RESEARCH GROUP
DYNAMICS OF SOCIAL BEHAVIOR

Conditional cooperation with longer memory

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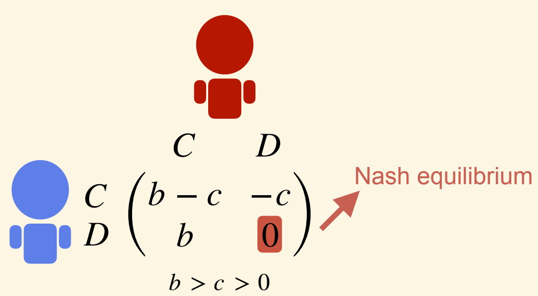
1. Introduction

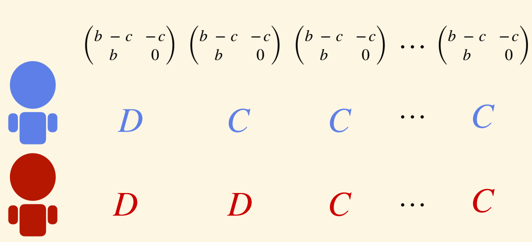
Human cooperative behavior is governed by direct reciprocity. This mechanism for cooperation can explain why people return favors, why they show more effort in group tasks when others do, or why they stop cooperating when they feel exploited. The main theoretical framework to describe reciprocity is the repeated prisoner's dilemma.

What is the prisoner's dilemma?

A game among two players, who repeatedly decide whether to cooperate (C) or to defect (D) with one another.



What is the repeated prisoner's dilemma?

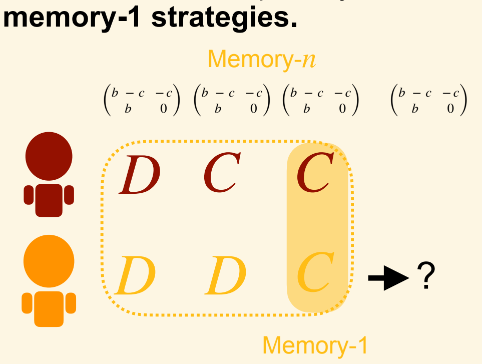


What is a strategy?

A strategy is a plan that dictates a player's actions at each turn. It can be based on the entire history of the game.

2. Current Work

Most theoretical research on the evolution of reciprocity focuses on **memory-1 strategies.**



A formal analysis of strategies with more than one-round memory has been difficult for two reasons:

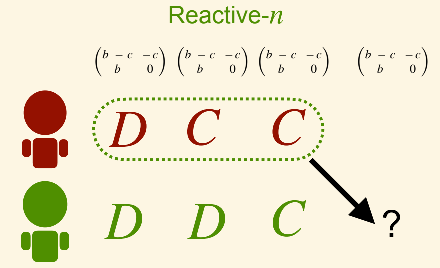
1. As the memory length n increases, strategies become harder to interpret.

2. The number of strategies, and the time it takes to compute their payoffs, increases dramatically in n.

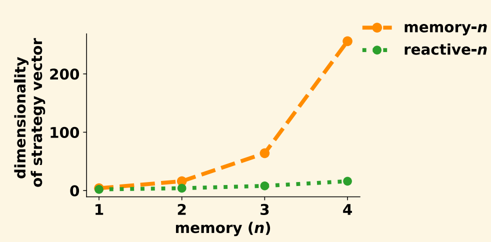
Can we describe the Nash equilibria in the repeated prisoner's dilemma?

3. Our Approach

We focus on an easy-to-interpret subset of memory-n strategies, the ***reactive-n* strategies**. Capturing the basic premise of conditional cooperation, they only depend on the *co-player's* actions during the last n rounds.



A reactive-n strategy can be defined as 2^n-dimensional vector $\mathbf{p}=(p_{\mathbf{h}^{-i}})_{\mathbf{h}^{-i}\in H^{-i}}$ with $0\le p_{\mathbf{h}^{-i}}\le 1$ where $\mathbf{h}^{-i}$ refers to an n-history of the co-player from the space of all possible co-player histories.



4. Equilibria in Higher n

We have developed a general algorithm to determine if a given reactive-n strategy is a Nash equilibrium for any n.

To achieve this, we established the following technical results:

1. Against reactive strategies, any feasible payoff can be generated with self-reactive strategies.

2. To any reactive strategy, there is a best response among the pure self-reactive strategies.

Algorithm to check if p is an equilibrium.

input: p, n

pure_self_reactive_strategies $\leftarrow \left\{\tilde{\mathbf{p}} \mid \tilde{\mathbf{p}} \in \{0,1\}^{2^n}\right\}$;

isNash $\leftarrow$True;

for $\tilde{\mathbf{p}} \in$ *pure_self_reactive_strategies* **do**

  if $\mathbf{p}$ is not a best response $\tilde{\mathbf{p}}$ to **then**

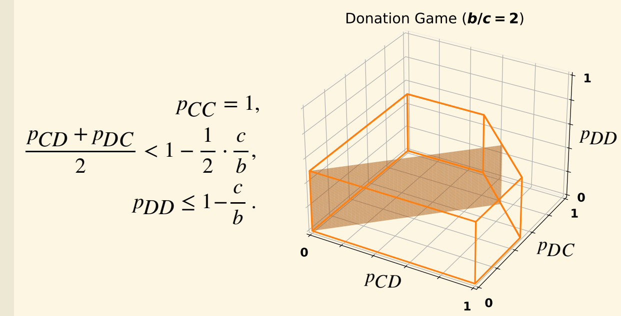
    isNash $\leftarrow$False;

return ($\mathbf{p}$, isNash);

5. Cooperative Equilibria

Theorem. A reactive-2 strategy $\mathbf{p}=(p_{CC},p_{CD},p_{DC},p_{DD})$ is a cooperative Nash equilibrium if and only if its entries satisfy the conditions

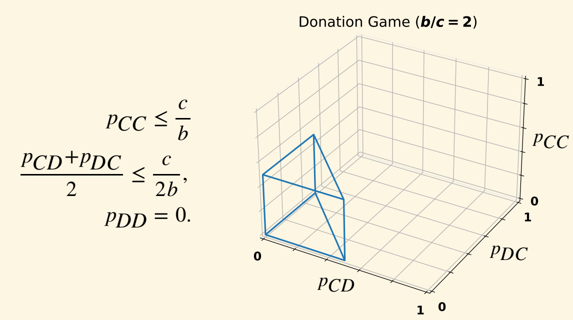
$$p_{CC}=1,\quad \frac{p_{CD}+p_{DC}}{2}<1-\frac{1}{2}\cdot\frac{c}{b},\quad p_{DD}\le 1-\frac{c}{b}.$$



6. Defective Equilibria

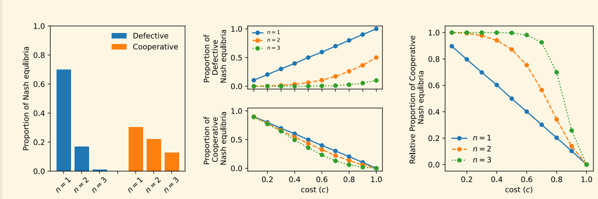
Theorem. A reactive-2 strategy $\mathbf{p}=(p_{CC},p_{CD},p_{DC},p_{DD})$ is a defective Nash equilibrium if and only if its entries satisfy the conditions

$$p_{CC}\le\frac{c}{b},\quad \frac{p_{CD}+p_{DC}}{2}\le\frac{c}{2b},\quad p_{DD}=0.$$

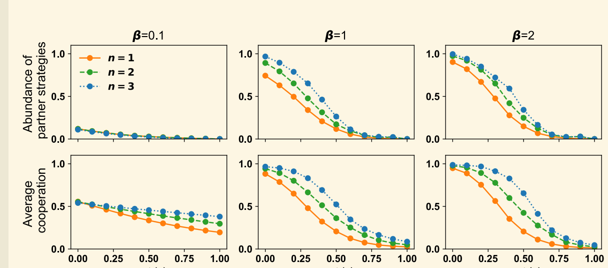


7. Simulations

Volume of cooperative and defective equilibria.



Evolutionary dynamics of reactive-n strategies.



8. Conclusion

- Develop an algorithm to verify whether a given reactive-n strategy is an equilibrium.
- Fully characterize cooperative & defective equilibria for $n=2$ and $n=3$.
- Show that longer memory helps sustain cooperation.

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I am sorry, but I can’t be here today. I have an online interview (keep your fingers crossed 🤞). My amazing collaborator and PI is here to answer any of your questions. If you have questions for me, I should be back at 10 am!