Conditional cooperation with longer memory

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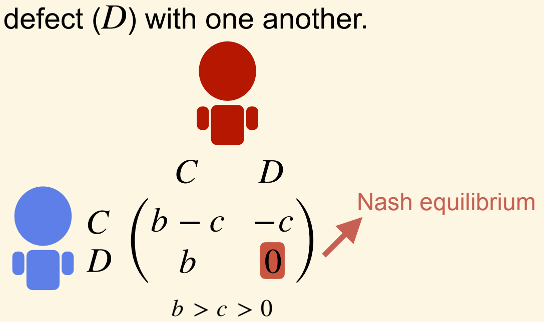
arXiv:2402.02437

1. Introduction

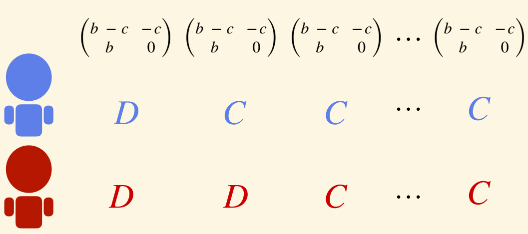
Human cooperative behavior is governed by direct reciprocity. This mechanism for cooperation can explain why people return favors, why they show more effort in group tasks when others do, or why they stop cooperating when they feel exploited. The main theoretical framework to describe reciprocity is the **repeated prisoner's dilemma.**

What is the prisoner's dilemma?

A two players game, who repeatedly decide whether to cooperate (C) or to defect (D) with one another.



What is the repeated prisoner's dilemma?

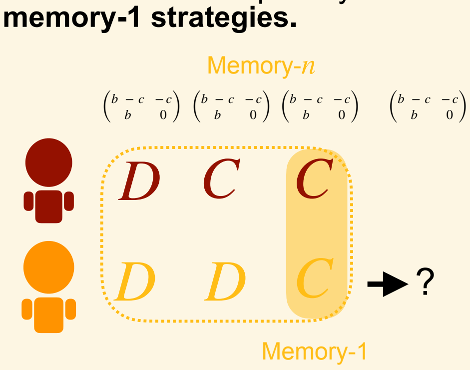


What is a strategy?

A strategy is a plan that dictates a player's actions at each turn and it can be based on the entire history of the game.

2. Current Work

Most theoretical research on the evolution of reciprocity focuses on **memory-1 strategies.**



A formal analysis of strategies with more than one-round memory has been difficult for two reasons:

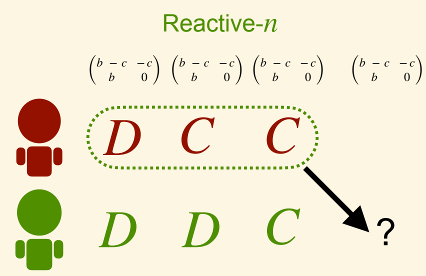
1. As the memory length n increases, strategies become harder to interpret.

2. The number of strategies, and the time it takes to compute their payoffs, increases dramatically in n.

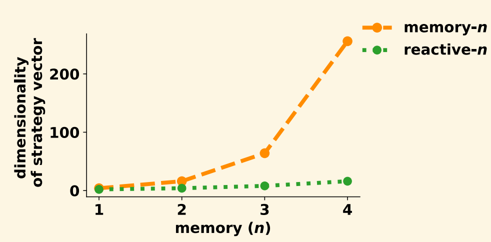
Can we say anything about Nash in the repeated prisoner's dilemma?

3. Our Approach

We focus on an easy-to-interpret subset of memory-n strategies, the ***reactive-n* strategies**. Capturing the basic premise of conditional cooperation, they only depend on the *co-player's* actions during the last n rounds.



Definition. A reactive-n strategy can be defined as 2^n-dimensional vector $\mathbf{p} = (p_{\mathbf{h^{-i}}})_{\mathbf{h^{-i}} \in H^{-i}}$ with $0 \le p_{\mathbf{h^{-i}}} \le 1$ where $\mathbf{h^{-i}}$ refers an n-history of the co-player from the space of all possible co-player histories.



4. Nash in Higher n

We have developed a general algorithm to determine if a given reactive-n strategy is a Nash equilibrium for any n.

To achieve this, we established the following technical results:

1. Against reactive strategies, any feasible payoff can be generated with self-reactive strategies.

2. To any reactive strategy, there is a best response among the pure self-reactive strategies.

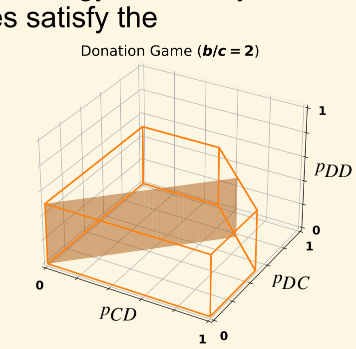
An algorithm to check p if is Nash.

input: p, n
pure self reactive strategies $\leftarrow \left\{ \tilde{\mathbf{p}} \mid \tilde{\mathbf{p}} \in \{0,1\}^{2^n} \right\}$;
isNash ←True;
for $\tilde{\mathbf{p}} \in$ *pure self reactive strategies* **do**
  if p is not a best response $\tilde{\mathbf{p}}$ **to then**
    isNash ←False;
return ($\mathbf{p}$, isNash);

5. Cooperative Nash

Theorem. A reactive-2 strategy can be defined as the vector $\mathbf{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$, and it is a cooperative Nash strategy if and only if, the strategy entries satisfy the conditions,

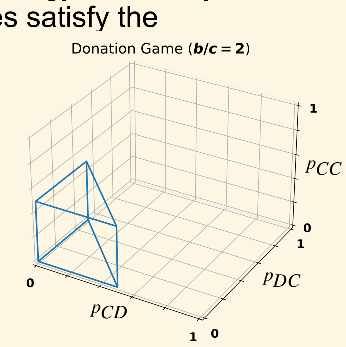
$$p_{CC} = 1,$$
$$\frac{p_{CD} + p_{DC}}{2} < 1 - \frac{1}{2} \cdot \frac{c}{b},$$
$$p_{DD} \le 1 - \frac{c}{b}.$$



6. Defective Nash

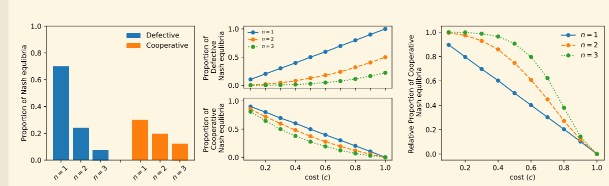
Theorem. A reactive-2 strategy can be defined as the vector $\mathbf{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$, and it is a defective Nash strategy if and only if, the strategy entries satisfy the conditions,

$$p_{CC} \le \frac{c}{b}$$
$$\frac{p_{CD} + p_{DC}}{2} \le \frac{c}{2b},$$
$$p_{DD} = 0.$$

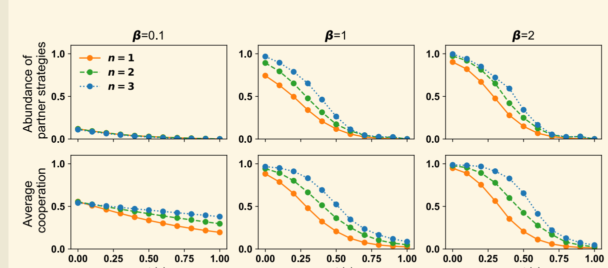


7. Simulations

Volume of cooperative and defective Nash.



Evolutionary dynamics of reactive-n strategies.



8. Conclusion

- Developed an algorithm to verify whether a given reactive-n strategy is Nash.
- We fully characterize cooperative & defective Nash strategies for $n = 2$ and $n = 3$.
- We showed that longer memory helps sustain cooperation.