1 Reactive defecting Nash strategies in the donation game

In the previous section, we have characterized the reactive partner strategies for a special case of the donation game and the general prisoner's dilemma. In the following, we apply the same methods to characterize defecting Nash equilibria. For the case of reactive-1 strategies, we obtain the following characterization.

Theorem 1 (Reactive-1 defecting Nash strategies in the donation game)
A reactive-1 strategy $\mathbf{p}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$p_C \leq \tfrac{c}{b} \qquad \textit{and} \qquad p_D = 0. \tag{1}$$

Theorem 2 (Reactive-2 defecting Nash strategies in the donation game)
A reactive-2 strategy $\mathbf{p}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$p_{CC} \leq \frac{c}{b}, \qquad \frac{p_{CD}+p_{DC}}{2} \leq \frac{c}{2b}, \qquad p_{DD} = 0. \tag{2}$$

Theorem 3 (Reactive-3 defecting Nash strategies in the donation game)
A reactive-3 strategy $\mathbf{p}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$\begin{aligned} p_{CCC} &\leq \frac{c}{b} \\ \frac{p_{CDC}+p_{DCD}}{2} &\leq \frac{1}{2}\cdot\frac{c}{b} \\ \frac{p_{CCD}+p_{CDC}+p_{DCC}}{3} &\leq \frac{2}{3}\cdot\frac{c}{b} \\ \frac{p_{CDD}+p_{DCD}+p_{DDC}}{3} &\leq \frac{1}{3}\cdot\frac{c}{b} \\ \frac{p_{CCD}+p_{CDD}+p_{DCC}+p_{DDC}}{4} &\leq \frac{1}{2}\cdot\frac{c}{b} \\ p_{DDD} &= 0. \end{aligned} \tag{3}$$

We repeat the same analysis for reactive counting strategies. We obtain the following results.

Theorem 4 (Reactive-2 defecting Nash counting strategies in the donation game)
A reactive-2 counting strategy $\mathbf{r}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$r_2 \leq \frac{c}{b}, \qquad r_1 \leq \frac{1}{2}\cdot\frac{c}{b}, \qquad r_0 = 0. \tag{4}$$

Theorem 5 (Reactive-3 defecting Nash counting strategies in the donation game)
A reactive-3 counting strategy $\mathbf{r}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$r_3 \leq \frac{c}{b}, \qquad r_2 \leq \frac{2}{3}\cdot\frac{c}{b}, \qquad r_1 \leq \frac{1}{3}\cdot\frac{c}{b}, \qquad r_0 = 0. \tag{5}$$

ToDo: proofs.

We can observe that for each value of n, the left-hand side of the conditions for cooperative and defective Nash are the same. Moreover, we see that the right-hand side of the defective Nash conditions is always strictly smaller than those of the cooperative Nash conditions. This means that within the space of feasible strategies, the volume of partner strategies is always larger than the volume of defective Nash strategies. We also verify this analytical results numerically, Figure **??**.

2 Evolutionary Simulations

We perform the evolutionary analysis of Figure 3 of the main text. We simulate the evolutionary process twenty times this time, Figure **??**. We can see that the results remain unchanged.

2.1 Invasion Analysis

One question that arises is that from these top strategies which are partner strategies, and why are some partner strategies selected more than others. For example in the case of reactive 1, we take a reactive strategies to be ($p_C > 0.95$) and ($p_D < \frac{c}{b}$), hwer we osbset strategies of ($p_D = 0.1$) and closer to the boundy. The reason for this is exaimpled by Figure **??**.

The self payoffs of the two strategies would have been the same if p was equal to 1. However, since the evolutionary process almost never samlples such a strategy a stochastic pcc makes the difference. The likelyhood of both strategies to make a mistake while at the outome is the same. Howver, the distributions of the playing with themselves is different. The numerical evalution of the stateionary distribution is shown in Figure A. We also use the open source package Axelrod to simulate the play betwwen the strategies. We run each match for 10^6 steps and we repeate the match twenty times. From Figure B we see that the simulation estimtes the stattionary distribution with the numerical approach.

From the simulation we can record how long it takes for each pair to return to mutual cooperation after a mistake has occure. This is shown in Figure C. We can see that the pair p has a smaller cycle which means it can correct the mistake faster.

We can look into some of the cyles. The most common cycles for each are those of the nature, cc—cc and then. SHow in Figure 3, A and B.

Now we ask the question, what if the lower probably of this path was to be a defection instead of a cooperation. What we see is that for p rime it takes one extra timestep tp correct the mistake. This comes from the effect that to go to mutual cooperation a defection in the second to last round will be at the memory of the strategy, and a more forgiving strategy achieves mutual cooperation again faster.

3 Errors

So far, we have considered the case where there cannot be a mistake in the actions taken by a player; the actions of the players are realized without error. Here, we discuss what happens in the case where such an error is possible. More specifically, we consider that ϵ is the probability that a player makes a mistake in the action taken.

4 Memory-n

So far in the evolutionary simulations we have considered reactive strategies, and we have shown that strategies with larger memory allow for more cooperative populations. We repeate the evolutionary simulations but this time using memory n strategies. We get results for memory-1, memory-2 and for memory counting strategies for n equal to 1, 2 and 3. We omit memory-3 strategies because it is numerically untractable to run such simulations. The results still hold. For more memory more cooperation evovles, howber, not in the case of counting strategies.

To understand why counting strategies do not allow for more cooperation we focus on reactive-2 strategies and reactive-2 counting strategies. We run an invasion analysis where we select the most abundant reactive-2 strategy and hte most abundant reacto-2 coutning strategy. The strategies and the mean invasion time are given by Figure 3. We also ask the question, how many reactive-2 mutants can a reactive-2 coutnign strategy repel.

5 Proofs

The proof for Theorem 1.

The proof for Theorem 2.

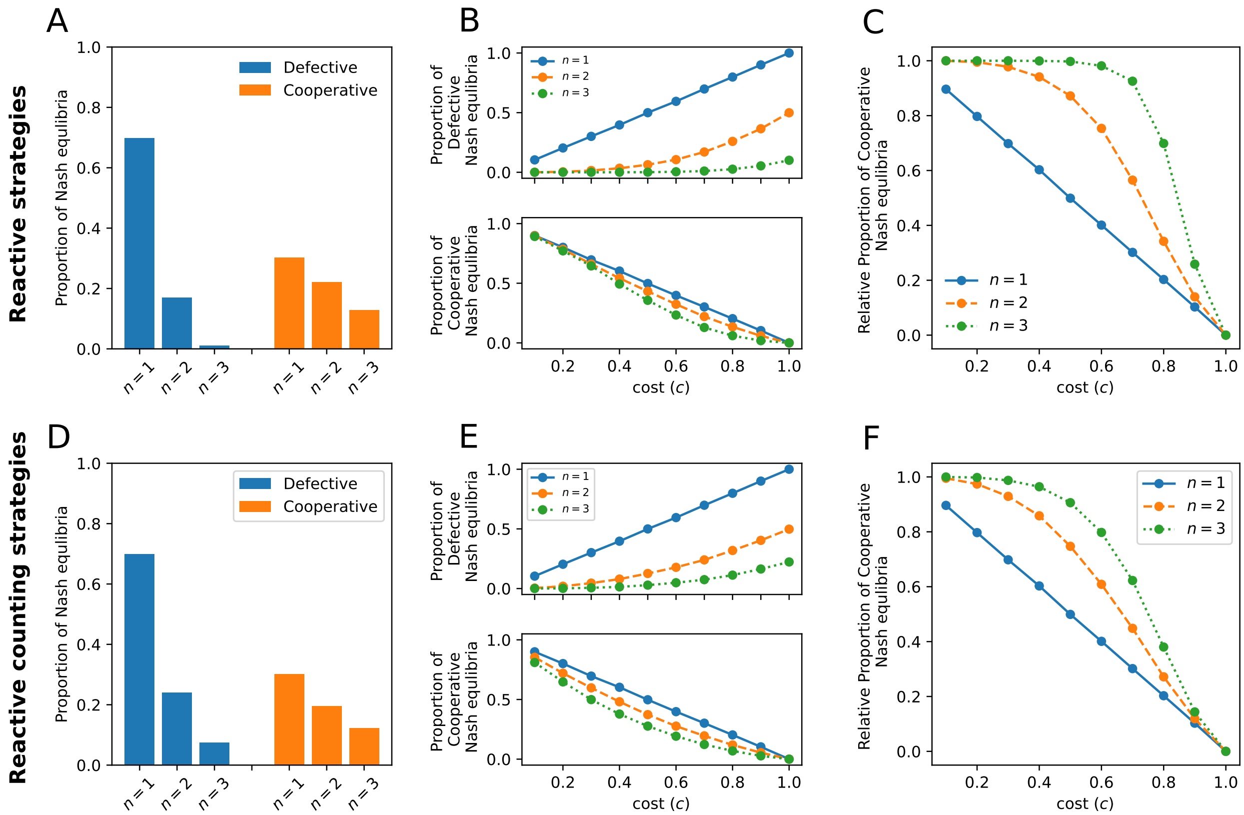

Figure S1: Volume of cooperative and defective Nash. We draw 10^4 random strategies from the feasible space of strategies and create two copies of each strategy. For one copy, we set the probability of cooperating after full cooperations of the co-player to 1. For the second copy, we set the probability of cooperating after full defections of the co-player to 0. We then checked if either copy is Nash: cooperative for the first copy and defective for the second. We set the benefit of cooperation to $b = 1$. **A, D** We plot the results for a given value of cost, $c = 0.5$. We do this for reactive strategies and reactive counting strategies. For $n = 1$, we can see the number of defective equilibria is higher than that of the cooperative. Howver, as we increase n this is not true anymore. The number of defective Nash decreases drastically. The number of partner strategies does as well, howber, we can see that it happends less quickly. **B** We plot the porpotion of equalibria over different values of cost, for reactive (B) and reactive counting strategies. As the cost increases so does the porpotion of defective equilibria, and the opposite is true for cooperative. As memory increases we obsvere again a significant drop in the porpotionf of defective, whereas a small descreses in the cooperative. The number of defective Nash strategies as a function of cost. **C, F** We plot the relative portoprtion of cooperation nash. For this we consider the number of cooperative and defecting Nash for each memory size, and we plot the fraction of cooperative. This increases as memory increases.

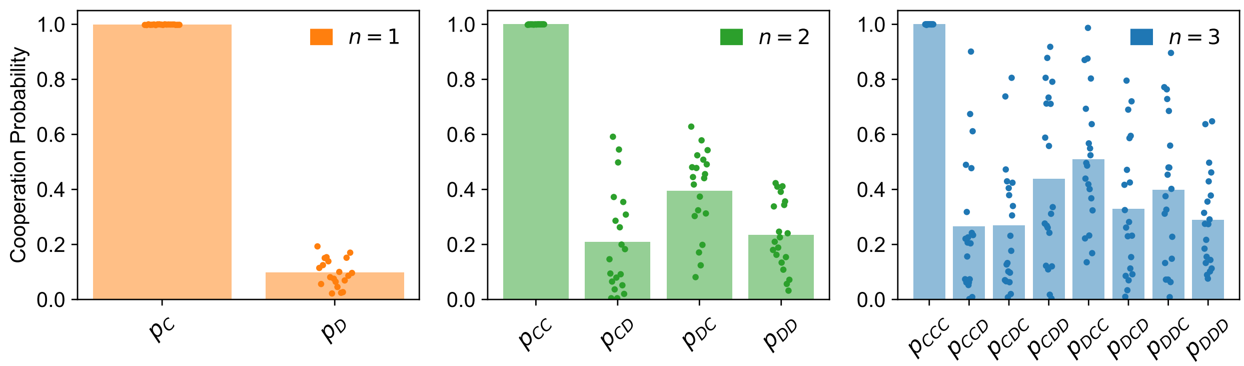

Figure S2: Evolutionary dynamics of reactive-n strategies. To explore the evolutionary dynamics among reactive-n strategies, we run simulations based on the method of Imhof and Nowak. This method assumes rare mutations. Every time a mutant strategy appears, it goes extinct or fixes before the arrival of the next mutant strategy. We run twenty independent simulations for reactive-n strategies. For each simulation, we record the most abundant strategy (the strategy that resisted most mutants). The respective average cooperation probabilities are in line with the conditions for partner strategies. Simulations are based on a donation game with $b = 1$, $c = 0.5$, a selection strength $\beta = 1$ and a population size $N = 100$, unless noted otherwise. For n equal to 1 and 2, simulations are run for $T = 10^7$ time steps. For $n = 3$ we use $T = 2 \cdot 10^7$ time steps.

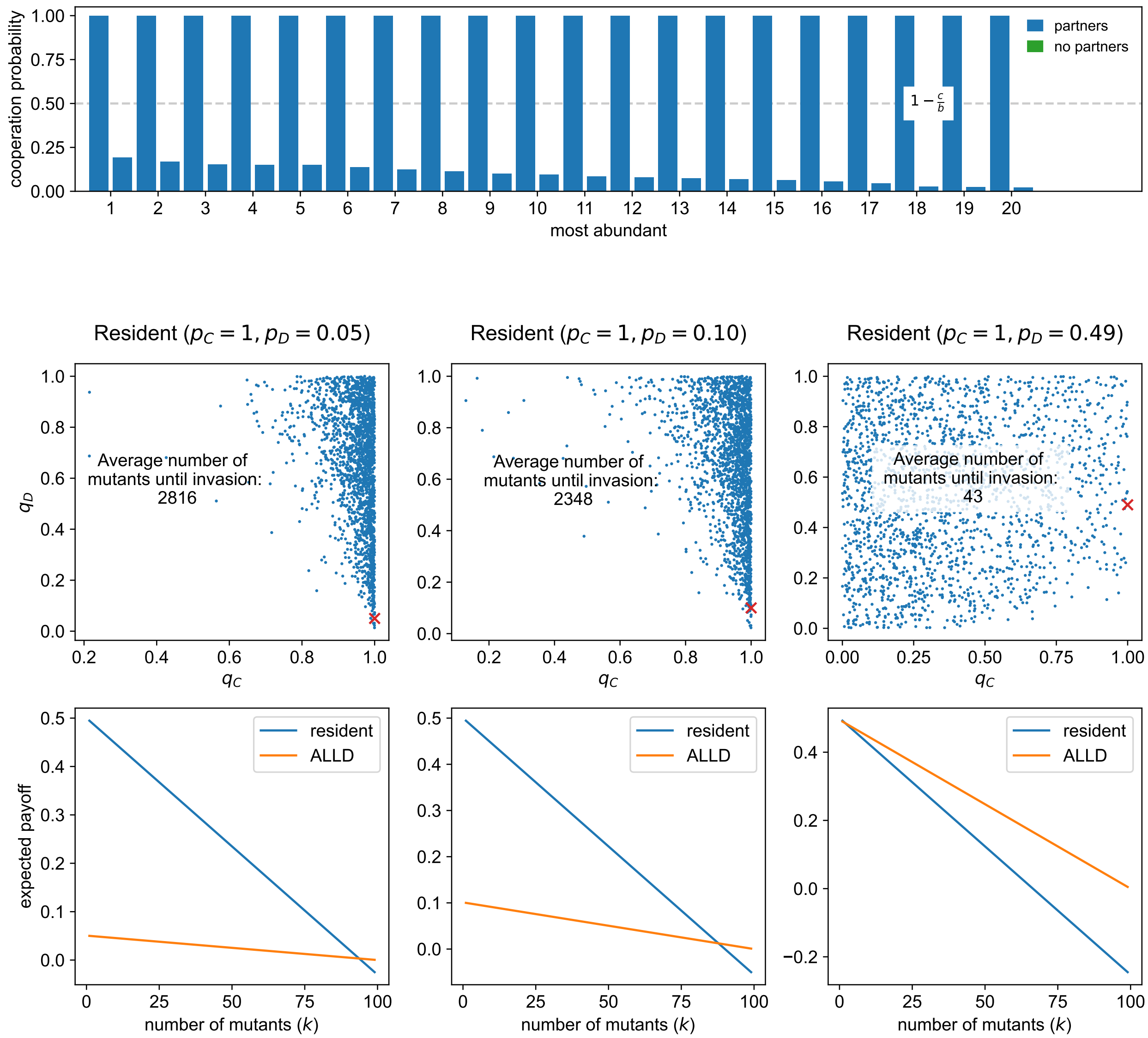

Figure S3: Invasion analysis for reactive-1. **A** The twenty most abundant reactive-1 strategies of Figure. We obsvere that all twenty of them are partner strategies, and than all have a very low p ¿ 0.2. **B** We perform an invision analysis. Namely we select one resident and we introduce mutatns until the mutant takes over and the resident is no more. We count how many time from 10 ** 3 a mutatn succeffully invaded the resident. We repeated this for three different resisndetd, from left to right. We Obseverd that a resident with a very low$p_d ismoreresistanttoinvasion, andthuswhyweobservethatthemostabundantstrategieshavealowpd.$**C**$Wecanexaiplethelowpdifwecon$ ALLD mutatns try to invavde. We can see that for the far left resident the expectedpayoff is almost always strivly higher than that of the ALLD mutatns, only whe 99 mutatn the payoff is the same. We repeated this for all three.

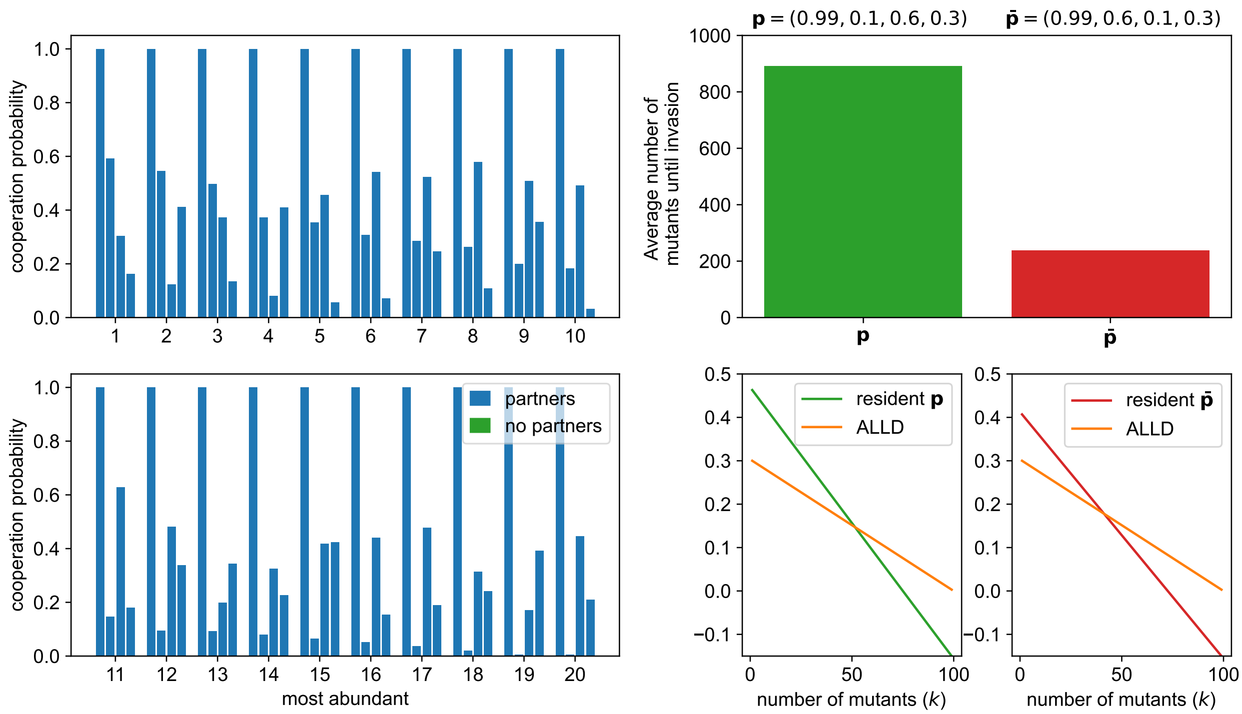

Figure S4: Invasion analysis for reactive-2. We repeate the same analysis for the reactive-2 strategies. The twenty top performing strategies are again partner strategies. Howver, what we observe here is that the partner strategies that are selected from the evolutionary process almost laways have a smaller pcd than pdc and a low pdd. We already discussed why a smaller pdd is selected. Howver, to understand the differencesbetween we will consider two examples of such strategies. Namely let $\mathbf{p} =$ and $\tilde{\mathbf{p}}$. We run an invasion analysis to see the difference and inddeed p is invaded much faster. The expected payoffs show that the slef payoffs od the strategies are different, and thus an alld recived a higher payoff with less mutants.

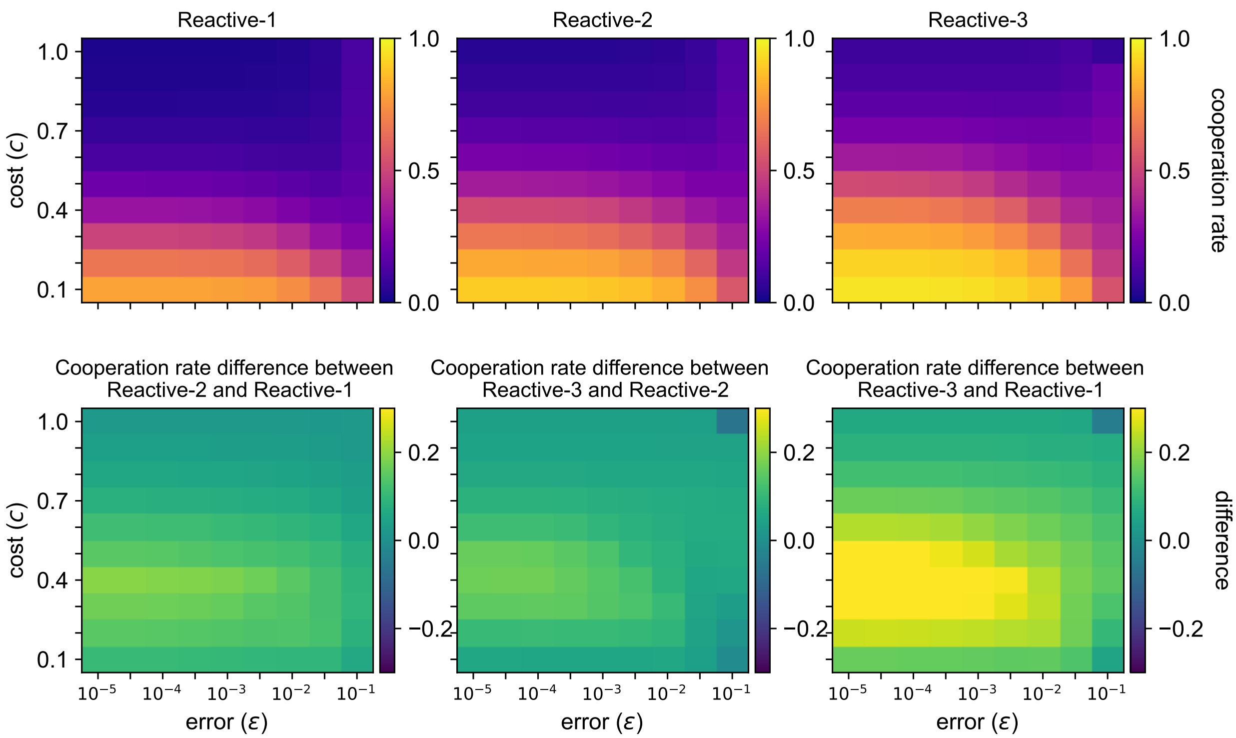

Figure S5: Cooperating rates with implementation errors. We simulate the evolutionary process, this time allowing for implementation errors. Specifically, we consider a probability ϵ that a player makes a mistake in the action taken. We calculate the average cooperation rate for different values of ϵ and c. **A** We plot the average cooperation rate for the different parameters when individuals use reactive-1, reactive-2, and reactive-3 strategies, respectively. **B** We plot the differences between the cooperation rates when individuals use different memory size strategies. From left to right, we show the differences between reactive-1 and reactive-2, reactive-2 and reactive-3, and reactive-1 and reactive-3 strategies.

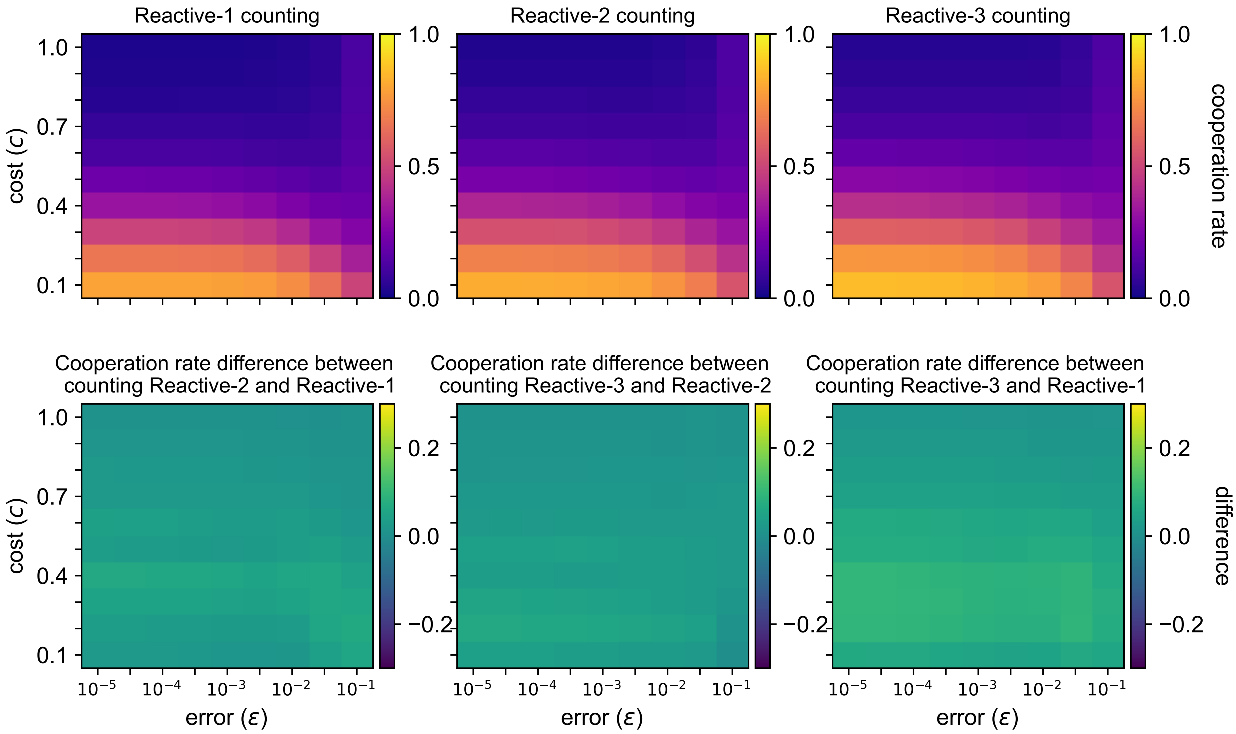

Figure S6: Cooperating rates with implementation errors.

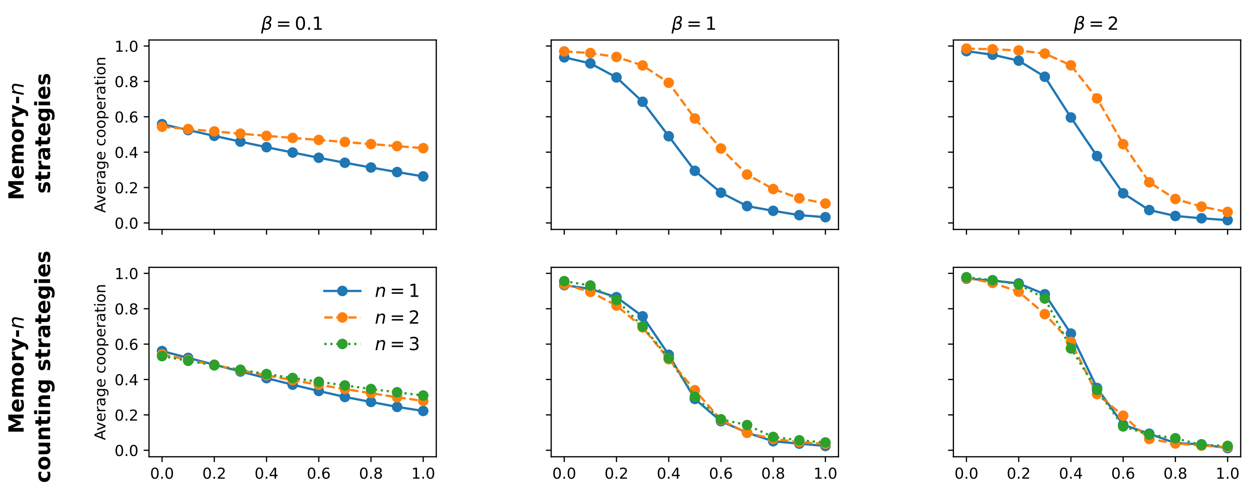

Figure S7: Memory-n simulations.