

Evolution of cooperation among individuals with limited payoff memory

Nikoleta E. Glynatsi, Christian Hilbe, Alex McAvoy

Abstract

Repeated games are vastly used in evolutionary game theory to explain cooperation in a variety of environments. Although existing models have shaped our understanding of human cooperation they often work with idealized assumptions. In an evolutionary process, individuals imitate other individuals of the population based on their fitness. It is commonly assumed that an individual computes their fitness after interacting with a representative sample of the population and remembering all the interactions they participated in. In real life we do not always remember all our interactions, instead we rather recall our most recent ones.

Here we introduce a framework that allows individuals to estimate their fitness based on a minimum of social information. We explore the difference between our framework and the classical framework using the iterated prisoner’s dilemma. We show that individuals relying on a single set of information tend to adopt less generous strategies and they achieve less cooperation than in the classical scenario. Although strategies in the latter scenario become more cooperative with increasing benefits, those in the former remain unchanged. We further explore scenarios where individuals are granted slightly more information, and our observations indicate that these individuals can attain higher levels of cooperation.

1 Introduction

Evolutionary game theory [1–4] describes the evolutionary dynamics of populations consisting of different types of interacting individuals. In the last two decades the interest of the field has shifted to the analysis of stochastic finite population dynamics in preference to traditional approaches such as the replicator dynamics [5, 6]. This is especially true in the topic of cooperation [7–12]. In stochastic evolutionary dynamics disadvantageous mutants have a small yet non-zero probability to reach fixation and this can lead to fundamental changes in the results. As shown in [3], using such dynamics a single cooperative strategy can invade a population of defectors without any special modifications in comparison to traditional approaches that one has to consider spatial structure [13] or when payoffs are subject to aggregate shocks [14].

Stochastic evolutionary dynamics model a finite population. Each time step of the process consists of three phases; (1) the *mutation phase* (2) the *game phase* (3) the *update phase*. In the mutation phase one

individual from the population is chosen to switch to a new mutant strategy with a probability μ . In the game phase individuals are randomly matched with other individuals in the population, and they engage in a repeated game where each subsequent turn occurs with a fixed probability δ . The updating phase depends on the process. Two classes of finite stochastic processes have been used extensively: (i) fitness-based processes in which an individual chosen proportional to fitness reproduces and the offspring replaces a randomly chosen individual [3] or (ii) *pairwise comparison processes* in which a pair of individuals is chosen and subsequently one of these individuals may adopt the strategy of the other [15].

Like all theoretical models, finite stochastic processes rest on a set of assumptions. Namely, in the game phase it is assumed that players use strategies with finite memory [16, 17]. For example, in a repeated game between two individuals the actions of the players in each turn are often determined by the action of the co-players in the previous turn [16]. This assumption is common as it allows for an explicit calculation of the players' payoffs [18]. In addition, it is assumed that individuals play many times and with all other players before reproduction takes place. So that in the updating phase the updating payoffs, or fitness, of individuals is given by the average payoffs of an infinitely repeated game against every member of the population.

These two assumptions create a curious inconsistency; a player with a finite memory (in the game stage) can recall infinite information regarding their interactions and each interaction's outcomes in the update stage. Prior research has examined the effects of constraining the interactions an individual has. Either by letting individuals have a stochastic number of interactions [19–21] or by only allowing for a single interaction [22]. However, these studies have not accounted for the use of strategies with finite memory. Moreover, no previous research has explored the effect of restricting not only the number of interactions, but also the amount of information available about each interaction. To this end, we propose a framework in which individuals, similar to the decisions at each turn, estimate their fitness based on a minimum of information.

We first consider two extreme scenarios, the classical scenario where an individual has a perfect memory of all their interactions and the alternative scenario of limited memory where individuals update their strategies only based on the very last payoff they obtained. We demonstrate the effects of limited payoff memory using a pairwise comparison process and the well studied game of the prisoner's dilemma [23]. We observe that individuals with limited payoff memory tend to adopt less generous strategies. We obtain similar results when we consider that individuals update their strategies based on more information. More specifically, up to the last two payoffs they obtained when interacting with up to two different members of the population.

2 Model Setup

A pairwise comparison process [15] starts with assigning all individuals of the population the same strategy. Thenceforth each elementary time step of the process consists of the mutation phase, the game phase and the update phase. In the game phase individuals are matched in pairs and that they participate in a repeated 2

person donation game; a special case of the prisoner's dilemma. In the donation game there are two actions: cooperation (C) and defection (D). By cooperating a player provides a benefit b to the other player at their cost c , with $0 < c < b$. Thus the payoffs for a player in each turn are given by,

$$\begin{array}{cc} & \begin{array}{cc} \text{cooperate} & \text{defect} \end{array} \\ \begin{array}{c} \text{cooperate} \\ \text{defect} \end{array} & \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix} \end{array} \quad (1)$$

Let $\mathbf{u} = (b - c, -c, b, 0)$ be the round payoffs in a vector format, and let $\mathcal{U} = \{r, s, t, p\}$ denote the set of feasible payoffs, where r denotes the payoff of mutual cooperation, s the sucker's payoff, t the temptation to defect payoff, and p the punishment payoff.

In repeated games there are infinite many strategies, however, similar to the literature we will assume that individuals use reactive strategies. A reactive strategy considers only the previous action of the other player, and thus, a reactive strategy s can be written as a three-dimensional vector $s = (y, p, q)$. The parameter y is the probability that the strategy opens with a cooperation and p, q are the probabilities that the strategy cooperates given that the opponent cooperated and defected equivalently. The play between a pair of reactive strategies $s_1 = (y_1, p_1, q_1)$ and $s_2 = (y_2, p_2, q_2)$ can be model as a Markov process with the transition matrix M [24],

$$M = \begin{bmatrix} p_1 p_2 & p_1 (1 - p_2) & p_2 (1 - p_1) & (1 - p_1) (1 - p_2) \\ p_2 q_1 & q_1 (1 - p_2) & p_2 (1 - q_1) & (1 - p_2) (1 - q_1) \\ p_1 q_2 & p_1 (1 - q_2) & q_2 (1 - p_1) & (1 - p_1) (1 - q_2) \\ q_1 q_2 & q_1 (1 - q_2) & q_2 (1 - q_1) & (1 - q_1) (1 - q_2) \end{bmatrix} \quad (2)$$

and the stationary vector $\mathbf{v}(s_1, s_2)$ which is the solution to $\mathbf{v}(s_1, s_2) \times M = \mathbf{v}(s_1, s_2)$.

In the update stage two individuals are randomly selected. From the two individuals, one serves as the 'learner' and the other as the 'role model'. The learner adopts the role model's strategy with a probability ρ given by,

$$\rho(\pi_L, \pi_{RM}) = \frac{1}{1 + e^{-\beta(\pi_{RM} - \pi_L)}}. \quad (3)$$

π_L and π_{RM} are the updating payoffs/fitness of the learner and the role model respectively. The updating payoffs are a measure of how successful individuals are in the current standing of the population. The parameter β is known as the selection strength, namely, it shows how important the payoff difference is when the learner is considering adopting the strategy of the role model.

For the results presented here we assume that mutations are rare ($\mu \rightarrow 0$). In fact, so rare that only two different strategies can be present in the population at any given time. However, in the Supplementary

In Information Section 9 we show that the main result holds for $\mu \neq 0$. The case of low mutation is vastly adopted because it allows us to explicitly calculate the fixation probability of a newly introduced mutant. More specifically, at each step one individual adopts a mutant strategy randomly selected from the set of feasible strategies. The fixation probability ϕ_M of the mutant strategy can be calculated explicitly as,

$$\phi_M = \frac{1}{1 + \sum_{i=1}^{N-1} \prod_k \frac{\lambda_k^-}{\lambda_k^+}}, \quad (4)$$

where k is the number of mutants and λ_k^- , λ_k^+ are the probabilities that the number of mutants decreases and increases respectively. Depending on the fixation probability ϕ_M the mutant either fixes (becomes the new resident) or goes extinct. Regardless, in the next elementary time step another mutant strategy is introduced to the population. We iterate this elementary population updating process for a large number of mutant strategies and we record the resident strategies at each time step. The probabilities λ_k^- and λ_k^+ depend on the fitness of the mutant and the resident strategies. In the next section we present how fitness is calculated in the cases of perfect and limited payoff memory.

2.1 Fitness based on Perfect and Limited Payoff Memory

In the perfect payoff memory case an individual updates based on the average payoff against each other member of the population, otherwise known as expected payoffs. The payoff of a reactive strategy s_1 against the reactive strategy s_2 in an infinitely repeated game ($\delta \rightarrow 1$) is explicitly calculated as,

$$\langle \mathbf{v}(s_1, s_2), \mathbf{u} \rangle.$$

In a population of size N there are k mutants and $N - k$ residents. Let $s_M = (y_M, p_M, q_M)$ and $s_R = (y_R, p_R, q_R)$ denote the strategies of a mutant and a resident, the expected payoffs π_R and π_M are given by,

$$\begin{aligned} \pi_R &= \frac{N-k-1}{N-1} \cdot \langle \mathbf{v}(s_R, s_R), \mathbf{u} \rangle + \frac{k}{N-1} \cdot \langle \mathbf{v}(s_R, s_M), \mathbf{u} \rangle, \\ \pi_M &= \frac{N-k}{N-1} \cdot \langle \mathbf{v}(s_M, s_R), \mathbf{u} \rangle + \frac{k-1}{N-1} \cdot \langle \mathbf{v}(s_M, s_M), \mathbf{u} \rangle. \end{aligned} \quad (5)$$

The probabilities that the number of mutants decreases and increases, λ_k^- and λ_k^+ , in the perfect payoff memory case are defined as,

$$\lambda_k^- = \rho(\pi_M, \pi_R) \quad \text{and} \quad \lambda_k^+ = \rho(\pi_R, \pi_M). \quad (6)$$

In the case of limited payoff memory we initially define the probability that a reactive strategy receives the payoff $u \in \mathcal{U}$ in the very last round of the game. This is given by Proposition 1 (see Supplementary Information Section 2.2.1 for proof).

Proposition 1. *Consider a repeated game, with continuation probability δ , between players with reactive strategies $s_1 = (y_1, p_1, q_1)$ and $s_2 = (y_2, p_2, q_2)$ respectively. Then the probability that the s_1 player receives the payoff $u \in \mathcal{U}$ in the very last round of the game is given by $v_u(s_1, s_2)$, as given by Equation (7).*

$$\begin{aligned}
v_r(s_1, s_2) &= (1-\delta) \frac{y_1 y_2}{1-\delta^2 l_1 l_2} + \delta \frac{\left(q_1 + l_1 ((1-\delta)y_2 + \delta q_2) \right) \left(q_2 + l_2 ((1-\delta)y_1 + \delta q_1) \right)}{(1-\delta l_1 l_2)(1-\delta^2 l_1 l_2)}, \\
v_s(s_1, s_2) &= (1-\delta) \frac{y_1 \bar{y}_2}{1-\delta^2 l_1 l_2} + \delta \frac{\left(q_1 + l_1 ((1-\delta)y_2 + \delta q_2) \right) \left(\bar{q}_2 + \bar{r}_2 ((1-\delta)y_1 + \delta p_1) \right)}{(1-\delta l_1 l_2)(1-\delta^2 l_1 l_2)}, \\
v_t(s_1, s_2) &= (1-\delta) \frac{\bar{y}_1 y_2}{1-\delta^2 l_1 l_2} + \delta \frac{\left(\bar{q}_1 + \bar{r}_1 ((1-\delta)y_2 + \delta p_2) \right) \left(q_2 + l_2 ((1-\delta)y_1 + \delta q_1) \right)}{(1-\delta l_1 l_2)(1-\delta^2 l_1 l_2)}, \\
v_p(s_1, s_2) &= (1-\delta) \frac{\bar{y}_1 \bar{y}_2}{1-\delta^2 l_1 l_2} + \delta \frac{\left(\bar{q}_1 + \bar{r}_1 ((1-\delta)y_2 + \delta p_2) \right) \left(\bar{q}_2 + \bar{r}_2 ((1-\delta)y_1 + \delta p_1) \right)}{(1-\delta l_1 l_2)(1-\delta^2 l_1 l_2)}.
\end{aligned} \tag{7}$$

In these expressions, we have used the notation $l_i := p_i - q_i$, $\bar{y}_i = 1 - y_i$, $\bar{q}_i := 1 - q_i$, and $\bar{l}_i := \bar{p}_i - \bar{q}_i = -l_i$ for $i \in \{1, 2\}$.

In the case of limited payoffs memory both the role model and the learner estimate their fitness after interacting with a single member of the population. At each time step there are five possible pairings. They interact with each other with a probability $\frac{1}{N-1}$, and they do not interact with other with a probability $1 - \frac{1}{N-1}$. In the latter case, each of them can interact with either a mutant or a resident. Both of them interact with a mutant with a probability $\frac{(k-1)(k-2)}{(N-2)(N-3)}$ and both interact with a resident with a probability $\frac{(N-k-1)(N-k-2)}{(N-2)(N-3)}$. The last two possible pairings are that either of them interacts with a resident whilst the other interacts with a mutant, and this happens with a probability $\frac{(N-k-1)(k-1)}{(N-2)(N-3)}$. We define the probability that the randomly chosen resident obtained a payoff of u_R in the last round of his respective game, and that the mutant obtained a payoff of u_M as $x(u_R, u_M)$.

$$\begin{aligned}
x(u_R, u_M) = & \frac{1}{N-1} \cdot v_{u_R}(s_R, s_M) \cdot 1_{(u_R, u_M) \in \mathcal{U}_F^2} \\
& + \left(1 - \frac{1}{N-1}\right) \left[\frac{k-1}{N-2} \frac{k-2}{N-3} v_{u_R}(s_R, s_M) v_{u_M}(s_M, s_M) + \frac{k-1}{N-2} \frac{N-k-1}{N-3} v_{u_R}(s_R, s_M) v_{u_M}(s_M, s_R) \right. \\
& \left. + \frac{N-k-1}{N-2} \frac{k-1}{N-3} v_{u_R}(s_R, s_R) v_{u_M}(s_M, s_M) + \frac{N-k-1}{N-2} \frac{N-k-2}{N-3} v_{u_R}(s_R, s_R) v_{u_M}(s_M, s_R) \right].
\end{aligned} \tag{8}$$

The probability that the number of mutants increases and decreases by one in the case of limited payoff memory are now given by,

$$\lambda_k^+ = \frac{N-k}{N} \cdot \frac{k}{N} \cdot \sum_{u_R, u_M \in \mathcal{U}} x(u_R, u_M) \rho(u_R, u_M) \quad \text{and} \quad \lambda_k^- = \frac{N-k}{N} \cdot \frac{k}{N} \cdot \sum_{u_R, u_M \in \mathcal{U}} x(u_R, u_M) \rho(u_M, u_R). \tag{9}$$

In this expression, $\frac{(N-k)}{N}$ is the probability that the randomly chosen learner is a resident, and $\frac{k}{N}$ is the probability that the role model is a mutant. The sum corresponds to the total probability that the learner adopts the role model's strategy over all possible payoffs u_R and u_M that the two player may have received in their respective last rounds.

3 Simulation Results

To assess the impact of updating payoffs, we simulate the evolutionary process, recording the strategies adopted by players over time based on perfect and limited payoff memory. We performed two separate runs for each approach varying the value of benefit b . Figure 1 depicts the evolving conditional cooperation probabilities p and q (note that we omit the opening move y , as the discount factor δ is relatively high). The figure suggests that when updating is based on perfect payoff memory players tend to be more generous and more cooperative.

Specifically, we observe that the resident population comprises either defectors or conditional cooperators $(1, q)$. The generosity level q adopted by the resident population depends on whether a defecting strategy can invade. Supplementary Information Section 2 and 3 show that in the perfect payoff memory framework, conditional cooperators of the form $(1, q < \frac{\epsilon}{b})$ can arise, however, in the case of limited payoff memory, only conditional cooperators of $(1, q < \frac{1}{2})$ can avoid invasion. The q -values of the resident strategies are generally higher in classical case, indicating that players are more likely to forgive a defection if their fitness depends on interacting with every member of the population. This effect becomes more pronounced as the benefit value increases, as the perfect memory condition on the left-hand side increases, while in the limited

memory case, it remains at $q \approx \frac{1}{2}$.

Higher q values lead to a more cooperative population. We compute the average cooperation rate for each simulation, which is the average cooperation rate within the resident population. In the case of perfect payoff memory, the average cooperation rate is consistently higher than that of the last round payoffs.

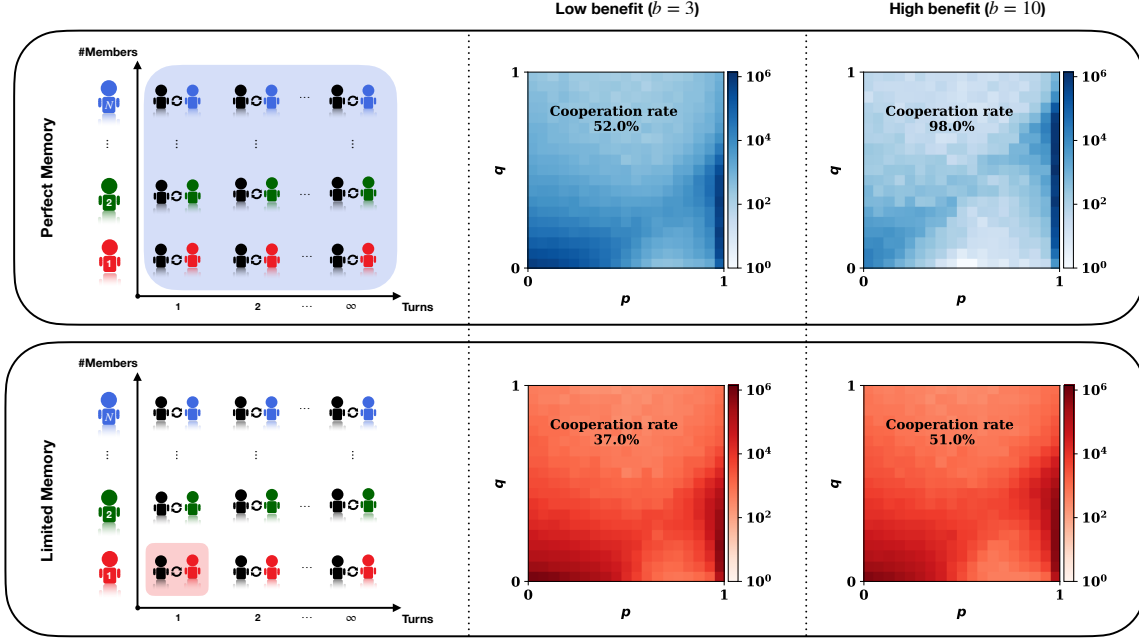


Figure 1: Evolutionary dynamics under perfect and limited payoff memory. (Schematic illustrations) On the left panels we show schematic illustrations of the perfect memory and the limited memory cases. The shaded background denotes the game phase information that an individual considers when updating strategies. In the case of perfect payoff memory the entire region is shaded and in the case of limited payoff memory only one turn with a single member of the population. **(Simulations)** We have run four simulations of the pairwise comparison process for $T = 10^7$ time steps. For each time step of the process we record the current resident population (y, p, q) . Since simulations are run for a relatively high continuation probability of $\delta = 0.999$, we do not report the players' initial cooperation probability y . The plots show how often the resident population chooses each combination (p, q) of conditional cooperation probabilities in the subsequent rounds. We also report the evolved cooperation rate which is calculated as the average cooperation rate within the resident population. **(Perfect Memory)** In the case of low benefit the resident population either consists of defectors (with $p \approx q \approx 0$) or of conditional cooperators. Conditional cooperators, or otherwise known as generous tit for tat, are a set of strategies that always cooperate following a cooperation ($p \approx 1$) and cooperate with a probability q given that the co-player has defected. q denotes the generosity of a player. The resident population applies a conditional cooperator strategy for which $q \leq 1 - c/b = 0.67$. In the case of high benefit the population mainly consists of conditional cooperators of the form ($p \approx 1, q \leq 1 - 1/10 = 0.9$). In the Supplementary Information Section 2 we show that a conditional cooperator needs to be of the form ($p \approx 1, q \leq 1 - c/b$) to not be invaded by defecting strategies. A higher generosity in the population results in a higher average cooperation rate. The average cooperation rate increases from 52% for $b = 3$ to 98% for $b = 10$. **(Limited Memory)** When players update their strategies based on their realized payoffs in the last round, there are two different predominant behaviors regardless of the benefit value. The resident population either consists of defectors (with $p \approx q \approx 0$) or of conditional cooperators. The maximum level of q , consistent with stable cooperation, is somewhat smaller compared to the perfect memory setting. Namely, in the Supplementary Information Section 3 we show that regardless of the value of benefit a conditional cooperator need to be of the form $q < \frac{1}{2}$ to not be invaded by defectors. The evolved cooperation rate only slightly increases from 37% ($b = 3$) to 51% ($b = 10$). Parameters: $N = 100$, $c = 1$, $\beta = 1$, $\delta = 0.999$.

We further investigate the impact of benefit and selection strength on generosity q and the cooperation

rate, as shown in Figure 2. According to Figure 2A, perfect memory consistently results in a higher cooperation rate, which increases with increasing benefit. On the other hand, the cooperation rate remains approximately 50% for limited payoff memory once $b = 5$. From Figure 2B we observe that for weak selection, $\beta < 1$, the two methods yield similar results, however, as β increases there is variation in the evolving populations. In the case of expected payoffs the resident populations become more cooperative, whereas in the case of limited payoff memory, the resident populations become more defective.

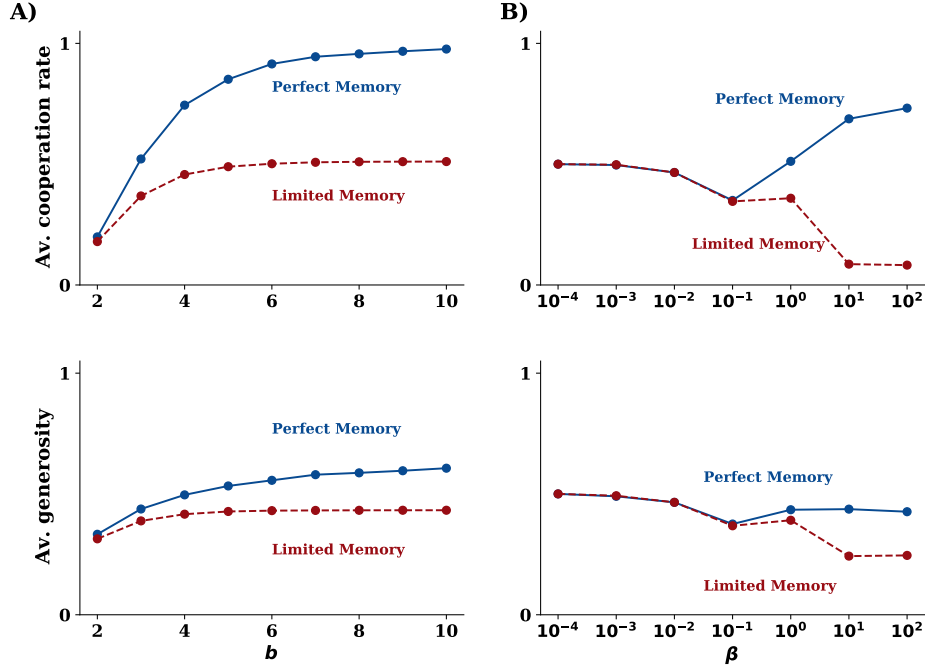


Figure 2: The evolution of cooperation and generosity for different values of benefit (A) and strength of selection (B). We report the average cooperation and the average reciprocity. The average cooperation rate is the average cooperation rate within the resident population. For the average reciprocity we select the residents that have a $p \approx 1$ and we take the average of their cooperation probability q . (A) We vary the benefit of cooperation b . In all cases, perfect memory updating payoffs appear to overestimate the average cooperation rate the population achieves. As expected in the case of limited memory the average generosity over the different values of benefit remains the same ($q \approx 0.5$), and as a result so does the average cooperation. (B) We vary the selection strength β . For weak selection, $\beta < 1$, the two methods yield similar results. However, as β increases in the case of limited memory payoffs the resident populations become more defective. Note that in the case of perfect payoff memory we see an increase in the cooperation rate even though the generosity remains stable. That is because the generosity does remain the same, however, now cooperative strategies remain fixed as the resident strategy for longer. Unless explicitly varied, the parameters of the simulation are $N = 100$, $b = 3$, $c = 1$, $\beta = 1$, $\delta = 0.99$. Simulations are run for $T = 5 \times 10^7$ time steps for each parameter combination.

The limited payoff memory framework can be expanded by enabling individuals to observe a greater number of rounds, interact with a larger number of members, or both (refer to Supplementary Information Sections 4-6). To gain further insight into the impact of limited payoff memory, we explore the scenarios of updating based on the last round with two members of the population, the last two rounds with another member of the population, and the last two rounds with two members of the population. To analyze the effects of this framework, we conduct numerical simulations using various fitness methods. The cooperation

rates for low and high benefits are presented in Figure 3.

We observe a significant rise in the cooperation rate with the introduction of slightly more information. Specifically, in scenarios involving two rounds or two interactions, the cooperation rates are almost identical. For a large population and a high continuation probability (δ), the conditional cooperators that are adopted by the resident population in these scenarios take on the same form ($1, q < \frac{\sqrt{2}}{2}$). When considering both sets of information (i.e., two rounds and two interactions), cooperation rates experience a greater increase, yet still remain lower compared to the classical scenario.

4 Conclusions

Cooperation can be seen as odd, why is it that we choose to help others at a personal cost? In spite of all the selfish genes', animal and human communities show signs of altruism and cooperation [25–27]. Evolutionary game theoretical models have helped us shape our understanding of the evolution of cooperation. In fact, the evolution of cooperation constitutes such a major focus of the field that evolutionary game theory seems to be reduced to the evolution of cooperation [28].

Evolutionary models in the past often feature a curious inconsistency. While these models depict how individuals make decisions in each round by assuming that they only retain memory of the previous round, they also assume that individuals possess a perfect memory when it comes to updating their strategies over time. To be precise, individuals are assumed to remember all of their past interactions and each interaction's outcome when updating strategies.

Here, we investigate the robustness of cooperation as models deviate from the assumption of perfect memory. While prior research has investigated the impact of constraining individuals' interactions, we take into account the limitation of not only interactions but also the information available for each outcome. Additionally, prior studies have only allowed for the adoption of simple strategies such as always cooperating or always defecting. In contrast, we enable the use of more intricate strategies where players can utilize the previous play of their co-player to make decisions.

In our framework, players update their strategies based on a combination of interactions and outcomes. The initial scenario we examined involved using one piece of information: the last round of one interaction. The outcomes suggest that cooperation faces difficulties in developing when the updating stage utilizes minimal social information. This effect is compounded as the benefit and strength of selection are independently increased. The findings indicate that cooperative players benefit from the ability to engage with all members of the population.

Furthermore, we investigated scenarios where the final two rounds, or the last two instances of interaction, were taken into account. We observed a statistically significant rise in the frequency of cooperative behavior. For a sizable population and a high likelihood of continued interactions, the two cases yield the same result as the overall payoff is influenced by two possible outcomes. Notably, the scenario involving

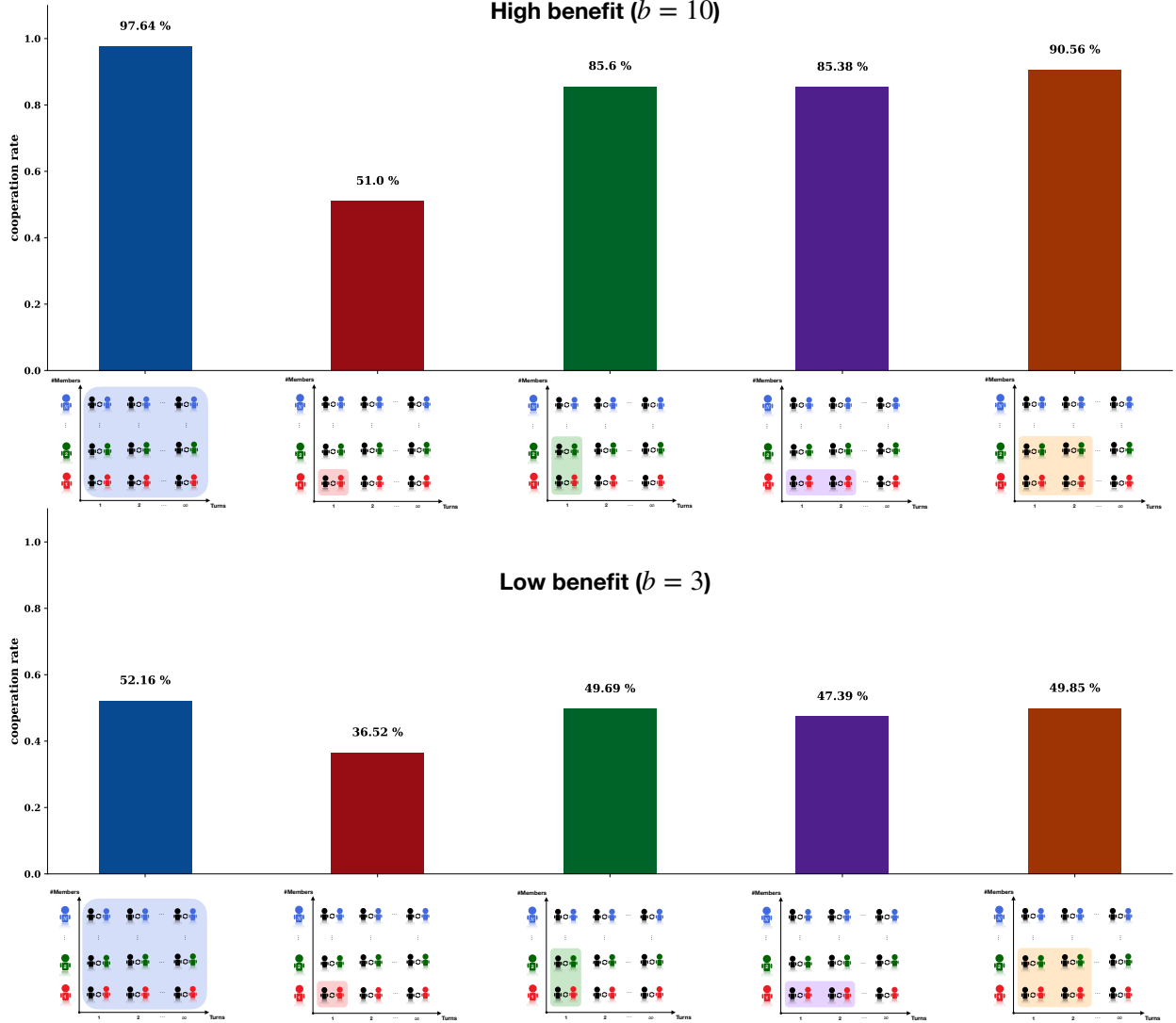


Figure 3: Average cooperation rates for different updating payoffs. From left to right, we present result on the following updating payoffs cases; (a) the expected payoffs (perfect memory), (b) the last round payoff from one interaction (limited memory), (c) the last round payoff from two interactions, (d) the last two rounds payoffs from one interaction, (e) the last two rounds payoffs from two interactions. For the updating payoffs (c) and (d) we have carried out an analysis which has shown that conditional cooperators should adopt a q value smaller than $\frac{\delta - 1 + \frac{\sqrt{2}}{2}}{\delta}$ and $\frac{\delta + \sqrt{\delta^2 + 1} - 1}{2\delta}$ respectively. Note that as $\delta \rightarrow 1$ both right hand sides tend to $\frac{\sqrt{2}}{2}$. Regardless the cooperation rate between for case (c) is slightly higher. In case (d) the cooperation rate is the second highest hinting that as we allow for more information the closer we move to the perfect payoff memory. We performed four pairwise non parametric tests (Mann-Whitney U) to compare the cooperation distributions of the residents in case (b) to cases (a), (c), (d), (e). In all four tests we reject the null hypothesis with $\alpha = 0.05$ and $p \approx 0$. Thus, there is significant difference between the cooperation rates. Unless explicitly varied, the parameters of the simulation are $N = 100$, $b = 3$, $c = 1$, $\beta = 1$, $\delta = 0.99$. Simulations are run for $T = 5 \times 10^7$ time steps for each parameter combination.

two rounds and two interactions yielded the highest cooperation rate among all the novel methodologies that we tested.

5 Acknowledgements

This work was supported by the European Research Council Starting Grant 850529: E-DIRECT.

References

- [1] John Maynard Smith. *Evolution and the Theory of Games*. Cambridge university press, 1982.
- [2] Josef Hofbauer, Karl Sigmund, et al. *Evolutionary games and population dynamics*. Cambridge university press, 1998.
- [3] Martin A Nowak, Akira Sasaki, Christine Taylor, and Drew Fudenberg. Emergence of cooperation and evolutionary stability in finite populations. *Nature*, 428(6983):646–650, 2004.
- [4] Christoph Hauert and György Szabó. Game theory and physics. *American Journal of Physics*, 73(5):405–414, 2005.
- [5] Josef Hofbauer, Peter Schuster, and Karl Sigmund. A note on evolutionary stable strategies and game dynamics. *Journal of Theoretical Biology*, 81(3):609–612, 1979.
- [6] Peter D Taylor and Leo B Jonker. Evolutionary stable strategies and game dynamics. *Mathematical biosciences*, 40(1-2):145–156, 1978.
- [7] Christian Hilbe, Martin A Nowak, and Karl Sigmund. Evolution of extortion in iterated prisoner’s dilemma games. *Proceedings of the National Academy of Sciences*, 110(17):6913–6918, 2013.
- [8] Christian Hilbe, Štěpán Šimsa, Krishnendu Chatterjee, and Martin A Nowak. Evolution of cooperation in stochastic games. *Nature*, 559(7713):246–249, 2018.
- [9] Nikoleta E Glynatsi and Vincent A Knight. Using a theory of mind to find best responses to memory-one strategies. *Scientific Reports*, 10(1):17287, 2020.
- [10] Hisashi Ohtsuki, Christoph Hauert, Erez Lieberman, and Martin A Nowak. A simple rule for the evolution of cooperation on graphs and social networks. *Nature*, 441(7092):502–505, 2006.
- [11] Martin A Nowak. Five rules for the evolution of cooperation. *science*, 314(5805):1560–1563, 2006.
- [12] Yohsuke Murase and Seung Ki Baek. Five rules for friendly rivalry in direct reciprocity. *Scientific reports*, 10(1):16904, 2020.

- [13] Martin A Nowak and Robert M May. Evolutionary games and spatial chaos. *Nature*, 359(6398):826–829, 1992.
- [14] Drew Fudenberg and Christopher Harris. Evolutionary dynamics with aggregate shocks. *Journal of Economic Theory*, 57(2):420–441, 1992.
- [15] Arne Traulsen, Jorge M Pacheco, and Martin A Nowak. Pairwise comparison and selection temperature in evolutionary game dynamics. *Journal of theoretical biology*, 246(3):522–529, 2007.
- [16] Martin A Nowak and Karl Sigmund. Tit for tat in heterogeneous populations. *Nature*, 355(6357):250–253, 1992.
- [17] Seung Ki Baek, Hyeong-Chai Jeong, Christian Hilbe, and Martin A Nowak. Comparing reactive and memory-one strategies of direct reciprocity. *Scientific reports*, 6(1):1–13, 2016.
- [18] Karl Sigmund. *The calculus of selfishness*. Princeton University Press, 2010.
- [19] Carlos P Roca, José A Cuesta, and Angel Sánchez. Time scales in evolutionary dynamics. *Physical review letters*, 97(15):158701, 2006.
- [20] Angel Sánchez and José A Cuesta. Altruism may arise from individual selection. *Journal of theoretical biology*, 235(2):233–240, 2005.
- [21] Arne Traulsen, Martin A Nowak, and Jorge M Pacheco. Stochastic payoff evaluation increases the temperature of selection. *Journal of theoretical biology*, 244(2):349–356, 2007.
- [22] Benno Woelfing and Arne Traulsen. Stochastic sampling of interaction partners versus deterministic payoff assignment. *Journal of Theoretical Biology*, 257(4):689–695, 2009.
- [23] Nikoleta E Glynatsi and Vincent A Knight. A bibliometric study of research topics, collaboration, and centrality in the iterated prisoner’s dilemma. *Humanities and Social Sciences Communications*, 8(1):1–12, 2021.
- [24] Martin Nowak and Karl Sigmund. Game-dynamical aspects of the prisoner’s dilemma. *Applied Mathematics and Computation*, 30(3):191–213, 1989.
- [25] Manfred Milinski. Tit for tat in sticklebacks and the evolution of cooperation. *nature*, 325(6103):433–435, 1987.
- [26] Benjamin Kerr, Margaret A Riley, Marcus W Feldman, and Brendan JM Bohannan. Local dispersal promotes biodiversity in a real-life game of rock–paper–scissors. *Nature*, 418(6894):171–174, 2002.

- [27] Gerald G Carter, Damien R Farine, Rachel J Crisp, Julia K Vrtilek, Simon P Ripperger, and Rachel A Page. Development of new food-sharing relationships in vampire bats. *Current Biology*, 30(7):1275–1279, 2020.
- [28] Arne Traulsen and Nikoleta E. Glynatsi. The future of theoretical evolutionary game theory. *Philosophical Transactions B*, 2022.