1 Reactive defecting Nash strategies in the donation game

In the previous section, we have characterized the reactive partner strategies for a special case of the donation game and the general prisoner's dilemma. In the following, we apply the same methods based on Section **??** to analyze defecting Nash equilibria. For the case of reactive-1 strategies, we obtain the following characterization.

Theorem 1 (Reactive-1 defecting Nash strategies in the donation game)
A reactive-1 strategy $\mathbf{p}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$p_C \leq \tfrac{c}{b} \qquad \textit{and} \qquad p_D = 0. \tag{1}$$

Theorem 2 (Reactive-2 defecting Nash strategies in the donation game)
A reactive-2 strategy $\mathbf{p}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$p_{CC} \leq \frac{c}{b}, \qquad \frac{p_{CD}+p_{DC}}{2} \leq \frac{c}{2b}, \qquad p_{DD} = 0. \tag{2}$$

Theorem 3 (Reactive-3 defecting Nash strategies in the donation game)
A reactive-3 strategy $\mathbf{p}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$\begin{aligned}
p_{CCC} &\leq \frac{c}{b} \\
\frac{p_{CDC}+p_{DCD}}{2} &\leq \frac{1}{2}\cdot\frac{c}{b} \\
\frac{p_{CCD}+p_{CDC}+p_{DCC}}{3} &\leq \frac{2}{3}\cdot\frac{c}{b} \\
\frac{p_{CDD}+p_{DCD}+p_{DDC}}{3} &\leq \frac{1}{3}\cdot\frac{c}{b} \\
\frac{p_{CCD}+p_{CDD}+p_{DCC}+p_{DDC}}{4} &\leq \frac{1}{2}\cdot\frac{c}{b} \\
p_{DDD} &= 0.
\end{aligned} \tag{3}$$

We repeat the same analysis for reactive counting strategies. We obtain the following results.

Theorem 4 (Reactive-2 defecting Nash counting strategies in the donation game)
A reactive-2 counting strategy $\mathbf{r}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$r_2 \leq \frac{c}{b}, \qquad r_1 \leq \frac{1}{2}\cdot\frac{c}{b}, \qquad r_0 = 0. \tag{4}$$

Theorem 5 (Reactive-3 defecting Nash counting strategies in the donation game)
A reactive-3 counting strategy $\mathbf{r}$ *is a defecting Nash strategy if and only if its entries satisfy the conditions*

$$r_3 \leq \frac{c}{b}, \qquad r_2 \leq \frac{2}{3}\cdot\frac{c}{b}, \qquad r_1 \leq \frac{1}{3}\cdot\frac{c}{b}, \qquad r_0 = 0. \tag{5}$$

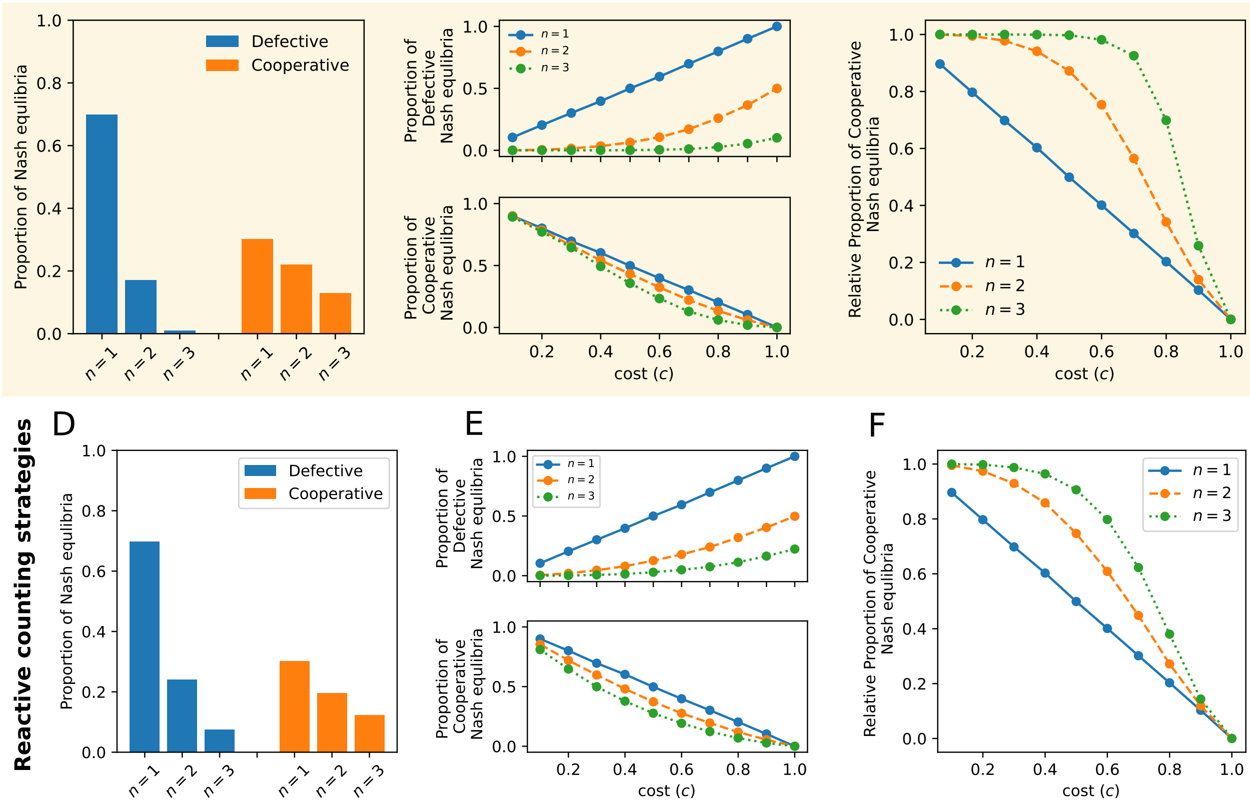

Figure S1: Volume of cooperative and defective Nash. We draw 10^4 random strategies from the feasible space of strategies and create two copies of each strategy. For one copy, we set the probability of cooperating after full cooperation of the co-player to 1. For the second copy, we set the probability of cooperating after full defections of the co-player to 0. We then checked if either copy is Nash: cooperative for the first and defective for the second. We set the benefit of cooperation to $b = 1$. **A** We plot the results for a given value of cost, $c = 0.5$. **B** The number of defective Nash strategies as a function of cost. **C** The number of cooperative Nash strategies as a function of cost.

We can observe that for each value of n, the left-hand side of the conditions for cooperative and defective Nash are the same. Moreover, it is clear that the right-hand side of the defective Nash conditions is always strictly smaller than those of the cooperative Nash conditions. This means that within the space of feasible strategies, the volume of partner strategies is always larger than the volume of defective Nash strategies. We verified these analytical results numerically as well.

We selected random strategies from the feasible space of strategies and created two copies of each strategy. For one copy, we set the probability of cooperating after full cooperation of the co-player to 1 (for example, for reactive-1, $p_C = 1$). For the second copy, we set the probability of cooperating after full defections of the co-player to 0 (for example, for reactive-2, $p_{DD} = 0$). We then checked if either copy is Nash: cooperative for the first and defective for the second. We repeated this process for 10^4 randomly selected strategies and plotted the relative volumes of cooperative and defective Nash equilibria (Figure S1). We also verified that this holds true for different values of cost.

2 Evolutionary Simulations

We perform the evolutionary analysis of Figure 3 of the main text. We simulate the evolutionary process twenty times this time, Figure S2. Now, one question that arises is how many of these strategies are actually partner strategies? And for the partner strategies, do we see all of them represented?

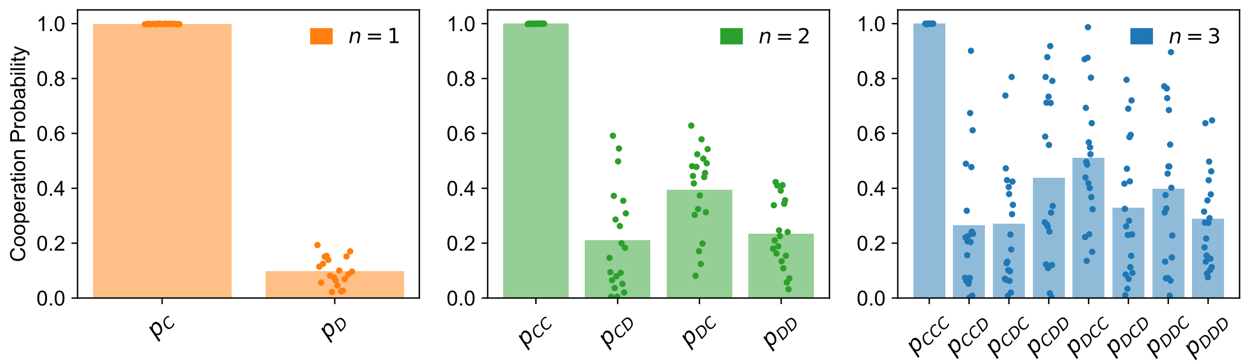

Figure S2: Evolutionary dynamics of reactive-n strategies. To explore the evolutionary dynamics among reactive-n strategies, we run simulations based on the method of Imhof and Nowak. This method assumes rare mutations. Every time a mutant strategy appears, it goes extinct or fixes before the arrival of the next mutant strategy. We run twenty independent simulations for reactive-n strategies. For each simulation, we record the most abundant strategy (the strategy that resisted most mutants). The respective average cooperation probabilities are in line with the conditions for partner strategies. Simulations are based on a donation game with $b = 1$, $c = 0.5$, a selection strength $\beta = 1$ and a population size $N = 100$, unless noted otherwise. For n equal to 1 and 2, simulations are run for $T = 10^7$ time steps. For $n = 3$ we use $T = 2 \cdot 10^7$ time steps.

Reactive-1 strategies. In the case of reactive-1, all the most abundant strategies are partner strategies. However, from Eq. and for a given value of c, the probability of defecting is... However, we observe only a lower value of p_{dd}, thus only some partner strategies are residents. This result is not explained by the theory. To better understand this result, we need to do an invasion analysis. For the invasion analysis, we consider a given resident strategy, and we sample 10^3 random strategies to estimate if this mutant will invade the resident. We record the number of mutants that took over. We repeat this 10^3 times. The results are shown in Fig. X, and what we observe is that a lower value of p_{dd} results in a more robust strategy to invasion, as only cooperative strategies on the GTFT can take over, in comparison with panel B where defective strategies are more likely to invade.

Reactive-12strategies. In the case of reactive-2, we observe that almost all strategies are partner strategies. However, what seems to be the case is that p_{dc} is higher on average than p_{cd}. In order to understand this, we again run an invasion analysis. This time we also consider two values of p_{dd}. The results are shown in Fig. Y.

3 Errors

So far, we have considered the case where there cannot be a mistake in the actions taken by a player; the actions of the players are realized without error. Here, we discuss what happens in the case where such an

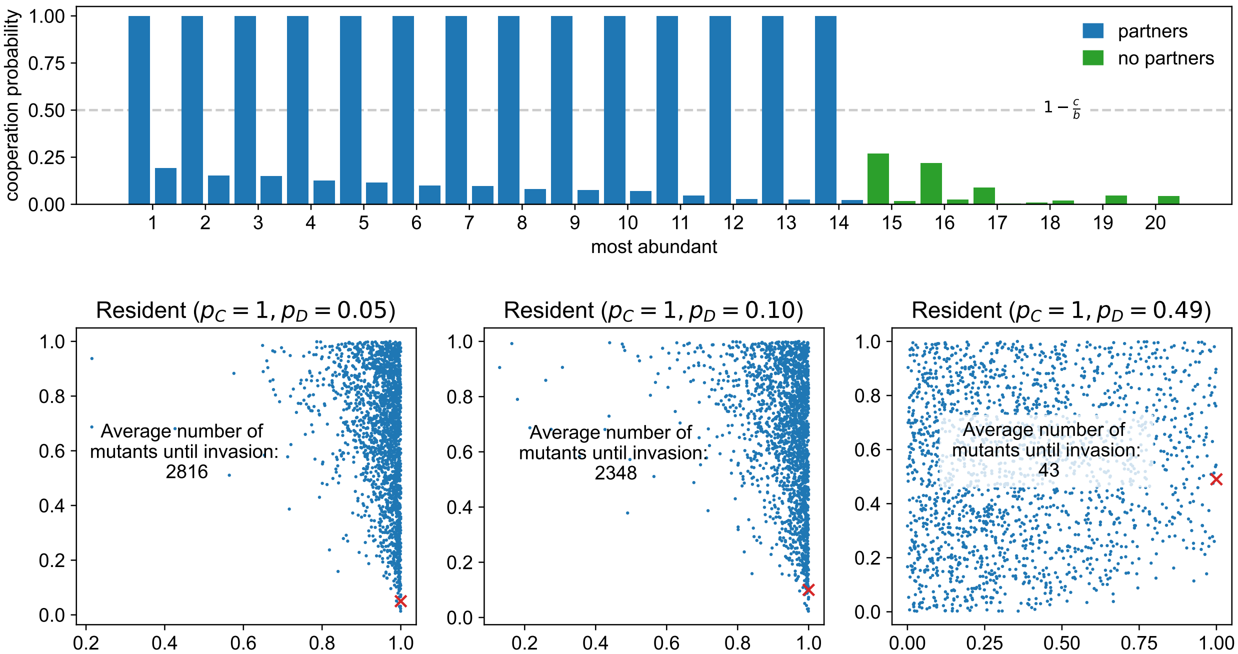

Figure S3: Invasion analysis for reactive-1.

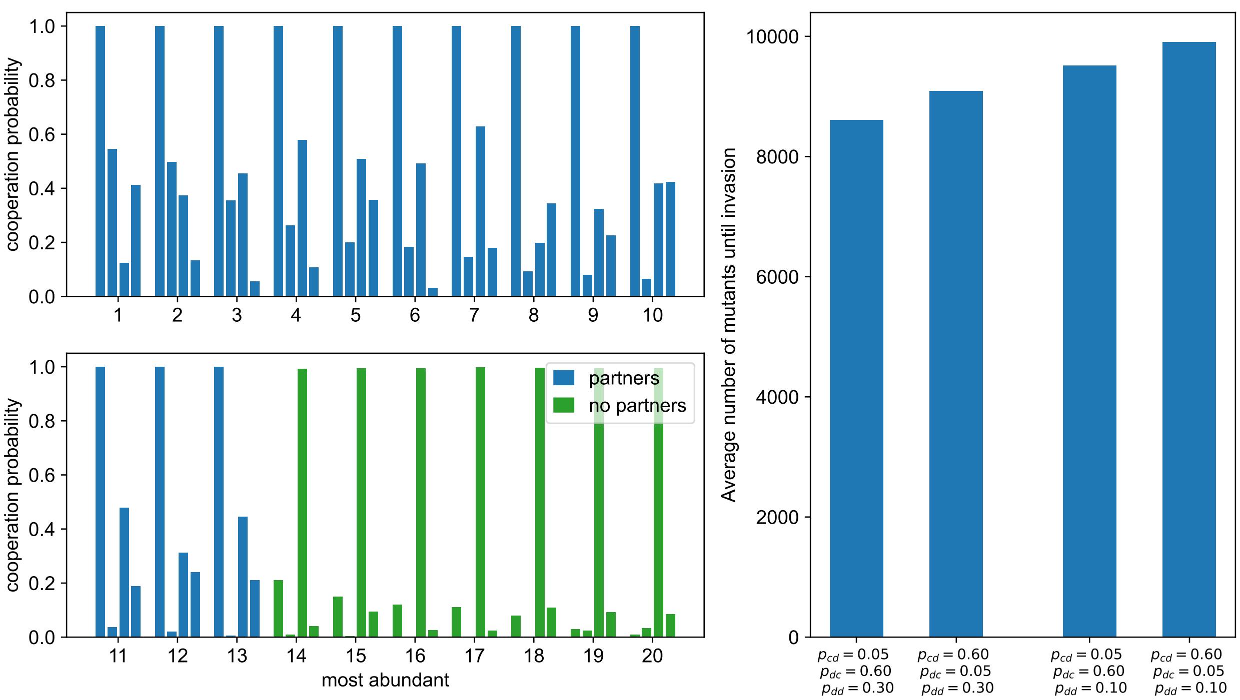

Figure S4: Invasion analysis for reactive-1.

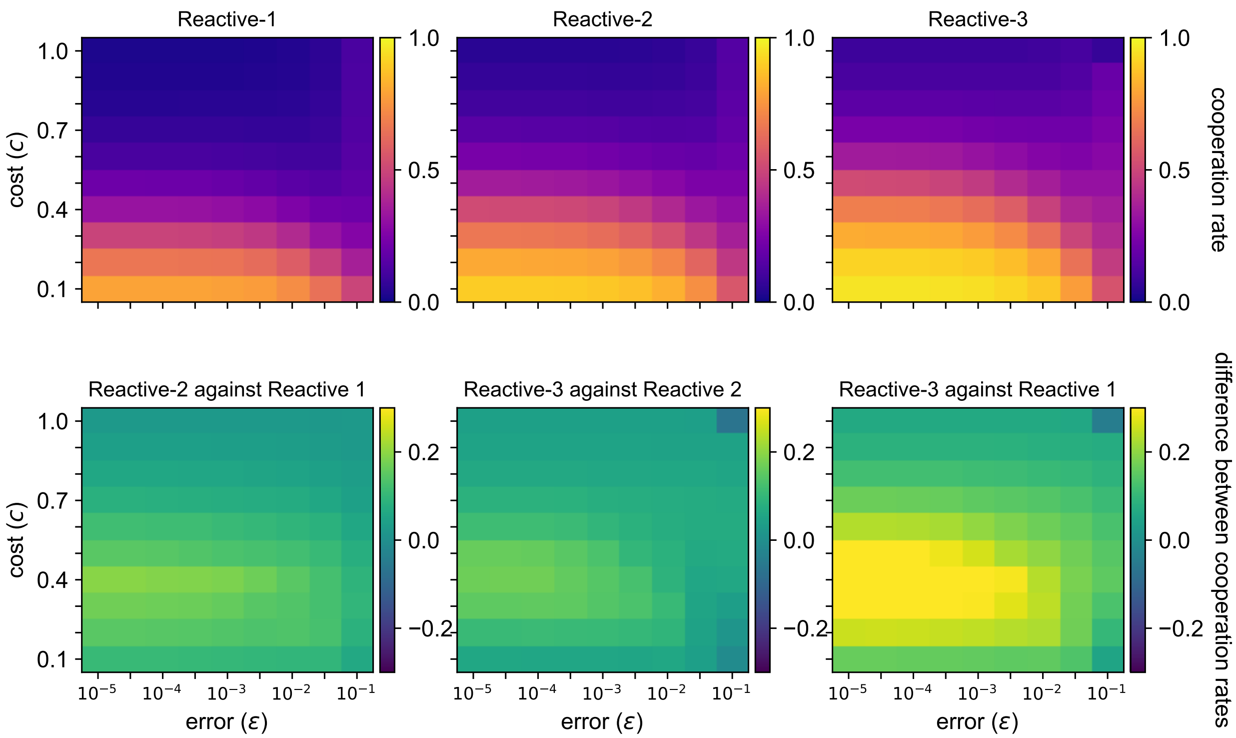

Figure S5: Cooperating rates with implementation errors. We simulate the evolutionary process, this time allowing for implementation errors. Specifically, we consider a probability ϵ that a player makes a mistake in the action taken. We calculate the average cooperation rate for different values of ϵ and c. **A** We plot the average cooperation rate for the different parameters when individuals use reactive-1, reactive-2, and reactive-3 strategies, respectively. **B** We plot the differences between the cooperation rates when individuals use different memory size strategies. From left to right, we show the differences between reactive-1 and reactive-2, reactive-2 and reactive-3, and reactive-1 and reactive-3 strategies.

error is possible. More specifically, we consider that ϵ is the probability that a player makes a mistake in the action taken.