1 Reactive defecting Nash strategies in the donation game

In the previous section, we have characterized the reactive partner strategies for a special case of the donation game and the general prisoner's dilemma. In the following, we apply the same methods based on Section **??** to analyze defecting Nash equilibria. For the case of reactive-1 strategies, we obtain the following characterization.

Theorem 1 (Reactive-1 defecting Nash strategies in the donation game)
A reactive-1 strategy $\mathbf{p}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$p_C \le \tfrac{c}{b} \qquad \textit{and} \qquad p_D = 0. \tag{1}$$

Theorem 2 (Reactive-2 defecting Nash strategies in the donation game)
A reactive-2 strategy $\mathbf{p}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$p_{CC} \le \frac{c}{b}, \qquad \frac{p_{CD}+p_{DC}}{2} \le \frac{c}{2b}, \qquad p_{DD} = 0. \tag{2}$$

Theorem 3 (Reactive-3 defecting Nash strategies in the donation game)
A reactive-3 strategy $\mathbf{p}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$\begin{aligned}
p_{CCC} &\le \frac{c}{b}\\
\frac{p_{CDC}+p_{DCD}}{2} &\le \frac{1}{2}\cdot\frac{c}{b}\\
\frac{p_{CCD}+p_{CDC}+p_{DCC}}{3} &\le \frac{2}{3}\cdot\frac{c}{b}\\
\frac{p_{CDD}+p_{DCD}+p_{DDC}}{3} &\le \frac{1}{3}\cdot\frac{c}{b}\\
\frac{p_{CCD}+p_{CDD}+p_{DCC}+p_{DDC}}{4} &\le \frac{1}{2}\cdot\frac{c}{b}\\
p_{DDD} &= 0.
\end{aligned} \tag{3}$$

We can observe that for each value of n, the left-hand side of the conditions for cooperative and defective Nash are the same. Moreover, it is clear that the right-hand side of the defective Nash conditions is always strictly smaller than those of the cooperative Nash conditions. This means that within the space of feasible strategies, the volume of partner strategies is always larger than the volume of defective Nash strategies. We verified these analytical results numerically as well.

We selected random strategies from the feasible space of strategies and created two copies of each strategy. For one copy, we set the probability of cooperating after full cooperation of the co-player to 1 (for example, for reactive-1, $p_C = 1$). For the second copy, we set the probability of cooperating after full defections of the co-player to 0 (for example, for reactive-2, $p_{DD} = 0$). We then checked if either copy is Nash: cooperative for the first and defective for the second. We repeated this process for 10^4 randomly selected

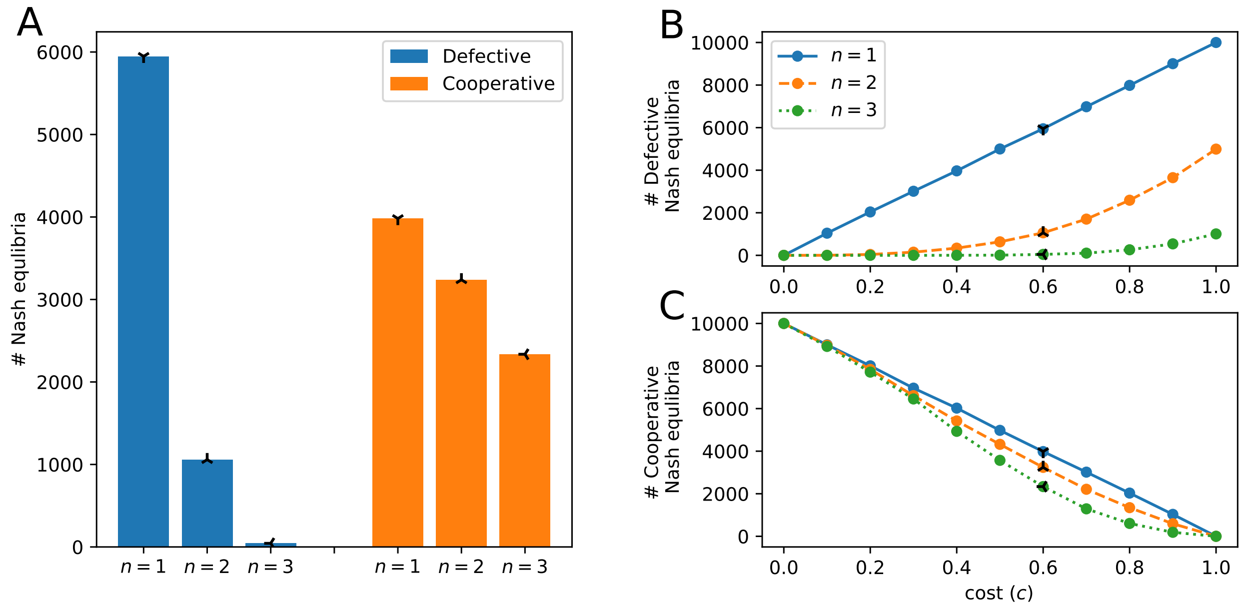

Figure S1: Volume of cooperative and defective Nash. We draw 10^4 random strategies from the feasible space of strategies and create two copies of each strategy. For one copy, we set the probability of cooperating after full cooperation of the co-player to 1. For the second copy, we set the probability of cooperating after full defections of the co-player to 0. We then checked if either copy is Nash: cooperative for the first and defective for the second. We set the benefit of cooperation to $b = 1$. **A** We plot the results for a given value of cost, $c = 0.5$. **B** The number of defective Nash strategies as a function of cost. **C** The number of cooperative Nash strategies as a function of cost.

strategies and plotted the relative volumes of cooperative and defective Nash equilibria (Figure S1). We also verified that this holds true for different values of cost.

Nash strategies are best response to themselves. This makes them robust against invasion. In the stochastic evolutionary processes considered in this work, this doesn't mean they cannot be invaded, but rather it makes it harder to do so. The volumes of cooperative Nash and defective Nash strategies can determine the evolutionary dynamics of cooperation. That is, they indicate the likelihood that a randomly selected strategy will produce sustained cooperation or defection. Therefore, since the volume of cooperative Nash increases with memory, and defective Nash decreases, we expect that as memory increases, the cooperation rate of the evolved population will also increase. This expectation is verified by the results in **Figure 4** of the paper.

2 Evolutionary Simulations

In this section, we will analyze the results of the evolutionary simulations to better understand the dynamics of the different strategies. As a reminder, in Fig. 3 of the manuscript panel, we simulated the evolutionary process twenty times, independent simulations for reactive-n strategies, and for reactive-n counting strategies. In panel A, we show for each of the three different memory values the strategies that were the most abundant from each simulation.

Now, one question that arises is how many of these strategies are actually partner strategies? And for the partner strategies, do we see all of them represented?

Reactive-1 strategies. In the case of reactive-1, all the most abundant strategies are partner strategies. However, from Eq. and for a given value of c, the probability of defecting is... However, we observe only a lower value of p_{dd}, thus only some partner strategies are residents. This result is not explained by the theory. To better understand this result, we need to do an invasion analysis. For the invasion analysis, we consider a given resident strategy, and we sample 10^3 random strategies to estimate if this mutant will invade the resident. We record the number of mutants that took over. We repeat this 10^3 times. The results are shown in Fig. X, and what we observe is that a lower value of p_{dd} results in a more robust strategy to invasion, as only cooperative strategies on the GTFT can take over, in comparison with panel B where defective strategies are more likely to invade.

Reactive-2 strategies. In the case of reactive-2, we observe that almost all strategies are partner strategies. However, what seems to be the case is that p_{dc} is higher on average than p_{cd}. In order to understand this, we again run an invasion analysis. This time we also consider two values of p_{dd}. The results are shown in Fig. Y.

3 Errors

So far, we have considered the case where there cannot be a mistake in the actions taken by a player; the actions of the players are realized without error. Here, we discuss what happens in the case where such an error is possible. More specifically, we consider that ϵ is the probability that a player makes a mistake in the action taken.