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Dynamics of Social Behavior
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Dr. May R. Berenbaum
Editor-in-Chief
PNAS

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Dear Prof. Dr. Berenbaum,

Please find enclosed the manuscript "**Conditional cooperation with longer memory**" by Nikoleta E. Glynatsi, Ethan Akin, Martin A. Nowak and Christian Hilbe, which we would like to submit to *PNAS*.

Direct reciprocity is a widespread mechanism for the evolution of cooperation. This mechanism explains why people return favors, show more effort in group tasks when others do, or cease cooperating when they feel exploited. Traditionally, most theoretical research on direct reciprocity focuses on strategies that only consider the outcome of the previous round. A systematic formal analysis of strategies with more than one-round memory is difficult and has never been attempted. Here we break new ground by providing the first exhaustive exploration of a strategy space with longer memory.

We focus on the reactive-n strategies. These strategies only depend on the co-player's last n actions, capturing the basic premise of conditional cooperation. While characterizing all Nash equilibria among the memory-n strategies has been challenging, we show that such a characterization is feasible – and surprisingly elegant – for reactive-n strategies.

Specifically, we derive an algorithm to verify whether a given reactive-n strategy is a *partner strategy*. Partner strategies are of particular interest because they can sustain full cooperation as a Nash equilibrium. We use this algorithm to fully characterize reactive-n partner strategies for n=2 and n=3, as well as for reactive-n counting strategies. A reactive-n counting strategy records how often the co-player has cooperated during the last n rounds. Our results build upon a technical result: to test whether a reactive-n strategy forms a Nash equilibrium, one only needs to consider deviations towards deterministic self-reactive-n strategies (strategies that only consider their own last n moves).

Furthermore, we perform extensive evolutionary simulations to study the impact of memory length on cooperation. These simulations reveal that longer memory increases the average cooperation rate because natural selection chooses partners. Interestingly, while this result holds for reactive-*n* strategies, it fails for reactive-*n* counting strategies. These evolutionary results highlight that paying attention to the exact sequence of moves is necessary for reaping the advantages of longer memory.

Overall, our results provide important insights into the logic of conditional cooperation when players have more than one-round memory. We demonstrate that partner strategies exist for all repeated prisoner's dilemmas and for all memory lengths. We believe that these results align well with the scope and objectives of the *Proceedings of the National Academy of Sciences*. In particular, they will certainly contribute to the ongoing scientific discourse on cooperative behaviors. Therefore, we would be delighted if you could consider this article for publication in your journal.

As handling editors, Kenneth Wachter, Marcus Feldman, and William Press would be excellent candidates. As potential referees, we would like to suggest the following:

- **Alexander J. Stewart** (ajs50@st-andrews.ac.uk, University of St Andrews) has characterized all Nash equilibria among the memory-1 strategies.
- **Feng Fu** (Feng.Fu@dartmouth.edu, Dartmouth University) has written many papers on repeated games and on evolutionary dynamics more generally.
- **Xingru Chen** (xingrucz@gmail.com, Beijing University of Posts and Telecommunications) is a mathematician with a strong interest in direct reciprocity.
- **Max Kleiman-Weiner** (maxkw@mit.edu, University of Washington) studies conditional cooperation from the viewpoint of computational cognitive science.
- **Christoph Hauert** (christoph.hauert@math.ubc.ca, University of British Columbia) is a leading expert on cooperation and evolutionary game theory.

Thank you for considering our submission.

With kind regards,
On behalf of the authors,

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