1 Reactive defecting Nash strategies in the donation game

In the previous section, we have characterized the reactive partner strategies for a special case of the donation game and the general prisoner's dilemma. In the following, we apply the same methods based on Section **??** to analyze defecting Nash equilibria. For the case of reactive-1 strategies, we obtain the following characterization.

Theorem 1 (Reactive-1 defecting Nash strategies in the donation game)
A reactive-1 strategy $\mathbf{p}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$p_C \le \tfrac{c}{b} \qquad \textit{and} \qquad p_D = 0. \tag{1}$$

Theorem 2 (Reactive-2 defecting Nash strategies in the donation game)
A reactive-2 strategy $\mathbf{p}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$p_{CC} \le \frac{c}{b}, \qquad \frac{p_{CD}+p_{DC}}{2} \le \frac{c}{2b}, \qquad p_{DD} = 0. \tag{2}$$

Theorem 3 (Reactive-3 defecting Nash strategies in the donation game)
A reactive-3 strategy $\mathbf{p}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$\begin{aligned} p_{CCC} &\le \frac{c}{b} \\ \frac{p_{CDC}+p_{DCD}}{2} &\le \frac{1}{2}\cdot\frac{c}{b} \\ \frac{p_{CCD}+p_{CDC}+p_{DCC}}{3} &\le \frac{2}{3}\cdot\frac{c}{b} \\ \frac{p_{CDD}+p_{DCD}+p_{DDC}}{3} &\le \frac{1}{3}\cdot\frac{c}{b} \\ \frac{p_{CCD}+p_{CDD}+p_{DCC}+p_{DDC}}{4} &\le \frac{1}{2}\cdot\frac{c}{b} \\ p_{DDD} &= 0. \end{aligned} \tag{3}$$

We repeat the same analysis for reactive counting strategies. We obtain the following results.

Theorem 4 (Reactive-2 defecting Nash counting strategies in the donation game)
A reactive-2 counting strategy $\mathbf{r}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$r_2 \le \frac{c}{b}, \qquad r_1 \le \frac{1}{2}\cdot\frac{c}{b}, \qquad r_0 = 0. \tag{4}$$

Theorem 5 (Reactive-3 defecting Nash counting strategies in the donation game)
A reactive-3 counting strategy $\mathbf{r}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$r_3 \le \frac{c}{b}, \qquad r_2 \le \frac{2}{3}\cdot\frac{c}{b}, \qquad r_1 \le \frac{1}{3}\cdot\frac{c}{b}, \qquad r_0 = 0. \tag{5}$$

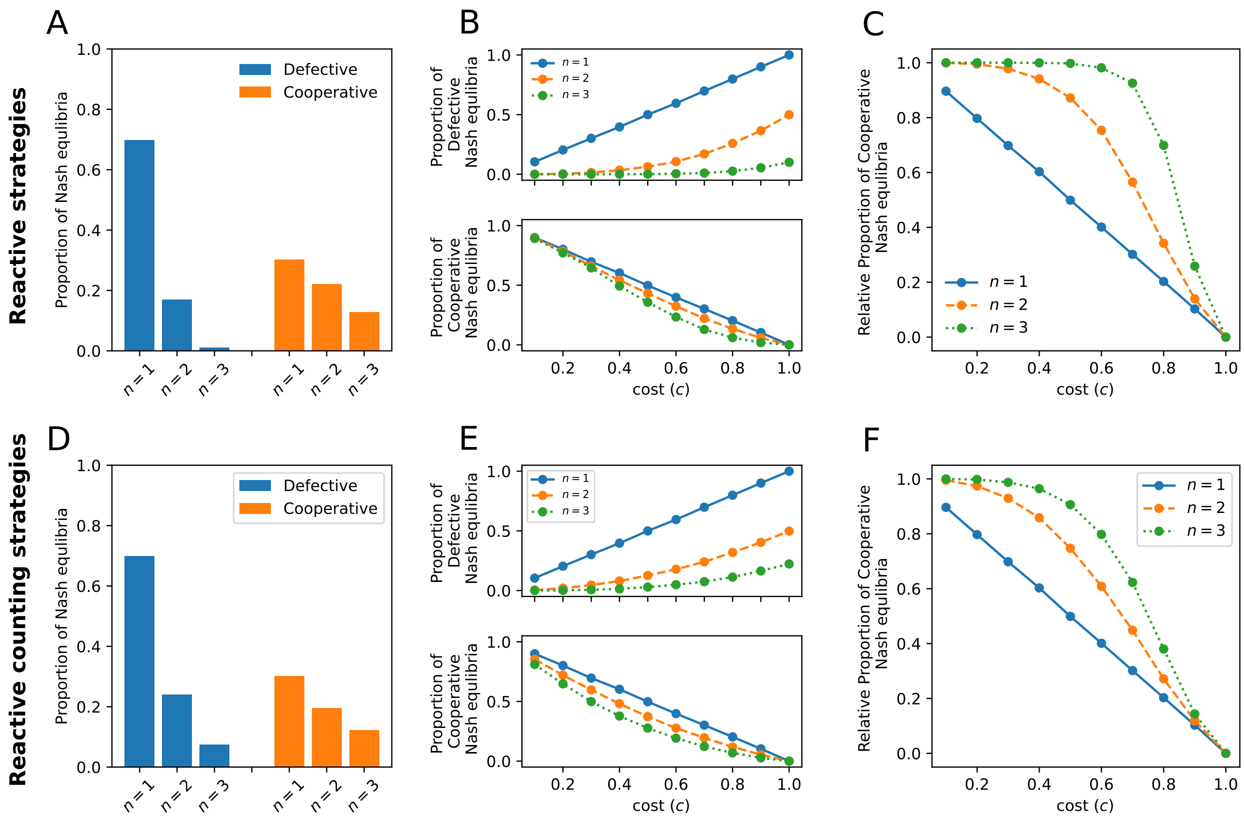

Figure S1: Volume of cooperative and defective Nash. We draw 10^4 random strategies from the feasible space of strategies and create two copies of each strategy. For one copy, we set the probability of cooperating after full cooperation of the co-player to 1. For the second copy, we set the probability of cooperating after full defections of the co-player to 0. We then checked if either copy is Nash: cooperative for the first and defective for the second. We set the benefit of cooperation to $b = 1$. **A** We plot the results for a given value of cost, $c = 0.5$. **B** The number of defective Nash strategies as a function of cost. **C** The number of cooperative Nash strategies as a function of cost.

We can observe that for each value of n, the left-hand side of the conditions for cooperative and defective Nash are the same. Moreover, it is clear that the right-hand side of the defective Nash conditions is always strictly smaller than those of the cooperative Nash conditions. This means that within the space of feasible strategies, the volume of partner strategies is always larger than the volume of defective Nash strategies. We verified these analytical results numerically as well.

We selected random strategies from the feasible space of strategies and created two copies of each strategy. For one copy, we set the probability of cooperating after full cooperation of the co-player to 1 (for example, for reactive-1, $p_C = 1$). For the second copy, we set the probability of cooperating after full defections of the co-player to 0 (for example, for reactive-2, $p_{DD} = 0$). We then checked if either copy is Nash: cooperative for the first and defective for the second. We repeated this process for 10^4 randomly selected strategies and plotted the relative volumes of cooperative and defective Nash equilibria (Figure S1). We also verified that this holds true for different values of cost.

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2 Evolutionary Simulations

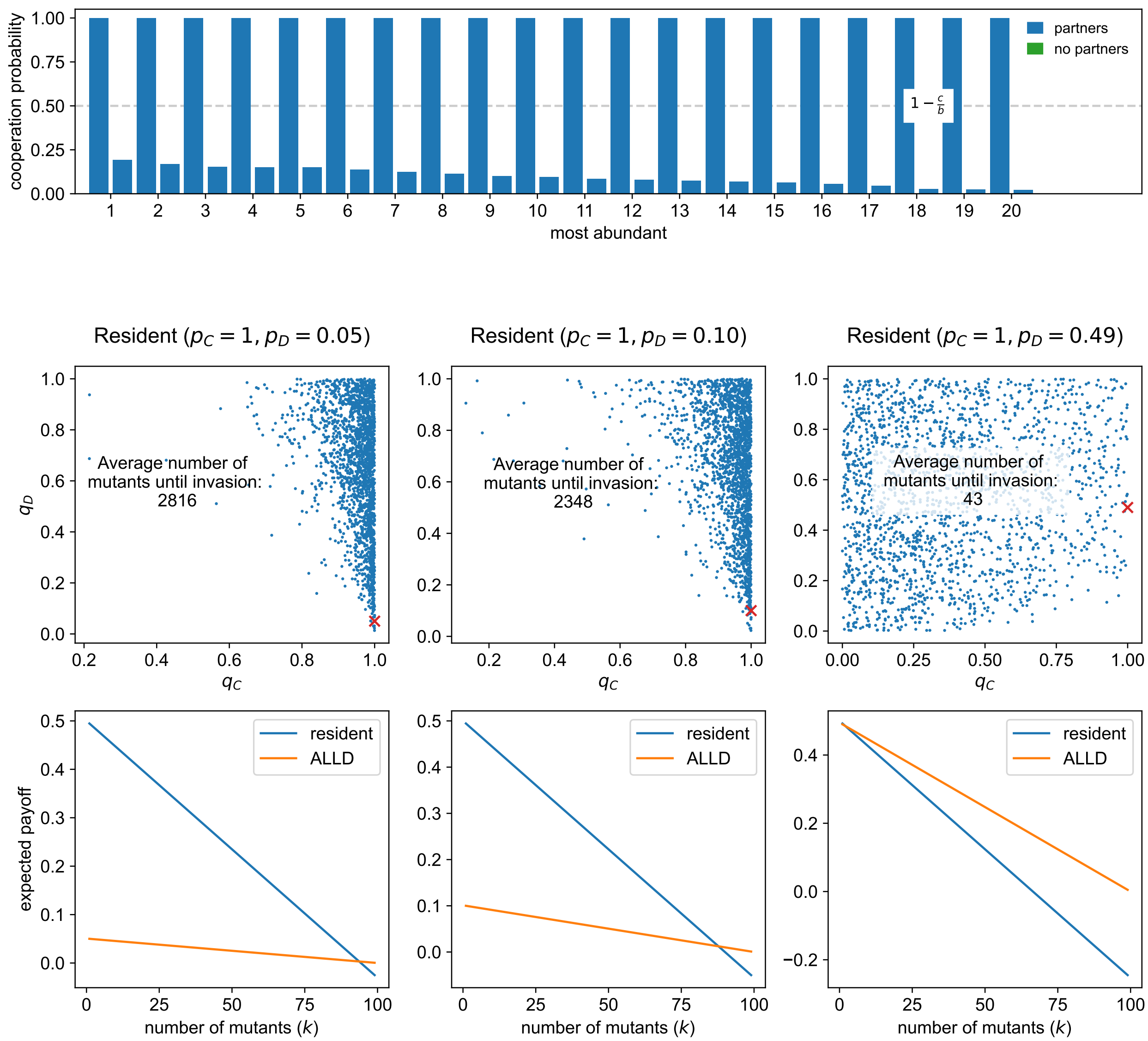

Figure S2: Invasion analysis for reactive-1.

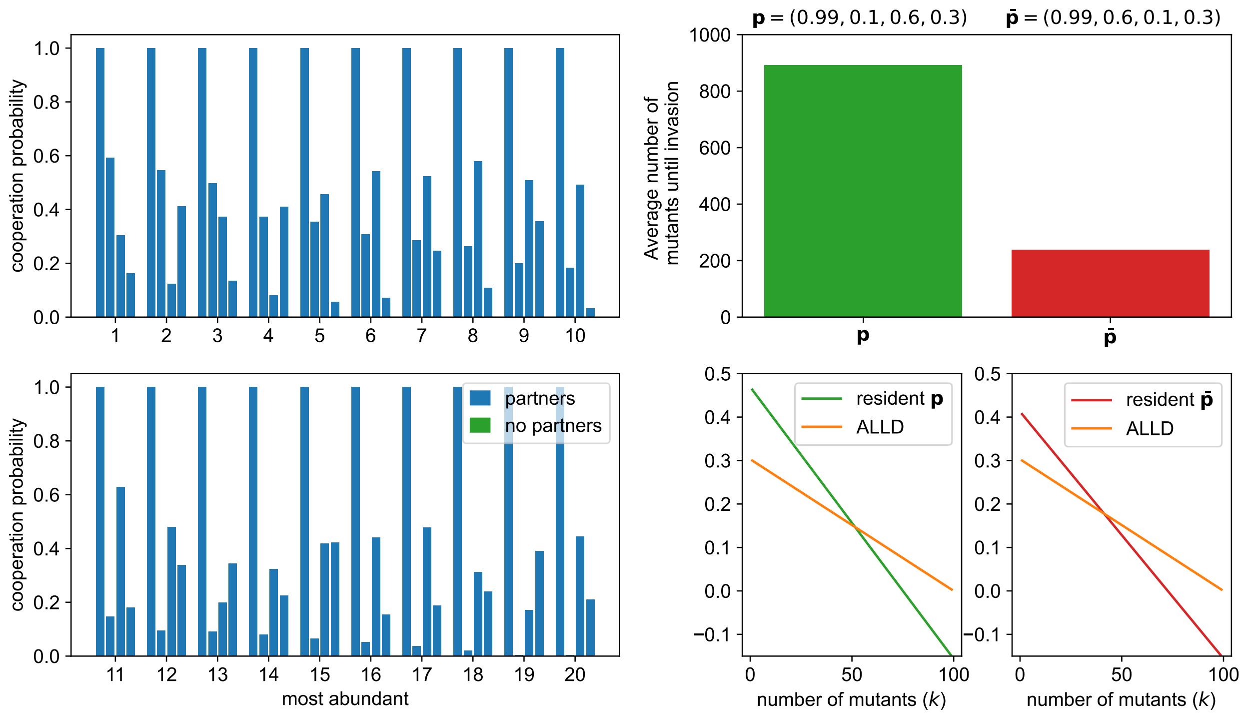

Figure S3: Invasion analysis for reactive-2.

3 Errors

So far, we have considered the case where there cannot be a mistake in the actions taken by a player; the actions of the players are realized without error. Here, we discuss what happens in the case where such an error is possible. More specifically, we consider that ϵ is the probability that a player makes a mistake in the action taken.

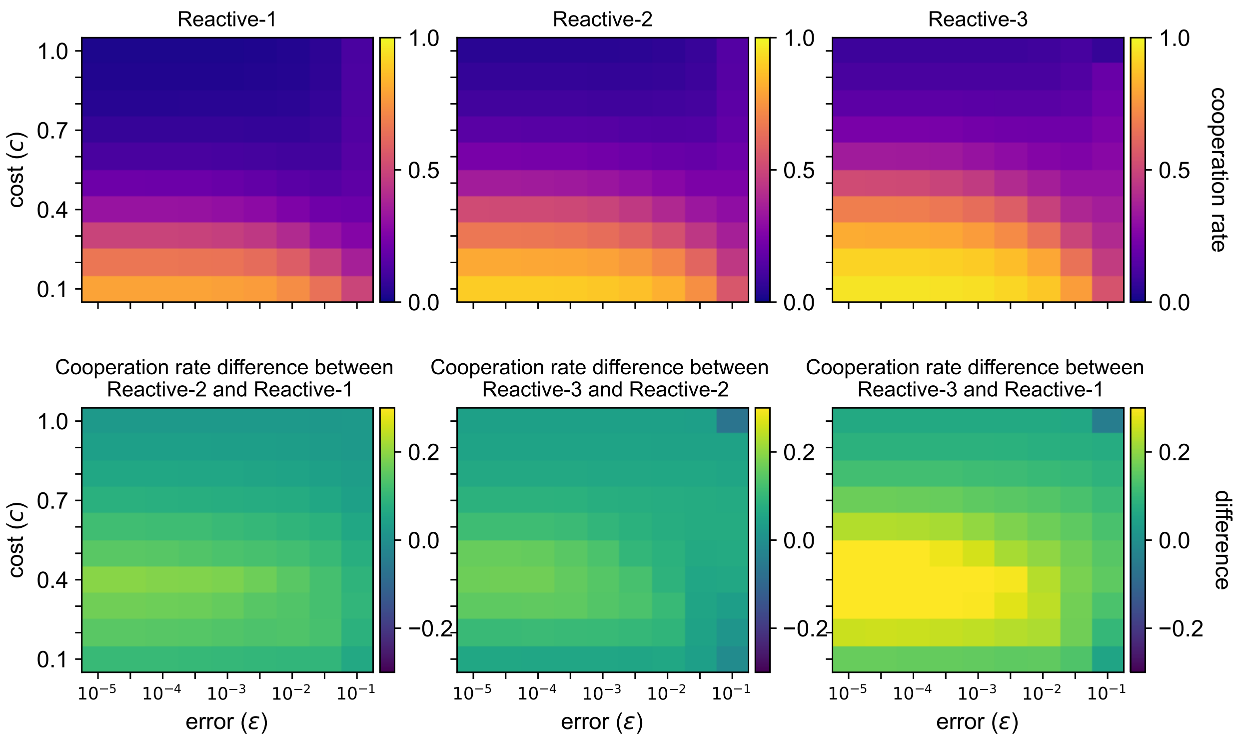

Figure S4: Cooperating rates with implementation errors. We simulate the evolutionary process, this time allowing for implementation errors. Specifically, we consider a probability ϵ that a player makes a mistake in the action taken. We calculate the average cooperation rate for different values of ϵ and c. **A** We plot the average cooperation rate for the different parameters when individuals use reactive-1, reactive-2, and reactive-3 strategies, respectively. **B** We plot the differences between the cooperation rates when individuals use different memory size strategies. From left to right, we show the differences between reactive-1 and reactive-2, reactive-2 and reactive-3, and reactive-1 and reactive-3 strategies.

4 Memory-n

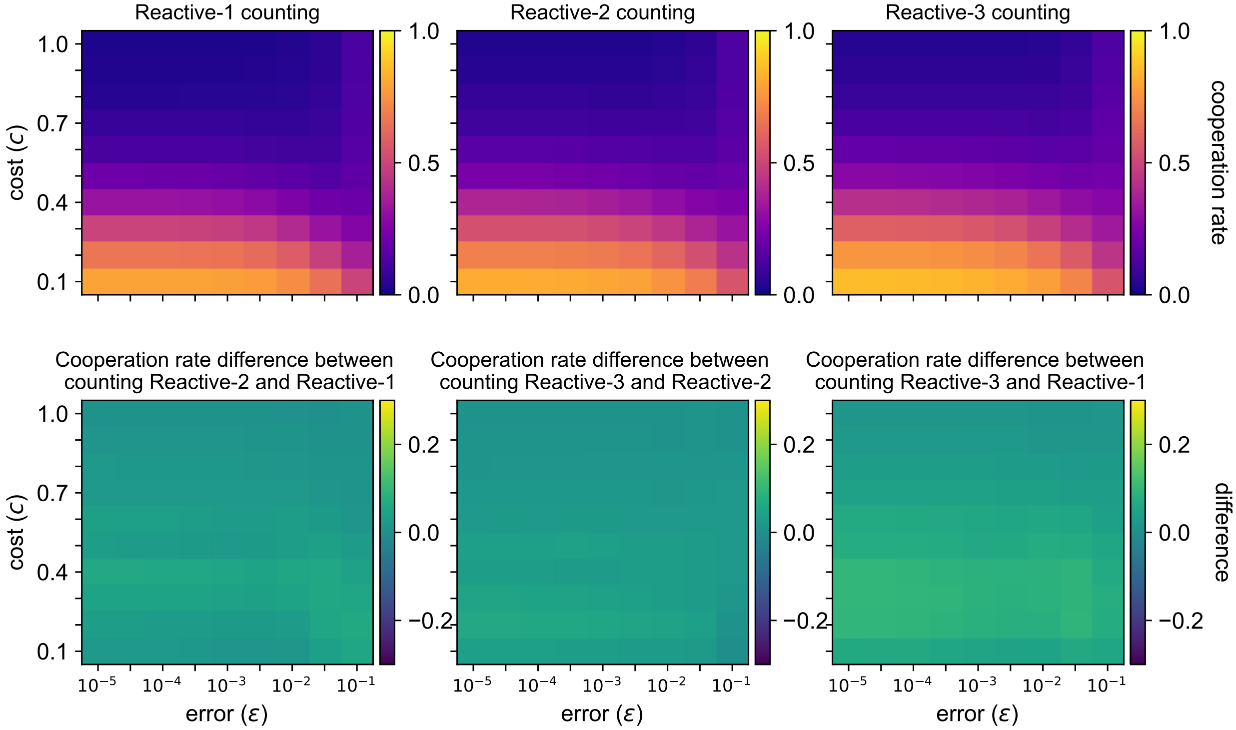

Figure S5: Cooperating rates with implementation errors.

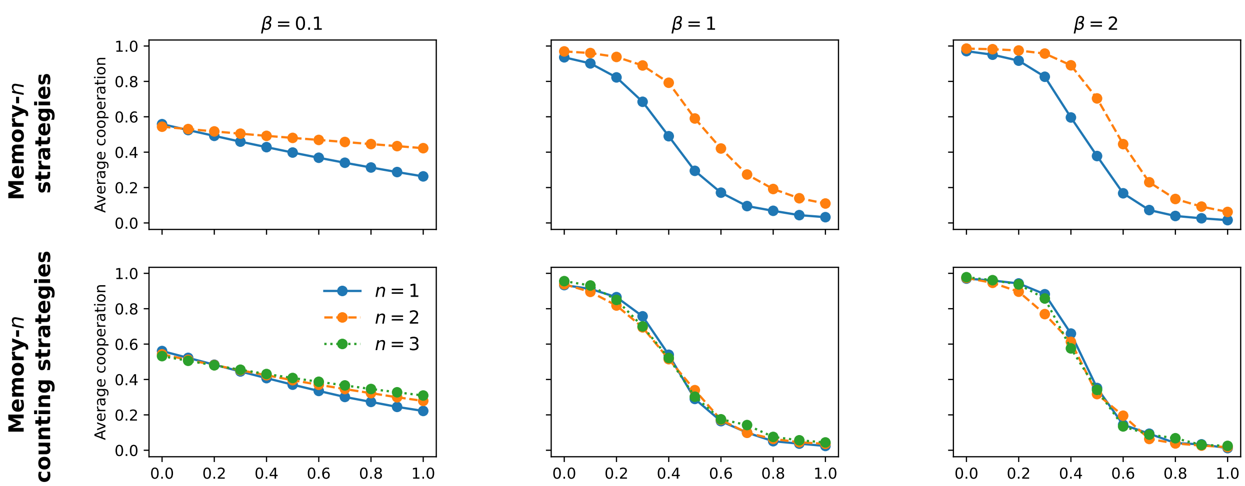

Figure S6: Memory-n simulations.