Good strategies with $n-$bit memory

@nikoletaglyn

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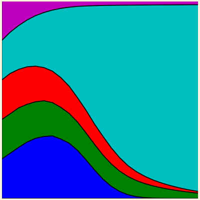
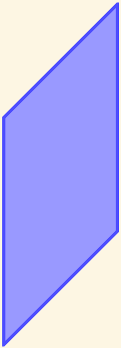
Christian Hilbe   Martin Nowak   Ethan Akin

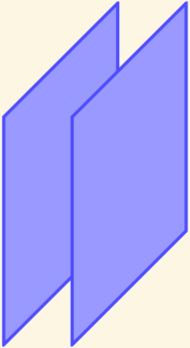


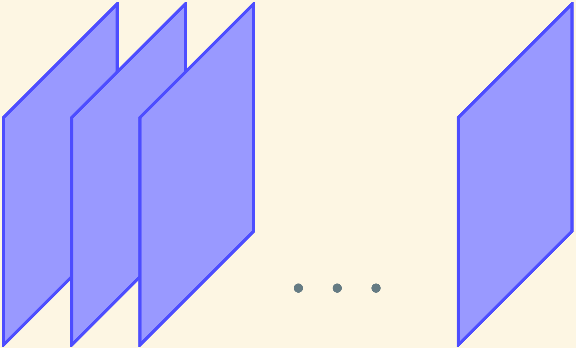
$$\begin{pmatrix} b - c & -c \\ b & 0 \end{pmatrix}$$

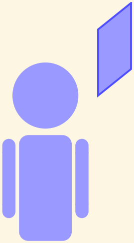
$$\begin{pmatrix} b - c & -c \\ b & 0 \end{pmatrix}$$

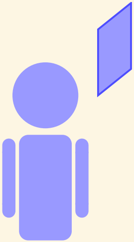
Nash ✓











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- Akin, E., 2016. The iterated prisoner's dilemma: good strategies and their dynamics.
- Stewart, A.J. and Plotkin, J.B., 2016. Small groups and long memories promote cooperation.
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CC
CD
DC
DD

$$p = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$$

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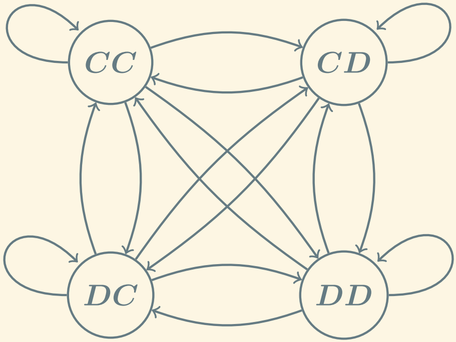
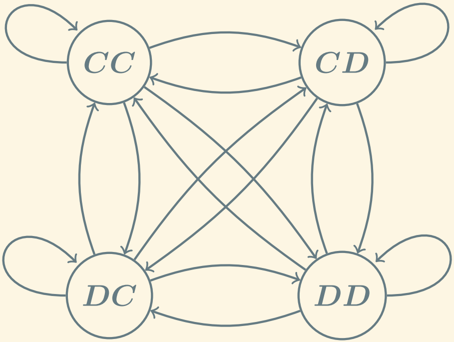
$q = (q_{CC}, q_{CD}, q_{DC}, q_{DD})$

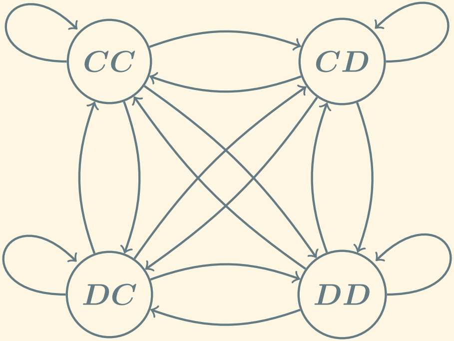


$p = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$ $q = (q_{CC}, q_{CD}, q_{DC}, q_{DD})$



$$p = (p_{CC}, p_{CD}, p_{DC}, p_{DD}) \qquad q = (q_{CC}, q_{CD}, q_{DC}, q_{DD})$$

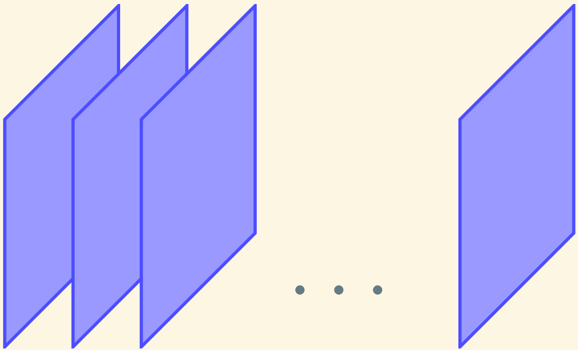
$$\mathbf{v}M = \mathbf{v}$$

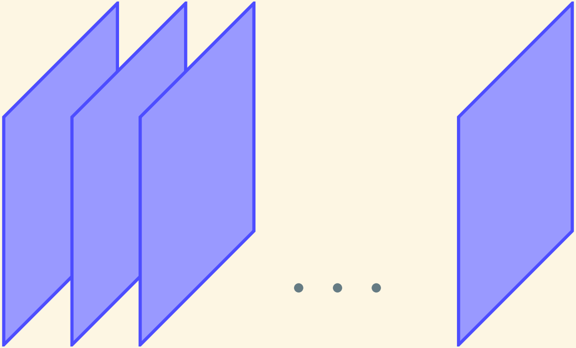


$$p = (p_{CC}, p_{CD}, p_{DC}, p_{DD}) \qquad q = (q_{CC}, q_{CD}, q_{DC}, q_{DD})$$

$$\mathbf{v}M = \mathbf{v}$$

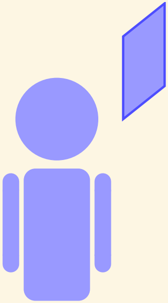
$$\mathbf{v} = (v_1, v_2, v_3, v_4)$$





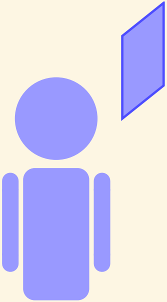
Nash?

Akin, E., 2016. The iterated prisoner's dilemma: good strategies and their dynamics.



$$\mathbf{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$$

Akin, E., 2016. The iterated prisoner's dilemma: good strategies and their dynamics.

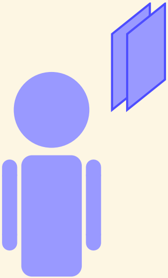


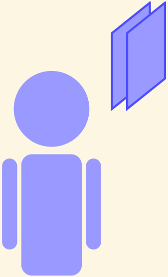
$$\mathbf{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$$

$$\mathbf{p} = (1, p_{CD}, p_{DC}, p_{DD})$$

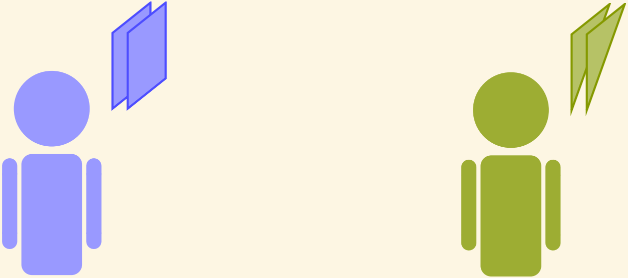
$$\frac{c}{b}\, p_{DC} \leq 1 - p_{CD} \tag{1}$$

$$\frac{c}{b-c}\, p_{DD} \leq 1 - p_{CD} \tag{2}$$

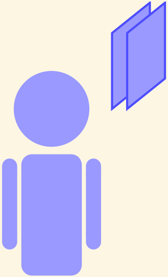




$$\mathbf{p} = (\mathbf{p}_{\mathrm{CC|CC}}, \mathbf{p}_{\mathrm{CC|CD}}, \mathbf{p}_{\mathrm{CC|DC}}, \cdots)$$



$$\mathbf{p} = (\mathbf{p}_{\textsc{cc}|\textsc{cc}}, \mathbf{p}_{\textsc{cc}|\textsc{cd}}, \mathbf{p}_{\textsc{cc}|\textsc{dc}}, \cdots)$$



$$\mathbf{p} = (\mathbf{p}_{\mathrm{CC|CC}}, \mathbf{p}_{\mathrm{CC|CD}}, \mathbf{p}_{\mathrm{CC|DC}}, \dots)$$

$$\hat{\mathbf{p}} = (\hat{\mathbf{p}}_{\mathrm{CC}}, \hat{\mathbf{p}}_{\mathrm{CC}}, \hat{\mathbf{p}}_{\mathrm{CC}}, \hat{\mathbf{p}}_{\mathrm{DD}})$$

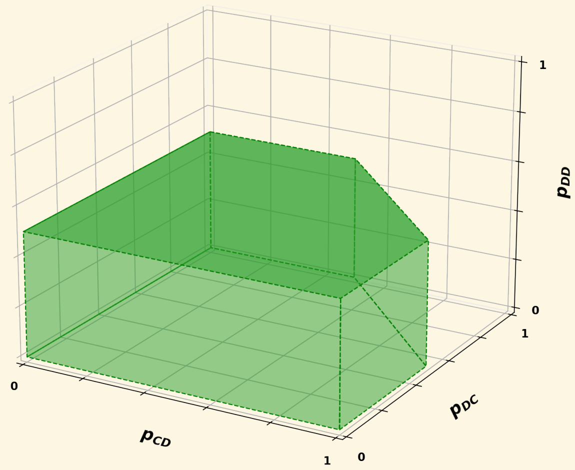
$$\hat{\mathbf{p}} = (\hat{p}_{CC}, \hat{p}_{CD}, \hat{p}_{DC}, \hat{p}_{DD})$$

$$\hat{\mathbf{p}} = (\hat{p}_{CC}, \hat{p}_{CD}, \hat{p}_{DC}, \hat{p}_{DD})$$

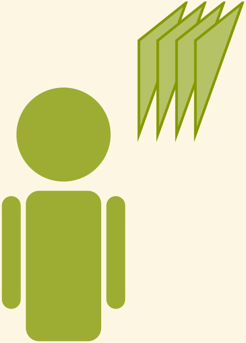
$$\hat{\mathbf{p}} = (1, \hat{p}_{CD}, \hat{p}_{DC}, \hat{p}_{DD})$$

$$\hat{p}_{DD} \leq 1 - \frac{c}{b} \tag{3}$$

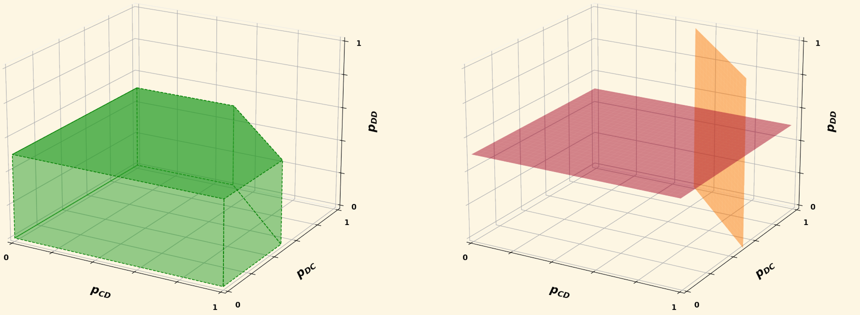
$$\frac{\hat{p}_{CD} + \hat{p}_{DC}}{2} \leq 1 - \frac{c}{2b} \tag{4}$$



0
1
p_{CD}
0
1
p_{DC}
0
1
p_{DD}



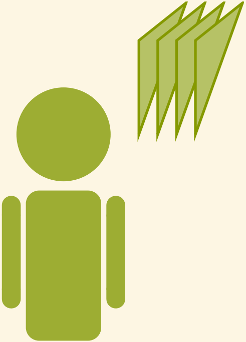
can we generalise to n-bits?



p_{DD}
p_{DC}
p_{CD}
0
1
p_{DD}
p_{DC}
p_{CD}
0
1

Lemma

An agreeable two-bit reactive strategy $\hat{p} = (\hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4)$ is of Nash type if and only if neither AllD nor the Alternator strategy yield a larger payoff.



can we generalise to n-bits?

1. **Self reactive**
2. **Memory** $n - 1$

Conjecture

For any strategy memory-n, for player q, p's score is exactly the same as if q had played a certain self reactive memory-n strategy.

extreme values 0 and 1, then the Markov equilibration time will not be long and thus not a consideration. In short, a deliberate X still has the upper hand.

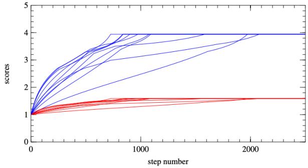


Fig. 3. Evolution of X's score (blue) and Y's score (red) in 10 instances. X plays a fixed extortionate strategy with extortion factor $\chi = 5$. Y evolves by making small steps in a gradient direction that increases his score. The 10 instances show different choices for the weights that Y assigns to different components of the gradient, i.e., how easily he can evolve along each. In all cases, X achieves her maximum possible (extortionate) score.

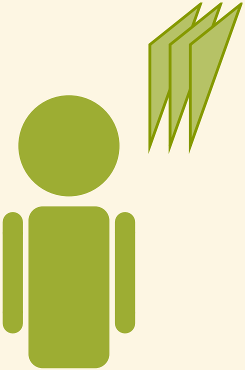
exactly evolution, on the hugely larger canvas of DNA-based life, that ultimately has produced X, the player with the mind.

Appendix A: Shortest-Memory Player Sets the Rules of the Game. In iterated play of a fixed game, one might have thought that a player Y with longer memory of past outcomes has the advantage over a more forgetful player X. For example, one might have thought that player Y could devise an intricate strategy that uses X's last 1,000 plays as input data in a decision algorithm, and that can then beat X's strategy, conditioned on only the last one iteration. However, that is not the case when the same game (same allowed moves and same payoff matrices) is indefinitely repeated. In fact, for any strategy of the longer-memory player Y, X's score is exactly the same as if Y had played a certain shorter-memory strategy (roughly, the marginalization of Y's long-memory strategy), disregarding any history in excess of that shared with X.

Let X and Y be random variables with values x and y that are the players' respective moves on a given iteration. Because their scores depend only on (x, y) separately at each time, a sufficient statistic is the expectation of the joint probability of (X, Y) over past histories H (of course in their proportion seen). Let $H = [H_0, H_1]$, where H_0 is the recent history shared by both X and Y, and H_1 is the older history seen only by Y. Then a straightforward calculation is,

1. **Self reactive** ✓
2. **Memory** $n-1$

1. **Self reactive** $\checkmark$
2. **Memory** $n-1$ **?**



Does the hypothesis that we only need to check self reactive memory-$(n-1)$ work for 3-bit reactive strategies?

Summary

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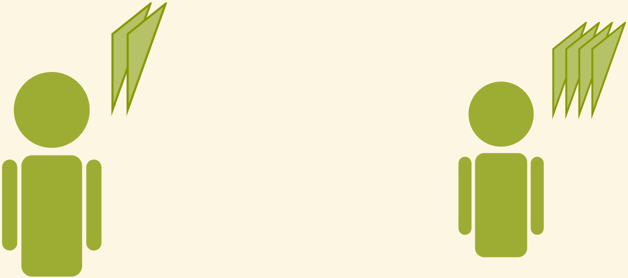

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http://web.evolbio.mpg.de/social-behaviour/

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- Ethan Akin