Prisoner's dilemma with stochastic payoffs

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1 Introduction

Evolutionary game theory is a mathematical framework for modeling evolution in biological, social and economical systems. The success of individuals is not constant, but depends on the composition of the population. Individuals interact according to rules of the game and their payoff is translated into reproductive fitness.

Most existing evolutionary game studies work with idealized assumptions. As stated in [1] on of these assumptions is that individuals play many times and with all other players before reproduction takes place, so that payoffs, equivalently fitness, are given by the mean distribution of types in the population. Both hypotheses, common in biological evolution, imply that selection occurs much more slowly than the interaction between individuals. Although recent experimental studies show that this may not always be the case in biology, it is clear that in cultural evolution or social learning the time scale of selection is much closer to the time scale of interaction. And these assumptions can change the outcome of the dynamics. The work of [1] we show that rapid selection affects evolutionary dynamics in such a dramatic way that for some games it even changes the stability of equilibria.

When modeling how individuals make decisions in each round, these models assume that players only remember the last round. However, when modeling how individuals update their strategies over time, individuals are assumed to have perfect memory. Here, we explore how robust our understanding of cooperation is as models deviate from the perfect memory assumption [2–4].

Evolutionary game dynamics have extensively been studied from mostly deterministic models based on rate equations to stochastic individual-based models. These more sophisticated models These more sophisticated models use a microscopic process as a starting point to determine how successful strategies spread. As stated in there are two classes have been used extensively: (i) fitness-based processes in which an individual chosen proportional to fitness reproduces and the offspring replaces a randomly chosen individualPairwise comparison processes in which a pair of individuals is chosen, and where subsequently one of these individuals may adopt the strategy of the other. This adoption occurs with a probability that depends on the payoff of both individuals, such that better players are more likely to be imitated than those who do worse.

We follow a Pairwise comparison processes. N individuals interact by playing a game and reproduce by selecting one individual, with probability proportional to the payoff, to duplicate and substitute a randomly chosen individual. The payoff of every player is set to zero after each reproduction event, and this two-step cycle is repeated until the population eventually stabilizes. This stochastic dynamics is discrete in both population and time, while keeping the population size constant over time. Interestingly, this microscopic dynamics leads to a difference equation that has been proposed as an adjusted [1] or discrete-time [2] analogous of the replicator equation, widely used in evolutionary game theory (see [9] for a recent, detailed discussion of this issue). Additionally, we note that for social applications, reproduction may be also interpreted as a learning process, in which individuals do not die but instead change the way they behave or their strategies.

As for the game, we consider the donation game where each player can cooperate by providing a benefit b to the other player at their cost c, with $0 < c < b$.

$$\begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix} \tag{1}$$

However, we will also consider the important case of symmetric 2×2 games, in which the payoffs are given by the following matrix:

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix} \tag{2}$$

For the donation game ($T = b, R = b - c, S = -c, P = 0$, and matrix (**??**)).

Let n be the number of individuals using strategy 1, also referred as type 1 individuals. After each reproduction event n may stay the same, increase by one, or decrease by one.

The individual chosen for reproduction A replaces B with probability p, which depends on the payoff difference between the two individuals. The composition of the population can only change if both individuals are of different types. We follow (Blume, 1993, Szabó and Tőke, 1998, Hauert and Szabó, 2005) in choosing the Fermi function from statistical physics for p

n both types of processes, the relative influence of the game is controlled by an external parameter, the so- called intensity of selection β. This parameter has strong parallels to the inverse temperature in statistical mechanics [16]

2 R.1

References

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