Applying modern data analysis techniques to tournament results of the Iterated Prisoner's Dilemma.

1 The Prisoner's Dilemma

The Prisoner's Dilemma is a two player repeated game commonly known in Game Theory as the model of cooperative behaviour. Each of the players has two strategies, to either cooperate (C) or defect (D). The decisions are made simultaneously and independently. The normal form representation of the game is given by matrix **??**.

$$\begin{bmatrix} (R, R) & (S, T) \\ (T, S) & (P, P) \end{bmatrix} \tag{1}$$

where the payoffs (R, P, S, T) are constrained by equations (**??**) and (**??**).

$$T > R > P > S, \tag{2}$$

$$2R > T + S. \tag{3}$$

In the one shot game the Nash equilibrium is given at (D, D) and both players received a payoff of P. Thus, at the point where both players defect. In the repeated form of the game the behaviour becomes more complicated. The optimal behaviour of a player in the repeated game, in the Iterated Prisoner's Dilemma, has been the focus of research for years.

An interesting approach was introduced in 1980 by, where a computer tournament was used to tackle the same question.

- The definition of a strategy
- The definition of a tournament.

2 Collecting data

For performing a large number of computer tournaments the open source package Axelrod Library is used in this work. The package was introduced in 2015, it is written in the programming language Python and it allow us to perform a number of tournaments with different strategies. The payoff values used in Axelrod are $(3, 1, 0, 5)$ and the following type of tournaments have been performed.

There are several tournament types introduced in the literature that have not been discussed.

1. **Standard tournament**. A round robin tournament where the number of turns for each match can vary between 200 and 1. The tournament is repeated between 10 and 100 times.
2. **Noisy tournament**. Similar to a standard tournament. A noisy tournament in a round robin tournament where noise is introduced. Noise is the probability that a players action is flipped. The probability of noise is ranging between 0 and 1.
3. **Probabilistic ending tournament**. Similar to a standard tournament however in a probabilistic ending tournament the number of turns is not specified. There is a probability (ranges between 0 and 1) that the match will end in the next round. Probabilistic ending tournament will be referred to as probend hereupon.
4. **Noisy and Probabilistic ending tournament**. A combination of noisy and probabilistic ending tournaments.

The process for generating the data set is described by Algorithm **??**. Every 20 iterations of a random seed a new sample is chosen. For each sample, for 20 repetitions random numbers of turns, repetitions of the tournament, the probability of noise in the tournament and the probability of the game ending in after each interaction are sampled.

For that set of parameters, four types of tournaments (as discussed above) are conducted. A standard one, a noisy one, a probend one and lastly a probend with noise.

Algorithm 1 Generating data

```
for seed in 10000 do
    if seed mod 20 = 0 then
        size ← random size
        players ← random players
    else
        turns ← random turns
        repetitions ← random repetitions
        noise ← random noise probability
        end ← random end probability
        standard results ← tournament(turns, players, repetitions)
        noise results ← tournament(turns, players, repetitions, noise)
        probend results ← tournament(players, repetitions, end)
        probend noise results ← tournament(players, repetitions, noise, end)
    end if
    return standard, noise, probend, probend noise results
end for
```

Once a tournaments is performed we export a summary of the performance of each strategy in the tournament. In the following sections we discuss how we manipulate the result set exported by Axelrod and hold a data analysis.

3 Data preparation

The data set used for the study is the combination of all of these summary data sets. The structure of the data set is shown in Table **??**. Each row represents a player in a given tournament.

Name is the name of the strategy. Stochastic, Memory depth and Use of arguments are characteristics of the given strategy. The performance of the strategy is reflected by columns Rate, Median score, Rank and Wins.

The environment and the type of tournaments are captured by columns Noise, Probend, Repetitions, Size and Turns. Note that when turns are non given the tournament is a probabilistic ending one, and vise versa. Similarly, when Noise argument in non given the tournament is known to be standard. Finally, when both Noise and Probend are non zero then the tournament is an probabilistic and noisy tournament

Name	Stochastic	Memory depth	Use of game	Use of length	CC rate	CD rate	Cooperation rating	DC rate	DD rate	Median score	Rank	Wins	Noise	Probend	Repetitions	Seed	Size	Turns
Adaptive	False	inf	True	False	0.195	0.214	0.409	0.279	0.310	2.430	14	77.0	0.391	NaN	25.0	0.0	101.0	148.0
Adaptive	False	inf	True	False	0.783	0.194	0.928	0.012	0.009	2.292	88	5.0	0.391	0.370	25.0	0.0	101.0	NaN

Table 1: Result Set

- the cooperation in each environment
- show that got a uniform distribution of our parameters

In this work each tournament type is studied individually. An individual data set containing the following information is constructed for each type.

- tournament index i
- tournament size N_i
- cooperation rating (C_r)
- rank (r_i)
- normalized rank $\bar{r_i} = \frac{r_i}{N_i}$
- maximum cooperation rating C_r^*
- minimum cooperation rating $\tilde{C_r}$
- normalised rank of maximum cooperator $\bar{r_C}$
- normalised rank of minimum cooperator $\bar{r_D}$
- mean cooperation rating $\bar{C_r}$
- median cooperation rating $\breve{C_r}$
- C ratio of the winner C_W
- C ratio of the looser C_L
- standard deviation of the cooperating ratio σ.

4 Analysis

- add r plots
- some more plots