A bibliometric comparison of collaboration and influence between game theoretic subfields.

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Abstract

1 Introduction

The focus of Section **??** has been to review academic publications on the topic of the IPD. Whilst in Section **??** several publications of specific interest were covered and the literature was manually partitioned in different sections, in the second part of this paper the publications are analysed using a large dataset of articles.

In Section 1.1 some background research on bibliometrics is discussed. The data collection process is covered in Section 1.2 and a preliminary analysis of the data is conducted in Section 1.3. In Section 1.4, graph theoretical methods will be used to ascertain the level of collaborative nature of the field and identify influence. This type of analysis has been carried out in [22]. The novelty here is to consider approaches not considered in [22] such as the centrality measures of the network, and apply them to a new dataset. A further comparison of the results are made, relative to two other sub fields of game theory: auction games [25] and the price of anarchy [30]. A temporal analysis is also considered. In Section 1.5, the results of the analysis are discussed.

The aim of Section **??** is to provide a concrete summary of the existing literature on the PD. This is done to provide a review which will allow the research community to understand overall trends in the field, and already existing results.

In Section **??** a comprehensive data set of literature regarding the PD, collected from the following sources, is presented and analysed. The data set has been archived and is available at [12].

- arXiv [23]; a repository of electronic preprints. It consists of scientific papers in the fields of mathematics, physics, astronomy, electrical engineering, computer science, quantitative biology, statistics, and quantitative finance, which all can be accessed online.
- PLOS [1]; a library of open access journals and other scientific literature under an open content license. It launched its first journal, PLOS Biology, in October 2003 and publishes seven journals, as of October 2015.
- IEEE Xplore Digital Library (IEEE) [17]; a research database for discovery and access to journal articles, conference proceedings, technical standards, and related materials on computer science, electrical engineering and electronics, and allied fields. It contains material published mainly by the Institute of Electrical and Electronics Engineers and other partner publishers.
- Nature [14]; a British multidisciplinary scientific journal, first published on 4 November 1869. It was ranked the world's most cited scientific journal by the Science Edition of the 2010 Journal Citation Reports and is ascribed an impact factor of 40.137, making it one of the world's top academic journals.

- Springer [24]; a leading global scientific publisher of books and journals. It publishes close to 500 academic and professional society journals.

The aim of the analysis is to review the amount of published academic articles as well as to measure and explore the collaborations within the field.

1.1 Background

As discussed in [34], bibliometrics (the statistical analysis of published works originally described by [28]) has been used to support historical assumptions about the development of fields [29], identify connections between scientific growth and policy changes [7], develop a quantitative understanding of author order [31], and investigate the collaborative structure of an interdisciplinary field [22]. Most academic research is undertaken in the form of collaborative effort and as [20] points out, it is rational that two or more people have the potential to do better as a group than individually. Collaboration in groups has a long tradition in experimental sciences and it has be proven to be productive according to [9]. The number of collaborations can be very different between research fields and understanding how collaborative a field is not always an easy task. Several studies tend to consider academic citations as a measure for these things. A blog post published by Nature [26] argues that depending on citations can often be misleading because the true number of citations can not be known. Citations can be missed due to data entry errors, academics are influenced by many more papers than they actually cite and several of the citations are superficial.

A more recent approach to measure collaborative behaviour is to use the co authorship network, as described in [22]. Using this approach has many advantages as several graph theoretic measures can be used as proxies to explain authors relationship. In [22], they analyse the development of the field "evolution of cooperation" using this approach. The topic "evolution of cooperation" is a multidisciplinary field which also includes a large number of publications on the PD. This paper builds upon the work done by [22] and extends their methodology. Though in [22], they considered a data set from a single source, Web of Science, the data set described here has been collected from five different sources. Moreover, the collaborative results of the analysis are compared to those of two different sub fields.

Co authorship networks have also been used in [34] for classifying topics of an interdisciplinary field. This was done using centrality measures, which will be covered in Section 1.2, here centrality measures are used in order to understand the influence an author can have and can receive by being part of an academic group. Furthermore, in [3] they look at the relationship between research impact and five classes of diversity: ethnicity, discipline, gender, affiliation, and academic age. These characteristics of the authors are not being captured here. In future work these characteristics would be included in the analysis.

1.2 Data Collection

Academic articles are accessible through scholarly databases. Several databases and collections today offer access through an open application protocol interface (API). An API allows users to query directly a journal's database and bypass the graphical user interface. Interacting with an API has two phases: requesting and receiving. The request phase includes composing a url with the details of the request. For example, `http://export.arxiv.org/api/query?search_query=abs:prisoner'sdilemma&max_results=1` represents a request message. The first part of the request is the address of the API. In this example the address corresponds to the API of arXiv. The second part of the request contains the search arguments. In this example it is requested that the word 'prisoners dilemma' exists within the article's title. The format of the request message is different from API to API. The receive phase includes receiving a number of raw metadata of articles that satisfies the request message. The raw metadata are commonly received in extensive markup language (xml) or Javascript object notation (json) formats [27]. Similarly to the request message, the structure of the received data differs from journal to journal.

The data collection is crucial to this study. To ensure that this study can be reproduced all code used to query the different APIs has been packaged as a Python library and is available online [13]. The software could be used for any type of projects similar to the one described here, documentation for it is available at: `http://arcas.readthedocs.io/en/latest/`. Project [13] allow users to collect articles from a list of APIs by specifying just a single keyword. Four prominent journals in the field and a pre print server were used as sources to collect data for this analysis: PLOS, Nature, IEEE, Springer and arXiv.

The following series of search terms were used to identify relevant articles:

- "prisoner's dilemma",
- "prisoners dilemma",
- "prisoner dilemma",
- "prisoners evolution",
- "prisoner game theory"

and articles for which any of these terms existed within the title, the abstract or the text are included in the analysis. More specifically, 23% of article considered here were included because any of the above terms existed within the abstract, 50% within the main text and 27% within the title. As will be described in Section 1.3, two other game theoretic sub fields, auction games and the price of anarchy were also considered in this work. For collecting data on these sub fields the search terms used were "auction game theory" and "price of anarchy". The three data sets are archived and available at [10, 11, 12]. Note that the latest data collection was performed on the 30$^{\text{th}}$ November 2018.

1.3 Preliminary Analysis

A summary of each of the three data sets used is presented in this section. The three data sets are:

- The main data set which contains articles on the prisoner's dilemma archived at [12].
- A data set which contains article on auction games archived at [10].
- A data set which contains articles on the price of anarchy archived at [11].

The main data set consists of 3077 articles with unique titles. In case of duplicates the preprint version of an article (collected from arXiv) was dropped. Of these 3077 articles, 77 have not been collected from the aforementioned APIs. These articles were of specific interest and manually added to the dataset throughout the writing of Section **??**. A similar approach was used in [22] where a number of articles of interest where manually added to the data set. A more detailed summary of the articles' provenance is given by Table 1. Only 3% of the data set consists of articles that were manually added and 34% of the articles were collected from arXiv.

The average number of publications per year is denoted as, $\mu_P = \frac{N_A}{N_Y}$, where N_A is the total number of articles and N_Y is the years of publication. The years of publication is calculated as the range between the collection date and the first published article, for each provenance, within the data. These averages are summarised in Table 2. Overall an average of 49 articles are published per year on the topic. The most significant contribution to this appears to be from arXiv with 16 articles per year, followed by Nature with 10 and Springer with 9.

Though the average publication offers insights about the publications of the fields, it remains a constant number. The data handled here is a time series starting in 1950, when the game was introduced, until 2018 (Figure 1). Two observations can be made from Figure 1.

provenance	# of Articles	Percentage%
Manual	77	2.50%
IEEE	295	9.59%
PLOS	482	15.66%
Springer	572	18.59%
Nature	673	21.87%
arXiv	1056	34.32%

Table 1: Articles' provenance for the main data set [12].

	Average Yearly publication
IEEE	5.0
PLOS	8.0
Springer	9.0
Nature	11.0
arXiv	16.0
Overall	49.0

Table 2: Average yearly publication (μ_P) for main data set [12].

1. There is a steady increase to the number of publications since the 1980s and the introduction of computer tournaments.

2. There is a decrease in 2017-2018. This is due to our data set being incomplete. Articles that have been written in 2017-2018 have either not being published or were not retrievable by the APIs at the time of writing this manuscript.

These observations can be confirmed by studying the time series. Using [18], an exponential distribution is fitted to the data from 1980-2016 (Figure 2). The fitted model can be used to forecast the behaviour of the field for the next 5 years. The forecasted periods are plotted in Figure 3. The time series has indicated a slight decrease however the model forecasts that the number of publications will keep increasing, thus demonstrating that the field of the iterated prisoner's dilemma continues to attract academic attention.

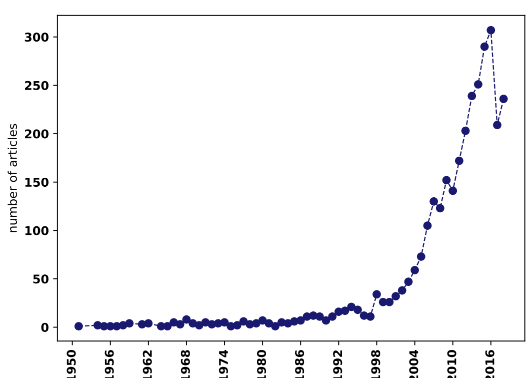


Figure 1: Line plot; number of articles published on the PD 1950-2019.

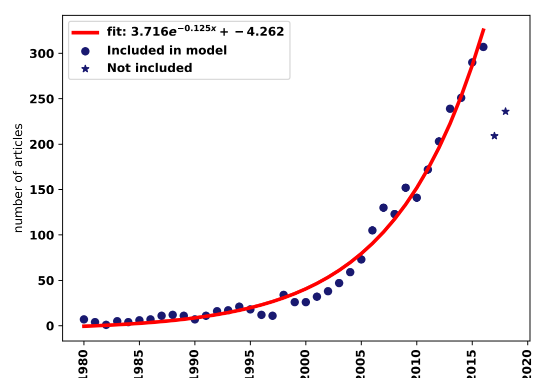


Figure 2: Scatter plot; number of articles published on the PD 1980-2019.

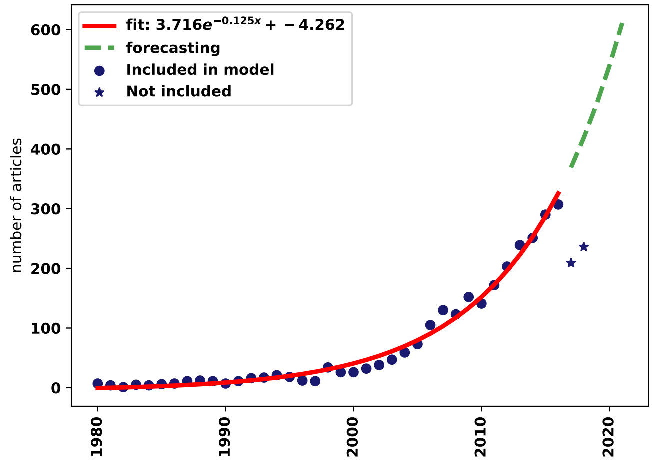


Figure 3: Forecast for 2017-2022.

To allow for a comparative analysis two sub fields of game theory have been chosen for this work; auction games and the price of anarchy.

- Auction theory is a branch of game theory which researches the properties of auction markets. Game theory has been used for years to study auctions and the behaviour of bidders [32]. The earliest entry in our data set [10] goes back to 1974 (Figure 4). Note that no articles have been added manually for auction games.
- Price of Anarchy is a concept in game theory which measures how the efficiency of a system degrades due to selfish behaviour of its agents. There is a variety of such measures however the price of anarchy has attracted a lot of attention since it's informal introduction in 1999 by [19]. Note that [19] has been manually added to the date set [11] (Figure 5).

A summary of both data sets, in comparison to that of [12], is given by Table 3.

	Num. Articles	Num. Authors	Manual (%)	PLOS (%)	Nature (%)	Springer (%)	IEEE (%)	arXiv (%)	Av. Yearly Publication
Prisoner's Dilemma	3077	5772	2.5	15.66	21.87	18.59	9.59	34.32	49.0
Auction Games	3444	5362	-	-	5.89	37.63	7.46	51.36	93.0
Price of Anarchy	748	1316	0.13	1.74	24.73	37.97	30.61	8.82	39.0

Table 3: Measures of all three data sets.

The IPD and auction theory are popular topics and have been studied for decades. A large number of articles have been collected for both topics, 3077 and 3444 respectively. Though, auction games have a larger number of articles, the IPD has almost 300 more authors.

Auction games have an overall average yearly publication of 93 articles per year compared to the PD with 49 per year. 50% of articles for auction games have been collected from the pre print server arXiv and no articles have been published in PLOS.

Compared to these two topics the price of anarchy is a fairly recent one. Only a total of 748 articles have been collected, however it has a large number of 1229 authors. On average each paper has two authors. It has an overall average yearly publication rate of 39 articles and the biggest contribution has been made to Springer (37%).

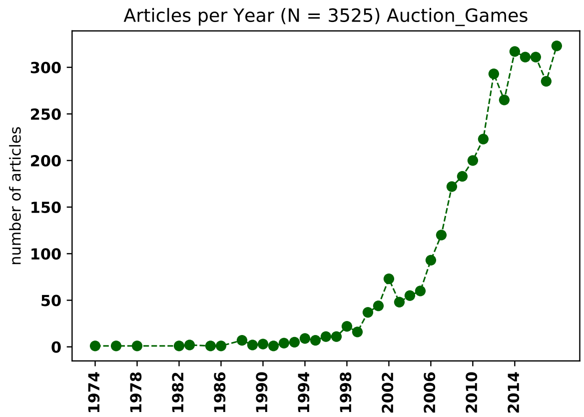


Figure 4: Line plot; number articles published on auction games 1974-2018.

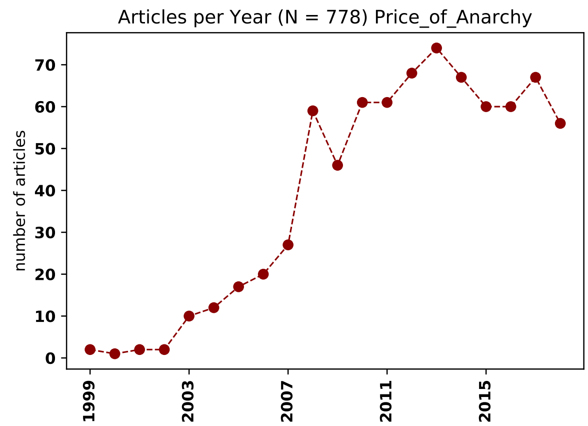


Figure 5: Line plot; number articles published on the price of anarchy.

1.4 Methodology

The relationship between the authors within a field will be modelled as a graph $G = (V_G, E_G)$ where V_G is the set of nodes and E_G is the set of edges. The set V_G represents the authors and an edge connects two authors if and only if those authors have written together. The co authorship network is constructed using the main data set described in Section 1.3 and the open source package [15]. The PD network is denoted as G_1 where the number of unique authors $|V(G_1)|$ is 5772 and $|E(G_1)|$ is 10397. All authors' names were formatted as their last name and first initial (i.e. Martin A. Nowak to Martin Nowak). This was done to avoid errors such as Martin A. Nowak and Martin Nowak being treated as a different person.

Collaborativeness will be analysed using measures such as, isolated nodes, connected components, clustering coefficient, communities, modularity and average degree. These measures show the number of connections authors can have and how strongly connected these people are. The number of isolated nodes is the number of nodes that are not connected to another node, thus the number of authors that have published alone. The average degree denotes the average number of neighbours for each nodes, i.e. the average number of collaborations between the authors.

A connected component is a maximal set of nodes such that each pair of nodes is connected by a path [8]. The number of connected components as well as the size of the largest connected component in the network are reported. The size of the largest connected component represents the scale of the central cluster of the entire network, as will discussed in the analysis section. Clustering coefficient and modularity are also calculated. The clustering coefficient, defined as 3 times the number of triangles on the graph divided by the number of connected triples of nodes, is a local measure of the degree to which nodes in a graph tend to cluster together in a clique [8]. It is precisely the probability that the collaborators of an author also write together.

In comparison, modularity is a global measure designed to measure the strength of division of a network into communities. The number of communities will be reported using the Clauset-Newman-Moore method [6]. Also the modularity index is calculated using the Louvain method described in [5]. The value of the modularity index can vary between $[-1, 1]$, a high value of modularity corresponds to a structure where there are dense connections between the nodes within communities but sparse connections between nodes in different communities. That means that there are many sub communities of authors that write together but not across communities.

Two further points are aimed to be explored in this work, (1) which people control the flow of information; as in which people influence the field the most and (2) which are the authors that gain the most from the influence of the field. To measure these concepts centrality measures are going to be used. Centrality measures are often used to understand different aspects of social networks [21]. Two centrality measures have been chosen for this paper and these are closeness and betweenness centrality.

1. In networks some nodes have a short distance to a lot of nodes and consequently are able to spread information on the network very effectively. A representative of this idea is **closeness centrality**, where a node is seen as centrally involved in the network if it requires only few intermediaries to contact others and thus is structurally relatively independent. Here, this is interpreted as a influence. Authors with a high value of closeness centrality, are the authors that spread scientific knowledge easier on the network and they have high influence.
2. Another centrality measure is the **betweenness centrality**, where the determination of an author's centrality is based on the quotient of the number of all shortest paths between nodes in the network that include the node in question and the number of all shortest paths in the network. In betweenness centrality the position of the node matters. Nodes with a higher value of betweenness centrality are located in positions that a lot of information pass through, this is interpreted as the gain from the influence, thus these authors gain the most from their networks.

In the next section all the metrics discussed here are calculated for the data sets in order to provide insights into the field.

1.5 Analysis of co authorship network

As mentioned previously, G_1 denotes the co authorship network of the IPD. A graphical representation is given by Figure 6a. It is evident that the network is disjoint, which is only natural as many authors write academic articles on their own. More specifically, a total of 176 authors only have single author publications, which corresponds to the 3.3 (%) of authors in G_1.

There are a total of 1356 connected components and the largest one has a size of 815 nodes. The largest connected component is shown in Figure 6b and is going to be refereed to as the main cluster of the network. There are total of 1369 communities in G_1. The network has a clustering coefficient of 0.708, thus authors are 70% likely to write with a collaborator's co author. The degree distribution, Figure 7, shows that the average degree is approximately 4. Thus authors are on average connected to 4 other authors, however there are authors with far more connections, the largest one being 58.

In [22] the collaborative metrics for the "evolution of cooperation" co authorship network were reported. Though their network is of smaller size (number of nodes $3670 < 5394$), the collaborative metrics are fairly similar between the two graphs (clustering coeff. 0.632 and modularity 0.950 close to 0.977), indicating that for the same field the same remarks can be made from a different co authors network. How do these compare to other fields and more specifically to other fields of game theory?

The auction games network G_2, and the price of anarchy network G_3 are given by Figures 6c and 6e and their respective largest cluster in Figures 6d and 6f. As stated before G_3 is the smallest network. G_2 appears to be very similar to G_1, however it's main cluster is larger in size.

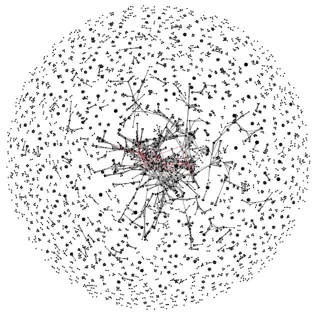
A summary of the collaborative metrics for all three co authorship networks is given by Table 4 and the degree distribution of all three networks is shown in Figure 7.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
Prisoner's Dilemma	5394	10397	176	3.3	1356	815	3.855	1369	0.977	0.708
Auction Games	5165	7861	256	5.0	1272	1348	3.044	1294	0.958	0.622
Price of Anarchy	1155	1953	4	0.3	245	222	3.382	253	0.965	0.712

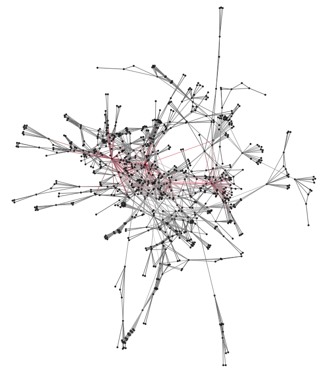
Table 4: Network metrics for G_1, G_2, G_3.

Using Table 4 and Figure 7 the following remarks can be made:

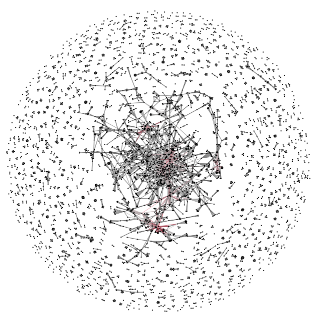
- Comparing to another well studied topic, auction games, the field of the IPD appears to be more collaborative. Due to the value of the average degree, authors in G_1 are known to have on average almost one more



(a) G_1 the co authorship network for the IPD.

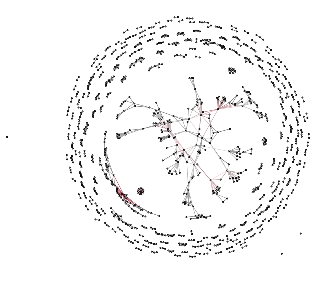


(b) $\bar{G}_1$ the largest connected component of G_1.



(c) G_2 the co authorship network for auction games.

(d) $\bar{G}_2$ the largest connected component of G_2.



(e) G_3 the co authorship network for price of anarchy.

(f) $\bar{G}_3$ the largest connected component of G_3.

Figure 6: Graphical representations of G_1, G_2, G_3 and their respective largest components.

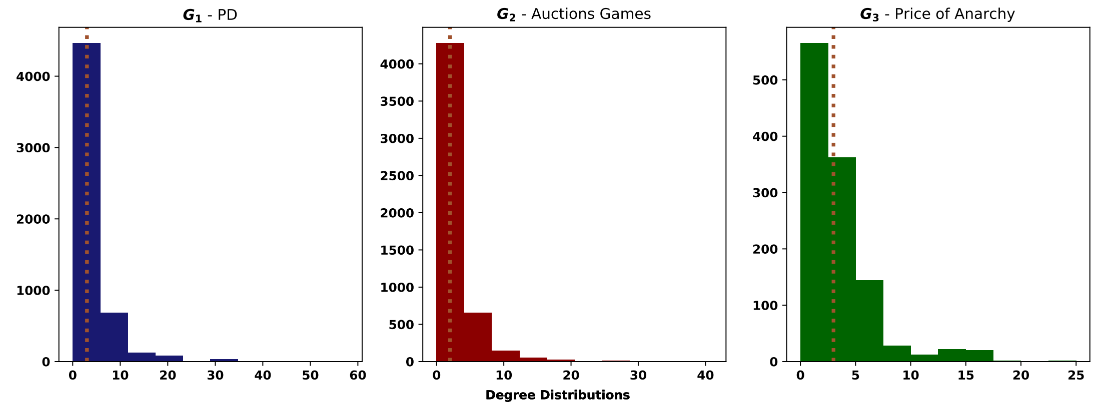


Figure 7: Degree distribution for networks G_1, G_2 and G_3. The descriptive statistics for each of the distribution are for G_1: mean = 3.85, median = 3, std = 4.25. For G_2: mean = 3.04, median = 2, std = 3.01 and for G_3: mean = 3.38, median = 3, std = 2.90. It is clear that these distributions are not normally distributed, which has also been verified using a statistical test. Moreover, the statistical difference of the medians has been tested using a Kruskal Wallis test. The medians of G_1 and G_3 are not significantly different, however they are are significantly larger than that of G_2.

collaboration than G_2. A slightly lower cluster coefficient (.622 < .702) of auction games indicate that it is less likely for authors in G_2 to collaborate with a co author. This could be an influence of the nature of the field itself were not only mathematics but software engineering skills are vital.

- Regarding the price of anarchy, the measures indicate that the field is not as mature as the other two sub fields. There are no isolated authors, which is more of an indication of the time the field has been active. As a more recent field there had been better communication tools that enable more collaborations between researchers. The average degree as well as the clustering coefficient (clustering coeff.= 0.713) of G_3 is comparable to those of the IPD.

These results can be extended to the main clusters of each network, as shown in Table 5. The metrics' values are fairly similar and the size of G_2's main cluster does not appear to gave any significant effect; all the same conclusions are made.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
Prisoner's Dilemma	815	2300	0	0.0	1	815	5.644	29	0.852	0.775
Auction Games	1348	3158	0	0.0	1	1348	4.685	28	0.860	0.699
Price of Anarchy	222	521	0	0.0	1	222	4.694	13	0.827	0.711

Table 5: Network metrics for largest components, $\bar{G}_1, \bar{G}_2, \bar{G}_3$.

The change of the networks over time is also studied by constructing the network cumulatively using a yearly interval. A total of 64 sub graphs over 64 periods, starting in 1950, were created and all the collaborative metrics for each sub graph have been calculated. Note that years 1952 and 1953 have no publications in our data set. The metrics of each network for each period are given by Table 6. Similar to the results of [22], it can been observed that the network G_1 grows over time and that the network always has modularity.

To better assess the change over time for each metric they have been plotted in Figure 8. The number of nodes, connected components and the size of largest component have been normalised such that the trend between the three networks can be compared.

- In Figure 8a the normalised number of nodes, which is calculated by dividing by the total number of nodes in each respective network, is shown. A steep increase in the size of all three networks is spotted soon after

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
1954 - 1950	3	0	3	100.0	3	1	0.000	-	-	0.000
1954 - 1955	2	0	2	100.0	2	1	0.000	-	-	0.000
1955 - 1956	3	0	3	100.0	3	1	0.000	-	-	0.000
1956 - 1957	4	0	4	100.0	4	1	0.000	-	-	0.000
1957 - 1958	6	0	6	100.0	6	1	0.000	-	-	0.000
1958 - 1959	7	0	7	100.0	7	1	0.000	-	-	0.000
1959 - 1961	7	0	7	100.0	7	1	0.000	-	-	0.000
1961 - 1962	8	0	8	100.0	8	1	0.000	-	-	0.000
1962 - 1964	9	0	9	100.0	9	1	0.000	-	-	0.000
1964 - 1965	10	0	10	100.0	10	1	0.000	-	-	0.000
1965 - 1966	17	3	11	64.7	14	2	0.353	14	0.666667	0.000
1966 - 1967	21	4	13	61.9	17	2	0.381	17	0.75	0.000
1967 - 1968	32	15	13	40.6	21	5	0.938	21	0.684444	0.135
1968 - 1969	36	17	16	44.4	24	6	0.944	24	0.629758	0.139
1969 - 1970	39	18	17	43.6	26	6	0.923	26	0.666667	0.128
1970 - 1971	51	28	18	35.3	31	6	1.098	31	0.826531	0.275
1971 - 1972	58	34	19	32.8	34	6	1.172	34	0.866782	0.345
1972 - 1973	59	35	18	30.5	34	6	1.186	34	0.873469	0.339
1973 - 1974	59	35	18	30.5	34	6	1.186	34	0.873469	0.339
1974 - 1975	60	35	19	31.7	35	6	1.167	35	0.873469	0.333
1975 - 1976	60	35	19	31.7	35	6	1.167	35	0.873469	0.333
1976 - 1977	68	37	23	33.8	41	6	1.088	41	0.885318	0.294
1977 - 1978	70	38	23	32.9	42	6	1.086	42	0.890582	0.286
1978 - 1979	73	42	23	31.5	42	6	1.151	42	0.893424	0.292
1979 - 1980	77	45	25	32.5	44	6	1.169	44	0.899753	0.307
1980 - 1981	80	50	26	32.5	45	6	1.250	45	0.8928	0.318
1981 - 1982	84	56	26	31.0	46	6	1.333	46	0.903061	0.350
1982 - 1983	87	57	27	31.0	48	6	1.310	48	0.906125	0.338
1983 - 1984	94	58	32	34.0	54	6	1.234	54	0.909037	0.313
1984 - 1985	95	58	33	34.7	55	6	1.221	55	0.909037	0.309
1985 - 1986	104	59	40	38.5	63	6	1.135	63	0.911807	0.283
1986 - 1987	116	61	48	41.4	73	6	1.052	73	0.916958	0.253
1987 - 1988	121	65	48	39.7	75	6	1.074	75	0.924497	0.268
1988 - 1989	134	76	47	35.1	80	6	1.134	80	0.937673	0.272
1989 - 1990	145	82	49	33.8	86	6	1.131	86	0.944676	0.272
1990 - 1991	158	88	53	33.5	94	6	1.114	94	0.950413	0.268
1991 - 1992	169	91	59	34.9	102	6	1.077	102	0.953025	0.251
1992 - 1993	186	104	62	33.3	110	6	1.118	110	0.95932	0.266
1993 - 1994	220	134	72	32.7	127	6	1.218	127	0.965471	0.317
1994 - 1995	239	144	74	31.0	137	6	1.205	137	0.969329	0.304
1995 - 1996	257	163	77	30.0	145	6	1.268	145	0.970831	0.318
1996 - 1997	279	178	81	29.0	156	6	1.276	156	0.974309	0.336
1997 - 1998	311	215	65	20.9	160	6	1.383	160	0.979773	0.354
1998 - 1999	329	239	58	17.6	162	6	1.453	162	0.981741	0.376
1999 - 2000	373	273	67	18.0	183	6	1.464	183	0.983778	0.387
2000 - 2001	400	320	54	13.5	184	7	1.600	184	0.983066	0.410
2001 - 2002	450	366	61	13.6	206	7	1.627	206	0.984547	0.418
2002 - 2003	509	414	58	11.4	229	7	1.627	229	0.987083	0.421
2003 - 2004	580	489	58	10.0	253	10	1.686	253	0.988052	0.429
2004 - 2005	679	599	57	8.4	284	19	1.764	284	0.98891	0.463
2005 - 2006	854	806	66	7.7	342	21	1.888	342	0.990724	0.496
2006 - 2007	1056	1117	76	7.2	402	24	2.116	402	0.989663	0.527
2007 - 2008	1255	1460	85	6.8	454	32	2.327	455	0.989734	0.549
2008 - 2009	1462	1759	104	7.1	520	56	2.406	521	0.987517	0.550
2009 - 2010	1700	2301	114	6.7	581	99	2.707	584	0.979084	0.571
2010 - 2011	2040	2954	121	5.9	665	121	2.896	668	0.980451	0.603
2011 - 2012	2422	3676	126	5.2	756	210	3.036	759	0.978972	0.629
2012 - 2013	2807	4398	138	4.9	843	330	3.134	849	0.977892	0.639
2013 - 2014	3199	5044	148	4.6	942	406	3.153	950	0.974719	0.651
2014 - 2015	3798	6221	159	4.2	1064	514	3.276	1074	0.976286	0.668
2015 - 2016	4472	8344	169	3.8	1184	614	3.732	1197	0.975044	0.690
2016 - 2017	4925	9235	173	3.5	1274	703	3.750	1288	0.975975	0.700
2017 - 2018	5385	10379	176	3.3	1356	815	3.855	1369	0.977065	0.708

Table 6: Collaborativeness metrics for cumulative graphs, $G \subseteq G_1$.

	# Nodes	# Edges	# Isolated nodes	% Isolated nodes	# Connected components	Size of largest component	Av. degree	# Communities	Modularity	Clustering coeff
1954 - 1950	1	0	1	100.0	1	1	0.000	-	-	0.000
1954 - 1955	1	0	1	100.0	1	1	0.000	-	-	0.000
1955 - 1956	1	0	1	100.0	1	1	0.000	-	-	0.000
1956 - 1957	1	0	1	100.0	1	1	0.000	-	-	0.000
1957 - 1958	1	0	1	100.0	1	1	0.000	-	-	0.000
1958 - 1959	1	0	1	100.0	1	1	0.000	-	-	0.000
1959 - 1961	1	0	1	100.0	1	1	0.000	-	-	0.000
1961 - 1962	1	0	1	100.0	1	1	0.000	-	-	0.000
1962 - 1964	1	0	1	100.0	1	1	0.000	-	-	0.000
1964 - 1965	1	0	1	100.0	1	1	0.000	-	-	0.000
1965 - 1966	2	1	0	0.0	1	2	1.000	1	0	0.000
1966 - 1967	2	1	0	0.0	1	2	1.000	1	0	0.000
1967 - 1968	5	8	0	0.0	1	5	3.200	1	0	0.867
1968 - 1969	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1969 - 1970	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1970 - 1971	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1971 - 1972	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1972 - 1973	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1973 - 1974	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1974 - 1975	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1975 - 1976	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1976 - 1977	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1977 - 1978	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1978 - 1979	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1979 - 1980	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1980 - 1981	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1981 - 1982	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1982 - 1983	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1983 - 1984	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1984 - 1985	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1985 - 1986	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1986 - 1987	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1987 - 1988	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1988 - 1989	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1989 - 1990	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1990 - 1991	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1991 - 1992	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1992 - 1993	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1993 - 1994	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1994 - 1995	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1995 - 1996	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1996 - 1997	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1997 - 1998	6	10	0	0.0	1	6	3.333	2	0.02	0.833
1998 - 1999	6	9	0	0.0	1	6	3.000	2	0.0493827	0.678
1999 - 2000	6	9	0	0.0	1	6	3.000	2	0.166667	0.900
2000 - 2001	7	21	0	0.0	1	7	6.000	1	0	1.000
2001 - 2002	7	21	0	0.0	1	7	6.000	1	0	1.000
2002 - 2003	7	21	0	0.0	1	7	6.000	1	0	1.000
2003 - 2004	10	13	0	0.0	1	10	2.600	2	0.37574	0.553
2004 - 2005	19	28	0	0.0	1	19	2.947	3	0.544005	0.730
2005 - 2006	21	32	0	0.0	1	21	3.048	4	0.543945	0.713
2006 - 2007	24	36	0	0.0	1	24	3.000	5	0.563272	0.678
2007 - 2008	32	59	0	0.0	1	32	3.688	3	0.627837	0.732
2008 - 2009	56	102	0	0.0	1	56	3.643	4	0.716792	0.699
2009 - 2010	99	238	0	0.0	1	99	4.808	7	0.781539	0.734
2010 - 2011	121	288	0	0.0	1	121	4.760	9	0.771858	0.713
2011 - 2012	210	610	0	0.0	1	210	5.810	13	0.780145	0.747
2012 - 2013	330	908	0	0.0	1	330	5.503	17	0.817439	0.753
2013 - 2014	406	1125	0	0.0	1	406	5.542	21	0.818674	0.749
2014 - 2015	514	1390	0	0.0	1	514	5.409	20	0.827712	0.757
2015 - 2016	614	1682	0	0.0	1	614	5.479	29	0.830544	0.765
2016 - 2017	703	1925	0	0.0	1	703	5.477	31	0.835941	0.774
2017 - 2018	815	2300	0	0.0	1	815	5.644	30	0.855152	0.775

Table 7: Collaborativeness metrics for cumulative graphs' main clusters, $G \subseteq \bar{G}_1$.

2000. This could indicate that more data is available in the sources used in this work following the year 2000. It is however, definitely not an effect of a single field, as it is true for all three sub fields considered here. The sudden increase following the year 2000, is also reported by the number of connected components and the size of the main cluster, Figures 8c, 8d. A connected components represents at least one publication which means that indeed more articles were gathered from 2000 onwards.

- Auction games have been throughout time, less collaborative compared to the IPD. The average degree (Figure 8b) and the clustering coefficient (Figure 8e) of the cumulative sub graphs have been lower than that of G_1. The only exception is during the years 2001-2008. For these year auction games appear to have had a more collaborative environment.
- In the price of anarchy cumulative graphs a sharp increase since the beginning of the field can be observed for all metrics. There are not many data points due to the recent development of the field, however these steep trends could be an indication that game theoretic and potentially all scientific research has over time been more collaborative. This could be due to logistic and technical solutions.
- The high values of modularity throughout time is not true only for the network reported in [22] but also for all three networks of this field. Indicating that authors tend to create communities and write only with people from their communities and not others.

The cumulative collaborative metrics have also been calculated for each main cluster, given in Table 7. As before, the conclusions do not appear to change.

The next results discussed here are on centrality measures. As a reminder, two centrality measures are reported, these are the closeness centrality and the betweenness centrality. Closeness centrality is a measure of how easy it is for an author to contact others, and consequently affect them; influence them. Thus closeness centrality here is a measure of influence. Betweenness centrality is a measure of how many paths pass through a specific node, thus the amount of information this person has access to. Betweenness centrality is used here as a measure of how much an author gains from the field. All centrality measure can have values ranging from 0 to 1.

For G_1 the most central author based on closeness and betweenness are given by Tables 9 and 10 respectively. The betweenness centrality of the most central authors in G_1 are rather low with the highest ranked author being Matjaz Perc with a between centrality of 0.008, Table 10. A publication of Perc's work has been briefly discussed in Section **??**, and the centrality measure suggest that the network is influenced by him. He is connected to a total of 58 nodes and he has published to all five of the different sources considered in the study. Though he also gains from his position in the network, the gain is minor. An author who is not in the top influencers but does indeed gain from his position in the network is Martin Nowak, who was extensively discussed in Section **??**.

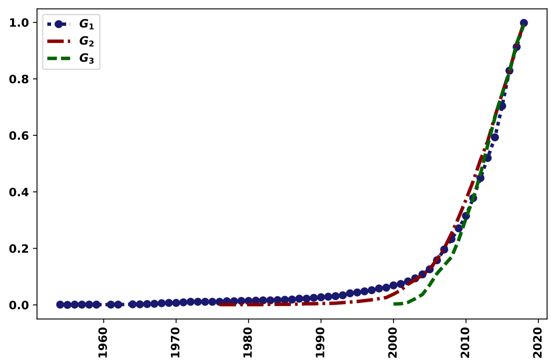
	Name	Closeness
1	Matjaz Perc	0.048447
2	Yamir Moreno	0.044840
3	Zhen Wang	0.044005
4	Long Wang	0.043770
5	Attila Szolnoki	0.043338
6	Luo-Luo Jiang	0.042148
7	Arne Traulsen	0.041790
8	Valerio Capraro	0.041257
9	Cheng-Yi Xia	0.040791
10	Angel Sanchez	0.040562

Figure 9: Ten most influenced authors in G_1.

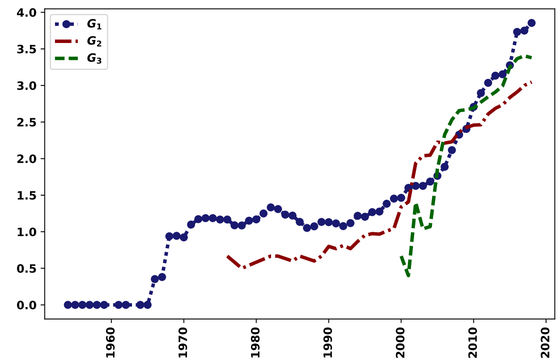
	Name	Betweeness
1	Matjaz Perc	0.008331
2	Zhen Wang	0.006356
3	Yamir Moreno	0.004806
4	Long Wang	0.003538
5	Martin Nowak	0.003230
6	Valerio Capraro	0.002739
7	Arne Traulsen	0.002479
8	Angel Sanchez	0.002319
9	Jianye Hao	0.002188
10	Franz Weissing	0.002186

Figure 10: Authors that gain the most influence in G_1.

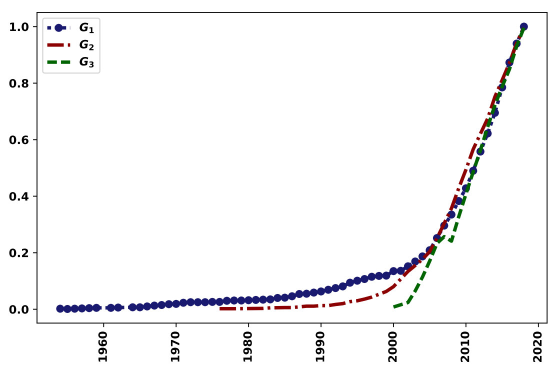
From Tables 9 and 10 it can be seen that authors in G_1 are more likely to affect their field instead of gaining from



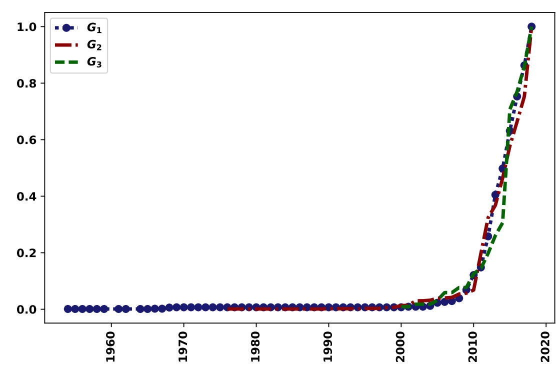
(a) % Nodes.



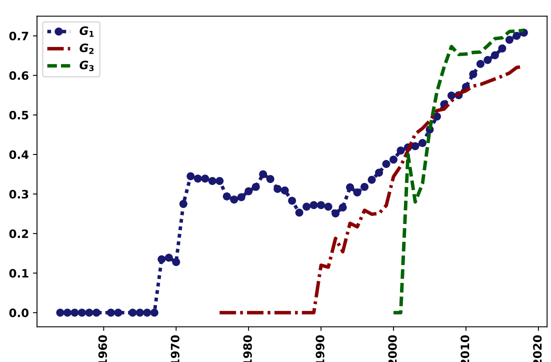
(b) Average Degree.



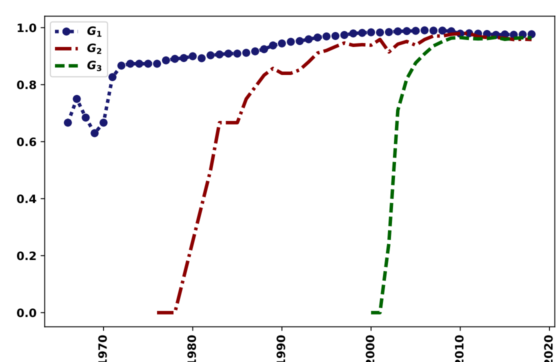
(c) % Connected components.



(d) % Size of largest connected component.



(e) Clustering coefficient.

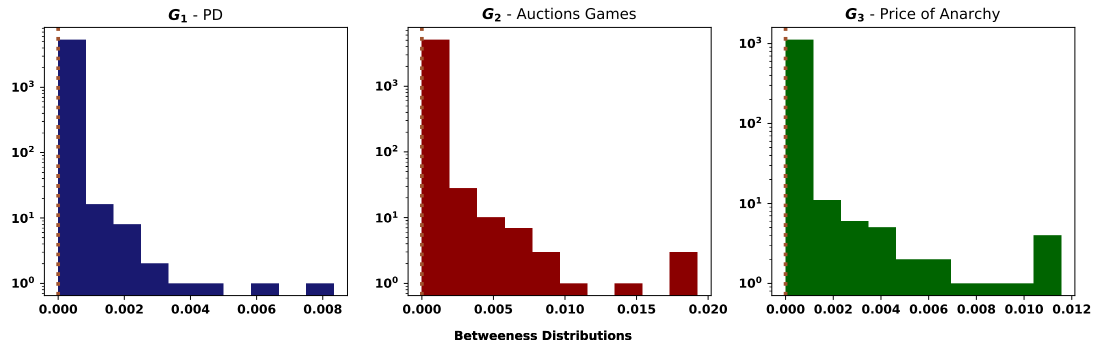


(f) Modularity.

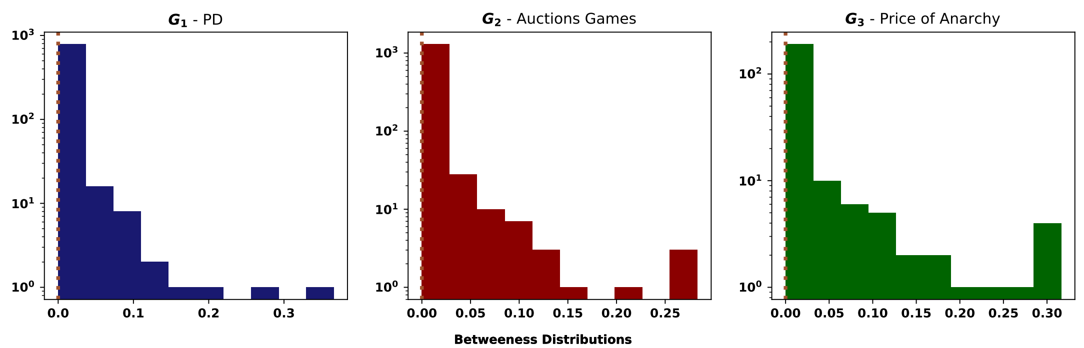
Figure 8: Collaborative metrics over time for cumulative networks for G_1, G_2 and G_3.

it. This can be better explored by considering the distributions of the centralities and by comparing them to other fields. The distributions for both centralities are plotted in Figures 11a and 12a, and in Figures 11b and 12b for their respective main clusters.

Regarding gaining from a network. An author is more likely to gain more from the influence of the field if they were authors in auction games or the price of anarchy. Though if it were an author in the main cluster it would make no statistical difference in which field they were to published. Overall, all the betweenness values are rather small and the distributions skewed to the left. This could imply that in all three networks, authors do not gain much from the influence of their fields.



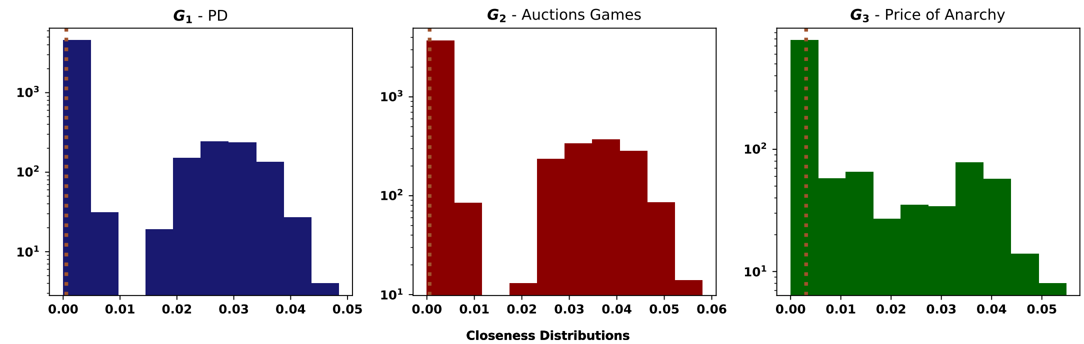
(a) Betweenness centrality distributions G_1, G_2, G_3. The descriptive statistics for each of the distribution are for G_1: mean= 0.000019, median= 0.0, std= 0.000207. For G_2: mean= 0.000086, median= 0.0, std= 0.000693 and for G_3: mean= 0.000151, median= 0.0, std= 0.000931. None of the three distributions is normally distributed and there is significant difference between the means (these have been tested using appropriate statistical difference). According to a Mann Whitney both G_2 and G_3 medians are significant larger than that of G_1 however there is not statistical difference between those two medians.



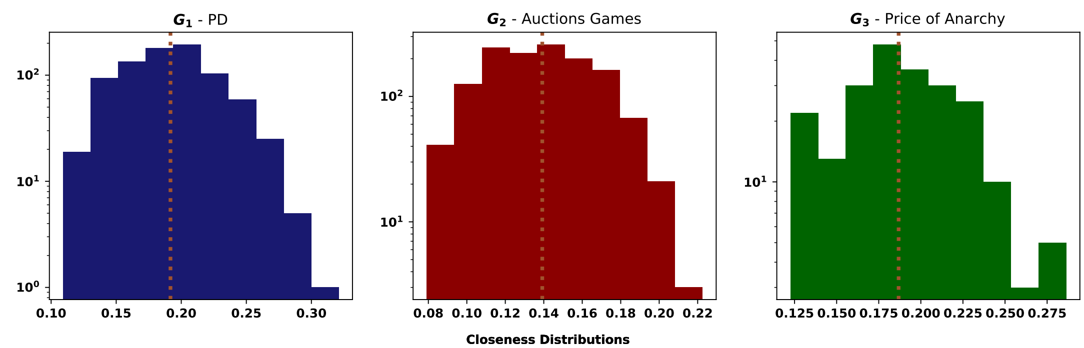
(b) Betweenness centrality distributions for G_1, G_2, G_3 respective main clusters.The descriptive statistics for each of the distribution are for G_1: mean= 0.0054, median= 0.0, std= 0.022. For G_2: mean= 0.0048, median= 0.0, std= 0.019 and for G_3: mean= 0.02, median= 0.0, std= 0.055. None of the three distributions is normally distributed. There is no statistical difference between the medians of G_1 and G_3. There is however, statistical difference between the median of G_2. These have been tested using a Kruskal Wallis test.

Figure 11

In relation to influencing your field. An author is most likely to influence their field if they write for the price of anarchy and authors that publish on auctions game are more likely to influence compared to authors in the IPD. Though if an author was to be placed in the main cluster of the respective field they would chose to be in either G_1 or G_3. In conclusion, authors regarding both influence metrics that have been defined here are more likely to gain more if they were to published on either topics of auction games or the price of anarchy. Though the value of gaining is actual small, you are more likely to influence your field more in another field compared to that of the PD.



(a) Closeness centrality distributions G_1, G_2, G_3. The descriptive statistics for each of the distribution are for G_1: mean= 0.0050, median= 0.00056, std= 0.010. For G_2: mean= 0.000086, median= 0.00058, std= 0.000693 and for G_3: mean= 0.000151, median= 0, std= 0.000931. None of the three distributions is normally distributed and the median of G_3 is statistically larger than that of G_2, which is larger than that of G_1.



(b) Closeness centrality distributions for G_1, G_2, G_3 respective main clusters. The descriptive statistics for each of the distribution are for G_1: mean= 0.19, median= 0.19, std= 0.035. For G_2: mean= 0.14, median= 0.14, std= 0.026 and for G_3: mean= 0.19, median= 0.19, std= 0.035. None of the three distributions is normally distributed. All medians are statistically different. The medians of G_1 and G_3 are greater than that of G_2.

Figure 12

2 Conclusion

This manuscript presented a literature review on the Iterated Prisoner's Dilemma. The opening sections focused on research trends and published works of the field. This was followed by a presentation of research and educational software. The later sections presented a meta analysis of publications with the aim of examining the collaborativeness of the authors and their influence in the research of the game.

The research trends covered in this manuscript included the early experiments using human subject research, the investigation of cooperative behaviour and the search of dominant strategies for the game. Human subject research had limitations, even so, it is still used by several researchers to date. Another framework of study are the computer tournaments introduced by Axelrod in 1980s. The search of strategies includes strategies that have been manually designed and strategies that have been found through training processes of structures such as finite state automata and neural networks. Moreover, cooperative behaviour and it's emergence under natural section was discussed in Section **??**. The results of several milestones have been summarised in this review. These included the emerge of cooperation in the PD in structured populations, the success and deficiency of the infamous Tit For Tat, and the training of complex strategies that evolved a handshake mechanism to combat invasion.

The meta analysis which was covered in the second part of this paper explored the number of publications, the authors collaborative behaviour and influence in the research field of the Iterated Prisoner's Dilemma. More than 3000 publications were automatically collected from five different sources using a bespoke piece of software written for this purpose [13]. A time series analysis predicted a continuous growth to the number of publications in the following years. Moreover, the authors of these papers were used to create a co authorship network which was studied to compare the field to two other prominent sub fields of game theory. The results of the analysis showed that the Iterated Prisoner's Dilemma field is more collaborative than the fields of auction games and the price of anarchy. However, authors are less likely to influence their peers.

Interesting areas of future work would include extending this analysis to more game theoretic sub fields, to evaluate whether the results remain the same. The study of the Prisoner's Dilemma is the study of cooperation, investigating the cooperative behaviours of authors is what this work has aimed to achieve.

3 Acknowledgements

A variety of software have been used in this work:

- The Axelrod library for IPD simulations [2].
- The Matplotlib library for visualisation [16].
- The Numpy library for data manipulation [33].
- The Networkx [15] package for analysing networks.
- Gephi [4] open source package for visualising networks.
- The louvain library for calculating the networks modularity `https://github.com/taynaud/python-louvain/issues`.

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