

Evolution of reciprocity with limited payoff memory

Nikoleta E. Glynatsi¹, Alex McAvoy^{2,3}, Christian Hilbe¹

¹Max Planck Research Group on the Dynamics of Social Behavior,
Max Planck Institute for Evolutionary Biology, Plön, Germany

²School of Data Science and Society, University of North Carolina at Chapel Hill,
Chapel Hill, NC 27599

³Department of Mathematics, University of North Carolina at Chapel Hill,
Chapel Hill, NC 27599

Abstract

Direct reciprocity can explain the evolution of cooperative behaviors in repeated interactions. According to this literature, individuals should naturally learn to adopt conditionally cooperative strategies if they have multiple encounters with their interaction partner. Corresponding models have greatly facilitated our understanding of cooperation, yet they often make strong assumptions on how individuals remember and process payoff information. For example, when strategies are updated through social learning, it is commonly assumed that individuals compare their respective average payoffs. This would require them to compute (or remember) their payoffs against all other population members. Instead, herein, we introduce a theoretical framework to study the evolution of reciprocity when individuals learn based on their most recent experiences. Even in the most extreme case that they only take into account their very last interaction, we find that cooperation can still evolve. However, such individuals adopt less generous strategies, and they tend to cooperate less often than in the classical setup with expected payoffs. Interestingly, once individuals remember the payoffs of two or three recent interactions, evolving cooperation rates quickly approach the classical limit. These findings contribute to a literature that explores which kind of cognitive capabilities are required for reciprocal cooperation. While our results suggest that some rudimentary form of payoff memory is necessary, it already suffices to remember a few interactions.

Keywords: Evolution of cooperation; direct reciprocity; repeated prisoner's dilemma; social learning; evolutionary dynamics

1 Introduction

Evolutionary game theory describes the dynamics of populations when an individual's fitness depends on the traits or strategies of other population members (1–3). This theory can be used to describe the dynamics of animal conflict (4), cancer cells (5), and of cooperation (6). Respective models translate strategic interactions into games (7). These games specify how individuals (players) interact, which strategies individuals can choose, and what fitness consequences (or payoffs) the different strategies have. In addition, these models also specify the mode by which successful strategies spread over time. In models of biological evolution, individuals with a high fitness produce more offspring; in models of cultural evolution, such individuals are imitated more often. Although biological and cultural evolution are sometimes treated as equivalent, there can be important differences (8, 9). For example, models of biological evolution do not require individuals to have any particular cognitive abilities. Here, it is the evolutionary process itself that biases the population towards strategies with higher fitness. In contrast, in models of cultural evolution, individuals need to be aware of the different strategies present in the population, and they need to identify those strategies with a higher payoff. As a consequence, evolutionary outcomes may depend on how easily different behaviors can be learned (10), and on how easy payoff comparisons are.

These difficulties to learn strategies by social imitation are particularly pronounced in models of direct reciprocity. This literature follows Trivers' insight that individuals have more of an incentive to cooperate in social dilemmas when they interact repeatedly (11). In repeated interactions, individuals can condition their behavior on their past experiences with their interaction partner. They may use strategies such as Tit-for-Tat (12, 13) or Generous Tit-for-Tat (14, 15) to preferentially cooperate with other cooperators. Such conditional strategies approximate human behavior fairly well (16–19) and they have also been documented in several other species (20–22) – although direct reciprocity is generally more difficult to demonstrate in animals (23–25). However, at the outset, it is not clear how easy it is to *learn* reciprocal strategies by social imitation. As one obstacle, individuals usually only observe other population members' actions (i.e., whether they cooperate or defect). In general, these observations do not suffice to infer the underlying strategies (i.e., the contingent rules that determine whether to cooperate or defect in any given situation). This problem is equivalent to the difficulties that arise when researchers need to estimate the participants' strategies in behavioral experiments (19). As another obstacle, even if others' strategies are perfectly observable, individuals might find it difficult to identify which ones have the highest payoff. After all, the payoff of a strategy of direct reciprocity is not determined by the outcome of any single round. Rather it is determined by how well this strategy fares over an entire sequence of rounds, against many different population members. In practice, such information might be both difficult to obtain and to process.

Most models of direct reciprocity abstract from these difficulties (26–41). They just assume individuals can easily copy the strategies of others. Similarly, they just assume that updating decisions are based on

the strategies' average (or expected) payoffs, which are based on all rounds and all interactions. These assumptions create a curious inconsistency in how models represent an individual's cognitive abilities. On the one hand, when playing the game, individuals are often assumed to have restricted memory. Respective studies typically assume that individuals make their decisions each round based on the outcome of the last round only (with only a few exceptions, see Refs. 42–46). Yet when learning new strategies, individuals are assumed to remember (or compute) each others' precise average payoff across many rounds and many interaction partners. Herein, we wish to explore whether this latter assumption is actually necessary for the evolution of reciprocity through social imitation. We ask whether individuals can learn to adopt reciprocal strategies even when learning is based on payoff information from a limited number of rounds.

To explore that question, we theoretically study imitation dynamics in the repeated prisoner's dilemma, using two extreme scenarios. The first scenario is the usual modeling approach. Here, individuals update their strategies based on their expected payoffs. We contrast this model with an alternative scenario where individuals update their strategies based on the very last (one-shot) payoff they obtained. We find that individuals with limited payoff memory tend to adopt less generous strategies. Yet moderate levels of cooperation can still evolve. Moreover, as we increase the individuals' payoff memory to include the last two or three one-shot payoffs, cooperation rates quickly approach the rates observed in the classical baseline case.

Overall, these findings suggest that while memory is important, already minimal payoff information may suffice for the evolution of direct reciprocity based on social learning. They also suggest that the classical model of reciprocity (based on expected payoffs) can often be interpreted as a useful approximation to more realistic models that include cognitive constraints.

2 Model and Methods

To explore the impact of limited payoff memory, we adapt existing models of the evolution of direct reciprocity through social learning. These models involve two different time scales. The short time scale describes the game dynamics. Here, individuals with fixed strategies are randomly matched to interact with each other in repeated social dilemmas. The long time scale describes the evolutionary dynamics. Here, individuals can update their repeated-game strategies based on the payoffs they yield. In the following, we introduce the basic setup of our model; all details and derivations are described in the electronic supplementary material.

Description of the game dynamics. We consider a well-mixed population consisting of N players. Players are randomly matched in pairs to participate in a repeated donation game (47). Each round, players can either cooperate (C) or defect (D). By cooperating, a player provides a benefit b to the other player at their own cost c , with $0 < c < b$. Thus, the players' payoffs in a single round are given by the matrix

$$\begin{array}{cc}
& \begin{array}{cc} \text{cooperate} & \text{defect} \end{array} \\
\begin{array}{c} \text{cooperate} \\ \text{defect} \end{array} & \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix}.
\end{array} \tag{1}$$

In particular, payoffs take the form of a prisoner's dilemma: Mutual cooperation yields a better payoff than mutual defection ($b - c > 0$), but each player individually prefers to defect independent of the co-player's action ($b > b - c$ and $0 > -c$). To incorporate that individuals interact repeatedly, we assume that after each round, there is a constant continuation probability δ of interacting for another round. For $\delta = 0$, we recover the case of a conventional (one-shot) prisoner's dilemma. Here, mutual defection is the only equilibrium. As δ increases, the game turns into a repeated game. Here, additional equilibria emerge, with some of them allowing for full cooperation (48–51).

In a one-shot donation game, players can only choose among two pure strategies (they can either cooperate or defect). In the repeated game, strategies become arbitrarily complex. Here, strategies are contingent rules, telling players what to do depending on the outcome of all their previous interactions. For simplicity, in the following we assume individuals use *reactive strategies* (15). A reactive strategy only depends on the other player's action in the previous round. Such strategies can be written as a three-dimensional tuple $s = (y, p, q)$. The first entry y is the probability that the player opens with cooperation in the first round. The two other entries are the probabilities that the player cooperates in all subsequent rounds, depending on whether the co-player cooperated (p) or defected (q) in the previous round. The set of reactive strategies is simple enough to facilitate an explicit mathematical analysis (1). Yet it is rich enough to capture several important strategies of repeated games. For example, it contains ALLD = $(0, 0, 0)$, the strategy that always defects. Similarly, it contains Tit-for-Tat, TFT = $(1, 1, 0)$, the strategy that copies the co-player's previous action (and that cooperates in the first round). Finally, it contains Generous Tit-for-Tat, GTFT = $(1, 1, q)$, where $q > 0$ reflects a player's generosity in response to a co-player's defection (14, 15).

In the short run, the players' strategies are taken to be fixed. Players use their strategies to decide whether to cooperate with any given interaction partner. In the long run, however, the players' strategies may change depending on the payoffs they yield, as we describe in the following.

Description of the evolutionary dynamics. Herein, we assume population members update their strategies based on social learning. To model these strategy updates, we consider a pairwise comparison process (52). This process assumes that at regular time intervals, one population member is randomly selected, and given the chance to revise its strategy. We refer to this player as the 'learner'. With probability μ (reflecting a mutation rate), the learner simply adopts a random strategy (all reactive strategies have the same probability to be chosen). With the converse probability $1 - \mu$, the learner randomly picks a 'role model' from the

population. The learner then compares its own payoff π_L from the repeated game to the role model's payoff π_R . The learner adopts the role model's strategy with a probability ρ described by a Fermi function (53, 54),

$$\rho(\pi_L, \pi_R) = \frac{1}{1 + e^{-\beta(\pi_R - \pi_L)}}. \quad (2)$$

The selection strength parameter $\beta \geq 0$ indicates how sensitive players are to payoff differences. For $\beta = 0$, payoff differences are irrelevant, and the learner simply adopts the role model's strategy with probability one half. As the selection strength β increases, players are increasingly biased to imitate the role model only if it has the higher payoff. In either case, we assume in the following that the role model's strategy can be identified. That is, if the learner decides to imitate the role model's strategy, the learner copies it exactly, as in previous works on direct reciprocity.

We deviate from previous models in how we interpret the payoffs π_L and π_R , which form the basis of the pairwise comparisons in Eq. (2). In previous work, these payoffs are taken to be the respective players' expected payoffs. We interpret that setup as a model with perfect payoff memory. There, the payoffs π_L and π_R represent an average over all possible repeated games the two individuals have played with all population members (**Fig. 1**, upper left panel). The use of expected payoffs is mathematically convenient, because explicit formulas for these payoffs are available (1). Herein, we compare this model of perfect payoff memory to a model with limited payoff memory. In that model, the players' payoffs π_L and π_R are taken to be the payoffs that each player received in their very last round prior to making social comparisons. That is, players only consider the very last repeated game they participated in, and there they only take into account the outcome of the very last round (**Fig. 1**, lower left panel). This assumption could reflect, for example, a strong recency bias in how individuals evaluate payoffs. In addition to this extreme case of limited payoff memory, later on we also explore cases in which players take into account the outcome of two, three, or four recent rounds.

Both in the case of perfect and limited memory, we iterate the elementary strategy updating step described above for many time steps. This gives rise to a stochastic process that describes which strategies players adopt over time. We explore the dynamics of this process mathematically and with computer simulations. For the results presented in the following, we assume that mutations are rare ($\mu \rightarrow 0$). This assumption is fairly common in evolutionary game theory, because it makes some computations more efficient (55–57), and because the results can be interpreted more easily. However, in Section 9 of the electronic supplementary material we show that our main results continue to hold for strictly positive mutation rates.

3 Results

Stability of cooperative populations. To get some intuition for the differences between perfect and limited payoff memory, we first analyze when cooperation is stable in either scenario. To this end, we consider a

resident population in which all players but one adopt a strategy of Generous Tit-for-Tat, $\text{GTFT} = (1, 1, q)$. The remaining mutant player adopts ALLD . We say *cooperation is stochastically stable* if the single mutant is more likely to imitate the residents than vice versa. For simplicity, we consider a large population ($N \rightarrow \infty$) and strong selection ($\beta \rightarrow \infty$). More general results are derived in the electronic supplementary material.

In the case of perfect payoff memory, it is straightforward to characterize when cooperation is stochastically stable. Here, we simply need to compute the players' expected payoffs. Because the population mostly consists of residents, and because residents mutually cooperate with each other, their expected payoff is $\pi_{\text{GTFT}} = b - c$. The mutant only interacts with residents, and hence receives a benefit in the first round, and in every subsequent round with probability q . As a result, the mutant's expected payoff is $\pi_{\text{ALLD}} = (1 - \delta + \delta q)b$. For perfect payoff memory, the requirement for cooperation to be stochastically stable reduces to the condition $\pi_{\text{GTFT}} > \pi_{\text{ALLD}}$. This yields

$$q < 1 - \frac{c}{\delta b}. \quad (3)$$

In particular, we recover the previous observation that $q = 1 - c/(\delta b)$ is the maximum generosity that cooperators should have (14, 15, 58). Because $q \geq 0$, we also conclude that cooperation can only be stable if $\delta > c/b$. Again, this condition for the feasibility of direct reciprocity is the condition found in the literature (6).

The logic of the case with limited payoff memory is somewhat different. Here we need to compute how likely each player obtains one of the four possible payoffs $\{b - c, -c, b, 0\}$ in the very last round of a game, before they make social comparisons. Because residents almost always interact with other residents, their last one-shot payoff is $\pi_{\text{GTFT}} = b - c$ almost surely. For the defecting mutant, there are two possibilities. (i) If the mutant's co-player happens to cooperate in the last round, the mutant receives $\pi_{\text{ALLD}} = b$. This case occurs with probability $1 - \delta + \delta q$. (ii) If the co-player defects in the last round, the mutant receives $\pi_{\text{ALLD}} = 0$. This occurs with the converse probability $\delta(1 - q)$. Because $b - c < b$, residents tend to imitate the mutant in the first case. Because $b - c > 0$, mutants tend to imitate the resident in the second case. Cooperation is stochastically stable if the first case is less likely than the second. This yields the condition

$$q < 1 - \frac{1}{2\delta}. \quad (4)$$

Interestingly, this condition no longer depends on the exact payoff values c and b . This independence arises because of our assumption of strong selection, in which case only the payoff ordering $b > c > 0$ matters. Because q is non-negative, condition (4) can only be satisfied if $\delta > 1/2$. That is, players need to interact in more than two rounds in expectation.

By comparing the two cases, we find that payoff memory affects whether a conditionally cooperative strategy $(1, 1, q)$ is viable. With perfect memory, the maximum generosity q needs to satisfy Eq. (3). In particular, this generosity can become arbitrarily large, provided the game's benefit-to-cost ratio b/c and the continuation probability δ are sufficiently large. In contrast, with limited payoff memory, the maximum

generosity is bounded by one half, and it is independent of the benefit-to-cost ratio.

Evolutionary dynamics of reciprocity. To explore whether the previous static observations describe the dynamics of evolving populations, we turn to simulations. We have run separate simulations for perfect and limited payoff memory, both for a low and a high benefit of cooperation ($b/c=3$ and $b/c=10$, respectively). In each case, we record which strategies (y, p, q) the players adopt over time. **Fig. 1** depicts the evolving conditional cooperation probabilities p and q (we omit the opening move y because we use a discount factor δ very close to one). In each case, we find that the players' strategies cluster in two regions of the strategy space. The first region corresponds to a neighborhood of ALLD with $(p, q) \approx (0, 0)$. The second region corresponds to a thin strip of cooperative strategies with $(p, q) = (1, q)$. Within this strip, we observe that most strategies satisfy the constraints on q imposed by the inequalities (3) and (4). That is, with perfect memory, most evolving strategies have $q < 1 - c/b$, whereas with limited payoff memory, most strategies have $q < 1/2$. In particular, in the latter case changes in the benefit parameter have no qualitative effect on the qualitative distribution of strategies.

In each case, the evolutionary dynamics follows a similar cyclic pattern (as previously described in Refs. 15, 30): Resident populations of defectors are most likely invaded by strategies close to TFT. Once the population adopts conditionally cooperative strategies $(1, 1, q)$, neutral drift may introduce larger values of generosity q . If the resident's generosity q violates the conditions (3) and (4), defectors can re-invade, and the cycle starts again. The overall cooperation rate, averaged over many time steps, depends on how much time the process spends near ALLD on the one hand, and near the strip of conditionally cooperative strategies on the other. For perfect payoff memory, the overall cooperation rate is substantial. It is 52% for low benefits, and 98% for high benefits. For limited payoff memory, the evolving cooperation rates are smaller but strictly positive, with 37% cooperation for low benefits and 51% cooperation for high benefits (**Fig. 1**).

To further investigate the influence of different parameters, we have systematically varied the benefit b and the selection strength β in Figure **Fig. 2**. According to **Fig. 2A**, perfect memory consistently results in a higher cooperation rate, and this relative advantage further increases with an increasing benefit b . Interestingly, for limited payoff memory, the cooperation rate remains stable at approximately 50% once $b \geq 5$. This again reflects our earlier observation that the feasibility of cooperation in this scenario is independent of the exact values of b and c , as described by Eq. (4). With respect to the effect of different selection strengths, **Fig. 2B** suggests that both perfect and limited payoff memory yields similar cooperation rates for weak selection $\beta < 1$. Beyond weak selection, increasing selection has a positive effect under perfect payoff memory, but a negative effect under limited payoff memory.

The effect of increasing individual payoff memory. So far, we have taken a rather extreme interpretation of limited payoff memory. In the respective scenario, we assumed that individuals update their strategies

based on their experience in a single round of the prisoner’s dilemma, against a single co-player. The limited payoff memory framework can be expanded in various ways. In particular, individuals may recall a larger number of rounds, they may recall their interactions with several co-players, or both. To gain further insights on the impact of payoff memory, we explore three additional scenarios. In the first scenario, players recall the payoffs they obtained in the last two rounds against a single co-player. In the second scenario, players recall their last-round payoffs against two co-players. Finally, in the last scenario, they recall the two last rounds against two co-players (further extensions are straightforward, but we do not explore them here).

For the first two scenarios, we can again derive an analytical condition for when cooperation is stochastically stable. To this end, we again assume populations are large and that selection is strong. For simplicity, we also assume that the game continues almost certainly after each round (i.e., δ approaches one). The details of this analysis can be found in the electronic supplementary material. Interestingly, we find in both scenarios that for $b > 2c$, cooperation is stochastically stable when $q < \frac{\sqrt{2}}{2} \approx 0.707$. Comparing this condition with the more stringent condition in Eq. (4) suggests that there are now more conditionally cooperative strategies that can sustain cooperation. Hence, cooperation should evolve more easily.

We explore this prediction with additional simulations, again for a low and a high benefit b , see **Fig. 3**. We observe that a minimal increase in the players’ payoff memory (compared to the baseline case with a single round recalled) can promote cooperation considerably. Specifically, in all three scenarios with extended memory, we now see similar cooperation rates, and they approach the rates observed under perfect memory. These results suggest that while it takes *some* payoff memory to sustain substantial cooperation rates, the requirements on memory seem to be rather modest.

4 Discussion

Cooperation can be seen as odd, why is it that we choose to help others at a personal cost? In spite of all the selfish genes’, animal and human communities show signs of altruism and cooperation (60–62). Evolutionary game theoretical models have helped us shape our understanding of the evolution of cooperation. In fact, the evolution of cooperation constitutes such a major focus of the field that evolutionary game theory seems to be reduced to the evolution of cooperation (63).

Evolutionary models in the past often feature a curious inconsistency. While these models depict how individuals make decisions in each round by assuming that they only retain memory of the previous round, they also assume that individuals possess a perfect memory when it comes to updating their strategies over time. To be precise, individuals are assumed to remember all of their past interactions and each interaction’s outcome when updating strategies.

Here, we investigate the robustness of cooperation as models deviate from the assumption of perfect memory. While prior research has investigated the impact of constraining individuals’ interactions, we take into account the limitation of not only interactions but also the information available for each outcome.

Additionally, prior studies have only allowed for the adoption of simple strategies such as always cooperating or always defecting. In contrast, we enable the use of more intricate strategies where players can utilize the previous play of their co-player to make decisions.

In our framework, players update their strategies based on a combination of interactions and outcomes. The initial scenario we examined involved using one piece of information: the last round of one interaction. The outcomes suggest that cooperation faces difficulties in developing when the updating stage utilizes minimal social information. This effect is compounded as the benefit and strength of selection are independently increased. The findings indicate that cooperative players benefit from the ability to engage with all members of the population.

Furthermore, we investigated scenarios where the final two rounds, or the last two instances of interaction, were taken into account. We observed a statistically significant rise in the frequency of cooperative behavior. For a sizable population and a high likelihood of continued interactions, the two cases yield the same result as the overall payoff is influenced by two possible outcomes. Notably, the scenario involving two rounds and two interactions yielded the highest cooperation rate among all the novel methodologies that we tested.

References

- [1] Hofbauer, J., Sigmund, K. *et al.* *Evolutionary games and population dynamics* (Cambridge university press, 1998).
- [2] Nowak, M. A., Sasaki, A., Taylor, C. & Fudenberg, D. Emergence of cooperation and evolutionary stability in finite populations. *Nature* **428**, 646–650 (2004).
- [3] Hauert, C. & Szabó, G. Game theory and physics. *American Journal of Physics* **73**, 405–414 (2005).
- [4] Maynard Smith, J. & Price, G. R. The logic of animal conflict. *Nature* **246**, 15–18 (1973).
- [5] Stein, A. *et al.* Stackelberg evolutionary game theory: how to manage evolving systems. *Philosophical Transactions of the Royal Society B* **378**, 20210495 (2023).
- [6] Nowak, M. A. Five rules for the evolution of cooperation. *Science* **314**, 1560–1563 (2006).
- [7] Smith, J. M. *Evolution and the Theory of Games* (Cambridge university press, 1982).
- [8] Wu, B., Bauer, B., Galla, T. & Traulsen, A. Fitness-based models and pairwise comparison models of evolutionary games are typically different—even in unstructured populations. *New Journal of Physics* **17**, 023043 (2015).
- [9] Smolla, M. *et al.* Underappreciated features of cultural evolution. *Philosophical Transactions of the Royal Society B* **376**, 20200259 (2021).
- [10] Chatterjee, K., Zufferey, D. & Nowak, M. A. Evolutionary game dynamics in populations with different learners. *Journal of Theoretical Biology* **301**, 161–173 (2012).
- [11] Trivers, R. L. The evolution of reciprocal altruism. *The Quarterly review of biology* **46**, 35–57 (1971).
- [12] Rapoport, A. & Chammah, A. M. *Prisoner’s Dilemma* (University of Michigan Press, Ann Arbor, 1965).
- [13] Axelrod, R. & Hamilton, W. D. The evolution of cooperation. *Science* **211**, 1390–1396 (1981).

- [14] Molander, P. The optimal level of generosity in a selfish, uncertain environment. *Journal of Conflict Resolution* **29**, 611–618 (1985).
- [15] Nowak, M. A. & Sigmund, K. Tit for tat in heterogeneous populations. *Nature* **355**, 250–253 (1992).
- [16] Fischbacher, U., Gächter, S. & Fehr, E. Are people conditionally cooperative? Evidence from a public goods experiment. *Economic Letters* **71**, 397–404 (2001).
- [17] Rand, D. G. & Nowak, M. A. Human cooperation. *Trends in Cogn. Sciences* **117**, 413–425 (2012).
- [18] Dal Bó, P. & Fréchette, G. R. Strategy choice in the infinitely repeated prisoner’s dilemma. *American Economic Review* **109**, 3929–3952 (2019).
- [19] Rossetti, C. & Hilbe, C. Direct reciprocity among humans. *Ethology* <https://doi.org/10.1111/eth.13407> (2023).
- [20] Carter, G. G. & Wilkinson, G. S. Food sharing in vampire bats, reciprocal help predicts donations more than relatedness or harassment. *Proceedings of the Royal Society B: Biological Sciences* **280**, 20122573 (2013).
- [21] Schweinfurth, M. K., Aeschbacher, J., Santi, M. & Taborsky, M. Male norway rats cooperate according to direct but not generalized reciprocity rules. *Animal Behaviour* **152**, 93–101 (2019).
- [22] Voelkl, B. *et al.* Matching times of leading and following suggest cooperation through direct reciprocity during V-formation flight in ibis. *Proceedings of the National Academy of Sciences USA* **112**, 2115–2120 (2015).
- [23] Clutton-Brock, T. Cooperation between non-kin in animal societies. *Nature* **462**, 51–57 (2009).
- [24] Silk, J. B. Reciprocal altruism. *Current Biology* **23**, 827–828 (2013).
- [25] Taborsky, M. Social evolution: Reciprocity there is. *Current Biology* **23**, 486–488 (2013).
- [26] Brauchli, K., Killingback, T. & Doebeli, M. Evolution of cooperation in spatially structured populations. *Journal of Theoretical Biology* **200**, 405–417 (1999).
- [27] Brandt, H. & Sigmund, K. The good, the bad and the discriminator - errors in direct and indirect reciprocity. *Journal of Theoretical Biology* **239**, 183–194 (2006).
- [28] Ohtsuki, H. & Nowak, M. A. Direct reciprocity on graphs. *Journal of Theoretical Biology* **247**, 462–470 (2007).
- [29] Szolnoki, A., Perc, M. & Szabó, G. Phase diagrams for three-strategy evolutionary prisoner’s dilemma games on regular graphs. *Physical Review E* **80**, 056104 (2009).
- [30] Imhof, L. A. & Nowak, M. A. Stochastic evolutionary dynamics of direct reciprocity. *Proceedings of the Royal Society B: Biological Sciences* **277**, 463–468 (2010).
- [31] van Segbroeck, S., Pacheco, J. M., Lenaerts, T. & Santos, F. C. Emergence of fairness in repeated group interactions. *Physical Review Letters* **108**, 158104 (2012).
- [32] Grujic, J., Cuesta, J. A. & Sanchez, A. On the coexistence of cooperators, defectors and conditional cooperators in the multiplayer iterated prisoner’s dilemma. *Journal of Theoretical Biology* **300**, 299–308 (2012).
- [33] Martinez-Vaquero, L. A., Cuesta, J. A. & Sanchez, A. Generosity pays in the presence of direct reciprocity: A comprehensive study of 2×2 repeated games. *PLoS One* **7**, e35135 (2012).
- [34] Stewart, A. J. & Plotkin, J. B. From extortion to generosity, evolution in the iterated prisoner’s dilemma. *Proceedings of the National Academy of Sciences USA* **110**, 15348–15353 (2013).
- [35] Pinheiro, F. L., Vasconcelos, V. V., Santos, F. C. & Pacheco, J. M. Evolution of all-or-none strategies in repeated public goods dilemmas. *PLoS Comput Biol* **10**, e1003945 (2014).
- [36] Stewart, A. J. & Plotkin, J. B. The evolvability of cooperation under local and non-local mutations. *Games* **6**, 231–250 (2015).
- [37] Baek, S. K., Jeong, H.-C., Hilbe, C. & Nowak, M. A. Comparing reactive and memory-one strategies of direct

- reciprocity. *Scientific reports* **6**, 1–13 (2016).
- [38] McAvoy, A. & Nowak, M. A. Reactive learning strategies for iterated games. *Proceedings of the Royal Society A* **475**, 20180819 (2019).
 - [39] Glynatsi, N. E. & Knight, V. A. Using a theory of mind to find best responses to memory-one strategies. *Scientific Reports* **10**, 17287 (2020).
 - [40] Schmid, L., Hilbe, C., Chatterjee, K. & Nowak, M. A. Direct reciprocity between individuals that use different strategy spaces. *PLoS Computational Biology* **18**, e1010149 (2022).
 - [41] Murase, Y., Hilbe, C. & Baek, S. K. Evolution of direct reciprocity in group-structured populations. *Scientific Reports* **12**, 18645 (2022).
 - [42] Hauert, C. & Schuster, H. G. Effects of increasing the number of players and memory size in the iterated prisoner’s dilemma: a numerical approach. *Proceedings of the Royal Society of London. Series B: Biological Sciences* **264**, 513–519 (1997).
 - [43] van Veelen, M., García, J., Rand, D. G. & Nowak, M. A. Direct reciprocity in structured populations. *Proceedings of the National Academy of Sciences USA* **109**, 9929–9934 (2012).
 - [44] Stewart, A. J. & Plotkin, J. B. Small groups and long memories promote cooperation. *Scientific reports* **6**, 1–11 (2016).
 - [45] Li, J. *et al.* Evolution of cooperation through cumulative reciprocity. *Nature Computational Science* **2**, 677–686 (2022).
 - [46] Murase, Y. & Baek, S. K. Grouping promotes both partnership and rivalry with long memory in direct reciprocity. *PLoS Computational Biology* **19**, e1011228 (2023).
 - [47] Sigmund, K. *The calculus of selfishness* (Princeton University Press, 2010).
 - [48] Friedman, J. A non-cooperative equilibrium for supergames. *Review of Economic Studies* **38**, 1–12 (1971).
 - [49] Akin, E. The iterated prisoner’s dilemma: Good strategies and their dynamics. In Assani, I. (ed.) *Ergodic Theory, Advances in Dynamics*, 77–107 (de Gruyter, Berlin, 2016).
 - [50] Hilbe, C., Traulsen, A. & Sigmund, K. Partners or rivals? Strategies for the iterated prisoner’s dilemma. *Games and Economic Behavior* **92**, 41–52 (2015).
 - [51] Stewart, A. J. & Plotkin, J. B. Collapse of cooperation in evolving games. *Proceedings of the National Academy of Sciences USA* **111**, 17558 – 17563 (2014).
 - [52] Traulsen, A., Pacheco, J. M. & Nowak, M. A. Pairwise comparison and selection temperature in evolutionary game dynamics. *Journal of theoretical biology* **246**, 522–529 (2007).
 - [53] Blume, L. E. The statistical mechanics of best-response strategy revision. *Games and Economic Behavior* **11**, 111–145 (1995).
 - [54] Szabó, G. & Tóke, C. Evolutionary Prisoner’s Dilemma game on a square lattice. *Physical Review E* **58**, 69–73 (1998).
 - [55] Fudenberg, D. & Imhof, L. A. Imitation processes with small mutations. *Journal of Economic Theory* **131**, 251–262 (2006).
 - [56] Wu, B., Gokhale, C. S., Wang, L. & Traulsen, A. How small are small mutation rates? *Journal of Mathematical Biology* **64**, 803–827 (2012).
 - [57] McAvoy, A. Comment on “Imitation processes with small mutations”. *J. Econ. Theory* **159**, 66–69 (2015).
 - [58] Schmid, L., Chatterjee, K., Hilbe, C. & Nowak, M. A unified framework of direct and indirect reciprocity. *Nature*

Human Behaviour **5**, 1292–1302 (2021).

- [59] Tkadlec, J., Hilbe, C. & Nowak, M. A. Mutation enhances cooperation in direct reciprocity. *Proceedings of the National Academy of Sciences USA* **120**, e2221080120 (2023).
- [60] Milinski, M. Tit for tat in sticklebacks and the evolution of cooperation. *Nature* **325**, 433–435 (1987).
- [61] Kerr, B., Riley, M. A., Feldman, M. W. & Bohannan, B. J. Local dispersal promotes biodiversity in a real-life game of rock–paper–scissors. *Nature* **418**, 171–174 (2002).
- [62] Carter, G. G. *et al.* Development of new food-sharing relationships in vampire bats. *Current Biology* **30**, 1275–1279 (2020).
- [63] Traulsen, A. & Glynatsi, N. E. The future of theoretical evolutionary game theory. *Philosophical Transactions B* (2022).

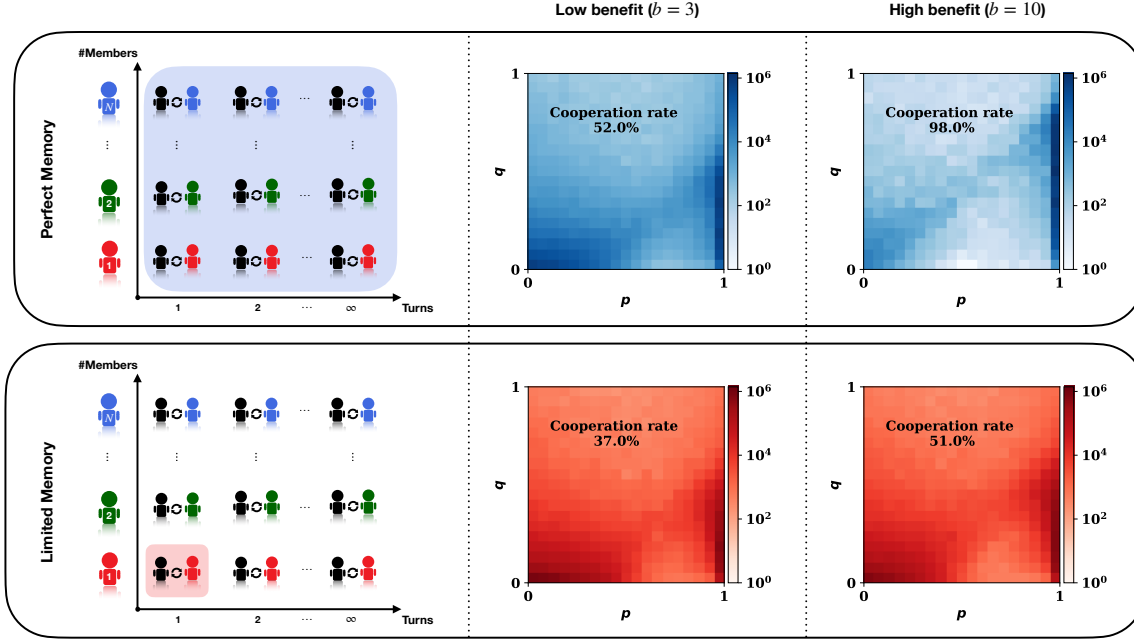


Figure 1: Evolutionary dynamics under perfect and limited payoff memory. (Schematic illustrations) On the left panels we show schematic illustrations of the perfect memory and the limited memory cases. The shaded background denotes the game phase information that an individual considers when updating strategies. In the case of perfect payoff memory the entire region is shaded and in the case of limited payoff memory only one turn with a single member of the population. **(Simulations)** We have run four simulations of the pairwise comparison process for $T = 10^7$ time steps. For each time step of the process we record the current resident population (y, p, q) . Since simulations are run for a relatively high continuation probability of $\delta = 0.999$, we do not report the players' initial cooperation probability y . The plots show how often the resident population chooses each combination (p, q) of conditional cooperation probabilities in the subsequent rounds. We also report the evolved cooperation rate which is calculated as the average cooperation rate within the resident population. **(Perfect Memory)** In the case of low benefit the resident population either consists of defectors (with $p \approx q \approx 0$) or of conditional cooperators. Conditional cooperators, or otherwise known as generous tit for tat, are a set of strategies that always cooperate following a cooperation ($p \approx 1$) and cooperate with a probability q given that the co-player has defected. q denotes the generosity of a player. The resident population applies a conditional cooperator strategy for which $q \leq 1 - c/b = 0.67$. In the case of high benefit the population mainly consists of conditional cooperators of the form ($p \approx 1, q \leq 1 - 1/10 = 0.9$). In the Supplementary Information Section 2 we show that a conditional cooperator needs to be of the form ($p \approx 1, q \leq 1 - c/b$) to not be invaded by defecting strategies. A higher generosity in the population results in a higher average cooperation rate. The average cooperation rate increases from 52% for $b = 3$ to 98% for $b = 10$. **(Limited Memory)** When players update their strategies based on their realized payoffs in the last round, there are two different predominant behaviors regardless of the benefit value. The resident population either consists of defectors (with $p \approx q \approx 0$) or of conditional cooperators. The maximum level of q , consistent with stable cooperation, is somewhat smaller compared to the perfect memory setting. Namely, in the Supplementary Information Section 3 we show that regardless of the value of benefit a conditional cooperator need to be of the form $q < \frac{1}{2}$ to not be invaded by defectors. The evolved cooperation rate only slightly increases from 37% ($b = 3$) to 51% ($b = 10$). Parameters: $N = 100$, $c = 1$, $\beta = 1$, $\delta = 0.999$.

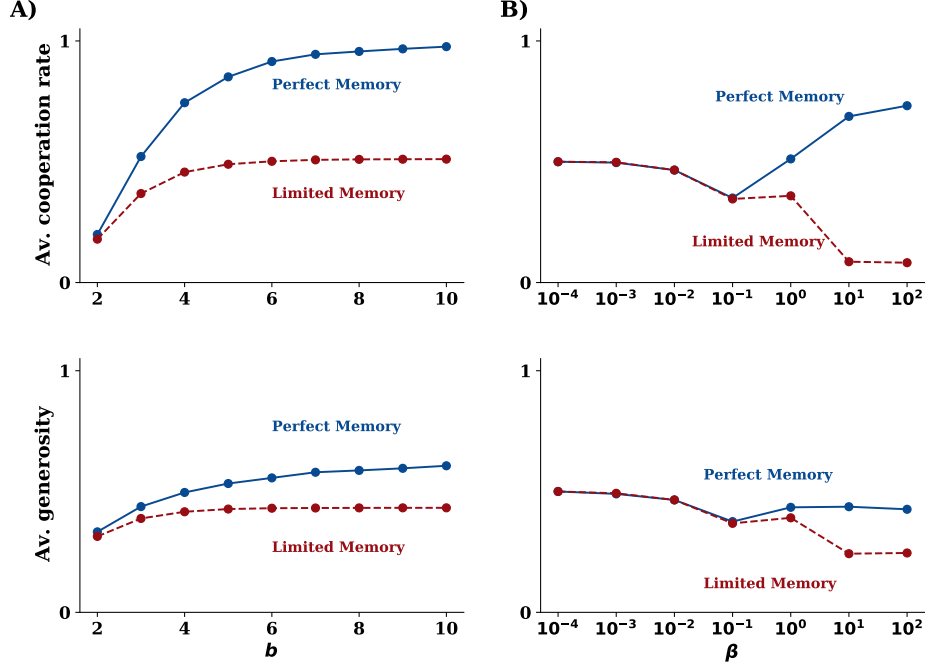


Figure 2: The evolution of cooperation and generosity for different values of benefit (A) and strength of selection (B). We report the average cooperation and the average reciprocity. The average cooperation rate is the average cooperation rate within the resident population. For the average reciprocity we select the residents that have a $p \approx 1$ and we take the average of their cooperation probability q . (A) We vary the benefit of cooperation b . In all cases, perfect memory updating payoffs appear to overestimate the average cooperation rate the population achieves. As expected in the case of limited memory the average generosity over the different values of benefit remains the same ($q \approx 0.5$), and as a result so does the average cooperation. (B) We vary the selection strength β . For weak selection, $\beta < 1$, the two methods yield similar results. However, as β increases in the case of limited memory payoffs the resident populations become more defective. Note that in the case of perfect payoff memory we see an increase in the cooperation rate even though the generosity remains stable. That is because the generosity does remain the same, however, now cooperative strategies remain fixed as the resident strategy for longer. Unless explicitly varied, the parameters of the simulation are $N = 100$, $b = 3$, $c = 1$, $\beta = 1$, $\delta = 0.99$. Simulations are run for $T = 5 \times 10^7$ time steps for each parameter combination.

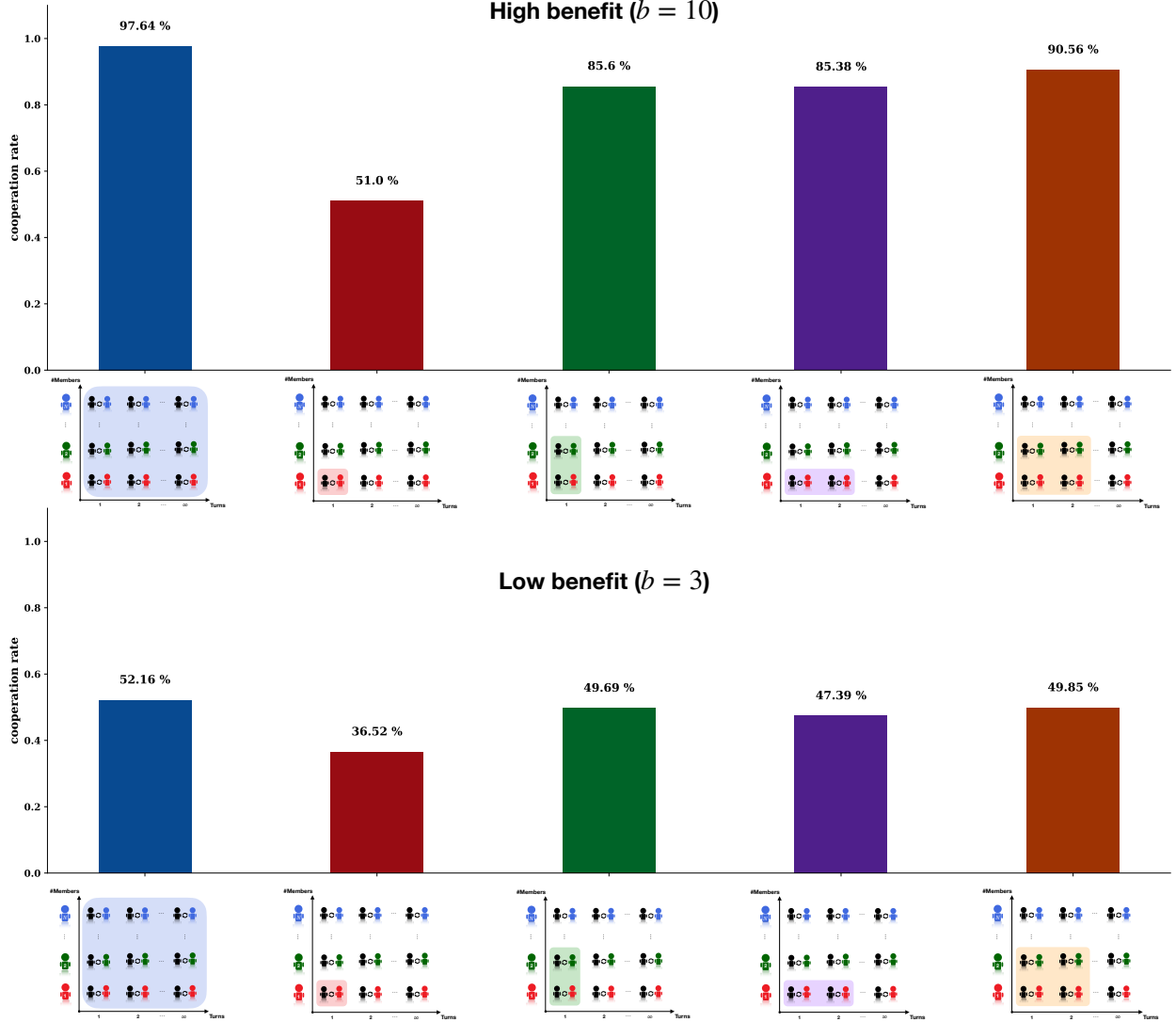


Figure 3: Average cooperation rates for different updating payoffs. From left to right, we present result on the following updating payoffs cases; (a) the expected payoffs (perfect memory), (b) the last round payoff from one interaction (limited memory), (c) the last round payoff from two interactions, (d) the last two rounds payoffs from one interaction, (e) the last two rounds payoffs from two interactions. For the updating payoffs (c) and (d) we have carried out an analysis which has shown that conditional cooperators should adopt a q value smaller than $\frac{\delta - 1 + \frac{\sqrt{2}}{2}}{\delta}$ and $\frac{\delta + \sqrt{\delta^2 + 1} - 1}{2\delta}$ respectively. Note that as $\delta \rightarrow 1$ both right hand sides tend to $\frac{\sqrt{2}}{2}$. Regardless the cooperation rate between for case (c) is slightly higher. In case (d) the cooperation rate is the second highest hinting that as we allow for more information the closer we move to the perfect payoff memory. We performed four pairwise non parametric tests (Mann-Whitney U) to compare the cooperation distributions of the residents in case (b) to cases (a), (c), (d), (e). In all four tests we reject the null hypothesis with $\alpha = 0.05$ and $p \approx 0$. Thus, there is significant difference between the cooperation rates. Unless explicitly varied, the parameters of the simulation are $N = 100$, $b = 3$, $c = 1$, $\beta = 1$, $\delta = 0.99$. Simulations are run for $T = 5 \times 10^7$ time steps for each parameter combination.