention since it's informal introduction in 1999 by [66]. Note that [66] has been manually added to the date set and it's its first entry (Figure 9).

A summary of both data sets collected on both topics, in comparison to that of [ref], is given by Table 4.

	Num. Articles	Num. Authors	Manual (%)	PLOS (%)	Nature (%)	Springer (%)	IEEE (%)	arXiv (%)	Av. Yearly Publication
Prisoner's Dilemma	3089	5811	2.88	15.6	21.79	18.52	9.55	34.19	49.0
Auction Games	3444	5362	-	-	5.89	37.63	7.46	51.36	93.0
Price of Anarchy	747	1315	0.13	1.74	24.63	38.02	30.66	8.84	39.0

Table 4: Measures of all three data sets.

The iterated prisoner's dilemma and auction theory are very well studied topics that have been publicising for decades. A large number of articles have been collected for both topics, 3089 and 34444 respectively. Though, auction games have a larger number of articles, the iterated prisoner's dilemma has almost 300 more authors.

Auction games have an overall average yearly publication of 93 articles per year compared to the prisoner's dilemma with 49 per year. 50% of articles for [ref] have been collected from the pre print server arXiv and no articles have been published in PLOS.

Compared to these two topics the price of anarchy is a fairly recent one. Only a total of 747 articles have been collected, however it has a large number of 1229 authors. On average each paper had had at least two authors. It has an overall average publication of 39 articles and the biggest contribution has been made to Springer.

3.4 Methodology

The relationship between the authors within a field will be modelled as a graph G with a set V_G of nodes and E_G of edges. The set V_G represents the authors and an edge connects two authors if and only if those authors have written together. The co authorship network is constructed using the main data set described in Section 3.3 and the open source package [47]. The prisoner's dilemma network is denoted as G_1 where the number of unique authors $|V(G_1)|$ is 5394 and $|E(G_1)|$ is 10397 . All authors' names were formatted as their last name and first initial (i.e. Martin A. Nowak to Martin Nowak). This was done to avoid errors such as Martin A. Nowak and Martin Nowak, being treated as a different person.

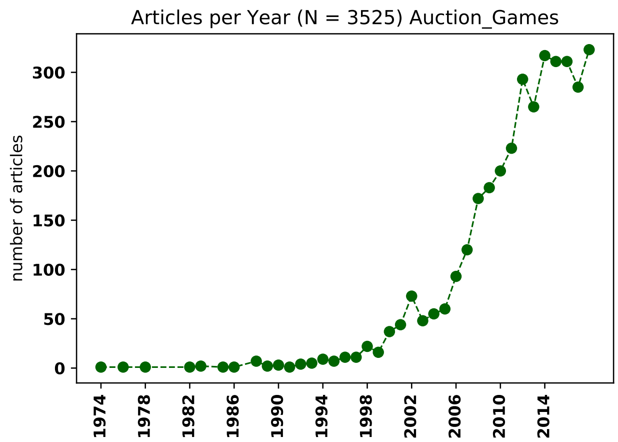


Figure 8: Line plot; # articles published on auction games 1974-2018.

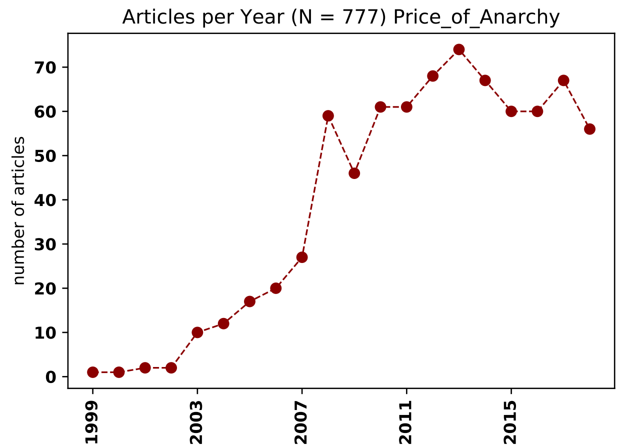


Figure 9: Line plot; # articles published on the price of anarchy.

Collaborativeness, will be analysed using measures such as, isolated nodes, connected components, clustering coefficient, communities, modularity and average degree. These measures show the number of connections authors can have and how strongly connected these people are. The number of isolated nodes is the number of nodes that are not connected to another node, thus the number of authors that have published alone. The average degree denotes the average number of neighbours for each nodes, i.e. the average number of collaborations between the authors.

A connected component is a maximal set of nodes such that each pair of nodes is connected by a path. The number of connected components as well as the size of the largest connected component in the network are reported. The size of the largest connected component represents the scale of the central cluster of the entire network, as it will discussed in the analysis section. Clustering coefficient and modularity and are also calculated. The clustering coefficient, defined as 3 times the number of triangle on the graph divided by the number of connected triples of nodes, is a local measure of the degree to which nodes in a graph tend to cluster together in a clique. It is precisely the probability that the collaborators of an author also write together.

In comparison, modularity is a global measure designed to measure the strength of division of a network into communities. The number of communities will be reported using the Clauset-Newman-Moore method [33]. Also the modularity index is calculated using the Louvain method described in [30]. The value of the modularity index can vary between $[-1, 1]$, a high value of modularity corresponds to a structure where there are dense connections between the nodes within communities but sparse connections between nodes in different communities. That means that authors in the network are mainly connected co-authors that they all have written together, and not to several different collaborators.

Networks are commonly dominated by one person who controls information flow and people that receive a great amount of information due to their position. Two further points are aimed to be explored in this work, (1) which people control the flow; as in which people influence the field the most and (2) which are the authors that gain the most from the influence of the field. To measure these concepts graph theoretic metrics, more specifically centrality measures are going to be used. Centrality measures are often used to understand different aspects fo social networks [68]. Two centrality measures have been chosen for this paper and these are closeness and betweenness centrality.

1. In networks some nodes have a short distance to other nodes and consequently are able to spread information on the network very effectively. A representative of this idea is **closeness centrality**, where a person is seen as centrally involved in the network if it requires only few intermediaries to contact others and thus is structurally relatively independent. Here, this is defined as influence. Authors with a high value of closeness centrality, are the authors that spread scientific knowledge easier on the network and they have high influence.

2. Another centrality measure is the **betweenness centrality**, where the determination of an author's centrality is based on the quotient of the number of all shortest paths between nodes in the network that include the regarded node and the number of all shortest paths in the network. In betweenness centrality the position of the node matters. Nodes with a higher value of betweenness centrality are located in positions that a lot of information pass through

them, this is defined as the gain from the influence, thus these authors gain the most from their networks.

In the next section all the metrics discussed here are calculated for the data sets in order to provide insights into the field.

3.5 Analysis of co authorship network

As mentioned previously, G_1 denotes the co authorship network of the iterated prisoner's dilemma. It's graphical representation is given by Figure 10a. It is evident that the network is disjoint, which is only natural as many authors write academic articles on their own. More specifically, a total of 176 authors, have had single author publications, which corresponds to the 3.3 (%) of authors in G_1.

There are a total of 1356 connected components and the largest one has a size of 815 nodes. The largest connected component is shown in Figure 10b and is going to be refereed to as the main cluster of the network. There are total of 1369 communities in G_1. The network has a clustering coefficient of 0.708 , thus authors are 70% likely to write with a collaborator's co author and the degree distribution, Figure 11, shows that the average degree is ≈ 4. Thus authors are on average connected to 4 other authors, however there are authors with far more connections, the largest one being 58.

In [71] the collaborative metrics for the "evolution of cooperation" co authorship network were reported. Though their network is of smaller size (number of nodes $3670 < 5394$), the collaborative metrics are fairly similar between the two graphs (clustering coeff. $0.632 \approx 0.708$ and modularity $0.950 \approx 0.977$), indicating that for the same multidisciplinary field the same remarks can be made from a different co authors network. But how does these compare to other fields and more specifically to other fields of game theory? The representation of the two graphs,

- G_2 for auction games and
- G_3 for the price of anarchy,

are given by Figures 10c and 10e and their respective clusters in Figures 10d and 10f. As stated before G_3 is the smallest network of all three, this is also clear from it's graphical representation. The G_2 network appears to be very similar to G_1, however it's main cluster is larger in size.

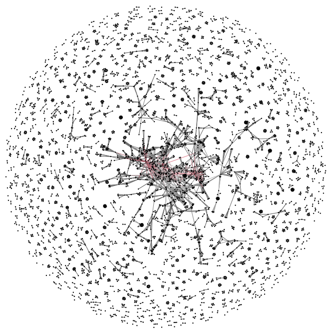
A summary of the collaborative metrics for all three co authorship networks is given by Table 5 and shown in Figure 11 are the degree distributions of all three networks. In G_1 and G_2 there are cases of high degree (> 20) but this could be an affect of the size of the data, networks and subsequently the size of the main clusters.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
Prisoner's Dilemma	5394	10397	176	3.3	1356	815	3.855	1369	0.977	0.708
Auction Games	5165	7861	256	5.0	1272	1348	3.044	1295	0.957	0.622
Price of Anarchy	1155	1953	4	0.3	245	222	3.382	253	0.965	0.712

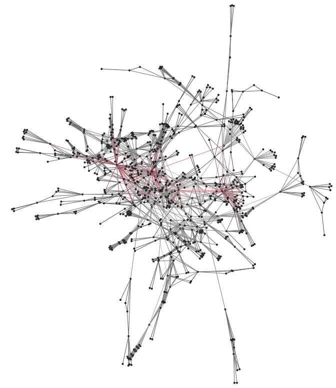
Table 5: Network metrics for G_1, G_2, G_3.

Regarding the three sub fields of game theory and using Table 5 and Figure 11 the following remarks can be made:

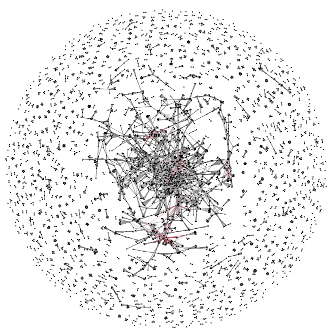
- All three networks have similar values of modularity index, and they are all very high (Table 5), indicating that the networks are partitioned in many communities. Note that the number of communities is very much similar to the number of connected components. This is all to expected. Due to the nature of our network, most connected components represent a single publication written by all the authors in the component (corresponding to a fully connected graph), and due to that density they are also a community on their own.
- Comparing to another well studied topic, auction games, the field of the iterated prisoner's dilemma appear to be more collaborative. Due to the value of the average degree, authors in G_1 are known to have on average almost one more collaboration than G_2. A slightly lower cluster coefficient ($.622 < .702$) of auction games indicate that is less likely for authors in G_2 to collaborate with a collaborators co author.



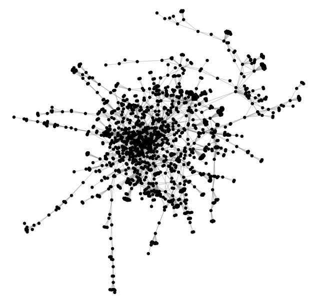
(a) G_1 network.



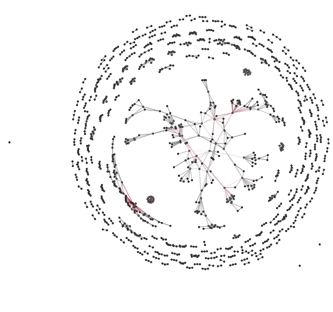
(b) G_1 larger connected component.



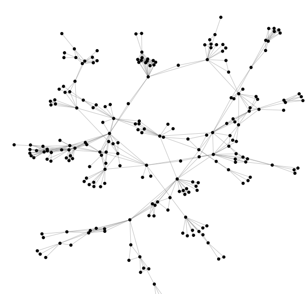
(c) G_2 network.



(d) G_2 larger connected component.



(e) G_3 network.



(f) G_3 larger connected component.

Figure 10: Graphical representations of G_1, G_2, G_3 and their respective main clusters.

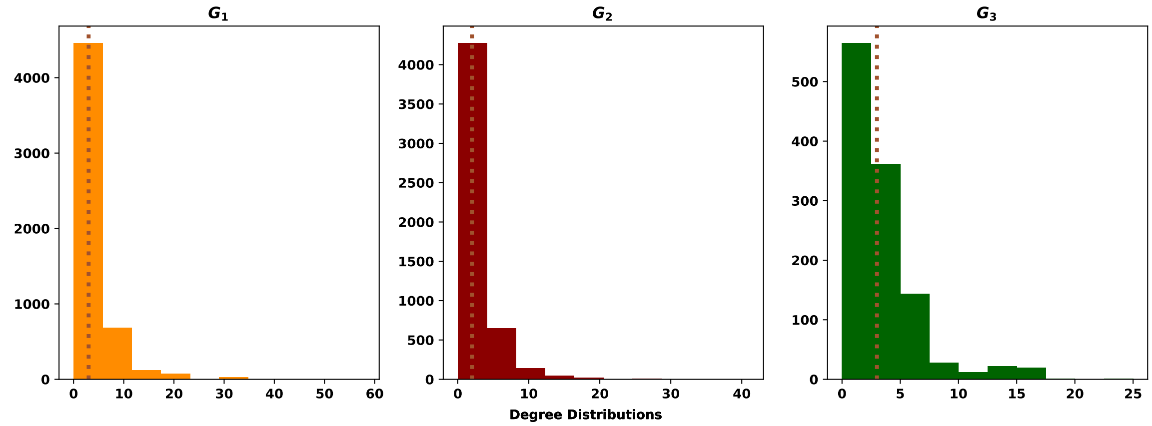


Figure 11: Degree distribution for networks G_1, G_2 and G_3. The descriptive statistics for each of the distribution are for G_1: mean = 3.85, median = 3, std = 4.25. For G_2: mean = 3.04, median = 2, std = 3.01 and for G_3: mean = 3.38, median = 3, std = 2.90. It is clear that these distributions are not normally distributed, which has also been verified using a statistical test. Moreover, the statistical difference of the medians has been tested using a Kruskal Wallis test. The medians of G_1 and G_3 are not significantly different, however they are are significantly large than that of G_2.

- Regarding the price of anarchy, the measures indicate that the field is not as mature as the other two sub fields. There are no isolated authors, which is more of an indication of the time the field has been active. As a more recent field there had been better communication tools that enable more collaborations between researches. The average degree as well as the clustering coefficient (clustering coeff.= 0.713) of G_3 is comparable to those of the iterated prisoner's dilemma.

These results can be extend to the main clusters of each network, Table 6. The metrics's values are fairly similar and the size of G_2's main cluster does not appear to gave any significant effect; all the same conclusions are made. Compared to auction games the iterated prisoner's dilemma is a more collaborative field, and it is fairly similar to the price of anarchy. However, our analysis suggests that the price of anarchy is still a maturing field.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
Prisoner's Dilemma	815	2300	0	0.0	1	815	5.644	29	0.855	0.775
Auction Games	1348	3158	0	0.0	1	1348	4.685	26	0.856	0.699
Price of Anarchy	222	521	0	0.0	1	222	4.694	12	0.817	0.711

Table 6: Network metrics for G_1, G_2, G_3.

The change of the networks over time is also studied by constructing the network cumulatively with a year interval. A total of 64 sub graphs over 64 periods, starting in 1950, were created and all the collaborative metrics for each sub graph have been calculated. Note that years 1952 and 1953 have no publications in our data set. The metrics of each network for each period are given by Table 7. Similar to the results of [71], it can been observed that the network G_1 grows over time and that the network always had a high value of modularity.

To better assess the change over time for each metric they have been plotted in Figure 12. The number of nodes, connected components and the size of largest component have been normalised such that the trend between the three networks can be compared.

- In Figure 12a the normalised number of nodes, which is calculated by dividing by the total number of nodes in each respective network, is shown. A steep increase in the size of all three networks is spotted soon after 2000. This could indicate that more data have been available in the sources used in this work following the year 2000. It is however, definitely not a effect of a single field, as it is true for all three sub fields considered here. The sudden increase following the year 2000, is also reported by the number of connected components and the size of the main cluster, Figures 12c, 12d. A connected components represents at least one publication which means that indeed more articles are being gathered from 2000 onwards.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
1954 - 1950	3	0	3	100.0	3	1	0.000	-	-	0.000
1954 - 1955	2	0	2	100.0	2	1	0.000	-	-	0.000
1955 - 1956	3	0	3	100.0	3	1	0.000	-	-	0.000
1956 - 1957	4	0	4	100.0	4	1	0.000	-	-	0.000
1957 - 1958	6	0	6	100.0	6	1	0.000	-	-	0.000
1958 - 1959	7	0	7	100.0	7	1	0.000	-	-	0.000
1959 - 1961	7	0	7	100.0	7	1	0.000	-	-	0.000
1961 - 1962	8	0	8	100.0	8	1	0.000	-	-	0.000
1962 - 1964	9	0	9	100.0	9	1	0.000	-	-	0.000
1964 - 1965	10	0	10	100.0	10	1	0.000	-	-	0.000
1965 - 1966	17	3	11	64.7	14	2	0.353	14	0.666667	0.000
1966 - 1967	21	4	13	61.9	17	2	0.381	17	0.75	0.000
1967 - 1968	32	15	13	40.6	21	5	0.938	21	0.684444	0.135
1968 - 1969	36	17	16	44.4	24	6	0.944	24	0.629758	0.139
1969 - 1970	39	18	17	43.6	26	6	0.923	26	0.666667	0.128
1970 - 1971	51	28	18	35.3	31	6	1.098	31	0.826531	0.275
1971 - 1972	58	34	19	32.8	34	6	1.172	34	0.866782	0.345
1972 - 1973	59	35	18	30.5	34	6	1.186	34	0.873469	0.339
1973 - 1974	59	35	18	30.5	34	6	1.186	34	0.873469	0.339
1974 - 1975	60	35	19	31.7	35	6	1.167	35	0.873469	0.333
1975 - 1976	60	35	19	31.7	35	6	1.167	35	0.873469	0.333
1976 - 1977	68	37	23	33.8	41	6	1.088	41	0.885318	0.294
1977 - 1978	70	38	23	32.9	42	6	1.086	42	0.890582	0.286
1978 - 1979	73	42	23	31.5	42	6	1.151	42	0.893424	0.292
1979 - 1980	77	45	25	32.5	44	6	1.169	44	0.899753	0.307
1980 - 1981	80	50	26	32.5	45	6	1.250	45	0.8928	0.318
1981 - 1982	84	56	26	31.0	46	6	1.333	46	0.903061	0.350
1982 - 1983	87	57	27	31.0	48	6	1.310	48	0.906125	0.338
1983 - 1984	94	58	32	34.0	54	6	1.234	54	0.909037	0.313
1984 - 1985	95	58	33	34.7	55	6	1.221	55	0.909037	0.309
1985 - 1986	104	59	40	38.5	63	6	1.135	63	0.911807	0.283
1986 - 1987	116	61	48	41.4	73	6	1.052	73	0.916958	0.253
1987 - 1988	121	65	48	39.7	75	6	1.074	75	0.924497	0.268
1988 - 1989	134	76	47	35.1	80	6	1.134	80	0.937673	0.272
1989 - 1990	145	82	49	33.8	86	6	1.131	86	0.944676	0.272
1990 - 1991	158	88	53	33.5	94	6	1.114	94	0.950413	0.268
1991 - 1992	169	91	59	34.9	102	6	1.077	102	0.953025	0.251
1992 - 1993	186	104	62	33.3	110	6	1.118	110	0.95932	0.266
1993 - 1994	220	134	72	32.7	127	6	1.218	127	0.965471	0.317
1994 - 1995	239	144	74	31.0	137	6	1.205	137	0.969329	0.304
1995 - 1996	257	163	77	30.0	145	6	1.268	145	0.970831	0.318
1996 - 1997	279	178	81	29.0	156	6	1.276	156	0.974309	0.336
1997 - 1998	311	215	65	20.9	160	6	1.383	160	0.979773	0.354
1998 - 1999	329	239	58	17.6	162	6	1.453	162	0.981741	0.376
1999 - 2000	373	273	67	18.0	183	6	1.464	183	0.983778	0.387
2000 - 2001	400	320	54	13.5	184	7	1.600	184	0.983066	0.410
2001 - 2002	450	366	61	13.6	206	7	1.627	206	0.984547	0.418
2002 - 2003	509	414	58	11.4	229	7	1.627	229	0.987083	0.421
2003 - 2004	580	489	58	10.0	253	10	1.686	253	0.988052	0.429
2004 - 2005	679	599	57	8.4	284	19	1.764	284	0.98891	0.463
2005 - 2006	854	806	66	7.7	342	21	1.888	342	0.990724	0.496
2006 - 2007	1056	1117	76	7.2	402	24	2.116	402	0.989663	0.527
2007 - 2008	1255	1460	85	6.8	454	32	2.327	455	0.989753	0.549
2008 - 2009	1462	1759	104	7.1	520	56	2.406	521	0.987517	0.550
2009 - 2010	1700	2301	114	6.7	581	99	2.707	584	0.979084	0.571
2010 - 2011	2040	2954	121	5.9	665	121	2.896	668	0.980396	0.603
2011 - 2012	2422	3676	126	5.2	756	210	3.036	759	0.979127	0.629
2012 - 2013	2807	4398	138	4.9	843	330	3.134	849	0.977978	0.639
2013 - 2014	3199	5044	148	4.6	942	406	3.153	951	0.973882	0.651
2014 - 2015	3798	6221	159	4.2	1064	514	3.276	1074	0.975594	0.668
2015 - 2016	4472	8344	169	3.8	1184	614	3.732	1197	0.9754	0.690
2016 - 2017	4925	9235	173	3.5	1274	703	3.750	1293	0.975251	0.700
2017 - 2018	5385	10379	176	3.3	1356	815	3.855	1369	0.97706	0.708

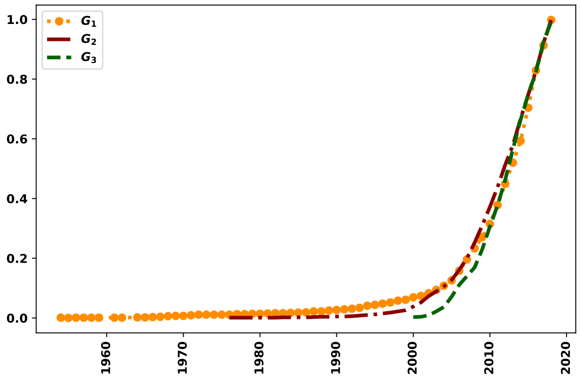
Table 7: Collaborativeness metrics for cumulative graphs.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
1954 - 1950	1	0	1	100.0	1	1	0.000	-	-	0.000
1954 - 1955	1	0	1	100.0	1	1	0.000	-	-	0.000
1955 - 1956	1	0	1	100.0	1	1	0.000	-	-	0.000
1956 - 1957	1	0	1	100.0	1	1	0.000	-	-	0.000
1957 - 1958	1	0	1	100.0	1	1	0.000	-	-	0.000
1958 - 1959	1	0	1	100.0	1	1	0.000	-	-	0.000
1959 - 1961	1	0	1	100.0	1	1	0.000	-	-	0.000
1961 - 1962	1	0	1	100.0	1	1	0.000	-	-	0.000
1962 - 1964	1	0	1	100.0	1	1	0.000	-	-	0.000
1964 - 1965	1	0	1	100.0	1	1	0.000	-	-	0.000
1965 - 1966	2	1	0	0.0	1	2	1.000	1	0	0.000
1966 - 1967	2	1	0	0.0	1	2	1.000	1	0	0.000
1967 - 1968	5	8	0	0.0	1	5	3.200	1	0	0.867
1968 - 1969	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1969 - 1970	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1970 - 1971	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1971 - 1972	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1972 - 1973	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1973 - 1974	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1974 - 1975	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1975 - 1976	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1976 - 1977	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1977 - 1978	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1978 - 1979	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1979 - 1980	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1980 - 1981	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1981 - 1982	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1982 - 1983	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1983 - 1984	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1984 - 1985	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1985 - 1986	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1986 - 1987	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1987 - 1988	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1988 - 1989	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1989 - 1990	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1990 - 1991	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1991 - 1992	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1992 - 1993	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1993 - 1994	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1994 - 1995	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1995 - 1996	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1996 - 1997	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1997 - 1998	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1998 - 1999	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1999 - 2000	6	10	0	0.0	1	6	3.333	2	0.02	0.833
2000 - 2001	7	21	0	0.0	1	7	6.000	1	0	1.000
2001 - 2002	7	21	0	0.0	1	7	6.000	1	0	1.000
2002 - 2003	7	21	0	0.0	1	7	6.000	1	0	1.000
2003 - 2004	10	13	0	0.0	1	10	2.600	2	0.37574	0.553
2004 - 2005	19	28	0	0.0	1	19	2.947	3	0.524235	0.730
2005 - 2006	21	32	0	0.0	1	21	3.048	4	0.530273	0.713
2006 - 2007	24	36	0	0.0	1	24	3.000	5	0.563272	0.678
2007 - 2008	32	59	0	0.0	1	32	3.688	4	0.627837	0.732
2008 - 2009	56	102	0	0.0	1	56	3.643	4	0.713331	0.699
2009 - 2010	99	238	0	0.0	1	99	4.808	8	0.781601	0.734
2010 - 2011	121	288	0	0.0	1	121	4.760	9	0.776054	0.713
2011 - 2012	210	610	0	0.0	1	210	5.810	12	0.779433	0.747
2012 - 2013	330	908	0	0.0	1	330	5.503	17	0.813721	0.753
2013 - 2014	406	1125	0	0.0	1	406	5.542	18	0.814914	0.749
2014 - 2015	514	1390	0	0.0	1	514	5.409	21	0.828125	0.757
2015 - 2016	614	1682	0	0.0	1	614	5.479	25	0.83153	0.765
2016 - 2017	703	1925	0	0.0	1	703	5.477	25	0.839089	0.774
2017 - 2018	815	2300	0	0.0	1	815	5.644	29	0.854812	0.775

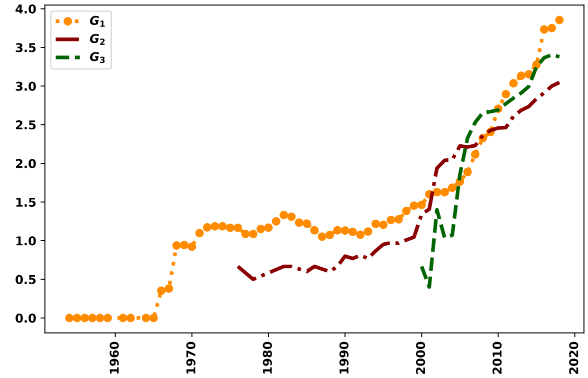
Table 8: Collaborativeness metrics for cumulative graphs' main clusters.

- Auction games have been through out time less collaborative compared to the iterated prisoner's dilemma. The average degree (Figure 12b) and the clustering coefficient (Figure 12e) of the cumulative sub graphs have been lower than that of G_1. The only exception is during years 2001-2008. For these year auction games appear to have had a more collaborative environment.
- In the price of anarchy cumulative graphs a sharp increase since the beginning of the field can be observed for all metrics. There are not many data points due to the recent development of the field, however these steep trends could be an indication that game theoretic and potentially all scientific research has over time been more collaborative. This could be due to logistic and techical solutions.
- The high values of modularity through out time is not true only for the network reported in [71] but also for all three networks of this field. This could indicate a limitation to the co authorship network. The measure is likely to be skewed, as each paper is more likely a connected component on it's own.

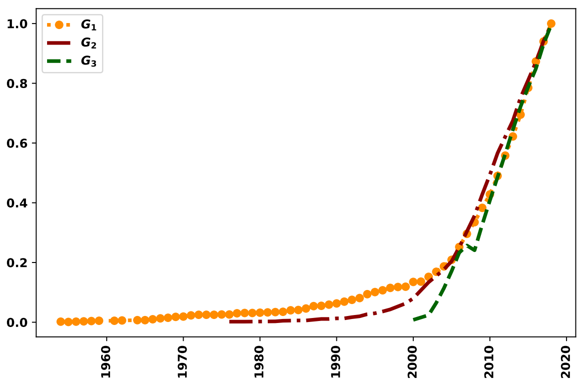
The cumulative collaborative metrics have also been calculated for each main cluster, given in Table 8. Summarily, the results do not appear to change over the main cluster.



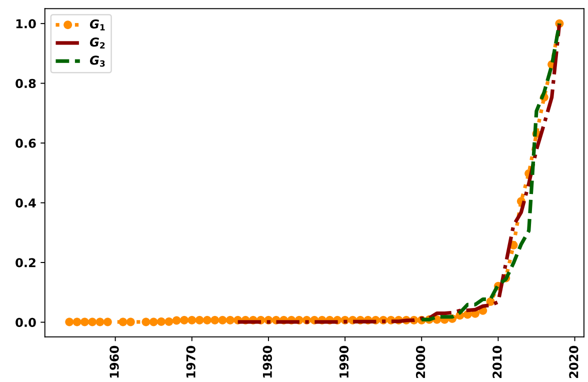
(a) % Nodes.



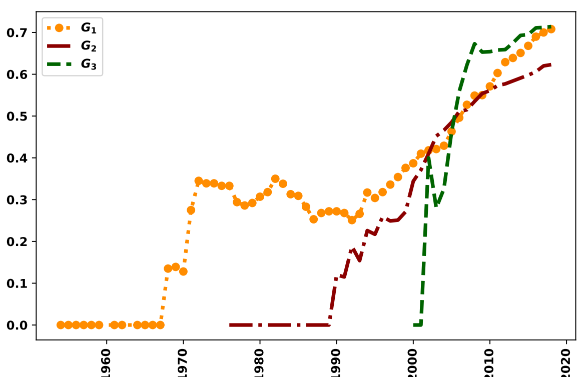
(b) Average Degree.



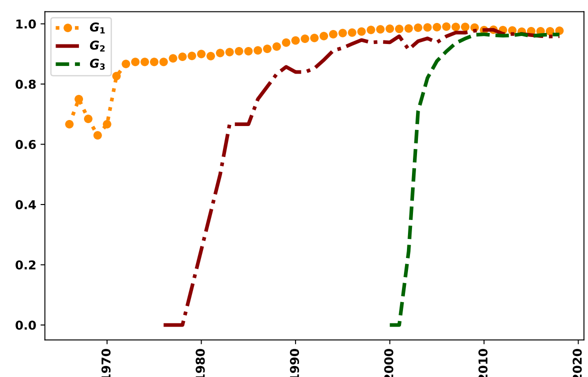
(c) % Connected components.



(d) % Size of largest connected component.



(e) Clustering coefficient.



(f) Modularity.

Figure 12: Collaborative metrics over time for cumulative networks for G_1, G_2 and G_3.

The next results discussed here are on centrality measures. As a reminder, two centrality measures are reported here, these are the closeness centrality and the betweenness centrality. Closeness centrality is a measure of how easy is for an author to contact others, and consequently affect them; influence them. Thus closeness centrality here is a measure of influence. Betweenness centrality is a measure of how many paths pass through a specific node, thus the amount of information this person has access to. Betweenness centrality is used here as a measure of how much an author gain from the field. All centrality measure can have values ranging from $[0, 1]$.

For G_1 the most central author based on closeness and betweenness are given by Tables 13 and 14 respectively. The betweenness centrality of the most central authors in G_1 are rather low with the highest ranked author being Matjaz Perc with a between centrality of 0.008, Table 14. A publication of Perc's work has been briefly discussed in Section, and the centrality measure suggest that the network is very influenced by him. He is connected to a total of 58 nodes and he has published to all five of the different sources considered in the study. Though he also gains from his position in the network, the gain is minor. An author who is not in the top influencers but does indeed gain from his position in the network is Martin Nowak. An author that his work has been discussed in Section 2.

	Name	Closeness
1	Matjaz Perc	0.048447
2	Yamir Moreno	0.044840
3	Zhen Wang	0.044005
4	Long Wang	0.043770
5	Attila Szolnoki	0.043338
6	Luo-Luo Jiang	0.042148
7	Arne Traulsen	0.041790
8	Valerio Capraro	0.041257
9	Cheng-Yi Xia	0.040791
10	Angel Sanchez	0.040562

Figure 13: Ten most influenced authors in G_1.

	Name	Betweeness
1	Matjaz Perc	0.008331
2	Zhen Wang	0.006356
3	Yamir Moreno	0.004806
4	Long Wang	0.003538
5	Martin Nowak	0.003230
6	Valerio Capraro	0.002739
7	Arne Traulsen	0.002479
8	Angel Sanchez	0.002319
9	Jianye Hao	0.002188
10	Franz Weissing	0.002186

Figure 14: Authors that gain the most influence in G_1.

From Tables 13 and 14 it can be seen that authors in G_1 are more likely to affect their field instead of gaining from it. This can be better explored by considering the distributions of the centralities and by comparing them to other fields. The distributions for both centralities are plotted in Figures 15a and 16a, and in Figures 15b and 16b for their respective main clusters.

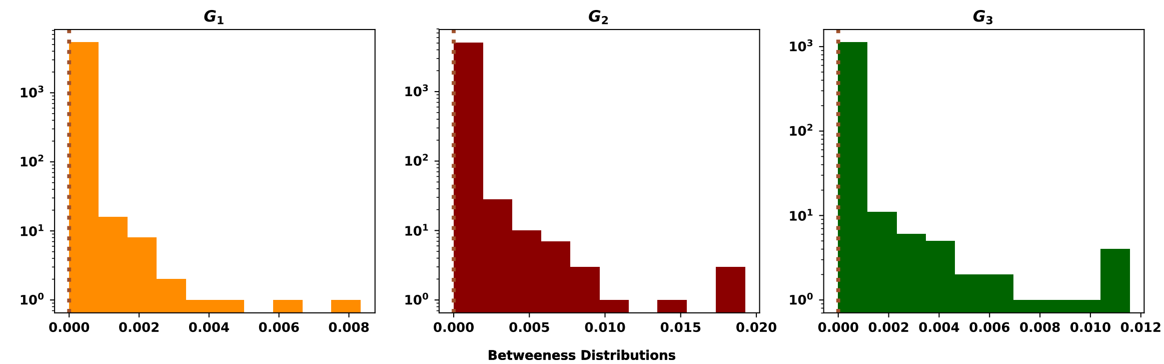
Regarding gaining from your network. An author is more likely to gain more from the influence of the field if they were authors in auction games or the price of anarchy. Though if it were an author in the main cluster it would make to statistical difference in which field they were to published. Overall, all the betweenness values are rather small and the distributions skewed to the left. This could imply that in all three networks, authors do not gain much from the influence of their fields.

In relation to influencing your field. An author is most likely to influence their field if the write for the price of anarchy and authors that publish on auctions game are more likely to influence compared to authors in the iterated prisoner's dilemma. Though if an author was to be placed in the main cluster of the respective field they would chose to be in either G_1 or G_3. In conclusion, authors regarding both influence metrics that have been defined here are more likely to gain more if they were to published on either topics of auction games or the price of anarchy. Though the value of gaining is actual very small, you are more likely to influence you field more in another compared to that of the prisoner's dilemma.

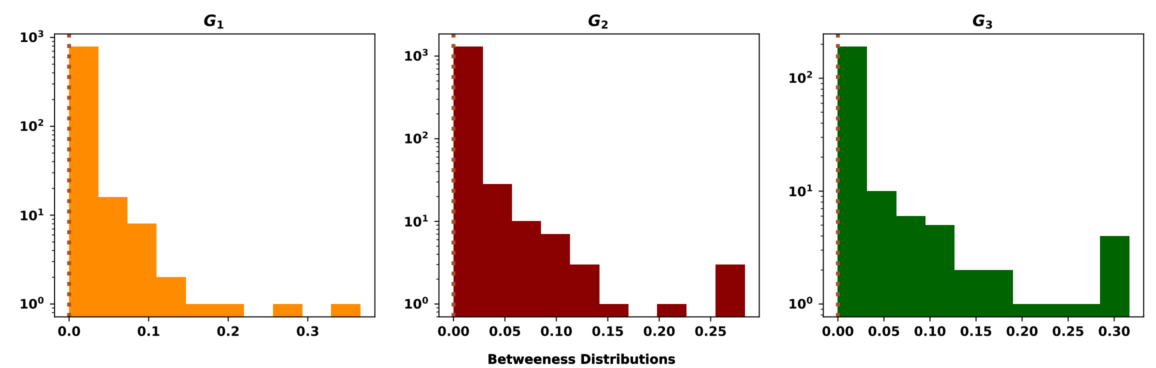
4 Acknowledges

A variety of software libraries have been used in this work:

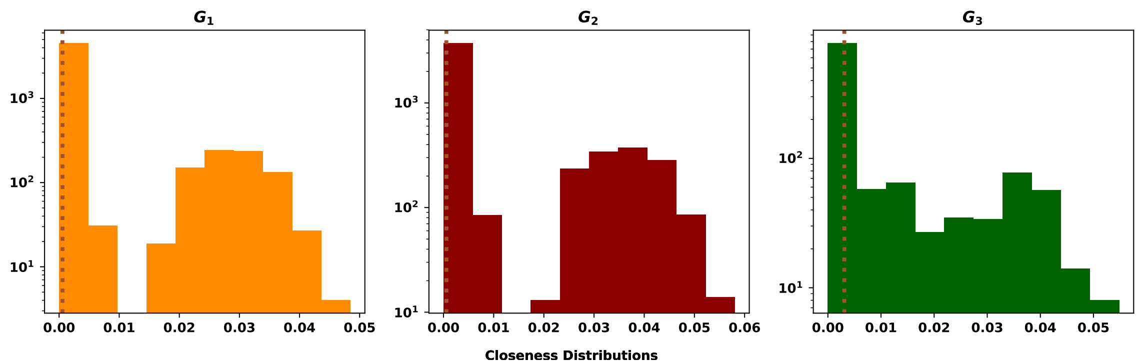
- Networkx [47], library for analysing networks.
- Gephi [26] open source package for visualising networks.



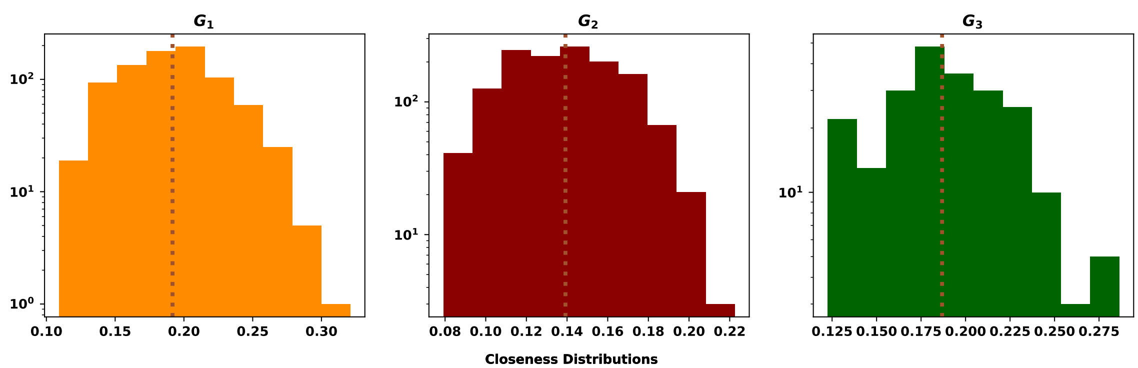
(a) Betweenness centrality distributions G_1, G_2, G_3. The descriptive statistics for each of the distribution are for G_1: mean= 0.000019, median= 0.0, std= 0.000207. For G_2: mean= 0.000086, median= 0.0, std= 0.000693 and for G_3: mean= 0.000151, median= 0.0, std= 0.000931. None of the three distributions is normally distributed and there is significant difference between the means (these have been tested using appropriate statistical difference). According to a Mann Whitney both G_2 and G_3 medians are significant larger than that of G_1 however there is not statistical difference between those two medians.



(b) Betweenness centrality distributions for G_1, G_2, G_3 respective main clusters.The descriptive statistics for each of the distribution are for G_1: mean= 0.0054, median= 0.0, std= 0.022. For G_2: mean= 0.0048, median= 0.0, std= 0.019 and for G_3: mean= 0.02, median= 0.0, std= 0.055. None of the three distributions is normally distributed. There is no statistical difference between the medians of G_1 and G_3. There is however, statistical difference between the median of G_2. These have been tested using a Kruskal Wallis test.



(a) Closeness centrality distributions G_1, G_2, G_3. The descriptive statistics for each of the distribution are for G_1: mean= 0.0050, median= 0.00056, std= 0.010. For G_2: mean= 0.000086, median= 0.00058, std= 0.000693 and for G_3: mean= 0.000151, median= 0, std= 0.000931. None of the three distributions is normally distributed and the median of G_3 is statistically larger than that of G_2, which is larger than that of G_1.



(b) Closeness centrality distributions for G_1, G_2, G_3 respective main clusters. The descriptive statistics for each of the distribution are for G_1: mean= 0.19, median= 0.19, std= 0.035. For G_2: mean= 0.14, median= 0.14, std= 0.026 and for G_3: mean= 0.19, median= 0.19, std= 0.035. None of the three distributions is normally distributed. All medians are statistically different. The medians of G_1 and G_3 are greater than that of G_2.

- louvain, library for calculating the networks modularity.

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