Two bits (and more) reactive strategies in repeated games

Nikoleta E. Glynatsi, Christian Hilbe, Martin Nowak

We explore infinitely repeated (2×2) symmetric games. Players use the set reactive strategies to make decision at each turn. We consider the repeated game where at each turn players, simultaneously and independently, decide to cooperate or to defect. In the case of two players, the payoffs are the following,

$$\begin{array}{c} \\ \text{cooperate} \\ \text{defect} \end{array} \begin{array}{c} \begin{array}{cc} \text{cooperate} & \text{defect} \end{array} \\ \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix} \end{array} \tag{1}$$

Strategies of the reactive set only take into account the actions of the co-player. The most well studied reactive strategies are those that take into account only the last turn of the opponent. Here will reefer to these as *one-bit reactive strategies*. A one-bit reactive strategy is written as $p = (p_C, p_D)$ where p_C is the probability of cooperating after the co-player has cooperated and p_D after they defected.

The play of reactive strategies can be modelled as a Markov chain. In the case of the one-bit reactive strategies, there are only 4 possibles states CC, CD, DC, DD and the transition matrix is given by,

$$\begin{bmatrix} p_1 q_1 & p_1 (1-q_1) & q_1 (1-p_1) & (1-p_1)(1-q_1) \\ p_2 q_1 & p_2 (1-q_1) & q_1 (1-p_2) & (1-p_2)(1-q_1) \\ p_1 q_2 & p_1 (1-q_2) & q_2 (1-p_1) & (1-p_1)(1-q_2) \\ p_2 q_2 & p_2 (1-q_2) & q_2 (1-p_2) & (1-p_2)(1-q_2) \end{bmatrix} \tag{2}$$

Here we exlore the case. Here we explore the cases where.

The players can be in

The transition matrix given by:

Computing the stationary distribution of this matrix (analytically) is not trivia.

0.1 Analytical analysis

$$y = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} \tag{3}$$

$$[p_1^2 q_1^2 \tag{4}$$
$$p_1^2 q_1 (1 - q_1) \tag{5}$$
$$p_1 q_1^2 (1 - p_1) \tag{6}$$
$$p_1 q_1 (p_1 - 1) (q_1 - 1) \tag{7}$$
$$p_1^2 q_1 (1 - q_1) \tag{8}$$
$$p_1^2 (q_1 - 1)^2 \tag{9}$$
$$p_1 q_1 (p_1 - 1) (q_1 - 1) \tag{10}$$
$$- p_1 (p_1 - 1) (q_1 - 1)^2 \tag{11}$$
$$p_1 q_1^2 (1 - p_1) \tag{12}$$
$$p_1 q_1 (p_1 - 1) (q_1 - 1) \tag{13}$$
$$q_1^2 (p_1 - 1)^2 \tag{14}$$
$$- q_1 (p_1 - 1)^2 (q_1 - 1) \tag{15}$$
$$p_1 q_1 (p_1 - 1) (q_1 - 1) \tag{16}$$
$$- p_1 (p_1 - 1) (q_1 - 1)^2 \tag{17}$$
$$- q_1 (p_1 - 1)^2 (q_1 - 1) \tag{18}$$
$$(p_1 - 1)^2 (q_1 - 1)^2] \tag{19}$$

0.2 Numerical analysis

For proof that our formulation is correct to the jupyter notebppk ‘Numerical simulanetlous’.

An evolutionary approach based on Nowak and Imphof gives the following results when we vary the benefit c.