A meta analysis of tournaments and an evaluation of performance in the Iterated Prisoner's Dilemma.

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Abstract

The Iterated Prisoner's Dilemma has been used for decades as a model of behavioural interactions. From the celebrated performance of Tit for Tat, to the introduction of the zero-determinant strategies, to the use of sophisticated structures such as neural networks, the literature has been exploring the performance of strategies in the game for years. Most of these strategies are now accessible due to an open source package; Axelrod-Python. This manuscript make use of Axelrod-Python to conduct a meta analysis of Iterated Prisoner's Dilemma tournaments. The aim is to evaluate the performance of 194 strategies over 45686 Iterated Prisoner's Dilemma tournaments and to explore the factors of their success.

1 Background

The Iterated Prisoner's Dilemma (IPD) is a repeated two player game that models behavioural interactions, and more specifically, interactions where self-interest clashes with collective interest. At each turn of the game both players, simultaneously and independently, decide between cooperation (C) and defection (D) whilst having memory of their prior interactions. The payoffs for each player, at each turn, is influenced by their own choice and the choice of the other player. The payoffs of the game are generally defined by:

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix}$$

where $T > R > P > S$ and $2R > T + S$. The most common values used in the literature [17] are $R = 3, P = 1, T = 5, S = 0$. These values are also used in this work.

Conceptualising strategies and understanding the best way of playing the game has been of interest to the scientific community since the formulation of the game in 1950 [29]. Following the computer tournaments of Axelrod in the 1980's [15, 16], a strategy's performance in a round robin computer tournament became a common evaluation technique for newly designed strategies. Today more than 200 strategies exist in the literature and several tournaments, excluding Axelrod's, have been undertaken [20, 33, 37, 57, 58].

In the 80's, Axelrod performed two computer tournaments [15, 16]. The contestants were strategies submitted in the form of computer code. They competed against all other entries, a copy of themselves and a random strategy. The winner was decided on the average score a strategy achieved. The winner of both tournaments was the simple strategy Tit For Tat which cooperated on the first turn and then simply copied the previous action of it's opponent. Due to the strategy's strong performance in both tournaments, and moreover in a series of evolutionary experiments [17], Tit For Tat was thought to be the most robust basic strategy in the IPD.

However, further research proved that the strategy had weakness, and more specifically, it was shown that the strategy suffered in environments with noise [20, 28, 47, 56]. This was mainly due to the strategy's lack of generosity and contrition. The strategy was quick to punish a defection, and in a noisy environment it could lead to a repeated cycle of defections and cooperations. Some new strategies, more robust in tournaments with noise, were soon introduced and became the new protagonists of the game. These include Nice and Forgiving [20], Pavlov [49] and Generous Tit For Tat [50].

In 2004, a 20th Anniversary Iterated Prisoner Dilemma Tournament took place with 233 entries. This time the winning strategy was not designed on a reciprocity based approach but on a mechanism of teams [26, 27, 54]. A team from Southampton University took advantage of the fact that a participant was allowed to submit multiple strategies. They submitted a total of 60 strategies that could recognised each other and colluded to increase one members score. This resulted with three of the strategies to be ranked in the top spots. The performance of the Southampton University team received mixed attention, though they had won the tournament as stated in [25] "technically this strategy violates the spirit of the Prisoner's Dilemma, which assumes that the two prisoners cannot communicate with one another".

Another set of IPD strategies that have received a lot of attention are the zero-determinants [52]. By forcing a linear relationship between the payoffs the zero-determinant strategies can ensure that they will never receive less than their opponents. The American Mathematical Society's news section stated that "the world of game theory is currently on fire". Zero-determinant strategies are indeed a set of mathematically unique strategies and robust in pairwise interactions, however, their simplicity and extortionate behaviour have been tested. In [33] a tournament containing over 200 strategies, including zero-determinants, was ran. None of the zero-determinant strategies ranked in top spots. Instead, the top ranked strategies were a set of trained strategies based lookup tables [14], hidden markov models [33] and finite state automata [45].

Though only a select pieces of work have been discussed, the IPD is rich, and new strategies and competitions are being published every year. The question, however, still remains the same: what is the best way to play the game?

This manuscript uses an open source package called Axelrod-Python [4] to simulate a large number of computer tournaments, using as many strategies as possible from the literature. The aim is to evaluate the performance of 194 strategies in IPD tournaments, and futhermore, explore the factors of their success. This is done not only for standard round robin tournaments, but also for tournaments with noise and tournaments with a probabilistic ending. The different tournament types as well as the data collection are covered in Section 2. Section 3, focuses on the best performing strategies for each type of tournament and overall. Section 4, explores the traits which contribute to good performance, and finally the results are summarised in Section 5.

This manuscripts uses several parameters. These are introduced in the following sections, however, the full set of parameters and their definitions are given in Appendix A. All the data is available at [32].

2 Data collection

For the purposes of this manuscript a data set containing results of IPD tournaments has been generated and is available at [32]. This was done using the open source package Axelrod-Python [4], and more specifically, version 3.0.0. Axelrod-Python allows for different types of IPD computer tournaments to be simulated whilst containing a list of over 180 strategies. Most of these are strategies described in the literature with a few exceptions being strategies that have been contributed specifically to the package. This paper make use of 194 strategies implemented in version 3.0.0. A list of the strategies is given in the Appendix B. Though Axelrod-Python features several tournament types, this work considers only standard, noisy, probabilistic

ending and noisy probabilistic ending tournaments.

Standard tournaments, are tournaments similar to that of Axelrod's in [15]. There are N strategies which all play an iterated game of n number of turns against each other. Note that self interactions are not included. Similarly, **noisy tournaments** have N strategies and p_n number of turns, but at each turn there is a probability p_n that a player's action will be flipped. **Probabilistic ending tournaments**, are of size N and after each turn a match between strategies ends with a given probability p_e. Finally, **noisy probabilistic ending** tournaments have both a noise probability p_n and an ending probability p_e. For smoothing the simulated results a tournament is repeated for k number of times. To evaluate the effect of smoothing this is allowed to vary. The winner of each tournament is based on the average score a strategy achieved and not by number of wins.

The process of collecting tournament results implemented in this manuscript is described by Algorithm 1. For each trial a random size N is selected, and from the 194 strategies a random list of N strategies is chosen. For the given list of strategies a standard, a noisy, a probabilistic ending and a noisy probabilistic ending tournament are performed and repeated k times. The parameters for the tournaments as well as the number of repetitions are selected once for each trial. The parameters and their respective minimum and maximum values are given by Table 1.

parameter	parameter explanation	min value	max value
N	number of strategies	3	195
k	number of repetitions	10	100
n	number of turns	1	200
p_n	probability of flipping action at each turn	0	1
p_e	probability of match ending in the next turn	0	1

Table 1: Data collection; parameters' values

The source code for the data collection as well as the source code for the analysis, which will be discussed in the following sections, have been written following best practices [5, 21]. It has been packaged and is available here.

Algorithm 1: Data collection Algorithm

---
foreach *seed* $\in [0, 11420]$ **do**

    $N \leftarrow$ randomly select integer $\in [N_{min}, N_{max}]$;

    players $\leftarrow$ randomly select N players;

    $k \leftarrow$ randomly select integer $\in [k_{min}, k_{max}]$;

    $n \leftarrow$ randomly select integer $\in [n_{min}, n_{max}]$;

    $p \leftarrow$ randomly select float $\in [p_{n\,min}, p_{n\,max}]$;

    $e \leftarrow$ randomly select float $\in [p_{e\,min}, p_{e\,max}]$;

    result standard $\leftarrow$ Axelrod.tournament(players, n, k);

    result noisy $\leftarrow$ Axelrod.tournament(players, n, p_n, k);

    result probabilistic ending $\leftarrow$ Axelrod.tournament(players, p_e, k);

    result noisy probabilistic ending $\leftarrow$ Axelrod.tournament(players, p_n, p_e, k);

return *result standard, result noisy, result probabilistic ending, result noisy probabilistic ending*;

A total of 11420 trials of Algorithm 1 have been run. For each trial the results for 4 different tournaments were collected, thus a total of 45686 (11420 × 4) tournament results have been retrieved. Each tournament outputs a result summary in the form of Table 2.

The result summary, Table 2, has N number of rows because each row contains information for each strategy that participated in the tournament. The information includes the strategy's rank, median score, the rate with which the strategy cooperated (C_r), its match win count and the probability that the strategy cooperated in the opening move. Moreover, the probabilities of a strategy being in any of the four states (CC, CD, DC, DD), and the rate of which the strategy cooperated after each state.

Rank	Name	Median score	Cooperation rating (C_r)	Win	Initial C	CC	CD	DC	DD	CC to C	CD to C	DC to C	DD to C
0	EvolvedLookerUp2 2 2	2.97	0.705	28.0	1.0	0.639	0.066	0.189	0.106	0.836	0.481	0.568	0.8
1	Evolved FSM 16 Noise 05	2.875	0.697	21.0	1.0	0.676	0.020	0.135	0.168	0.985	0.571	0.392	0.07
2	PSO Gambler 1 1 1	2.874	0.684	23.0	1.0	0.651	0.034	0.152	0.164	1.000	0.283	0.000	0.136
3	PSO Gambler Mem1	2.861	0.706	23.0	1.0	0.663	0.042	0.145	0.150	1.000	0.510	0.000	0.122
4	Winner12	2.835	0.682	20.0	1.0	0.651	0.031	0.141	0.177	1.000	0.441	0.000	0.462
...	...	...	...	...	...	...	...	...	...	...	...	...	...

The header "Rates" spans the columns CC, CD, DC, DD, CC to C, CD to C, DC to C, DD to C.

Table 2: Output result of a single tournament.

A measure that has been manually included is the **normalised rank**. The normalised rank, denoted as r, is calculated as a strategy's rank divided by the tournament's size (N). The normalised rank will be used in the next section to evaluate the performance of strategies.

Table A summary of the number of entries each strategy has is given by Table. Each strategy participated on average in 5180 tournaments of each type. The strategy that participated in the most tournaments () is and the strategy with the minimum enties is.

	count	mean	std	min	25%	50%	75%	max
entries	195.0	5154.0	188.0	4693.0	5041.0	5147.0	5286.0	5639.0

3 Top ranked strategies

This section evaluates the performance of 194 IPD strategies. The performance of each strategy is evaluated in four tournament types, which were presented in Section 2, followed by an evaluation of their performance over all the 45686 simulated tournaments of this work.

Each strategy participated in multiple tournaments of the same type (on average 5180). For example Tit For Tat has participated in a total of 5114 tournaments of each type. The strategy's normalised rank distribution in these is given in Figure 1. A value of $r = 0$ corresponds to a strategy winning the tournament where a value of $r = 1$ corresponds to the strategy coming last. Because of the strategies' multiple entries their performance is evaluated based on the **median normalised rank** denoted as $\bar{r}$.

The top 15 strategies for each tournament type based on $\bar{r}$ are given in Table 3.

In standard tournaments 10 out of the 15 top strategies are introduced in [33]. These are strategies based on finite state automata (FSM), hidden markov models (HMM), artificial neural networks (ANN), lookup tables (LookerUp) and stochastic lookup tables (Gambler) that have been trained using reinforcement learning algorithms (evolutionary and particle swarm algorithms). They have been trained to perform well against the strategies in [4] in a standard setting, thus their performance in the specific setting was anticipated.
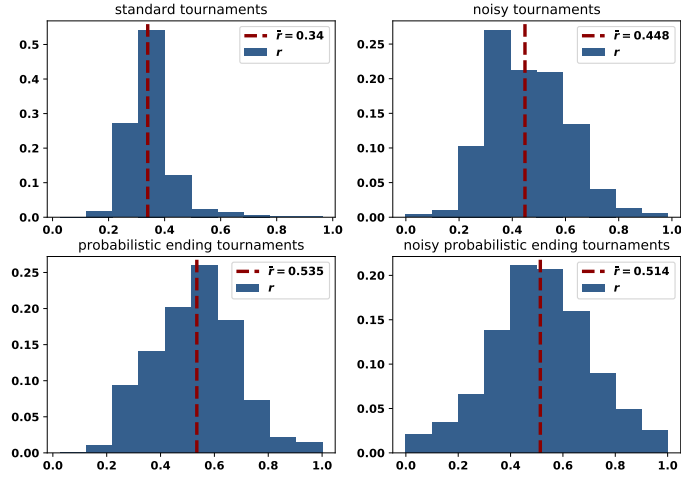
Figure 1: Tit For Tat's r distribution in tournaments. The best performance of the strategy has been in standard tournaments where it achieved a $\bar{r}$ of 0.339.

	Standard		Noisy		Probabilistic ending		Noisy probabilistic ending	
	Name	$\bar{r}$	Name	$\bar{r}$	Name	$\bar{r}$	Name	$\bar{r}$
0	Evolved HMM 5	0.00667	Grumpy	0.14020	Fortress4	0.01266	Alternator	0.30370
1	Evolved FSM 16	0.00995	e	0.19388	Defector	0.01429	phi	0.30978
2	EvolvedLookerUp2 2 2	0.01064	Tit For 2 Tats	0.20617	Better and Better	0.01587	e	0.31250
3	Evolved FSM 16 Noise 05	0.01667	Slow Tit For Two Tats	0.20962	Tricky Defector	0.01875	pi	0.31686
4	PSO Gambler 2 2 2	0.02143	Cycle Hunter	0.21538	Fortress3	0.02174	Limited Retaliate	0.35263
5	Evolved ANN	0.02878	Risky QLearner	0.22222	Gradual Killer	0.02532	Anti Tit For Tat	0.35431
6	Evolved ANN 5	0.03390	Retaliate 3	0.22887	Aggravater	0.02778	Retaliate 3	0.35563
7	PSO Gambler 1 1 1	0.03704	Cycler CCCCCD	0.23507	Raider	0.03077	Limited Retaliate 3	0.35563
8	Evolved FSM 4	0.04891	Retaliate 2	0.23913	Cycler DDC	0.04545	Retaliate	0.35714
9	PSO Gambler Mem1	0.05036	Defector Hunter	0.24038	Hard Prober	0.05128	Retaliate 2	0.35767
10	Winner12	0.06011	Retaliate	0.24177	SolutionB1	0.06024	Limited Retaliate 2	0.36134
11	Fool Me Once	0.06140	Hard Tit For 2 Tats	0.25000	Meta Minority	0.06077	Hopeless	0.36842
12	DBS	0.07143	ShortMem	0.25286	Bully	0.06081	Arrogant QLearner	0.40651
13	DoubleCrosser	0.07200	Limited Retaliate 3	0.25316	Fool Me Forever	0.07080	Cautious QLearner	0.40909
14	BackStabber	0.07519	Limited Retaliate	0.25706	EasyGo	0.07101	Fool Me Forever	0.41764

Table 3: Top performances for each tournament type based on $\bar{r}$.

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DoubleCrosser, and Fool Me Once, are strategies not from the literature but from [4]. DoubleCrosser is a strategy that makes use of the number of turns because is set to defect on the last two rounds. The strategy was expected to not perform as well in tournaments where the number of turns is not specified, but the strategy did not perform well in tournaments with noise either. Finally, Winner 12 [44] and DBS [13] are both from the the literature. DBS is strategy specifically designed for noisy environments, however, it ranks highly only in standard ones.

Figure 2 gives the distributions of r for the top ranked strategies. The distributions are skewed towards zero and the highest median is at 0.075. This indicates that the top ranked strategies are very likely to perform well in a standard tournament despite the tournament size, the opponents, the turns etc.
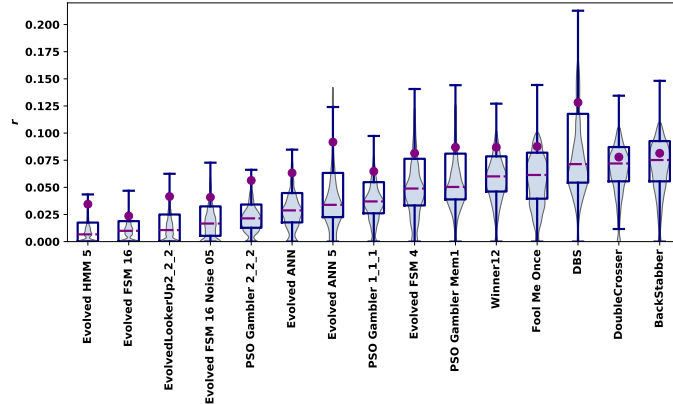
Figure 2: r distributions of top 15 strategies in standard tournaments.

The top strategies in noisy tournaments are shown in Figure 3. These include deterministic strategies, such as Tit For 2 Tats [16], Hard Tit For 2 Tats [58] and Cycler CCCCCD, and strategies which decide their actions based on the cooperations to defections ratio, such as ShortMem [23], Grumpy, e, π and ϕ (all from [4]). The Retaliate and Limited Retaliate strategies are implemented in [4] by the same contributor. The strategies are designed to defect if the opponent has tricked the strategy more often than $x\%$ of the times that the strategy has done the same. Finally, in 4th and 9th are Hunter strategies which trying to extort, equivalently, strategies that play cyclically and defectors.

From Figure 3, it is evident that the normalised rank distributions in noisy environments are more variant with higher medians compared to standard tournaments. The distributions are bimodal. This indicates that although the top ranked strategies mainly performed well, there are several tournaments that they ranked in the bottom half.

The 15 top ranked strategies in probabilistic ending tournaments include Fortress 3, Fortress 4 (both introduced in [11]), Raider [12] and Solution B1 [12], which are strategies based on finite state automata introduced by Daniel and Wendy Ashlock. These strategies have been evolved using reinforcement learning, however, there were trained to maximise their payoffs in tournaments with fixed turns (150 specifically) and not in probabilistic ending ones. In probabilistic ending tournaments it appears that the top ranks are mostly occupied by defecting strategies. These include Better and Better, Gradual Killer, Hard Prober (all from [1]), Bully (Reverse Tit For Tat) [48] and Defector. Thus, it's surprisingly that EasyGo and Fool Me Forever are ranked 14th and 15th. These strategies are actually the same; they will defect until their opponent defect, then they will cooperate until the end. Both strategies have repeatedly ranked highly as shown in Figure 4, and there are cases for which they were the winners of the tournament.

The distributions of the normalised rank in probabilistic ending tournaments, shown in Figure 4, are less variant than those of noisy tournaments. The medians are lower than 0.1 and the distributions are skewed
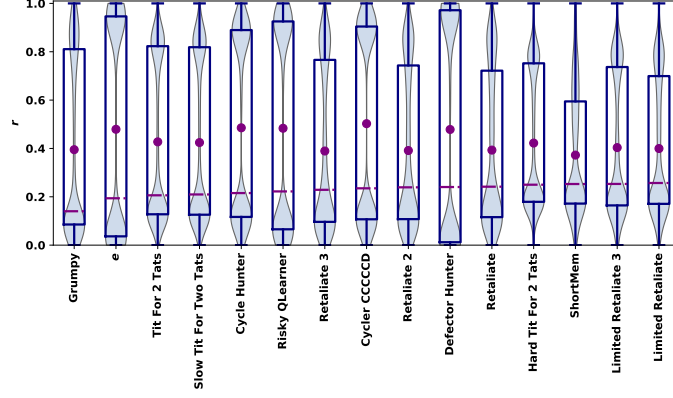
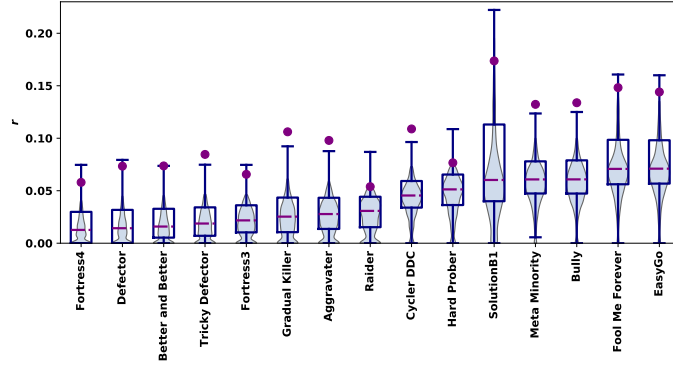Figure 3: r distributions for best performed strategies in noisy tournaments.

Figure 4: r distributions for best performed strategies in probabilistic ending tournaments.

towards 0. Though the large difference between the means and the medians indicates some outliers, the strategies have overall performed well in the probabilistic ending tournaments that they participated.

In tournaments with both noise and an unspecified number of turns several of the top ranked strategies are strategies that were highly ranked in noisy tournaments. This demonstrates that, e, π, ϕ and the Retaliate family are robust in noisy environments regardless of the number of turns. However, strategies from the top ranks in probabilistic ending tournaments did not rank highly here.

Up till now, the performances of the 194 strategies have been evaluated for individual tournament types. The data set considered in this work, described in Section 2, contains a total of 45686 tournament results. For this part of the manuscript the strategies are ranked based on the median normalised rank they achieved over the entire data set. The top 15 strategies are given in Table 4 and their normalised rank distributions are given in Figure 6.

The top ranks include strategies that have been previously mentioned. The top ranks are overtaken by the set of Retaliate strategies followed by BackStabber and DoubleCrosser. DoubleCrosser performed well in standard tournaments and the strategy is just an extension of BackStabber. PSO Gambler and Evolved HMM 5 are trained strategies introduced in [33] and Nice Meta Winner and NMWE Memory One are strategies based on teams. Grudger is a strategy from Axelrod's original tournament and Forgetful Fool Me Once is based on the same approach as Grudger.

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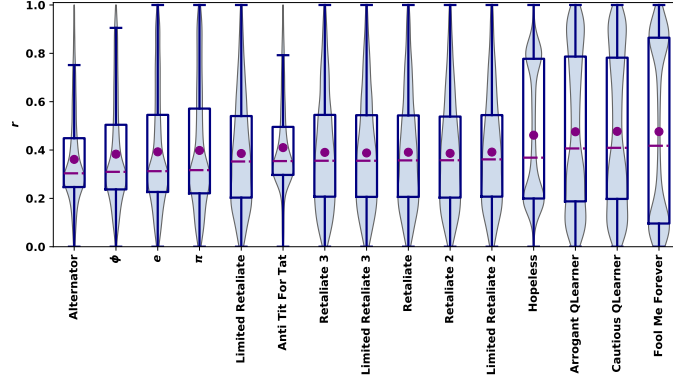
Figure 5: r distributions for best performed strategies in noisy probabilistic ending tournaments.

Name	Normalized_Rank
Limited Retaliate 3	0.28609
Retaliate 3	0.29630
Retaliate 2	0.30250
Limited Retaliate 2	0.30328
Limited Retaliate	0.31000
Retaliate	0.31707
BackStabber	0.32381
DoubleCrosser	0.33136
Nice Meta Winner	0.34921
PSO Gambler 2_2_2 Noise 05	0.35146
Grudger	0.35156
Evolved HMM 5	0.35714
NMWE Memory One	0.35714
Nice Meta Winner Ensemble	0.35870
Forgetful Fool Me Once	0.35884

Table 4: Top performances over all the tournaments

Figure 6 gives the normalised rank distributions of these strategies. The distributions of the Retaliate strategies have no statistical difference. Thus, in an IPD tournament where the type is not specified, playing as any of the Retaliate strategies will have the result.
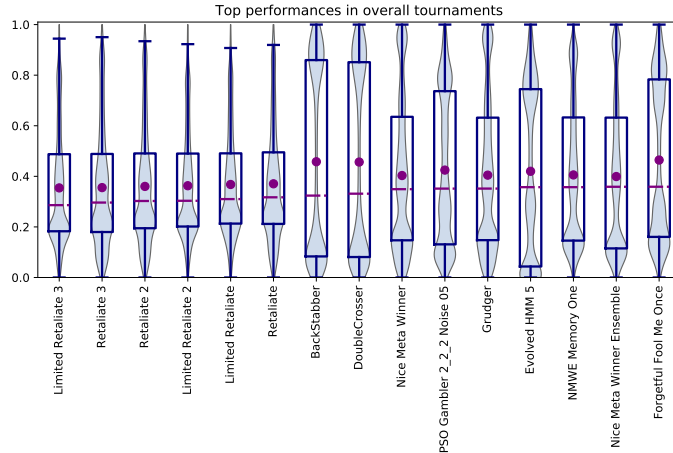
Figure 6: r distributions for best performed strategies in the data set [32].

This section presented the winning strategies in a series of IPD tournaments. In standard tournaments the top spots were dominated by complex strategies that had been trained using reinforcement learning techniques. In noisy environments, whether the number of turns was fixed or not, the winning strategies were deterministic strategies designed to defect if the opponent tricked them more than a current amount of the times that they had tricked their opponent. In probabilistic ending tournaments most of the winning strategies were defecting strategies. A few trained finite state automata, designed by the same authors, also performed well in this setting. Finally the performance of all 194 strategies over the 45686 tournaments in this manuscript was assessed on $\bar{r}$. The top ranked strategies were a mixture of behaviours that did well in standard tournaments and tournaments with noise, as well as a few strategies based on teams.

The results of this section imply that successful strategies for specific settings exist for an IPD tournament. The top ranked strategies in both standard tournaments and tournaments with probabilistic ending, managed to rank in the top 10% of the tournament most of the times. Strategies successful in noisy environments demonstrated that they were not affected by the number of turns. However, there has been not a single strategy that has shown to perform well in more than one setting. The aim of the next section is to understand which are the factors that made these strategies successful. In each setting separately but also overall.

4 Evaluation of performance

The aim of this section is to explore the factors that contribute to a strategy's successful performance. The factors explored are measures regarding a strategy's behaviour, alogn with measures regarding the tournaments the strategies competed in. These are given in Table 5.

Axelrod-Python makes use of classifiers to classify strategies according to various dimensions. These determine whether a strategy is stochastic or deterministic, whether it makes use of the number of turns or the game's payoffs. The memory usage measure is calculated as the memory size of strategy (which is specified in the strategies implementation in [4]) divide by the number of turns. For tournaments with a probabilistic

measure	measure explanation	source	value type	min value	max value
stochastic	If a strategy is stochastic	strategy classifier from [4]	boolean	False	True
makes use of game	If a strategy makes used of the game information	strategy classifier from [4]	boolean	False	True
makes use of length	If a strategy makes used of the number of turns	strategy classifier from [4]	boolean	False	True
memory usage	The memory size of a strategy divided by the number of turns	memory size from [4]	float	0	1
SSE	A measure of how far a strategy is from ZD behaviour	method described in [39]	float	0	1
max cooperating rate ($C_{\max}$)	The biggest cooperating rate in the tournament	result summary	float	0	1
min cooperating rate ($C_{\min}$)	The smallest cooperating rate in the tournament	result summary	float	0	1
median cooperating rate (C_{median})	The median cooperating rate in the tournament	result summary	float	0	1
mean cooperating rate (C_{mean})	The mean cooperating rate in the tournament	result summary	float	0	1
$C_r / C_{\max}$	A strategy's cooperating rate divided by the maximum	result summary	float	0	1
$C_r / C_{\min}$	A strategy's cooperating rate divided by the minimum	result summary	float	0	1
$C_r / C_{\mathrm{median}}$	A strategy's cooperating rate divided by the median	result summary	float	0	1
C_r / C_{mean}	A strategy's cooperating rate divided by the mean	result summary	float	0	1
C_r	The cooperating ratio of a strategy	result summary	float	0	1
CC to C rate	The probability a strategy will cooperate after a mutual cooperation	result summary	float	0	1
CD to C rate	The probability a strategy will cooperate after being betrayed by the opponent	result summary	float	0	1
DC to C rate	The probability a strategy will cooperate after betraying the opponent	result summary	float	0	1
DD to C rate	The probability a strategy will cooperate after a mutual defection	result summary	float	0	1
p_n	The probability of a player's action being flip at each interaction	trial summary	float	0	1
n	The number of turns	trial summary	integer	1	200
p_e	The probability of a match ending in the next turn	trial summary	float	0	1
N	The number of strategies in the tournament	trial summary	integer	3	195
k	The number of repetitions of a given tournament	trial summary	integer	10	100

Table 5: The measures which are included in the performance evaluation analysis.

ending the number of turns has been not collected, so the memory usage measure is not used for probabilistic ending tournaments. The SSE is a measure of extortionate behaviour introduced in [39]. A value of 1 indicates no extortionate behaviour at all whereas a value of 0 indicates that a strategy is always trying to extortion it's opponent. The rest of the factors considered are the CC to C, CD to C, DC to C, and DD to C rates as well as cooperating ratio of a strategy. The minimum, maximum, medium and median cooperating ratios of each tournament are also included, and finally the number of turns, the number of strategies and the probabilities of noise and the game ending are also included.

Table 6 shows the correlation coefficients between the measures of Table 5 the median score and the median normalised rank. Note that the correlation for the classifiers is not included because they are binary variables and they will be evaluated using a different method. The correlation coefficients for all the measures in Table 5 against themselves have also been calculated and a graphical representation can be found in the Appendix C.

In standard tournaments the measures CC to C, C_r, $C_r/C_{\max}$ and the cooperating ratio compared to C_{median} and C_{mean} have a moderate negative effect on the normalised rank, and a moderate positive on the median score. The SSE error and the DD to C have the opposite effects. Thus, in standard tournaments behaving cooperatively corresponds to a more successful performance. Even though being nice pays off, that's not true against defective strategies. Cooperating after a mutual defection lowers a strategy's success. Figure 7 confirms that the winners of standard tournaments always cooperate after a mutual cooperation and almost always defects after a mutual defection.
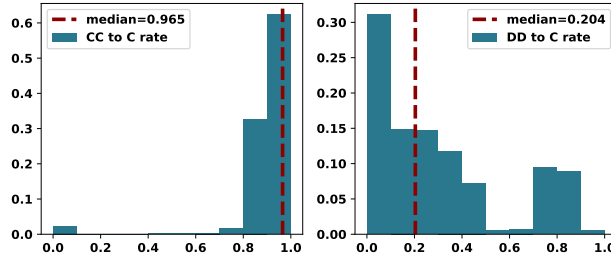

Figure 7: Distributions of CC to C and DD to C for the winners in standard tournaments.

	Standard		Noisy		Probabilistic ending		Noisy probabilistic ending		Overall	
	r	median score	r	median score	r	median score	r	median score	r	median score
CC to C rate	-0.501	0.501	0.414	-0.504	0.408	-0.323	0.260	0.022	0.108	0.081
CD to C rate	0.226	-0.199	0.456	-0.330	0.320	-0.017	0.205	-0.220	0.281	-0.177
C_r	-0.323	0.384	0.711	-0.678	0.714	-0.832	0.579	-0.135	0.360	-0.124
C_r / C_{max}	-0.323	0.381	0.616	-0.551	0.714	-0.833	0.536	-0.116	0.395	-0.265
C_r / C_{mean}	-0.331	0.358	0.731	-0.740	0.721	-0.861	0.649	-0.621	0.428	-0.439
C_r / C_{median}	-0.331	0.353	0.652	-0.669	0.712	-0.852	0.330	-0.466	0.294	-0.405
C_r / C_{min}	0.109	-0.080	-0.358	0.250	-0.134	0.150	-0.368	0.113	-0.161	-0.190
C_{max}	-0.000	0.049	0.000	0.023	-0.000	0.046	0.000	-0.004	0.000	0.280
C_{mean}	-0.000	0.229	-0.000	0.271	0.000	0.200	0.000	0.690	-0.000	0.553
C_{median}	0.000	0.209	-0.000	0.240	-0.000	0.187	-0.000	0.673	-0.000	0.544
C_{min}	0.000	0.084	0.000	-0.017	-0.000	0.007	-0.000	0.041	0.000	-0.250
DC to C rate	0.127	-0.100	0.509	-0.504	-0.018	0.033	0.341	-0.016	0.173	-0.088
DD to C rate	0.412	-0.396	0.533	-0.436	-0.103	0.176	0.378	-0.263	0.237	-0.239
N	0.000	-0.009	-0.000	0.002	-0.000	0.003	-0.000	0.001	-0.000	-0.001
k	0.000	-0.002	-0.000	0.003	-0.000	0.001	-0.000	-0.008	0.000	-0.001
n	0.000	-0.125	-0.000	-0.024	-	-	-	-	0.000	-0.074
p_e	-	-	-	-	0.000	0.165	0.000	-0.058	0.000	0.055
p_n	-	-	-0.000	0.207	-	-	-0.000	-0.650	-0.000	-0.256
Make use of game	-0.003	-0.022	0.025	-0.082	-0.053	-0.108	0.013	-0.016	-0.004	-0.053
Make use of length	-0.158	0.124	0.005	-0.123	-0.025	-0.090	0.014	-0.016	-0.044	-0.028
SSE	0.473	-0.452	0.463	-0.337	-0.156	0.223	0.305	-0.259	0.233	-0.167
memory usage	-0.082	0.095	-0.007	-0.017	-	-	-	-	-0.046	0.040
stochastic	0.006	-0.024	0.022	-0.026	0.002	-0.130	0.021	-0.013	0.013	-0.048

Table 6: Correlations table between the measures of Table 5 the normalised rank and the median score.

Compared to standard tournaments, in both noisy and in probabilistic ending tournaments the higher the rates of cooperation the lower a strategy's success and median score. A strategy would want to cooperate less than both the mean and median cooperator in such settings. In probabilistic ending tournaments the correlations coefficients have a larger values, indicating a stronger effect. Thus a strategy will be punished more by it's cooperative behaviour in probabilistic ending environments. The distributions of the C_r of the winners in both tournaments is given by Figure 8. It confirms that the winners in noisy tournaments cooperated less than 35% of the times and in probabilistic ending tournaments less than 10%.
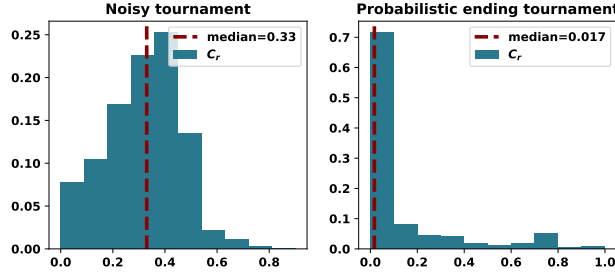
Figure 8: C_r distributions of the winners in noisy and in probabilistic ending tournaments.

In noisy probabilistic ending tournaments and in over all the tournaments' results, the only measures that had a moderate affect are $C_r/C_{\mathrm{mean}}, C_r/C_{\mathrm{max}}$ and C_r. In such environments cooperative behaviour appears to be punished by not as much as in noisy and probabilistic ending tournaments.

The effect of the measures on performances is further tested using a random forest classification [22] approach. Initially, the performances are clustered based on their normalised rank and the median score by a $k-$means algorithm [8]. The number of clusters is not deterministically chosen but it is based on the silhouette coefficients [55]. Consider the case of standard tournaments, the chosen number of clusters is 2 and Figure 9 illustrates the trials of clustering the performances in 2, 3 and 4 clusters respectively. The number of chosen

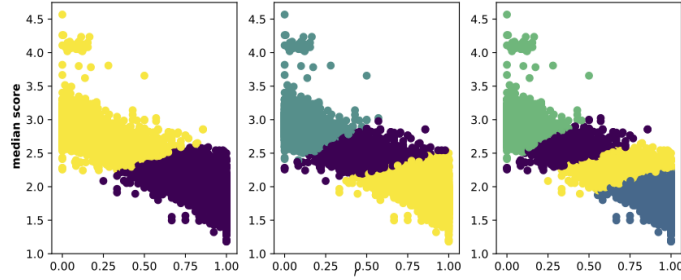clusters for each type and overall are given in Table 7.

Figure 9: Clustering trials for standard tournaments. A number of 2 clusters has been chosen with a silhouette score of 0.66 against 0.511 and 0.50 respectively.

Tournament type	Number of clusters	Silhouette coefficient
standard	2	0.648
noisy	3	0.493
probabilistic ending	2	0.680
noisy probabilistic ending	4	0.415
over 45686 tournaments	3	0.443

Table 7: Number of clusters for each type and over all the tournament results and the respective silhouette coefficients.

A random forest approach is applied to the data to predict the cluster to which a strategy's performance has been assigned to. The random forest method constructs many individual decision trees and the predictions from all trees are pooled to make the final prediction. The random forest models are trained on a training set of 70% of the tournaments results. The accuracy of each model based on R^2 are given by Table 8. The out of the bag error [34] has also been calculated. The models fit well, and a high value of both the accuracy measure on the test data and the OOB error indicate that the model is not over fitting.

Tournament type	R^2 training data	R^2 test data	R^2 OOB score
standard	0.998545	0.989890	0.982331
noisy	0.996677	0.950572	0.935383
probabilistic ending	0.999592	0.995128	0.992819
noisy probabilistic ending	0.990490	0.813905	0.791418
over 45686 tournaments	0.993396	0.913409	0.898059

Table 8: Accuracy metrics for random forest models.

The importance that the measures of Table 5 had on the classification task; to which cluster a performance was assigned to, have been calculated and are given by Figure 15. The classifiers which where not included in the previous analysis appear to have no effect, and several of the measures that are highted by the importance are inline with the correlation results. The most important measures based on the random forest analysis were C_r/C_{median} and C_r/C_{mean}.

The effect of both these measures can be further explored. Their distributions for the winners of the tournaments are given by Figures 16, 17, 18, 19 and 20. The results suggest that if a strategy with a theory of mind competed in a standard tournament, then the strategy would want to be the mean/median cooperator of that tournament. In comparison, if the same strategy competed in a tournament with noisy or the strategy had no information regarding the setting of the tournament, then it would want to cooperate 67% of the
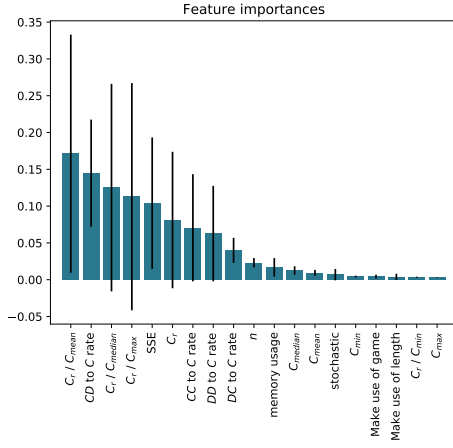
Figure 10: Standard tournaments
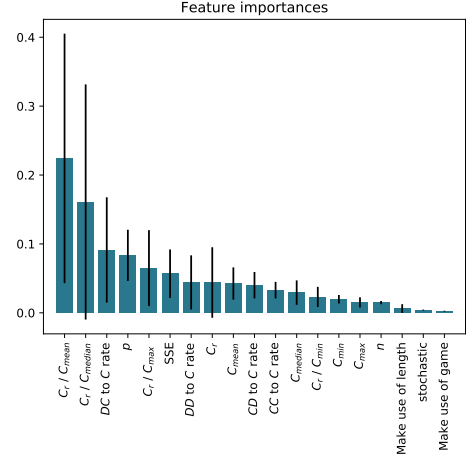
Figure 11: Noisy tournaments
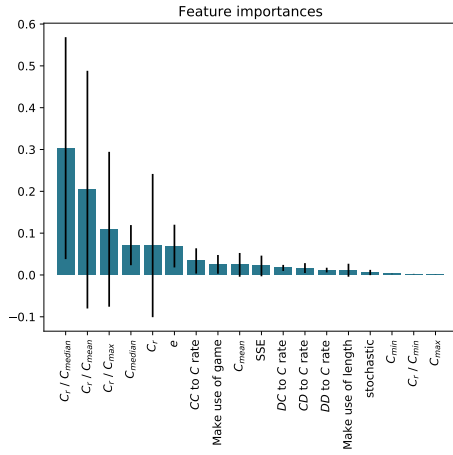
Figure 12: Probabilistic ending tournaments
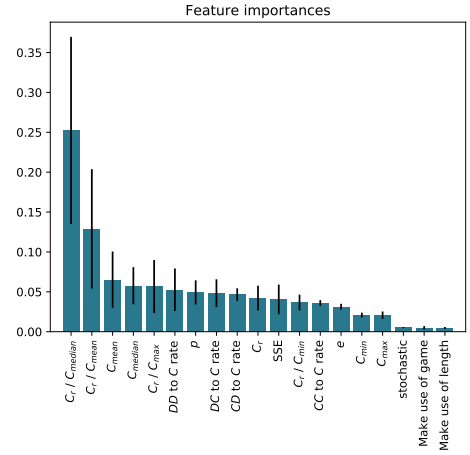
Figure 13: Noisy probabilistic ending tournaments
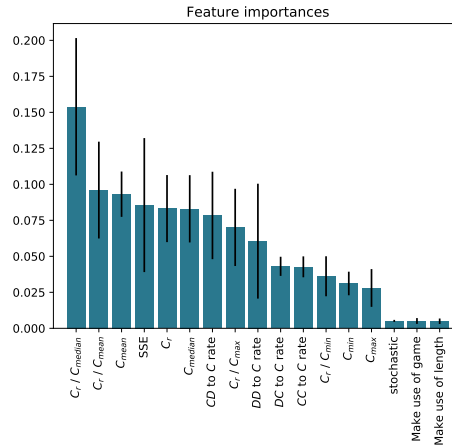
Figure 14: overall ending tournaments

Figure 15: Importance of features in IPD tournaments.

times the mean or median cooperator did. In probabilistic ending tournament it has already been mentioned that defecting strategies prevail. This result is once again confirmed in this section.
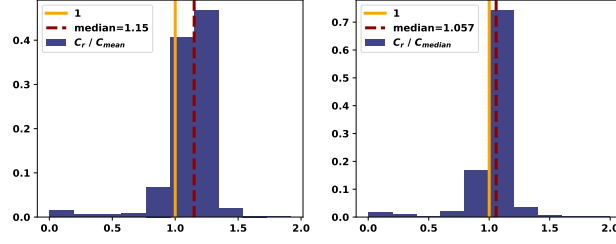

Figure 16: Distributions of C_r/C_{median} and C_r/C_{median} for winners of standard tournaments.
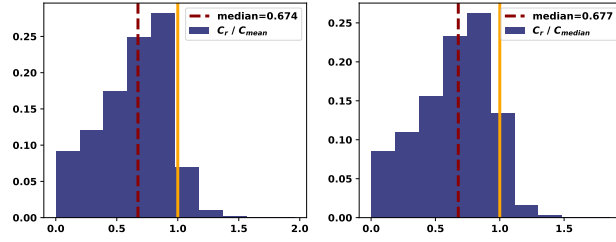
Figure 17: Distributions of C_r/C_{median} and C_r/C_{median} for winners of noisy tournaments.
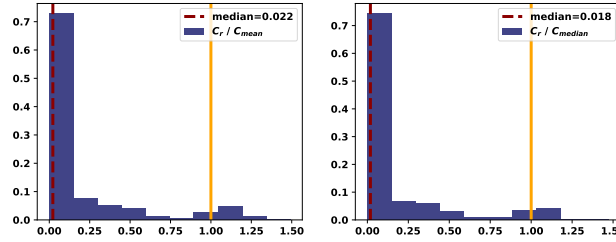
Figure 18: Distributions of C_r/C_{median} and C_r/C_{median} for winners of probabilistic ending tournaments.
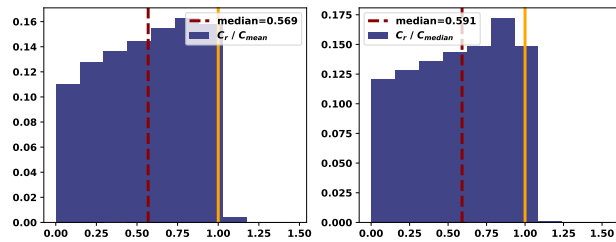
Figure 19: Distributions of C_r/C_{median} and C_r/C_{median} for winners of noisy probabilistic ending tournaments.

In this section the effect of several measures, regarding a strategy's behaviour and the tournament, on it's performance were presented. This was done using two approaches. Correlation coefficients and a random forest analysis. The results of these are discussed in the following section.
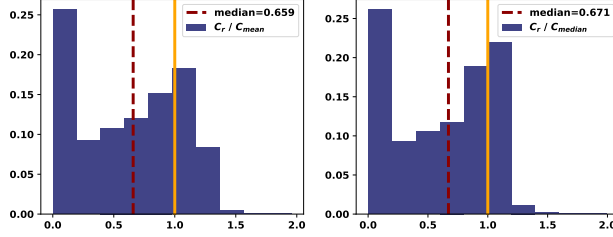
Figure 20: Distributions of C_r/C_{median} and C_r/C_{median} for winners of over all the tournaments.

5 Conclusion

This manuscript has explored the performance of 194 strategies of the Iterated Prisoner's Dilemma in a large number of computer tournaments. The results of the analysis demonstrated that, although for specific tournament type, such as standard, noisy and probabilistic ending tournaments, dominant strategies exist there is not a single dominant strategy if the environments vary. Moreover, a strategy with a theory of mind should aim to adapt its behaviour based on the mean and median cooperators.

The 194 strategies used in this manuscript have been mainly for the literature, and they have been accessible due to an open source software called Axelrod-Python. The software was used to generate a total of 45686 computer tournaments results with different number of strategies and different participants each time. The data collection was described in Section 2. In Section 3, the tournaments results were used to present the top performances. The data set contained results from four different settings, and these were also studied individually. In standard tournaments complex strategies trained using reinforcement learning ranked in the top places. In probabilistic ending tournaments, several trained strategies based on finite state automata also performed well. The rest top strategies, in this setting, were defecting strategies. Finally, in tournaments with noise, the Retaliate set of strategies demonstrated their robustness in tournaments with probabilistic ending and not.

Section 4, covered an analysis of performance based on several measures associated with a strategy and with the environments it was competing. The results of this analysis showed that a strategy's characteristics such as whether or not it's stochastic, and the information it used regarding the game had no effect on the strategy's success. The most important factors have been those that compared the strategy's behaviour to it's environment. The cooperating ratio of the strategy compared to the mean and median cooperator was highlighted as the most important feature in the analysis. More specifically, if a strategy were to enter a tournament with a theory of mind of it's environment it would choose to be the median cooperator in standard tournaments, the defector in probabilistic ending tournaments and to cooperate 60% of the times the median cooperator did in noisy and noisy probabilistic tournaments. Lastly, if a strategy was aware of the opponents but not of the setting on the tournament, a strategy would be more likely to be successful if it were to identify the median cooperator and cooperated 67% of the times that they did.

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6   Acknowledgements

A   A summary of parameters

measure	measure explanation
stochastic	If a strategy is stochastic
makes use of game	If a strategy makes used of the game information
makes use of length	If a strategy makes used of the number of turns
memory usage	The memory size of a strategy divided by the number of turns
SSE	A measure of how far a strategy is from extortionate behaviour
$(C_{\max})$	The biggest cooperating rate in the tournament
$(C_{\min})$	The smallest cooperating rate in the tournament
(C_{median})	The median cooperating rate in the tournament
(C_{mean})	The mean cooperating rate in the tournament
C_r / $C_{\max}$	A strategy's cooperating rate divided by the maximum
C_r / $C_{\min}$	A strategy's cooperating rate divided by the minimum
C_r / C_{median}	A strategy's cooperating rate divided by the median
C_r / C_{mean}	A strategy's cooperating rate divided by the mean
C_r	The cooperating ratio of a strategy
CC to C rate	The probability a strategy will cooperate after a mutual cooperation
CD to C rate	The probability a strategy will cooperate after being betrayed by the opponent
DC to C rate	The probability a strategy will cooperate after betraying the opponent
DD to C rate	The probability a strategy will cooperate after a mutual defection
p	The probability of a player's action being flip at each interaction
n	The number of turns
e	The probability of a match ending in the next turn
N	The number of strategies in the tournament

Table 9: The measures which are included in the performance evaluation analysis.

B   List of strategies

The strategies used in this study which are from Axelrod version 3.0.0 [4].

1. ϕ [4]
2. π [4]
3. e [4]
4. ALLCorALLD [4]
5. Adaptive [42]
6. Adaptive Pavlov 2006 [38]
7. Adaptive Pavlov 2011 [42]
8. Adaptive Tit For Tat: 0.5 [59]
9. Aggravater [4]
10. Alexei [3]
11. Alternator [17, 46]
12. Alternator Hunter [4]
13. Anti Tit For Tat [35]
14. AntiCycler [4]
15. Appeaser [4]
16. Arrogant QLearner [4]
17. Average Copier [4]
18. Backstabber [4]
19. Better and Better [1]
20. Bully [48]
21. Calculator [1]
22. Cautious QLearner [4]
23. Champion [16]
24. CollectiveStrategy [2]
25. Contrite Tit For Tat [63]
26. Cooperator [17, 46, 52]
27. Cooperator Hunter [4]
28. Cycle Hunter [4]

29. Cycler CCCCCD [4]
30. Cycler CCCD [4]
31. Cycler CCCDCD [4]
32. Cycler CCD [46]
33. Cycler DC [4]
34. Cycler DDC [46]
35. DBS [13]
36. Davis [15]
37. Defector [17, 46, 52]
38. Defector Hunter [4]
39. Double Crosser [4]
40. Desperate [61]
41. DoubleResurrection [7]
42. Doubler [1]
43. Dynamic Two Tits For Tat [4]
44. EasyGo [42, 1]
45. Eatherley [16]
46. Eventual Cycle Hunter [4]
47. Evolved ANN [4]
48. Evolved ANN 5 [4]
49. Evolved ANN 5 Noise 05 [4]
50. Evolved FSM 16 [4]
51. Evolved FSM 16 Noise 05 [4]
52. Evolved FSM 4 [4]
53. Evolved HMM 5 [4]
54. EvolvedLookerUp1 1 1 [4]
55. EvolvedLookerUp2 2 2 [4]
56. Eugine Nier [3]

57. Feld [15]
58. Firm But Fair [30]
59. Fool Me Forever [4]
60. Fool Me Once [4]
61. Forgetful Fool Me Once [4]
62. Forgetful Grudger [4]
63. Forgiver [4]
64. Forgiving Tit For Tat [4]
65. Fortress3 [11]
66. Fortress4 [11]
67. GTFT [31, 49]
68. General Soft Grudger [4]
69. Gradual [19]
70. Gradual Killer [1]
71. Grofman[15]
72. Grudger [15, 18, 19, 61, 42]
73. GrudgerAlternator [1]
74. Grumpy [4]
75. Handshake [53]
76. Hard Go By Majority [46]
77. Hard Go By Majority: 10 [4]
78. Hard Go By Majority: 20 [4]
79. Hard Go By Majority: 40 [4]
80. Hard Go By Majority: 5 [4]
81. Hard Prober [1]
82. Hard Tit For 2 Tats [58]
83. Hard Tit For Tat [60]
84. Hesitant QLearner[4]
85. Hopeless [61]
86. Inverse [4]
87. Inverse Punisher [4]

C   Correlation coefficients

A graphical representation of the correlation coefficients for the measures in Table 5.
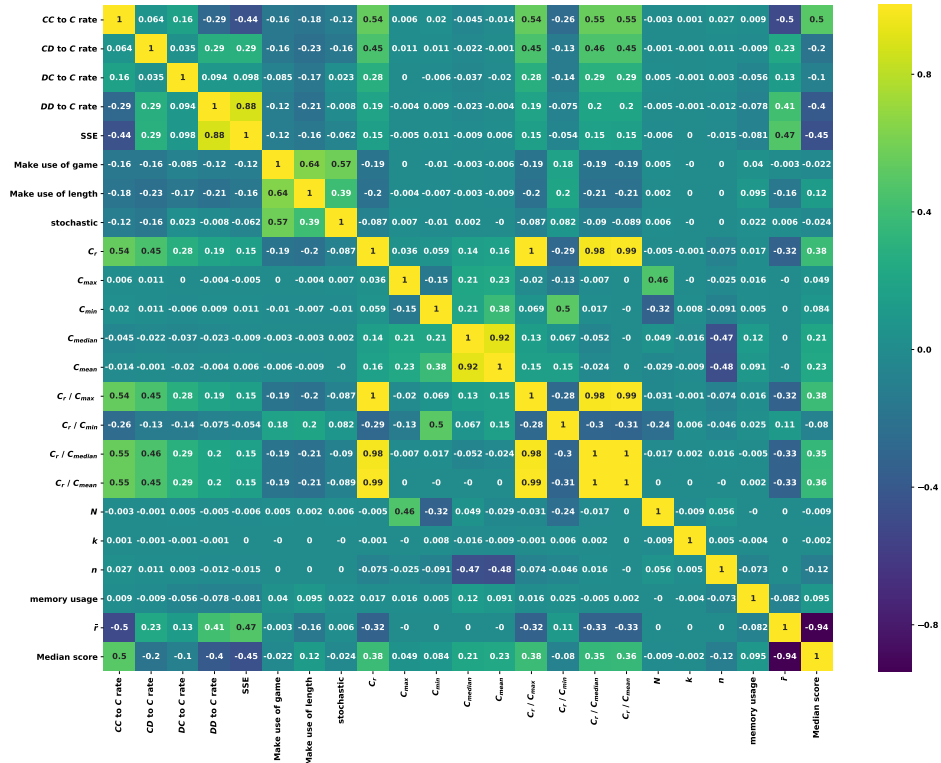

Figure 21: Correlation coefficients of measures in Table 5 for standard tournaments
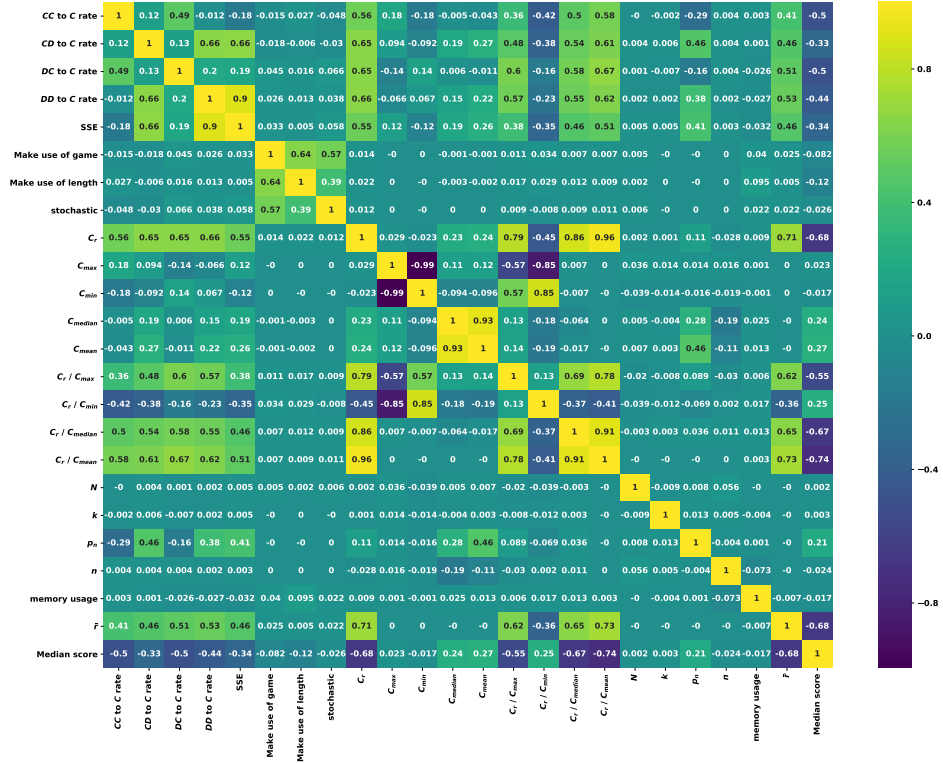
22

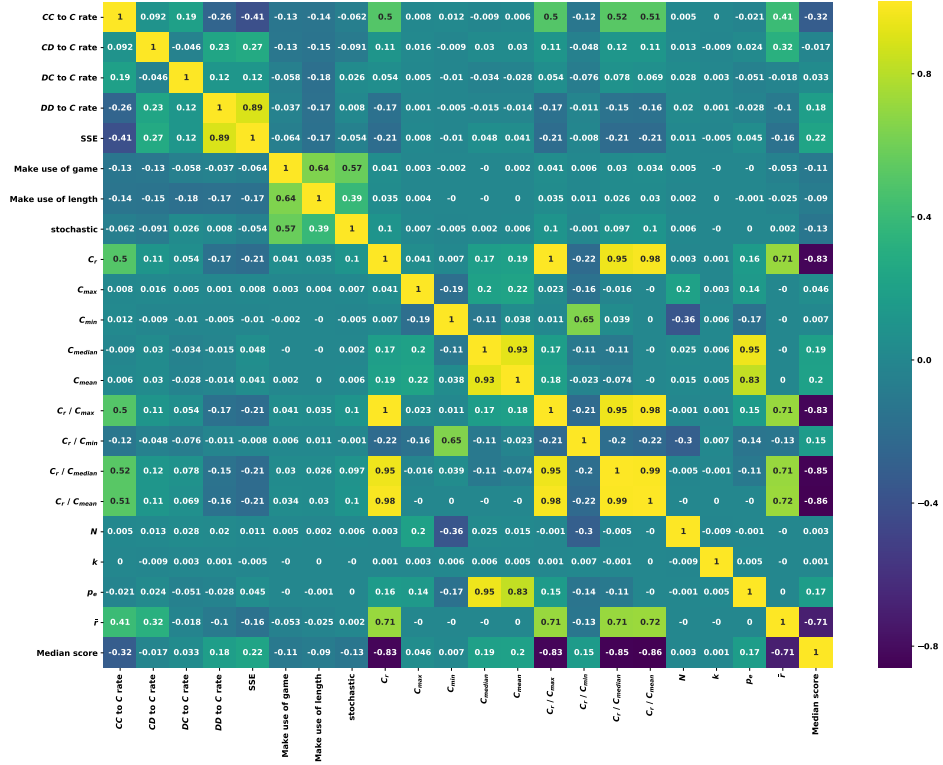Figure 22: Correlation coefficients of measures in Table 5 for noisy tournaments

Figure 23: Correlation coefficients of measures in Table 5 for probabilistic ending tournaments
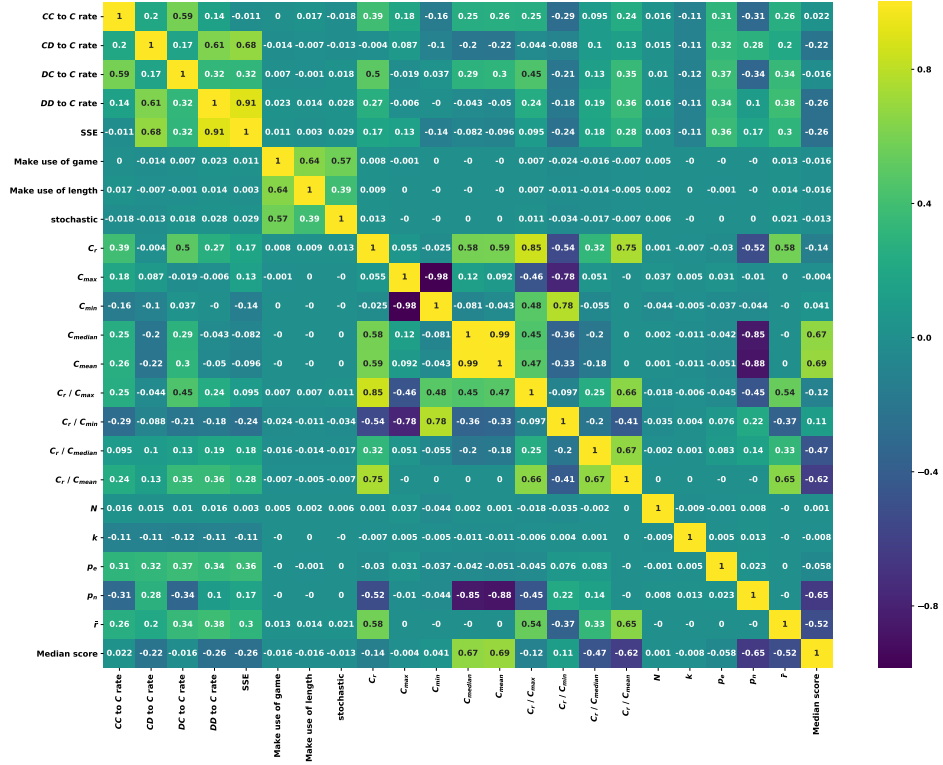
Figure 24: Correlation coefficients of measures in Table 5 for noisy probabilistic ending tournaments