Properties of winning Iterated Prisoner's Dilemma strategies.

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Abstract

From the celebrated performance of Tit for Tat, to the introduction of the zero-determinant strategies, to the use of sophisticated structures such as neural networks, the literature has been exploring the performance of Iterated Prisoner's Dilemma strategies for decades. The results of the literature, however, have been relying on a small number of somewhat arbitrarily selected strategies in a very small number of tournaments, casting doubt on the generalizability of results from statistical perspective. This manuscript evaluates 195 strategies in 45686 tournaments, presents the top performing strategies, and analyzes their salient features. The results imply that there is not a single strategy that performs well in any Iterated Prisoner's Dilemma interaction. There are several properties, however, that heavily influence the best performing strategies, these are: be nice, be provocable, be a little envious, be clever and adapt to the environment.

1 Background

The Iterated Prisoner's Dilemma (IPD) is a repeated two player game that models behavioural interactions, and more specifically, interactions where self-interest clashes with collective interest. At each turn of the game both players, simultaneously and independently, decide between cooperation (C) and defection (D) whilst having memory of their prior interactions. The payoffs for each player, at each turn, is influenced by their own choice and the choice of the other player. The payoffs of the game are generally defined by:

$$\begin{pmatrix} R & S \\ T & P \end{pmatrix}$$

where $T > R > P > S$ and $2R > T + S$. The most common values used in the literature [13] are $R = 3, P = 1, T = 5, S = 0$. These values are also used in this work.

Conceptualising strategies and understanding the best way of playing the game has been of interest to the scientific community since the formulation of the game in 1950 [20]. Following the computer tournaments of Axelrod in the 1980's [11, 12], a strategy's performance in a round robin computer tournament became a common evaluation technique for newly designed strategies. Many tournaments have followed Axelrod's [15, 24, 27, 40, 41] and today more than 200 strategies exist in the literature.

Axelrod performed two computer tournaments [11, 12] in the 80's. The winner of both tournaments was the simple strategy Tit For Tat which cooperated on the first turn and then simply copied the previous action of its opponent. Axelrod concluded that the strategy's robustness was due to four properties, which he adapted in four suggestions on doing well in an IPD:

- Do not be envious

- Be "nice"; Do not be the first to defect
- Reciprocate both cooperation and defection; Be provocable
- Do not be too clever

As a result of the strategy's strong performance in both tournaments, and moreover in a series of evolutionary experiments [13], Tit For Tat was often claimed to be the most robust basic strategy in the IPD.

There are strategies which have built upon Tit For Tat, and the reciprocity based approach. In [14] Gradual was introduced which was constructed to have the same qualities as those of Tit for Tat except one, Gradual had a memory of the game since the beginning of it. Gradual recorded the number of defections by the opponent and punished them with a growing number of defections. It would then enter a calming state in which it would cooperates for two rounds. A strategy with the same intuition as Gradual is Adaptive Tit for Tat [42]. Adaptive Tit for Tat does not keep a permanent count of past defections, it maintains a continually updated estimate of the opponent's behaviour, and uses this estimate to condition its future actions.

Other works have built upon the limitations of the strategy, and others have shown that suggestions made by Axelrod were incomplete. In [15, 19, 32, 39] it was shown that Tit For Tat suffered in environments with noise. This was mainly due to the strategy's lack of generosity and contrition. The strategy was quick to punish a defection, and in a noisy environment it could lead to a repeated cycle of defections and cooperations. Some new strategies, more robust in tournaments with noise, were soon introduced and became the new protagonists of the game. These include Nice and Forgiving [15], Pavlov [34] and Generous Tit For Tat [35]. In [37] a set of envious IPD strategies that have received a lot of attention were introduced called the zero-determinant strategies (ZDs). By forcing a linear relationship between the payoffs ZDs can ensure that they will never receive less than their opponents. ZDs are indeed a set of mathematically unique strategies and robust in pairwise interactions, however, their superiority in tournament settings have been tested. In [24] a series of clever strategies were introduced, and a tournament containing over 200 strategies was ran and none of the aforementioned strategies ranked in top spots. Instead, the top ranked strategies were the set of clever trained strategies based on lookup tables [10], hidden markov models [24] and finite state automata [31].

Though only select pieces of work have been discussed, there is a board number of strategies in the literature, and new strategies and competitions are being published every year [23]. The question, however, still remains the same: what is the best way to play the game?

Compared to other works, whereas a few selected strategies are evaluated on a small number of tournaments, this manuscript evaluates the performance of 195 strategies in 45686 tournaments. These tournaments come in a number of variations including tournaments with noise, probabilistic match length and both noise and probabilistic match length. The later part of the paper, evaluates the impact of features on the performance of the strategies using modern machine learning techniques. These features include measures regarding a strategy's behaviour and measures regarding the tournaments. The outcomes of our work reinforce the discussion started by Axelrod, and it concludes that the properties of a successful strategy in the IPD are:

- ~~Do not be envious~~
- Be "nice"; Do not be the first to defect
- Reciprocate both cooperation and defection; Be provocable
- ~~Do not be too clever~~
- Adapt to the environment; Adjust to the mean cooperator

The different tournament types as well as the data collection, which is made possible due to an open source package called Axelrod-Python, are covered in Section 2. The data set generated for this work has been made publicly available [22] and can be used for further analysis and insights. Section 3, focuses on the best performing strategies for each type of tournament and overall. Section 4, explores the traits which contribute to good performance, and finally the results are summarised in Section 5. This manuscripts uses several parameters. These are introduced in the following sections, however, the full set of parameters and their definitions are given in Appendix A.

2 Data collection

For the purposes of this manuscript a data set containing results of IPD tournaments has been generated and is available at [22]. This was done using the open source package Axelrod-Python library [3] (APL), and more specifically, version 3.0.0. APL allows for different types of IPD computer tournaments to be simulated whilst containing a large list of strategies. Most of these are strategies described in the literature with a few exceptions being strategies that have been contributed specifically to the package. This paper makes use of 195 strategies implemented in version 3.0.0. A list of the strategies is given in the Appendix **??**. Although APL features several tournament types, this work considers standard, noisy, probabilistic ending and noisy probabilistic ending tournaments.

Standard tournaments, are tournaments similar to that of Axelrod's in [11]. There are N strategies which all play an iterated game of n number of turns against each other. Note that self interactions are not included. Similarly, **noisy tournaments** have N strategies and n number of turns, but at each turn there is a probability p_n that a player's action will be flipped. **Probabilistic ending tournaments**, are of size N and after each turn a match between strategies ends with a given probability p_e. Finally, **noisy probabilistic ending** tournaments have both a noise probability p_n and an ending probability p_e. For smoothing the simulated results a tournament is repeated for k number of times. This was allowed to vary in order to evaluate the effect of smoothing. The winner of each tournament is based on the average score a strategy achieved and not by the number of wins.

The process of collecting tournament results implemented in this manuscript is described by Algorithm 1. For each trial a random size N is selected, and from the 195 strategies a random list of N strategies is chosen. For the given list of strategies a standard, a noisy, a probabilistic ending and a noisy probabilistic ending tournament are performed and repeated k times. The parameters for the tournaments, as well as the number of repetitions, are selected once for each trial. The parameters and their respective minimum and maximum values are given by Table 1.

parameter	parameter explanation	min value	max value
N	number of strategies	3	195
k	number of repetitions	10	100
n	number of turns	1	200
p_n	probability of flipping action at each turn	0	1
p_e	probability of match ending in the next turn	0	1

Table 1: Data collection; parameters' values

The source code for the data collection, as well as the source code for the analysis, which will be discussed in the following sections, have been written following best practices [4, 16] and is available here.

A total of 11420 trials of Algorithm 1 have been run. For each trial the results for 4 different tournaments were collected, thus a total of 45686 (11420×4) tournament results have been retrieved. Each tournament outputs

Algorithm 1: Data collection Algorithm

foreach *seed* $\in [0, 11420]$ **do**

$N \leftarrow$ randomly select integer $\in [N_{min}, N_{max}]$;

players $\leftarrow$ randomly select N players;

$k \leftarrow$ randomly select integer $\in [k_{min}, k_{max}]$;

$n \leftarrow$ randomly select integer $\in [n_{min}, n_{max}]$;

$p_n \leftarrow$ randomly select float $\in [p_{n\,min}, p_{n\,max}]$;

$p_e \leftarrow$ randomly select float $\in [p_{e\,min}, p_{e\,max}]$;

result standard $\leftarrow$ Axelrod.tournament(players, n, k);

result noisy $\leftarrow$ Axelrod.tournament(players, n, p_n, k);

result probabilistic ending $\leftarrow$ Axelrod.tournament(players, p_e, k);

result noisy probabilistic ending $\leftarrow$ Axelrod.tournament(players, p_n, p_e, k);

return *result standard, result noisy, result probabilistic ending, result noisy probabilistic ending*;

a result summary in the form of Table 2. Each strategy have participated on average in 5154 tournaments of each type. The strategy with the maximum participation in each tournament type is Inverse Punisher with 5639 entries. The strategy with the minimum entries is EvolvedLookerUp 1 1 1 which was selected in 4693 trials.

The result summary, Table 2, has N number of rows because each row contains information for each strategy that participated in the tournament. The information includes the strategy's rank, median score, the rate with which the strategy cooperated (C_r), its match win count and the probability that the strategy cooperated in the opening move. Moreover, the probabilities of a strategy being in any of the four states (CC, CD, DC, DD), and the rate of which the strategy cooperated after each state. A feature that has been manually included is the **normalised rank**. The rank of a given strategy, denoted as R, can vary between 0 and $N-1$. Thus, the normalised rank, denoted as r, is calculated as a strategy's rank divided by the tournament's size $N-1$. In the next section the performance of these strategies is evaluated based on their normalised rank.

						Rates							
Rank	Name	Median score	Cooperation rating (C_r)	Win	Initial C	CC	CD	DC	DD	CC to C	CD to C	DC to C	DD to C
0	EvolvedLookerUp2 2 2	2.97	0.705	28.0	1.0	0.639	0.066	0.189	0.106	0.836	0.481	0.568	0.8
1	Evolved FSM 16 Noise 05	2.875	0.697	21.0	1.0	0.676	0.020	0.135	0.168	0.985	0.571	0.392	0.07
2	PSO Gambler 1 1 1	2.874	0.684	23.0	1.0	0.651	0.034	0.152	0.164	1.000	0.283	0.000	0.136
3	PSO Gambler Mem1	2.861	0.706	23.0	1.0	0.663	0.042	0.145	0.150	1.000	0.510	0.000	0.122
4	Winner12	2.835	0.682	20.0	1.0	0.651	0.031	0.141	0.177	1.000	0.441	0.000	0.462
…	…	…	…	…	…	…	…	…	…	…	…	…	…

Table 2: Output result of a single tournament.

3 Top ranked strategies

This section evaluates the performance of 195 IPD strategies. The performance of each strategy is evaluated in four tournament types, which were presented in Section 2, followed by an evaluation of their performance over all the 45686 simulated tournaments of this work.

Each strategy participated in multiple tournaments of the same type (on average 5154). For example Tit For Tat has participated in a total of 5114 tournaments of each type. The strategy's normalised rank distribution in these is given in Figure 1. A value of $r = 0$ corresponds to a strategy winning the tournament where a value of $r = 1$ corresponds to the strategy coming last. Because of the strategies' multiple entries their performance is evaluated based on the **median normalised rank** denoted as $\bar{r}$.

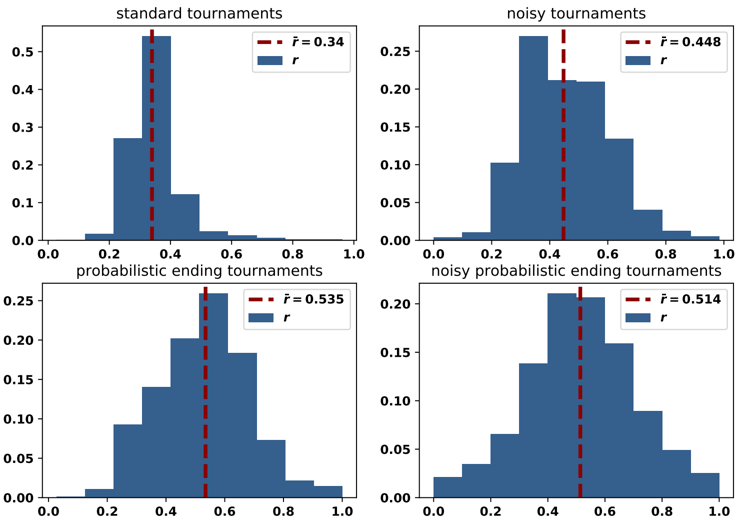


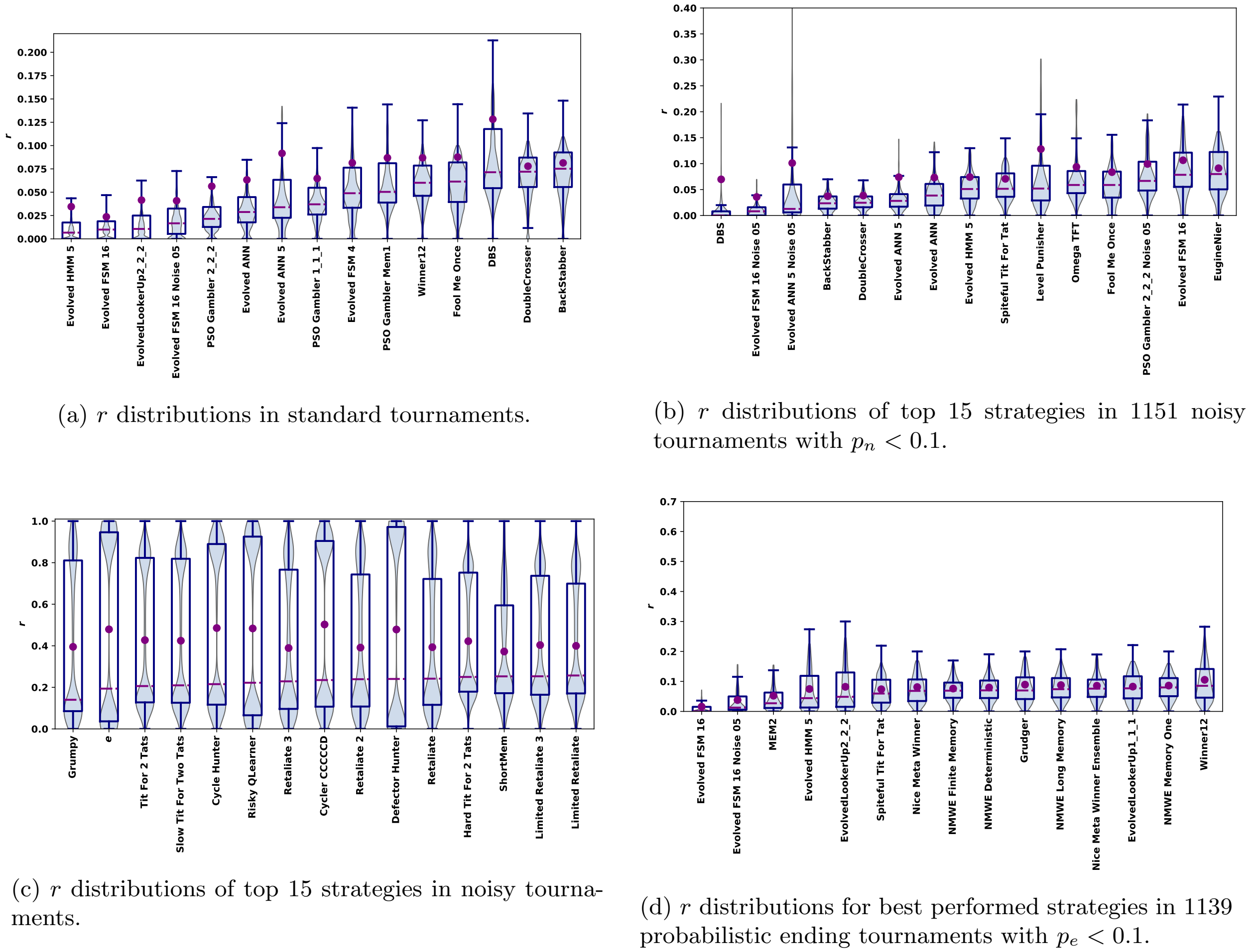
Figure 1: Tit For Tat's r distribution in tournaments. Lower values of r correspond to better performances. The best performance of the strategy has been in standard tournaments where it achieved a $\bar{r}$ of 0.34.

The top 15 strategies for each tournament type based on $\bar{r}$ are given in Table 3. The data collection process was design such as that the probabilities of noise and ending of the match to vary between 0 and 1. However, commonly used values of these probabilities are values less than 0.1. Thus, Table 3 also includes the top 15 strategies in noisy tournaments with $p_n < 0.1$ and probabilistic ending tournaments with $p_e < 0.1$.

	Standard		Noisy		Noisy ($p_n < 0.1$)		Probabilistic ending		Probabilistic ending ($p_e < 0.1$)		Noisy probabilistic ending	
	Name	$\bar{r}$	Name	$\bar{r}$	Name	$\bar{r}$	Name	$\bar{r}$	Name	$\bar{r}$	Name	$\bar{r}$
0	Evolved HMM 5	0.007	Grumpy	0.140	DBS	0.000	Fortress4	0.013	Evolved FSM 16	0.000	Alternator	0.304
1	Evolved FSM 16	0.010	e	0.194	Evolved FSM 16 Noise 05	0.008	Defector	0.014	Evolved FSM 16 Noise 05	0.013	ϕ	0.310
2	EvolvedLookerUp2 2 2	0.011	Tit For 2 Tats	0.206	Evolved ANN 5 Noise 05	0.013	Better and Better	0.016	MEM2	0.027	e	0.312
3	Evolved FSM 16 Noise 05	0.017	Slow Tit For Two Tats	0.210	BackStabber	0.024	Tricky Defector	0.019	Evolved HMM 5	0.044	π	0.317
4	PSO Gambler 2 2 2	0.021	Cycle Hunter	0.215	DoubleCrosser	0.025	Fortress3	0.022	EvolvedLookerUp2 2 2	0.049	Limited Retaliate	0.353
5	Evolved ANN	0.029	Risky QLearner	0.222	Evolved ANN 5	0.028	Gradual Killer	0.025	Spiteful Tit For Tat	0.060	Anti Tit For Tat	0.354
6	Evolved ANN 5	0.034	Retaliate 3	0.229	Evolved ANN	0.038	Aggravater	0.028	Nice Meta Winner	0.068	Limited Retaliate 3	0.356
7	PSO Gambler 1 1 1	0.037	Cycler CCCCCD	0.235	Spiteful Tit For Tat	0.051	Raider	0.031	NMWE Finite Memory	0.069	Retaliate 3	0.356
8	Evolved FSM 4	0.049	Retaliate 2	0.239	Evolved HMM 5	0.051	Cycler DDC	0.045	NMWE Deterministic	0.070	Retaliate	0.357
9	PSO Gambler Mem1	0.050	Defector Hunter	0.240	Level Punisher	0.052	Hard Prober	0.051	Grudger	0.070	Retaliate 2	0.358
10	Winner12	0.060	Retaliate	0.242	Omega TFT	0.059	SolutionB1	0.060	NMWE Long Memory	0.074	Limited Retaliate 2	0.361
11	Fool Me Once	0.061	Hard Tit For 2 Tats	0.250	Fool Me Once	0.059	Meta Minority	0.061	Nice Meta Winner Ensemble	0.076	Hopeless	0.368
12	DBS	0.071	Limited Retaliate 3	0.253	PSO Gambler 2 2 2 Noise 05	0.067	Bully	0.061	EvolvedLookerUp1 1 1	0.077	Arrogant QLearner	0.407
13	DoubleCrosser	0.072	ShortMem	0.253	Evolved FSM 16	0.078	EasyGo	0.071	NMWE Memory One	0.080	Cautious QLearner	0.409
14	BackStabber	0.075	Limited Retaliate	0.257	EugineNier	0.080	Fool Me Forever	0.071	Winner12	0.085	Fool Me Forever	0.418

Table 3: Top performances for each tournament type based on $\bar{r}$. The results of each type are based on 11420 unique tournaments of each type. The results for noisy tournaments with $p_n < 0.1$ are based on 1151 tournaments, and for probabilistic ending tournaments with $p_e < 0.1$ on 1139. The top ranks indicate that trained strategies perform well in a variety of environments, but so do simple deterministic strategies. The normalised medians are close to 0 for most environments, except environments with noise not restricted to 0.1. These two tournaments have the highest medians, implying that strategies from the collection of this work can not perform well in environments with such high values of noise.

Distributions of these top ranked strategies are given by Figure 2.



(a) r distributions in standard tournaments.

(b) r distributions of top 15 strategies in 1151 noisy tournaments with $p_n < 0.1$.

(c) r distributions of top 15 strategies in noisy tournaments.

(d) r distributions for best performed strategies in 1139 probabilistic ending tournaments with $p_e < 0.1$.

Figure 2: r distributions of the top 15 strategies in different environments. A lower value of $\bar{r}$ corresponds to a more successful performance. A strategy's r distribution skewed towards zero indicates that the strategy ranked highly in most tournaments it participated in.

In standard tournaments 10 out of the 15 top strategies are introduced in [24]. These are strategies based on finite state automata (FSM), hidden markov models (HMM), artificial neural networks (ANN), lookup tables (LookerUp) and stochastic lookup tables (Gambler) that have been trained using reinforcement learning algorithms (evolutionary and particle swarm algorithms). They have been trained to perform well against the strategies in APL in a standard tournament, thus their performance in the specific setting was anticipated. DoubleCrosser, BackStabber and Fool Me Once, are strategies not from the literature but from the APL. DoubleCrosser is an extension of BackStabber and both strategies make use of the number of turns because they are set to defect on the last two rounds. It should be noted that these strategies can be characterised as "cheaters". The source code of the strategies allows them to known the number of turns in a match (if they are specified). These strategies were expected to not perform as well in tournaments where the number of turns is not specified. Finally, Winner 12 [30] and DBS [9] are both from the the literature. DBS is a strategy specifically designed for noisy environments and it ranks highly in standard tournaments as well. Figure 2a gives the distributions of r for the top ranked strategies. The distributions are skewed towards zero and the highest median, of the top 15 strategies, is at 0.075. This indicates that the top ranked strategies perform well in any given standard tournament, despite the opponents and the number of turns.

In the case of noisy tournaments with $p_n < 0.1$ the top performed strategies include strategies specifically designed for noisy tournaments such as DBS, Evolved FSM 16 Noise 05, Evolved ANN 5 Noise 05, PSO Gambler 2 2 2 Noise 05 and Omega Tit For Tat [28], strategies which performed well in standard tournaments such as BackStabber, DoubleCrosser, Evolved ANN, FSM and HMM and deterministic strategies such as Spiteful Tit For Tat [1], Level Punisher [5], Eugine Nier [2]. The r distributions of these strategies are given in Figure 2b where it is evident that these strategies overall performed well in noisy tournaments. In comparison, the top ranked strategies in noisy environments when $p_n \in [0, 1]$ include strategies which decide their actions based on the cooperations to defections ratio, such as ShortMem [18], Grumpy [3] and e [3], and the Retaliate strategies which are designed to defect if the opponent has tricked them more often than x% of the times that they have done the same. The overall performance of these strategies is bimodal, Figure 2c, indicating that although the top ranked strategies mainly performed well, there are several tournaments that they ranked in the bottom half. The bimodality of the r distributions can be explained by Figure 3 which gives the r distributions for the top 6 strategies over the noise probability p_n. It is evident that the highly ranked strategies were highly ranked due to the their performance in tournaments with $p_n > 0.5$, and that in tournaments with a noise probability lower than 0.5 they performed poorly.

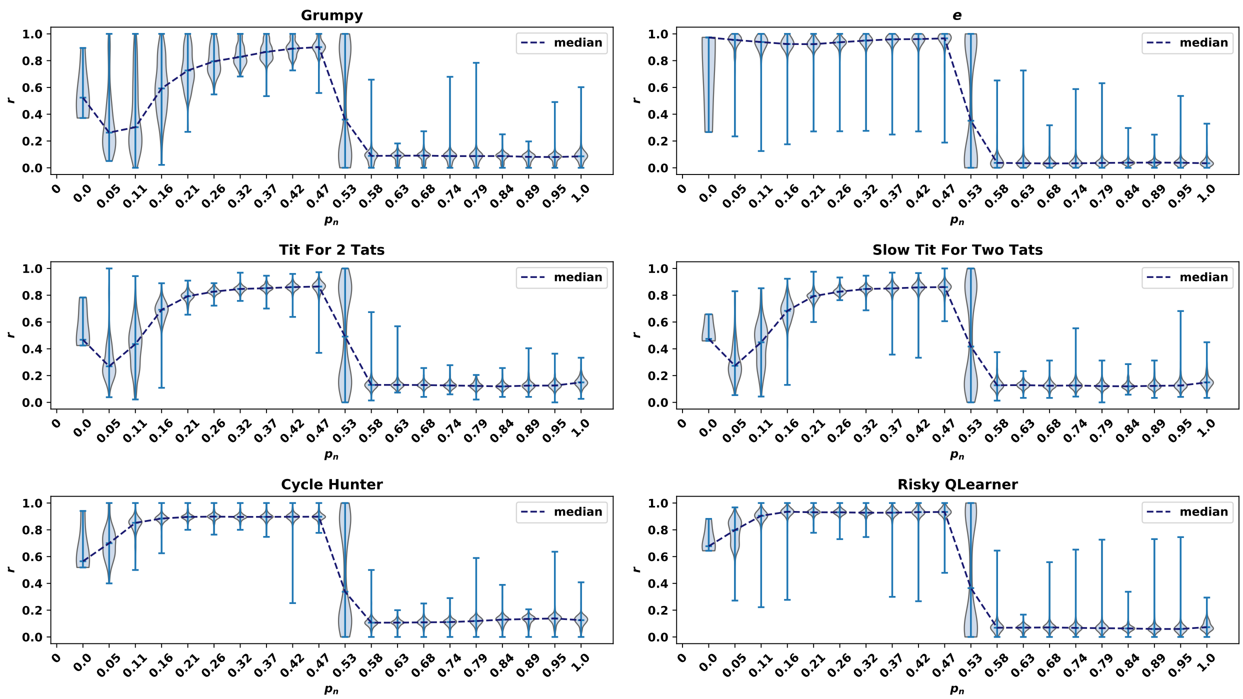


Figure 3: r distributions for top 6 strategies in noisy tournaments over the probability of noisy (p_n).

In probabilistic ending tournaments with $p_e < 0.1$ the effective strategies are Meta strategies which include Nice Meta Winner, NMWE Finite Memory, NMWE Deterministic, trained strategies which performed well in standard tournaments, Grudger [3] and Spiteful Tit for Tat [1]. The Meta strategies [3] are a set of rules that create a team of strategies and play as an ensemble or some other combination of their team members. The distributions of the normalised rank in the 1139 probabilistic ending tournaments with $p_e < 0.1$, are shown in Figure 2d. The medians of the top 15 strategies are lower than 0.1 and the distributions are skewed towards 0. The strategies have overall performed well in the probabilistic ending tournaments that they participated.

In probabilistic ending tournaments with $p_e \in [0, 1]$ the top ranks are mostly occupied by defecting strategies such as Better and Better, Gradual Killer, Hard Prober (all from [3]), Bully (Reverse Tit For Tat) [33] and Defector, and a series of strategies based on finite state automata introduced by Daniel Ashlock and Wendy

Ashlock; Fortress 3, Fortress 4 (both introduced in [7]), Raider [8] and Solution B1 [8]. As stated in the Folk Theorem [21] defecting strategies do better when the likelihood of the game ending in the next turn increased. This is demonstrated by Figure 4, which gives the distributions of r for the top 6 strategies in probabilistic ending tournaments over p_e. It shows that the 6 strategies start of with a high median rank, however, their ranked decreased as the the probability of the game ending increased and at the point of $p_e = 0.1$.

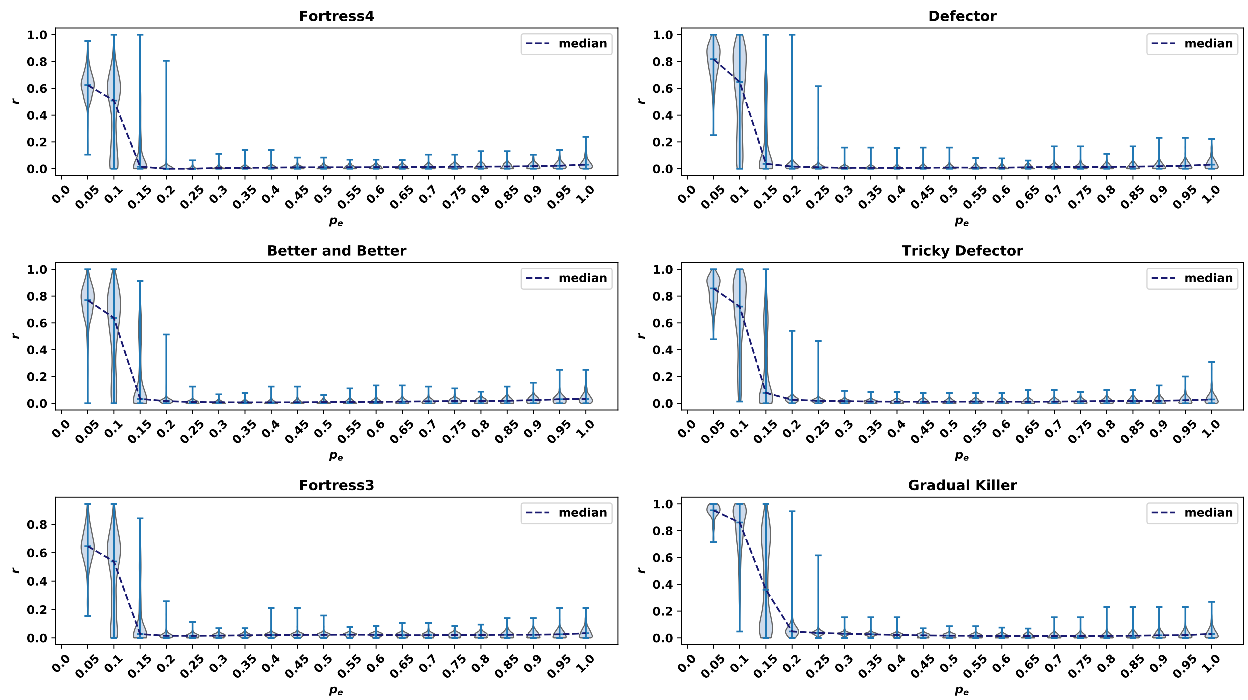


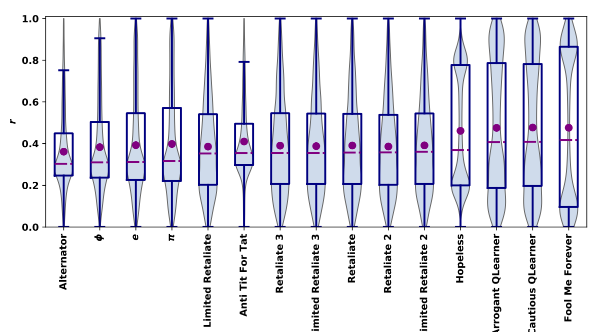
Figure 4: r distributions for top 6 strategies in probabilistic ending tournaments over p_e.

In tournaments with both noise and an unspecified number of turns several of the top ranked strategies are strategies that were highly ranked in noisy tournaments. However, strategies from the top ranks in probabilistic ending tournaments did not rank highly here. The distributions of r shown in Figure 5a have the largest median values compared to the top rank strategies of the other tournament types. A subset of noisy probabilistic ending tournaments has been considered such that $p_e < 0.1$ and $p_n < 0.1$. The top ranked strategies are given in Table 4 and it is shown that the Meta strategies which performed well in noisy tournaments with $p_n < 0.1$, perform well once again even the number of turns is not specified. Moreover, several strategies that did well in probabilistic ending tournaments such as Fortress 4, MEM2 and Spiteful Tit For Tat.

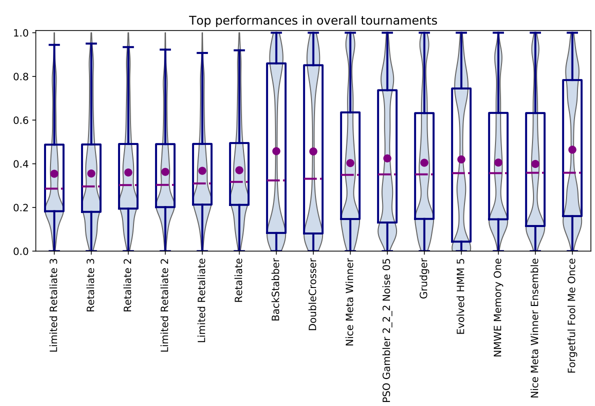
This section presented the winning strategies in a series of IPD tournaments. In standard tournaments the top spots were dominated by complex strategies that had been trained using reinforcement learning techniques. In noisy environments where the a noise probability strictly less than 0.1 was considered the successful strategies were strategies based on the behaviour of many strategies and strategies specifically designed for noisy tournaments. In probabilistic ending tournaments most of the highly ranked strategies were defecting strategies and trained finite state automata, all by the authors of [7, 8]. These strategies ranked high due to their performance in tournaments where the probability of the game ending after each turn was bigger than 0.1. In probabilistic tournaments with p_e less than 0.1 the highly ranked strategies were trained and Meta strategies. Finally the performance of all 195 strategies over the 45686 tournaments in this manuscript was assessed on $\bar{r}$. The top ranked strategies were a mixture of behaviours that did well in standard tournaments, tournaments with noise and a few Meta strategies. The results of this section imply that successful strategies

	Probabilistic ending ($p_n < 0.1$ & $p_e < 0.1$)		Overall	
	Name	$\bar{r}$	Name	$\bar{r}$
0	Raider	0.022	Limited Retaliate 3	0.286
1	MEM2	0.037	Retaliate 3	0.296
2	Prober 3	0.039	Retaliate 2	0.302
3	Evolved FSM 16 Noise 05	0.048	Limited Retaliate 2	0.303
4	Hard Prober	0.072	Limited Retaliate	0.310
5	Spiteful Tit For Tat	0.078	Retaliate	0.317
6	Better and Better	0.089	BackStabber	0.324
7	Grudger	0.091	DoubleCrosser	0.331
8	Fortress4	0.096	Nice Meta Winner	0.349
9	Meta Winner Memory One	0.099	PSO Gambler 2 2 2 Noise 05	0.351
10	NMWE Long Memory	0.099	Grudger	0.352
11	Nice Meta Winner	0.104	Evolved HMM 5	0.357
12	NMWE Deterministic	0.109	NMWE Memory One	0.357
13	NMWE Memory One	0.112	Nice Meta Winner Ensemble	0.359
14	Nice Meta Winner Ensemble	0.115	Forgetful Fool Me Once	0.359

Table 4: Top performances in 117 probabilistic ending tournaments with $p_e < 0.1$ and $p_n < 0.1$.



(a) r distributions for best performed strategies in noisy probabilistic ending tournaments.



(b) r distributions for best performed strategies in the data set [22].

for specific settings exist for an IPD tournament. The top ranked strategies in both standard tournaments and tournaments with probabilistic ending, managed to rank in the top 10% of the tournament most of the times. Strategies in noisy environments demonstrated that no strategy can be consistently successful, expected if the value of noise is constrained to less than a 0.1. Trained strategies, meta strategies and some deterministic defecting strategies have demonstrated good performance over several tournament types, however, there exists no one single strategy which appears to be dominant in an IPD competition. The aim of the next section is to understand which are the factors that made these strategies successful, in each setting separately but also overall.

Be envious and be clever.

4 Evaluation of performance

The aim of this section is to explore the features that contribute to a strategy's successful performance. The features explored are measures regarding a strategy's behaviour, along with measures regarding the tournaments the strategies competed in. A summary of the features is given in Table 5.

feature	feature explanation	source	value type	min value	max value
stochastic	If a strategy is stochastic	strategy classifier from APL	boolean	Na	Na
makes use of game	If a strategy makes used of the game information	strategy classifier from APL	boolean	Na	Na
makes use of length	If a strategy makes used of the number of turns	strategy classifier from APL	boolean	Na	Na
memory usage	The memory size of a strategy divided by the number of turns	memory size from APL	float	0	1
SSE	A measure of how far a strategy is from ZD behaviour	method described in [29]	float	0	1
max cooperating rate (C_{max})	The biggest cooperating rate in a given tournament	result summary	float	0	1
min cooperating rate (C_{min})	The smallest cooperating rate in a given tournament	result summary	float	0	1
median cooperating rate (C_{median})	The median cooperating rate in a given tournament	result summary	float	0	1
mean cooperating rate (C_{mean})	The mean cooperating rate in a given tournament	result summary	float	0	1
C_r / C_{max}	A strategy's cooperating rate divided by the maximum	result summary	float	0	1
C_{min} / C_r	A strategy's cooperating rate divided by the minimum	result summary	float	0	1
C_r / C_{median}	A strategy's cooperating rate divided by the median	result summary	float	0	1
C_r / C_{mean}	A strategy's cooperating rate divided by the mean	result summary	float	0	1
C_r	The cooperating ratio of a strategy	result summary	float	0	1
CC to C rate	The probability a strategy will cooperate after a mutual cooperation	result summary	float	0	1
CD to C rate	The probability a strategy will cooperate after being betrayed by the opponent	result summary	float	0	1
DC to C rate	The probability a strategy will cooperate after betraying the opponent	result summary	float	0	1
DD to C rate	The probability a strategy will cooperate after a mutual defection	result summary	float	0	1
p_n	The probability of a player's action being flip at each interaction	trial summary	float	0	1
n	The number of turns	trial summary	integer	1	200
p_e	The probability of a match ending in the next turn	trial summary	float	0	1
N	The number of strategies in the tournament	trial summary	integer	3	195
k	The number of repetitions of a given tournament	trial summary	integer	10	100

Table 5: The features which are included in the performance evaluation analysis.

APL classifies strategies along several dimensions. These determine whether a strategy is stochastic or deterministic, whether it makes use of the number of turns or the game's payoffs. The memory usage feature is calculated as the memory size of strategy (which is specified in the strategies implementation in the APL) divide by the number of turns. For example, Winner12 has a memory size of 2 and participated in a tournament where n was 101. In the given tournament Winner12 has a memory usage of 0.014925. There are strategies with an infinite memory size, for example Evolved FSM 16 Noise 05. These strategies have a memory usage of 1. For tournaments with a probabilistic ending the number of turns was not collected, so the memory usage feature is not used for probabilistic ending tournaments. The SSE is a feature introduced in [29] which shows how close a strategy is to behaving as a ZDs, and subsequently, in an extortionate way. The method identifies the ZDs closest to a given strategy and calculates the algebraic distance between them, defined as SSE. A SSE value of 1 indicates no extortionate behaviour at all whereas a value of 0 indicates that a strategy is behaving a ZDs. The rest of the features considered are the CC to C, CD to C, DC to C, and DD to C rates as well as cooperating ratio of a strategy. The minimum, maximum, medium and median cooperating ratios of each tournament are also included, and finally the number of turns, the number of strategies, the number of repetitions and the probabilities of noise and the game ending are also included.

Table 6 shows the correlation coefficients between the features of Table 5 the median score and the median normalised rank. Note that the correlation for the classifiers is not included because they are binary variables and they will be evaluated using a different method. The correlation coefficients for all the features in Table 5 against themselves have also been calculated and a graphical representation can be found in the Appendix B.

	Standard		Noisy		Probabilistic ending		Noisy probabilistic ending		Overall	
	r	median score	r	median score	r	median score	r	median score	r	median score
CC to C rate	-0.501	0.501	0.414	-0.504	0.408	-0.323	0.260	0.022	-0.501	0.501
CD to C rate	0.226	-0.199	0.456	-0.330	0.320	-0.017	0.205	-0.220	0.226	-0.199
C_r	-0.323	0.384	0.711	-0.678	0.714	-0.832	0.579	-0.135	-0.323	0.384
C_r / C_{max}	-0.323	0.381	0.616	-0.551	0.714	-0.833	0.536	-0.116	-0.323	0.381
C_r / C_{mean}	-0.331	0.358	0.731	-0.740	0.721	-0.861	0.649	-0.621	-0.331	0.358
C_r / C_{median}	-0.331	0.353	0.652	-0.669	0.712	-0.852	0.330	-0.466	-0.331	0.353
C_r / C_{min}	0.109	-0.080	-0.358	0.250	-0.134	0.150	-0.368	0.113	0.109	-0.080
C_{max}	-0.000	0.049	0.000	0.023	-0.000	0.046	0.000	-0.004	-0.000	0.049
C_{mean}	-0.000	0.229	-0.000	0.271	0.000	0.200	0.000	0.690	-0.000	0.229
C_{median}	0.000	0.209	-0.000	0.240	-0.000	0.187	-0.000	0.673	0.000	0.209
C_{min}	0.000	0.084	0.000	-0.017	-0.000	0.007	-0.000	0.041	0.000	0.084
DC to C rate	0.127	-0.100	0.509	-0.504	-0.018	0.033	0.341	-0.016	0.127	-0.100
DD to C rate	0.412	-0.396	0.533	-0.436	-0.103	0.176	0.378	-0.263	0.412	-0.396
N	0.000	-0.009	-0.000	0.002	-0.000	0.003	-0.000	0.001	0.000	-0.009
k	0.000	-0.002	-0.000	0.003	-0.000	0.001	-0.000	-0.008	0.000	-0.002
n	0.000	-0.125	-0.000	-0.024	-	-	-	-	0.000	-0.125
p_e	-	-	-	-	0.000	0.165	0.000	-0.058	-0.001	0.001
p_n	-	-	-0.000	0.207	-	-	-0.000	-0.650	0.002	-0.000
Make use of game	-0.003	-0.022	0.025	-0.082	-0.053	-0.108	0.013	-0.016	-0.003	-0.022
Make use of length	-0.158	0.124	0.005	-0.123	-0.025	-0.090	0.014	-0.016	-0.154	0.117
SSE	0.473	-0.452	0.463	-0.337	-0.156	0.223	0.305	-0.259	0.473	-0.452
memory usage	-0.082	0.095	-0.007	-0.017	-	-	-	-	-0.084	0.095
stochastic	0.006	-0.024	0.022	-0.026	0.002	-0.130	0.021	-0.013	0.006	-0.024

Table 6: Correlations table between the features of Table 5 the normalised rank and the median score.

In standard tournaments the features CC to C, C_r, C_r/C_{max} and the cooperating ratio compared to C_{median} and C_{mean} have a moderate negative effect on the normalised rank, and a moderate positive on the median score. The SSE error and the DD to C have the opposite effects. Thus, in standard tournaments behaving cooperatively corresponds to a more successful performance. Even though being nice pays off, that's not true against defective strategies. Cooperating after a mutual defection lowers a strategy's success. Figure 6 confirms that the winners of standard tournaments always cooperate after a mutual cooperation and almost always defects after a mutual defection.

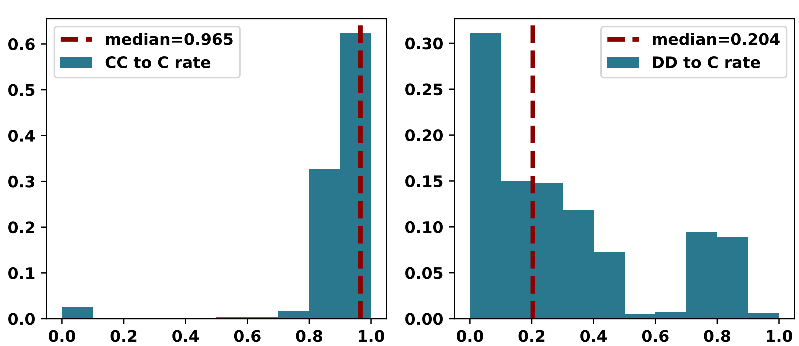


Figure 6: Distributions of CC to C and DD to C for the winners in standard tournaments.

Compared to standard tournaments, in both noisy and in probabilistic ending tournaments the higher the rates of cooperation the lower a strategy's success and median score. A strategy would want to cooperate less than both the mean and median cooperator in such settings. In probabilistic ending tournaments the correlation coefficients have a larger values, indicating a stronger effect. Thus a strategy will be punished more by its cooperative behaviour in probabilistic ending environments, this was seen in Section 4 as well. The distributions of the C_r of the winners in both tournaments is given by Figure 7. It confirms that the winners in noisy tournaments cooperated less than 35% of the times and in probabilistic ending tournaments less than 10%. In noisy probabilistic ending tournaments and in over all the tournaments' results, the only features that had a moderate affect are $C_r/C_{\text{mean}}, C_r/C_{\text{max}}$ and C_r. In such environments cooperative behaviour appears to be punished by not as much as in noisy and probabilistic ending tournaments.

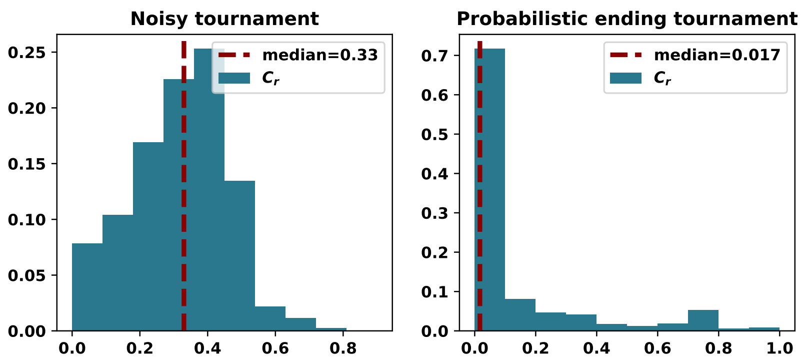


Figure 7: C_r distributions of the winners in noisy and in probabilistic ending tournaments.

A multivariate linear regression is fitted to model the relationship of Table's 5 and the normalised rank. Based on the correlation matrices. Appendix B the depended variables are correlated. The median cooperating ration C_median and C_r/C_median are highly correlated with C_mean and C_r/C_mean. So these features are dropped from the analysis. The C_r is also correlated and drop. Finally for some environments the C_max and C_min are correlated and dropped for the models of those types. More specifically, the list of features their p values and $p-$values of each feature as well as their coefficients are given by Table 7. The coefficient of determination, R squared, for each model is also reported in Table 7.

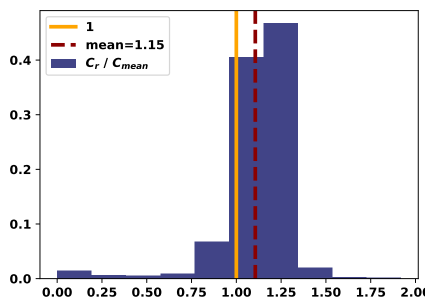
	Standard		Noisy		Probabilistic ending		Noisy probabilistic ending		Overall	
	R adjusted: 0.785		R adjusted: 0.895		R adjusted: 0.894		R adjusted: 0.872		R adjusted: 0.802	
	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value	Coefficient	p-value
CC to C rate	0.499	0.0	0.074	0.000	-0.039	0.0	-0.027	0.0	0.006	0.0
CD to C rate	0.330	0.0	-0.140	0.000	0.183	0.0	0.134	0.0	0.135	0.0
C_r / C_{max}	-	-	0.587	0.000	-	-	0.263	0.0	0.249	0.0
C_r / C_{mean}	-0.406	0.0	0.192	0.000	0.492	0.0	0.263	0.0	0.343	0.0
C_{min} / C_r	0.596	0.0	-0.272	0.000	0.014	0.0	-0.182	0.0	-0.044	0.0
DC to C rate	0.262	0.0	0.015	0.000	-0.034	0.0	0.042	0.0	0.068	0.0
DD to C rate	-0.026	0.0	0.029	0.000	0.098	0.0	0.220	0.0	0.097	0.0
SSE	0.500	0.0	0.141	0.000	-0.142	0.0	-0.101	0.0	0.031	0.0
memory usage	0.017	0.0	0.000	0.012	-	-	-	-	-	-

Table 7: Results of multivariate linear regressions with r as the depended variable.

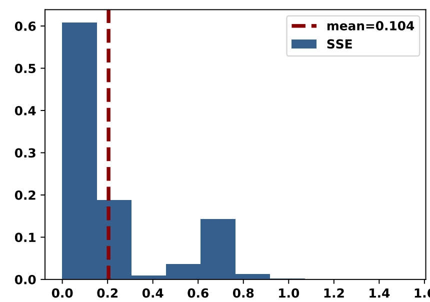
The results of the linear regression verify the results of the previous approaches. The only features that do no have a significant effect on the models are the number of repetitions and memory usage in noisy tournaments, and the number of turns, number of repetitions and thr probability p_e in probabilistic ending tournaments. Over all the models, the features that are the most important predictors are C_r, C_r / C_{max} and C_r / C_{mean}.

The features C_r/C_{median} and C_r/C_{mean} have been repeatably highlighted as importance features in explaining the variability of the performances. Thus, the effect of these two measures is further explored. In

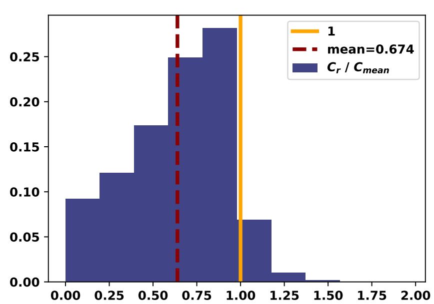
Figure ?? the distributions of C_r/C_{mean} and C_r/C_{median} are given for the winners in standard tournaments. A value of $C_r/C_{\text{mean}} = 1$ imply that the cooperating ratio of the winner was the same as the mean/median cooperating ratio of the tournament. In standard tournaments, the mean for both ratios is 1. Therefore, an effective strategy in standard tournaments was the mean/median cooperator of its respective tournament. In comparison, Figure ?? shows the distributions of the features for the winners in noisy tournaments where the mean is at 0.67. Thereupon the winners cooperated 67% of the times the mean/median cooperator did. This analysis is applied to the rest of the tournaments and the distributions are given by Figures ??, ?? and ??. In a tournament with noisy and a probabilistic ending the winners cooperated 60%, whereas in settings that the type of the tournament can vary between the types considered in this work the winners cooperated 67% of the times the mean or median cooperator did. Finally, in probabilistic ending tournament it has already been mentioned that defecting strategies prevail and this result is once again confirmed in this section.



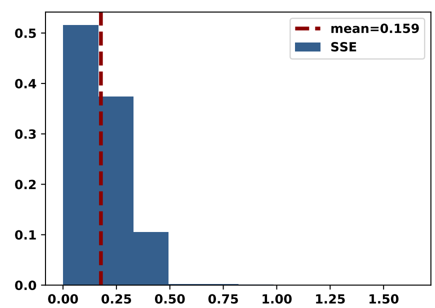
(a) Distributions of C_r/C_{mean} for winners of standard tournaments.



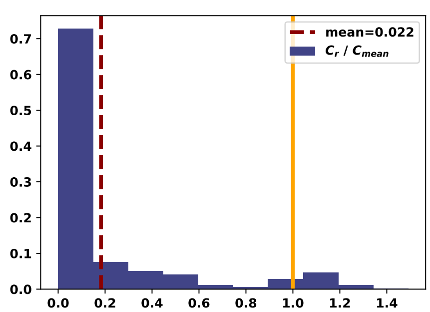
(b) Distributions of SSE for winners of standard tournaments.



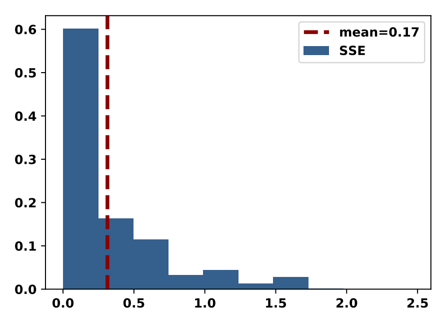
(a) Distributions of C_r/C_{mean} for winners of noisy tournaments.



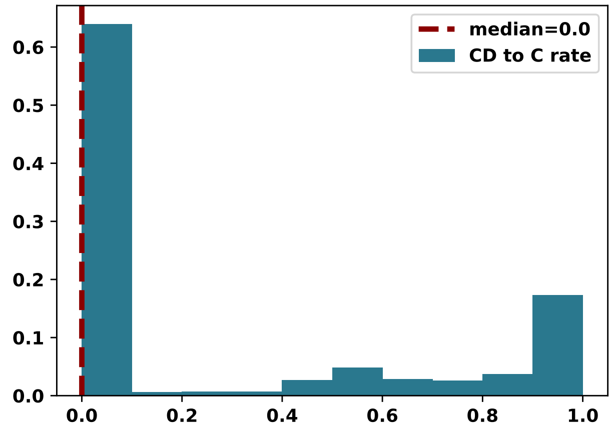
(b) Distributions of SSE for winners of noisy tournaments.

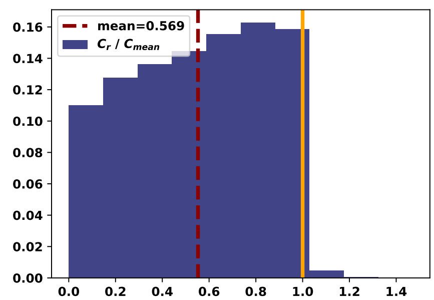


(a) Distributions of C_r/C_{mean} for winners of probabilistic ending tournaments.

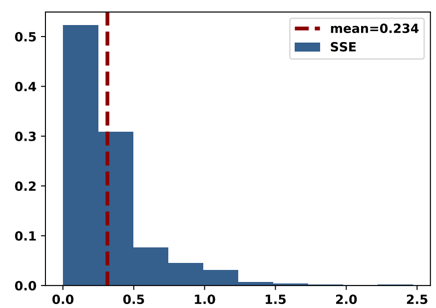


(b) Distributions of SSE for winners of probabilistic ending tournaments.

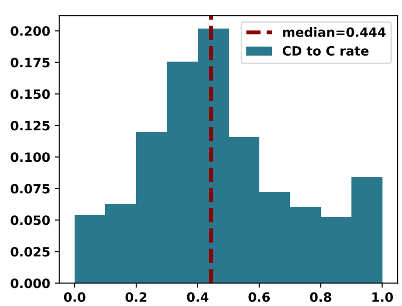


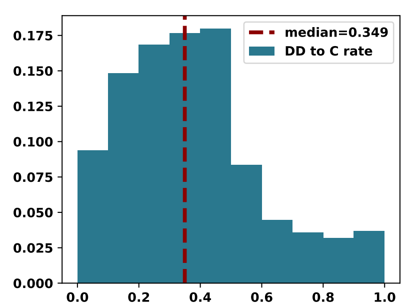


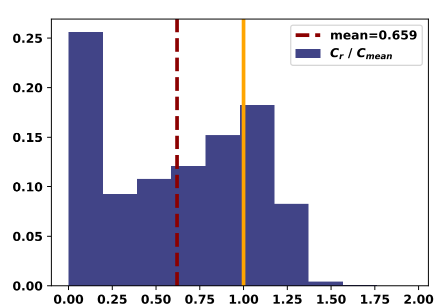
(a) Distributions of C_r/C_{mean} for winners of noisy probabilistic ending tournaments.

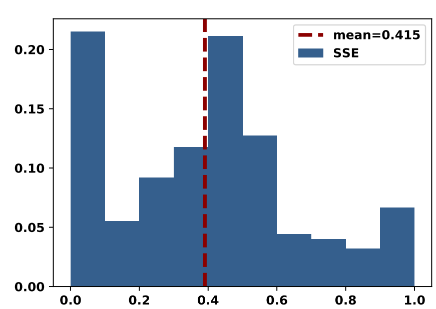


(b) Distributions of SSE for winners of noisy probabilistic ending tournaments.









In this section the effect of several features, regarding a strategy's behaviour and the tournament in which it participated on its performance were presented. This was done using three methods. Correlation coefficients, a random forest analysis and a linear regression. The results of these are summarised in the following section.

5 Conclusion

This manuscript has explored the performance of 195 strategies of the Iterated Prisoner's Dilemma in a large number of computer tournaments. The results of the analysis demonstrated that, although for specific tournament types such as standard, probabilistic ending tournaments and noisy tournaments with $p_n < 0.1$, dominant types of strategies exist there is not a single dominant strategy, from the collection on strategies considered in this work, in an IPD competition if the environments vary. Moreover, a strategy with a theory of mind should aim to adapt its behaviour based on the mean and median cooperators and should in general not be too cooperative.

The 195 strategies used in this manuscript have been mainly for the literature, and they have been accessible due to an open source software called the Axelrod-Python library. The software was used to generate a total of 45686 computer tournaments results with different number of strategies and different participants each time. The data collection was described in Section 2. In Section 3, the tournaments results were used to present the top performances. The data set contained results from four different settings, and these were also studied individually. In standard tournaments complex strategies trained using reinforcement learning ranked in the top spots. Some of these strategies ranked again in the top spots in probabilistic ending tournaments when a p_e of less 0.1 was considered and in noisy tournaments when p_n was less than 0.1. In probabilistic ending tournaments p_e was designed to vary between 0 and 1. It was demonstrated that for values larger than 0.1, as stated in the Folk Theorem, defecting strategies were winning the tournaments because there was a high likelihood of the game ending in the next turn. In tournaments with noise the median ranks of the top 15 strategies had the highest values and the r distributions were bimodal. The top rank strategies were performing both well and bad, and this indicates that in noisy tournaments there are not strategies that can guarantee winning. However, if the probability of noise was constrained at 0.1 then strategies designed for noisy tournaments indeed performed well. Overall, the top ranked strategies differed from one tournament type to another and the mechanism behind the winning strategies were all different, with some exceptions which include trained (Evolved) and Meta strategies.

Section 4 covered an analysis of performance based on several features associated with a strategy and with the environments in which it was competing. The results of this analysis showed that a strategy's characteristics such as whether or not it's stochastic, and the information it used regarding the game had no effect on the strategy's success. The most important features have been those that compared the strategy's behaviour to its environment. The cooperating ratio of the strategy compared to the mean and median cooperator was highlighted as the most important feature in the analysis. More specifically, if a strategy were to enter a tournament with a theory of mind of its environment it would choose to be the median cooperator in standard tournaments, to cooperate 10% of the time the median cooperator did in probabilistic ending tournaments and 60% in noisy and noisy probabilistic tournaments. Lastly, if a strategy was aware of the opponents but not of the setting of the tournament, a strategy would be more likely to be successful if it were to identify the median cooperator and cooperated 67% of the times that they did.

The data set described in this work contains the largest number of IPD tournaments, to the authors knowledge, and it available at [22]. Further data mining could be applied and provide new insights in the field.

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6 Acknowledgements

A variety of software have been used in this work:

- The Axelrod-Python library for IPD simulations [3].
- The Matplotlib library for visualisation [26].
- The Numpy library for data manipulation [43].
- The scikit-learn library for data analysis [36].

A Parameters Summary

All the parameters used in this manuscript alongside their explanation are given by Table 8.

B Correlation coefficients

A graphical representation of the correlation coefficients for the features in Table 5.

C Evaluation based on clustering and random forest.

The second method to evaluate the features importance in a strategy's success is a combination of a clustering task and a random forest algorithm. Initially the performances are clustered into different clusters based on them being successful or not. The performances are clustered into successful and unsuccessful clusters based on 4 different approaches. More specifically:

Feature	Explanation
SSE	A measure of how far a strategy is from extortionate behaviour defined in [29].
C_{max}	The biggest cooperating rate in the tournament.
C_{min}	The smallest cooperating rate in the tournament.
C_{median}	The median cooperating rate in the tournament.
C_{mean}	The mean cooperating rate in the tournament.
C_r / C_{max}	A strategy's cooperating rate divided by the maximum cooperating rate in the tournament.
C_{min} / C_r	The minimum in the tournament divided by a strategy's cooperating rate.
C_r / C_{median}	A strategy's cooperating rate divided by the median cooperating rate in the tournament.
C_r / C_{mean}	A strategy's cooperating rate divided by the mean cooperating rate in the tournament.
C_r	The cooperating rate of a strategy.
CC to C rate	The probability a strategy will cooperate after a mutual cooperation.
CD to C rate	The probability a strategy will cooperate after being betrayed by the opponent.
DC to C rate	The probability a strategy will cooperate after betraying the opponent.
DD to C rate	The probability a strategy will cooperate after a mutual defection.
p_n	The probability of a player's action being flipped at each interaction.
n	The number of turns in a match.
p_e	The probability of a match ending in the next turn.
N	The number of strategies in the tournament.
k	The number that a given tournament is repeated.

Table 8: The features which are included in the performance evaluation analysis.

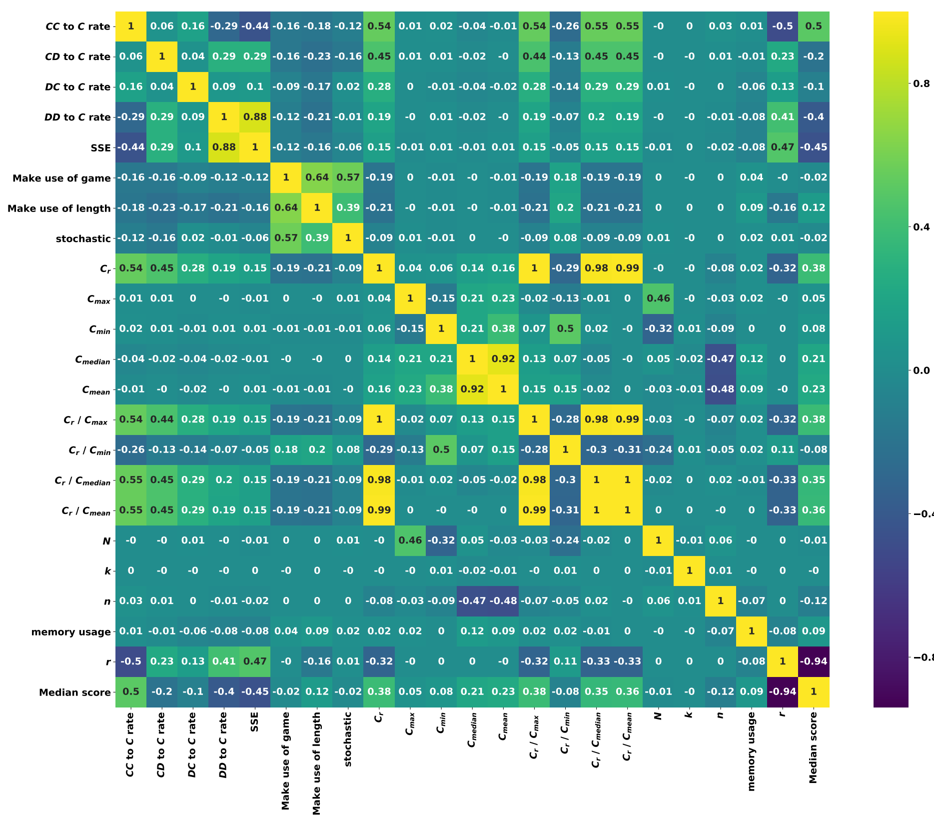


Figure 13: Correlation coefficients of features in Table 5 for standard tournaments

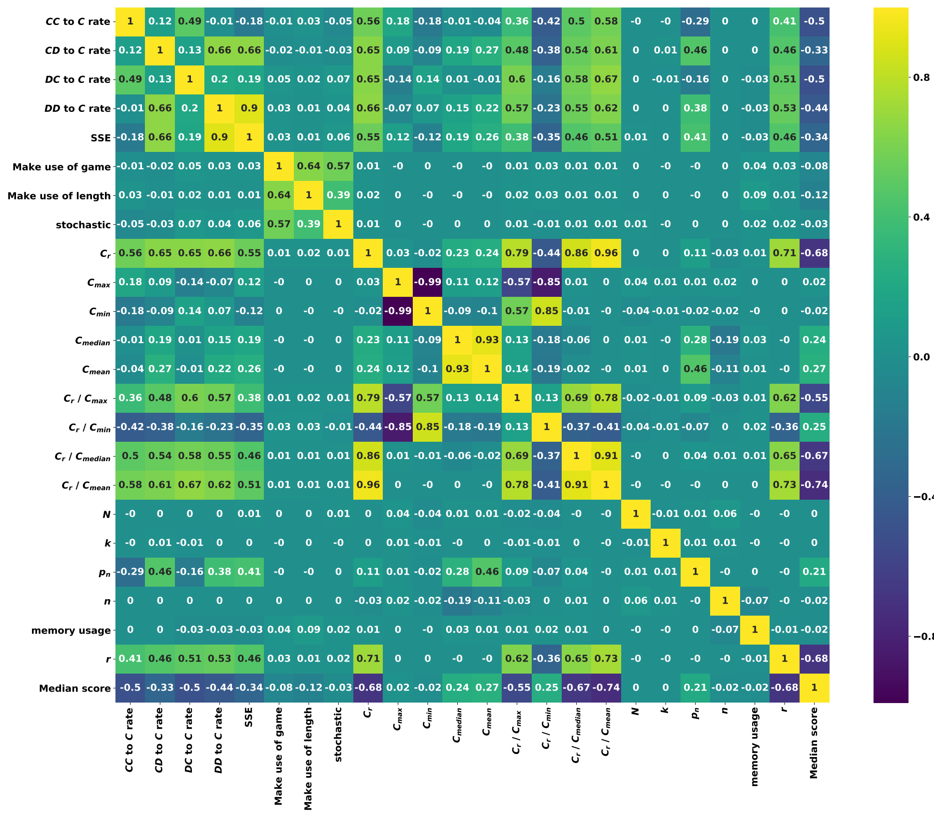


Figure 14: Correlation coefficients of features in Table 5 for noisy tournaments

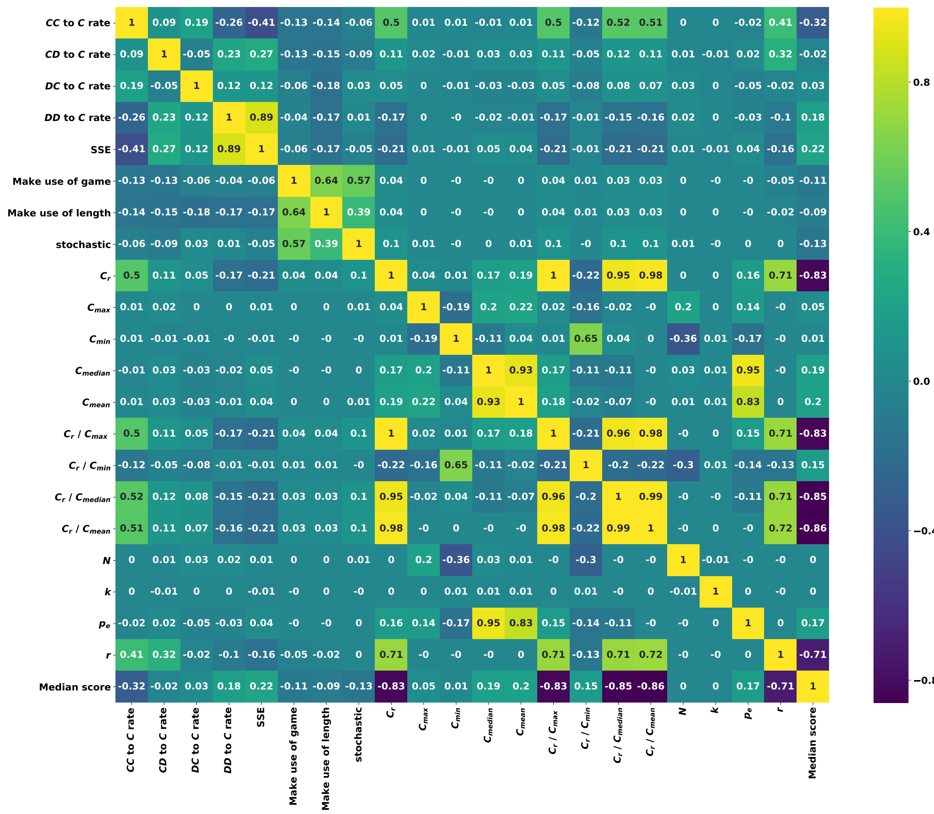


Figure 15: Correlation coefficients of features in Table 5 for probabilistic ending tournaments

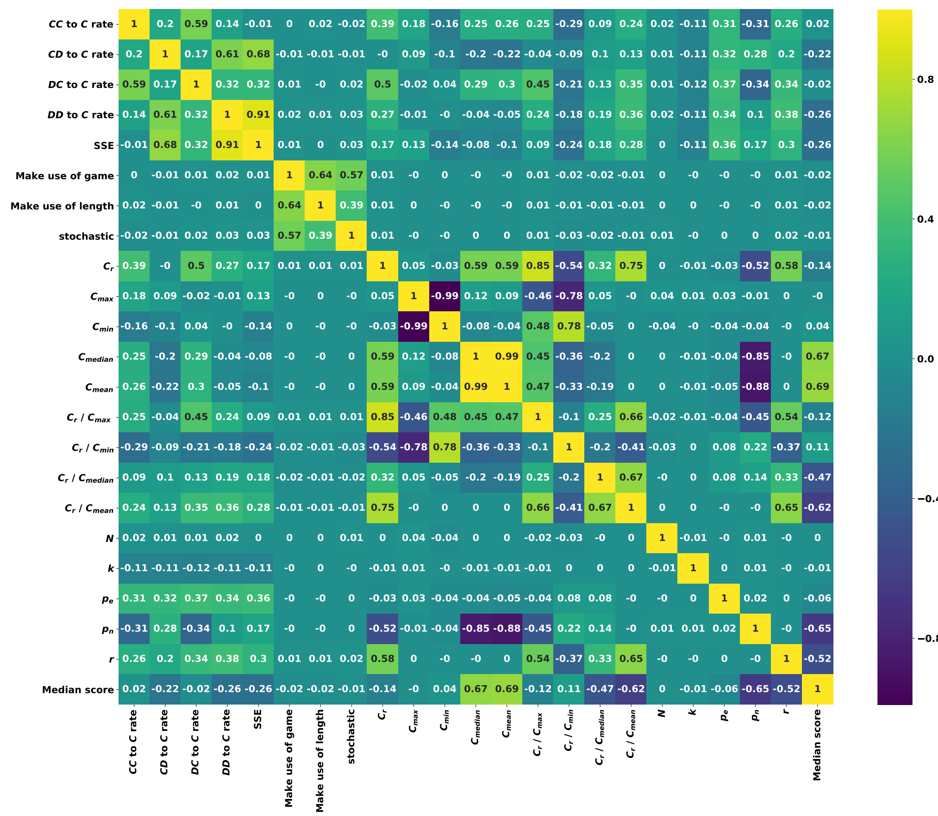


Figure 16: Correlation coefficients of features in Table 5 for noisy probabilistic ending tournaments

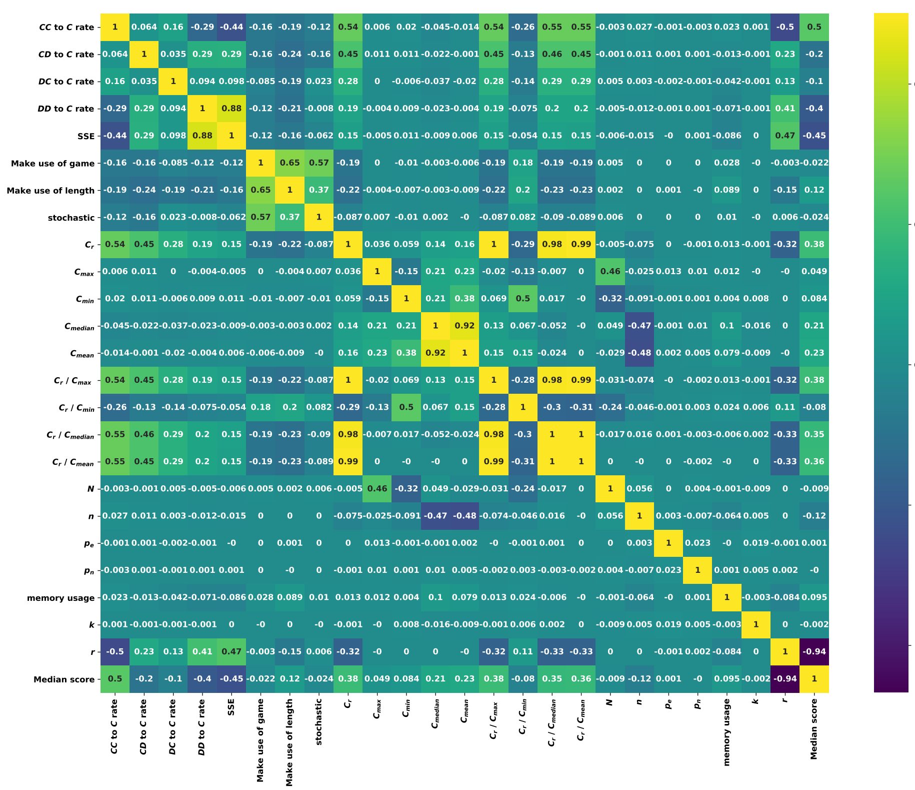


Figure 17: Correlation coefficients of features in Table 5 for data set

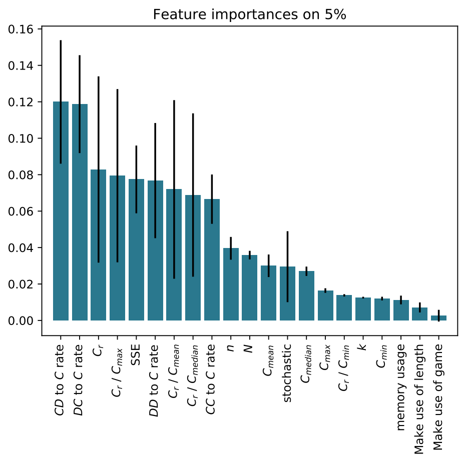
- **Approach 1:** The performances are divided into two clusters based on whether their performance was in the top 5% of their respective tournaments. Thus, whether r was smaller or larger than 0.05.
- **Approach 2:** The performances are divided into two clusters based on whether their performance was in the top 25% of their respective tournaments. Thus, whether r was smaller or larger than 0.25.
- **Approach 3:** The performances are divided into two clusters based on whether their performance was in the top 50% of their respective tournaments. Thus, whether r was smaller or larger than 0.50.
- **Approach 4:** The performances are clustered based on their normalised rank and their median score by a $k-$means algorithm [6]. The number of clusters is not deterministically chosen but it is based on the silhouette coefficients [38].

Once the performances have been assigned to a cluster for each approach a random forest algorithm [17] is applied. The problem is a supervised problem where the random forest algorithm predicts the cluster to which a performance has been assigned to using the features of Table 5. The random forest models are trained on a training set of 70% of the tournaments results. The accuracy of each model based on R^2 and the number of clusters for each tournament type (because in the case of Approach 4 it is not deterministically chosen) are given by Table 9. The out of the bag error (OOB) [25] has also been calculated. The models fit well, and a high value of both the accuracy measures on the test data and the OOB error indicate that the model is not over fitting.

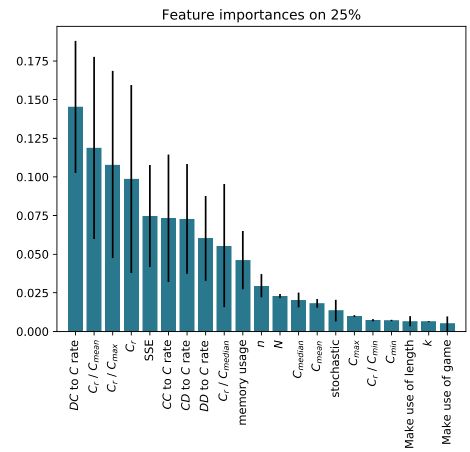
Tournament type	Clustering Approach	Number of clusters	R^2 training data	R^2 test data	R^2 OOB score
standard	Approach 1	2	0.998831	0.987041	0.983708
	Approach 2	2	0.998643	0.978626	0.969202
	Approach 3	2	0.998417	0.985217	0.976538
	Approach 4	2	0.998794	0.990677	0.982959
noisy	Approach 1	2	0.997786	0.972229	0.968332
	Approach 2	2	0.997442	0.963254	0.955219
	Approach 3	2	0.997152	0.953164	0.940528
	Approach 4	3	0.996923	0.950728	0.935444
probabilistic ending	Approach 1	2	0.997909	0.981490	0.978120
	Approach 2	2	0.997883	0.973492	0.967150
	Approach 3	2	0.990448	0.890068	0.875822
	Approach 4	2	0.999636	0.995183	0.992809
noisy probabilistic ending	Approach 1	2	0.995347	0.957846	0.952353
	Approach 2	2	0.992813	0.909346	0.898613
	Approach 3	2	0.990579	0.824794	0.806540
	Approach 4	4	0.989465	0.841652	0.824052
over 45686 tournaments	Approach 1	2	0.997271	0.972914	0.969198
	Approach 2	2	0.996323	0.951194	0.940563
	Approach 3	2	0.993707	0.906941	0.891532
	Approach 4	3	0.993556	0.913335	0.898453

Table 9: Accuracy metrics for random forest models.

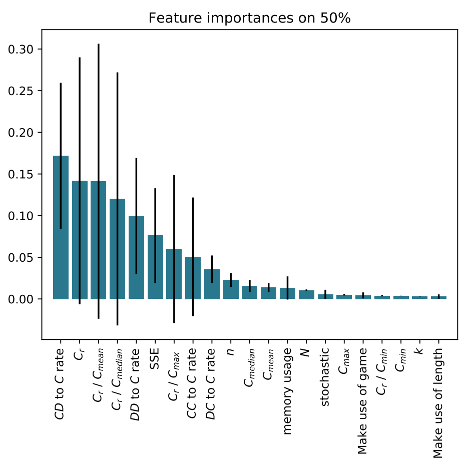
The importance that the features of Table 5 had on each random forest model are given by Figures 18, 19, 20, 21 and 22. These show that the classifiers stochastic, make use of game and make use of length have no significant effect, and several of the features that are highlighted by the importance are inline with the correlation results. Moreover, the smoothing parameter k appears to no have a significant effect either. The most important features based on the random forest analysis were C_r/C_{median} and C_r/C_{mean}.



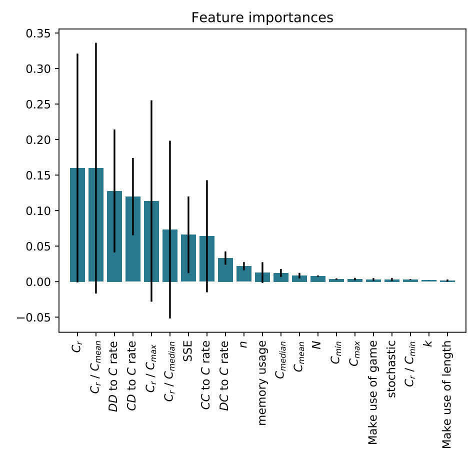
(a) Importance of features for clusters on 5% performance.



(b) Importance of features for clusters on 25% performance.

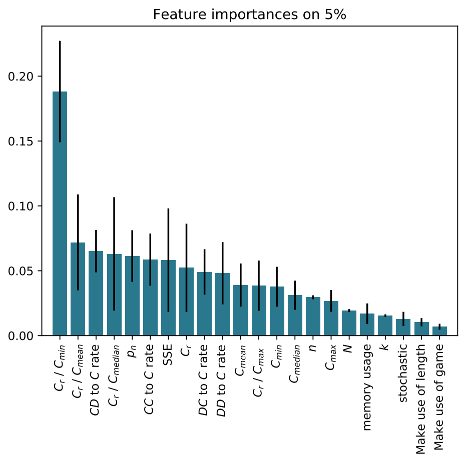


(c) Importance of features for clusters on 50% performance.

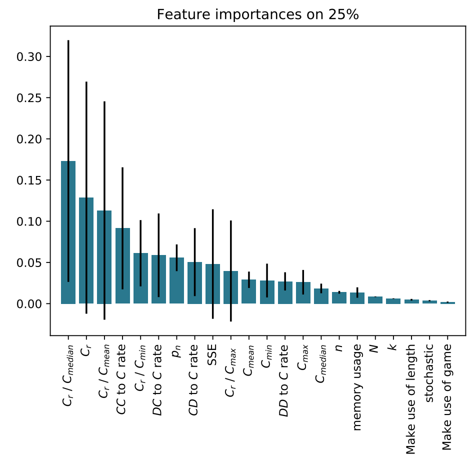


(d) Importance of features for clusters based on $k$means algorithm.

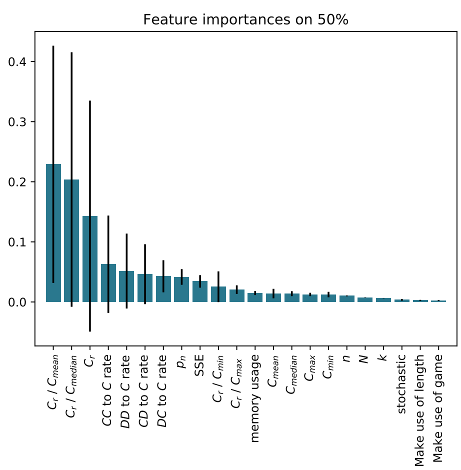
Figure 18: Importance of features in standard tournaments for different clustering methods.



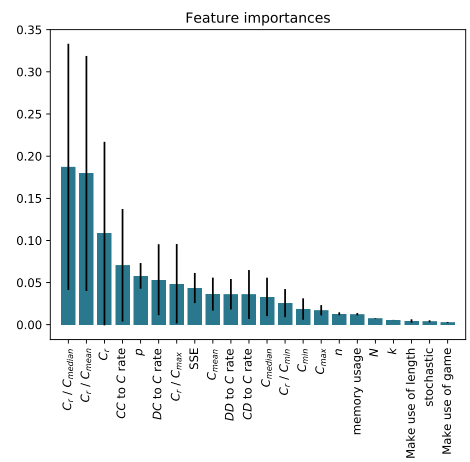
(a) Importance of features for clusters on 5% performance.



(b) Importance of features for clusters on 25% performance.

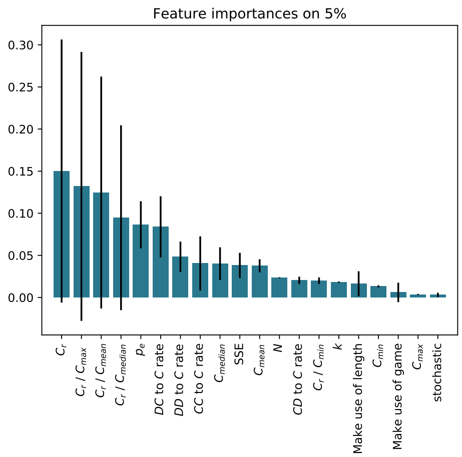


(c) Importance of features for clusters on 50% performance.

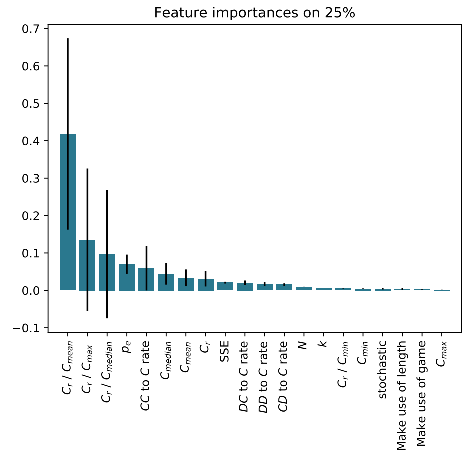


(d) Importance of features for clusters based on $k$means algorithm.

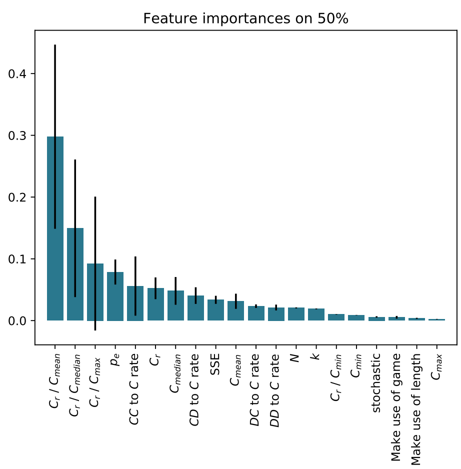
Figure 19: Importance of features in noisy tournaments for different clustering methods.



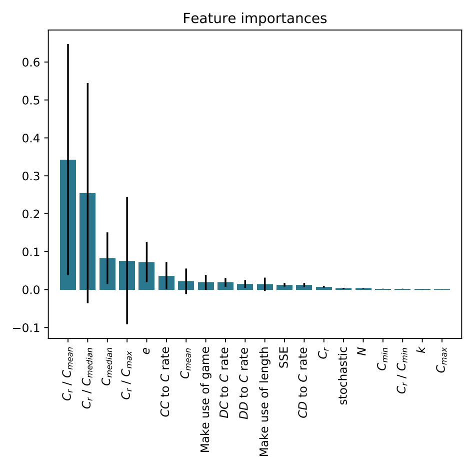
(a) Importance of features for clusters on 5% performance.



(b) Importance of features for clusters on 25% performance.

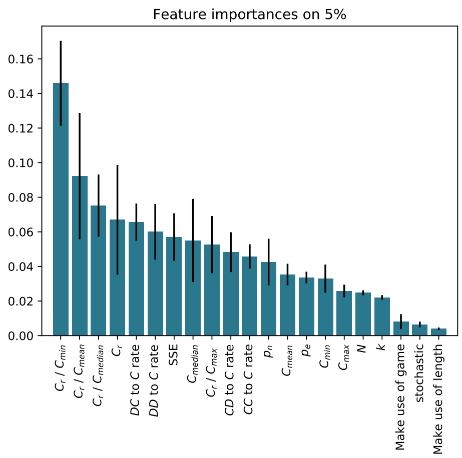


(c) Importance of features for clusters on 50% performance.

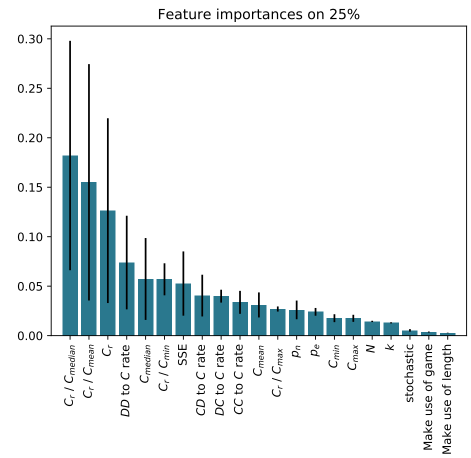


(d) Importance of features for clusters based on $k$means algorithm.

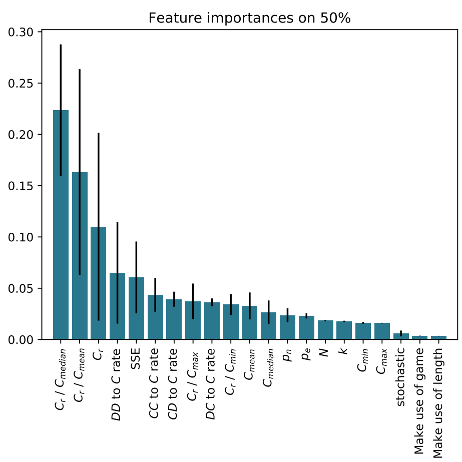
Figure 20: Importance of features in probabilistic ending tournaments for different clustering methods.



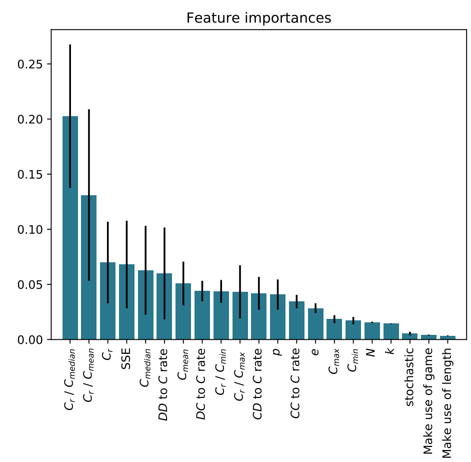
(a) Importance of features for clusters on 5% performance.



(b) Importance of features for clusters on 25% performance.

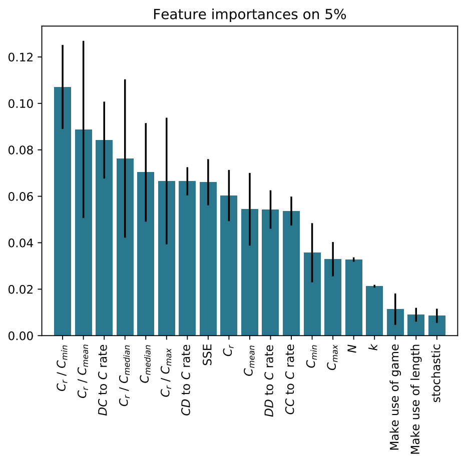


(c) Importance of features for clusters on 50% performance.

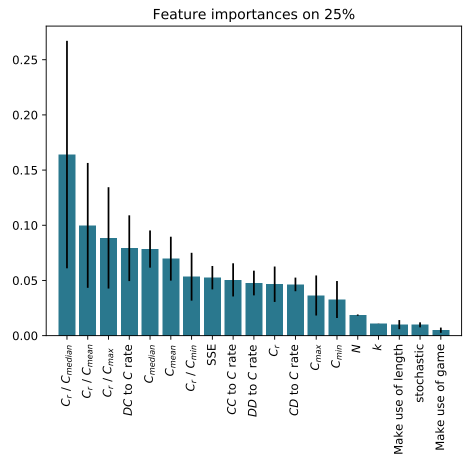


(d) Importance of features for clusters based on $k$means algorithm.

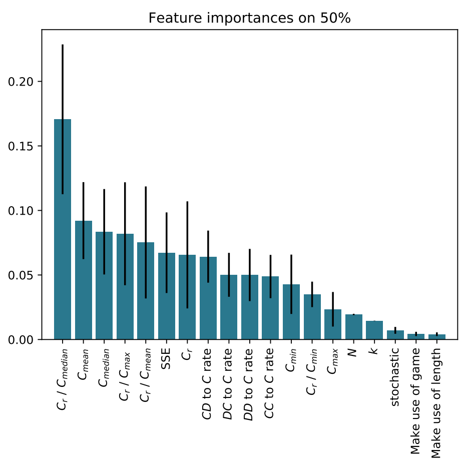
Figure 21: Importance of features in noisy probabilistic ending tournaments for different clustering methods.



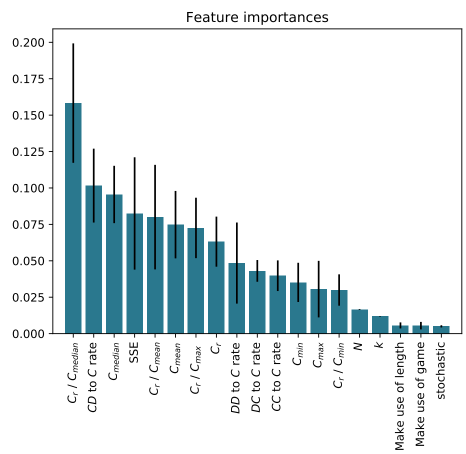
(a) Importance of features for clusters on 5% performance.



(b) Importance of features for clusters on 25% performance.



(c) Importance of features for clusters on 50% performance.



(d) Importance of features for clusters based on $k$means algorithm.

Figure 22: Importance of features over all the tournaments for different clustering methods.