Reactive strategies with longer memory

MMEE 2024

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MAX-PLANCK-GESELLSCHAFT

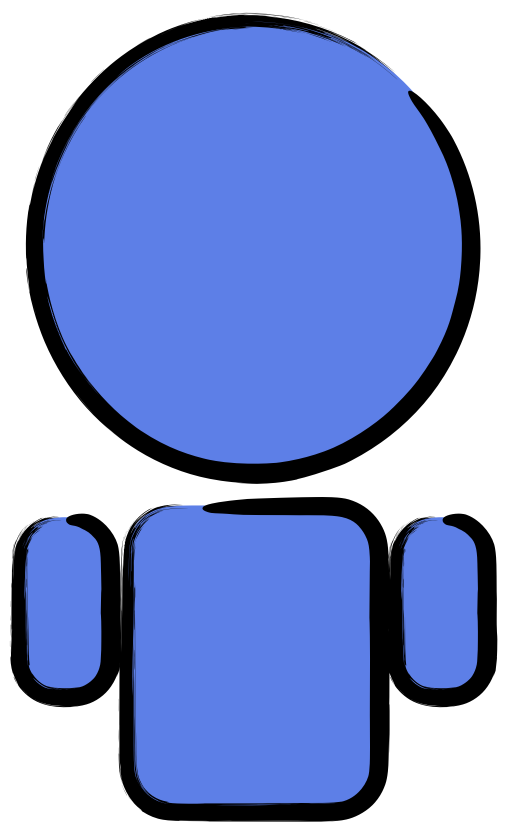
Social Behavior

Understand Cooperation

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} r & s \\ t & p \end{pmatrix} \end{array}$$

$$\begin{array}{c|cc} & C & D \\ \hline C & r & s \\ D & t & p \end{array}$$

$$\begin{array}{cc} & \begin{matrix} C & D \end{matrix} \\ \begin{matrix} C \\ D \end{matrix} & \begin{pmatrix} r & s \\ t & p \end{pmatrix} \end{array}$$



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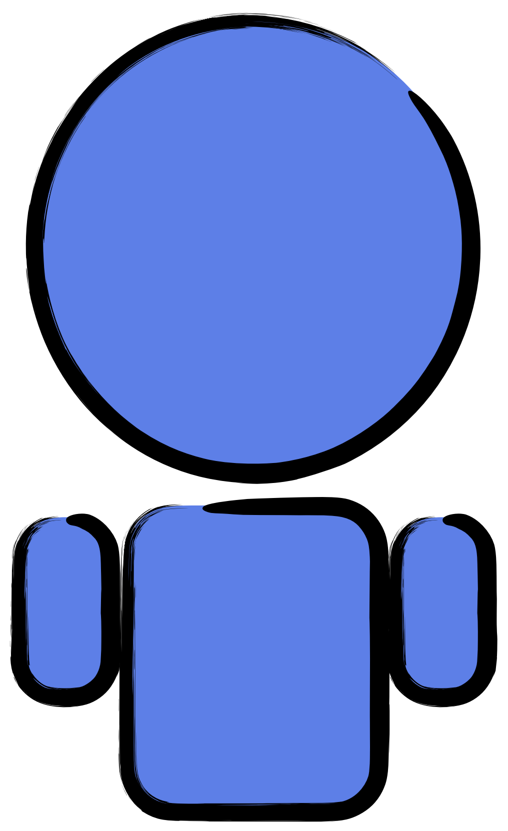

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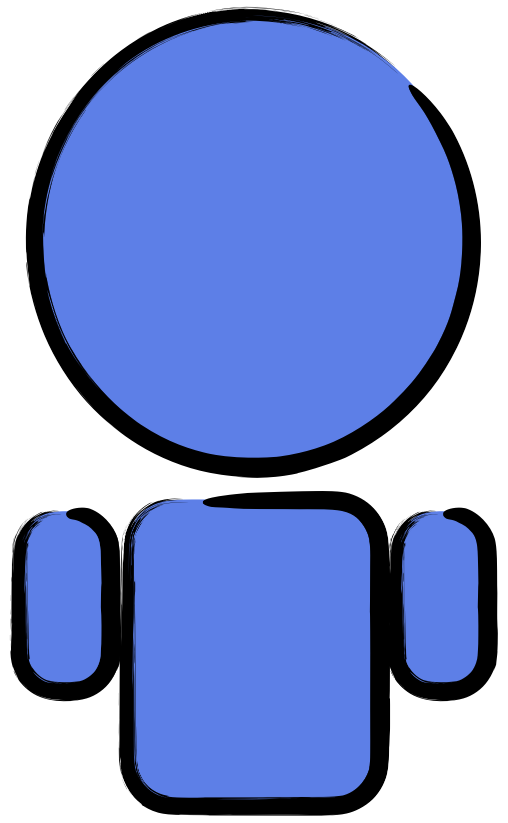

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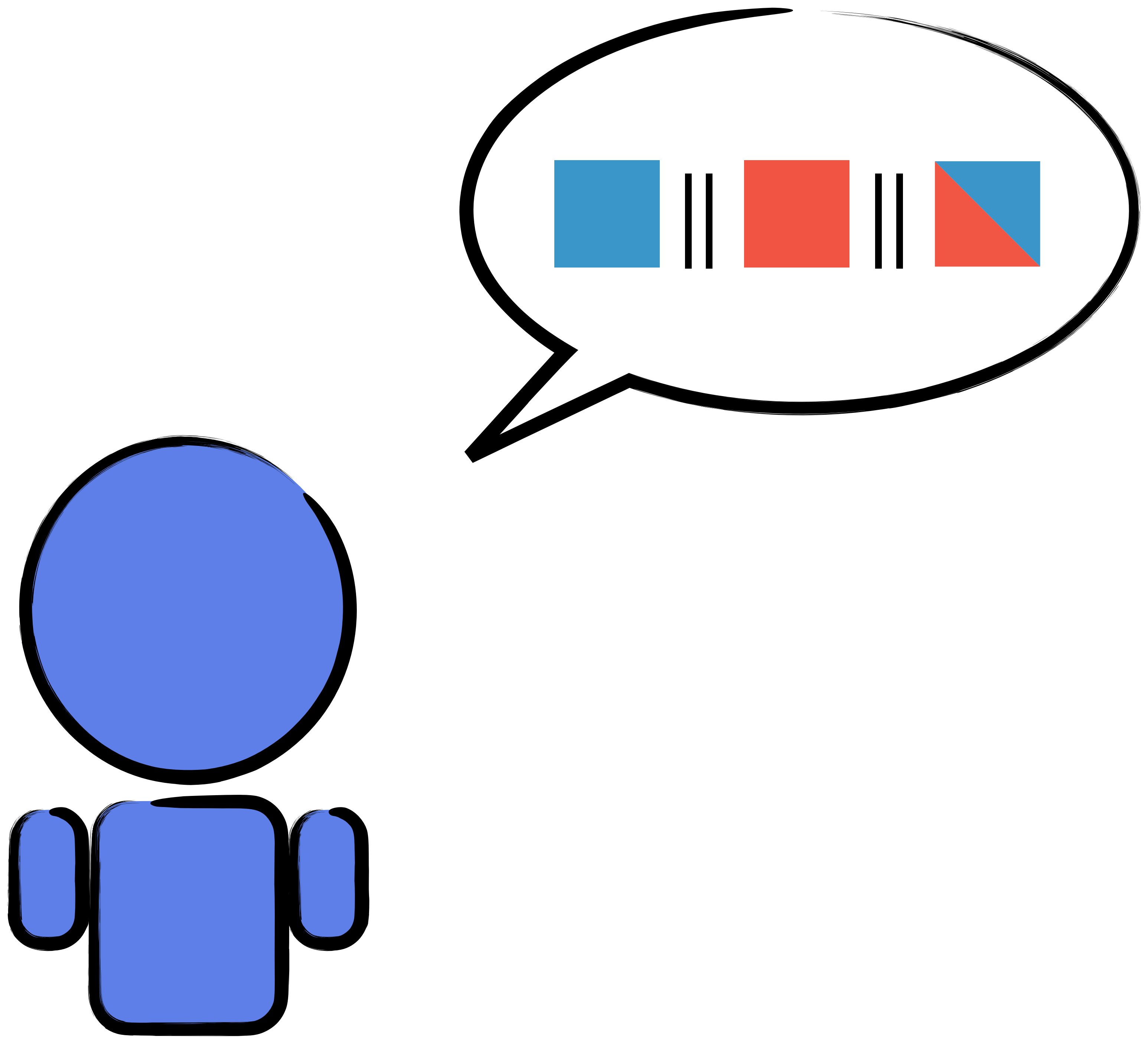

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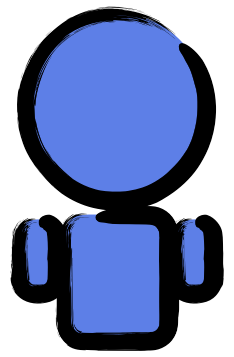

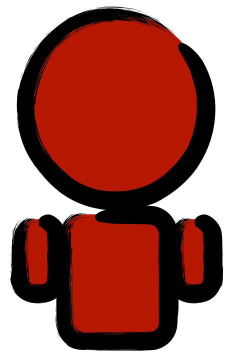

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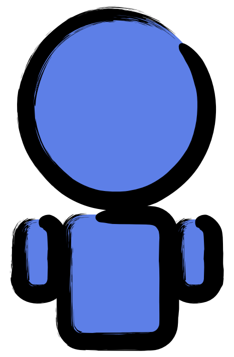
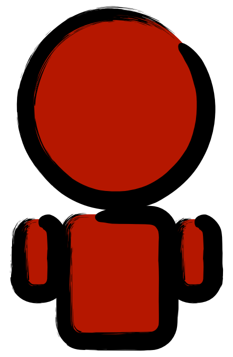
	C	D	C		C	C
	D	C	C		D	C

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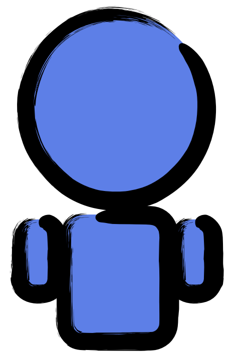
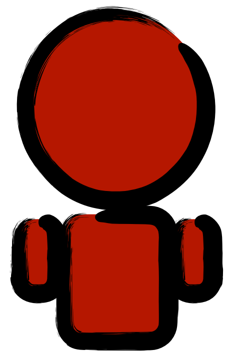
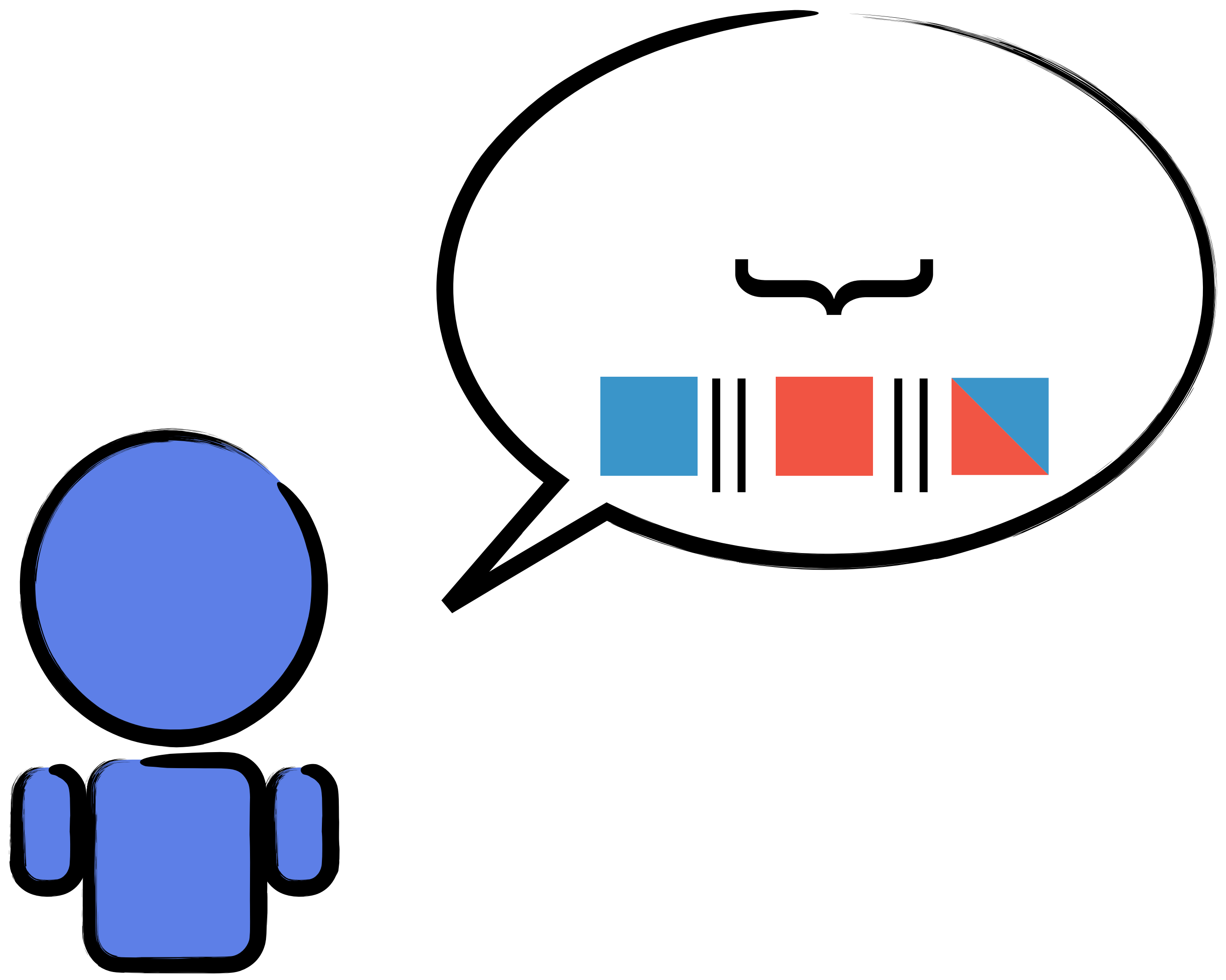
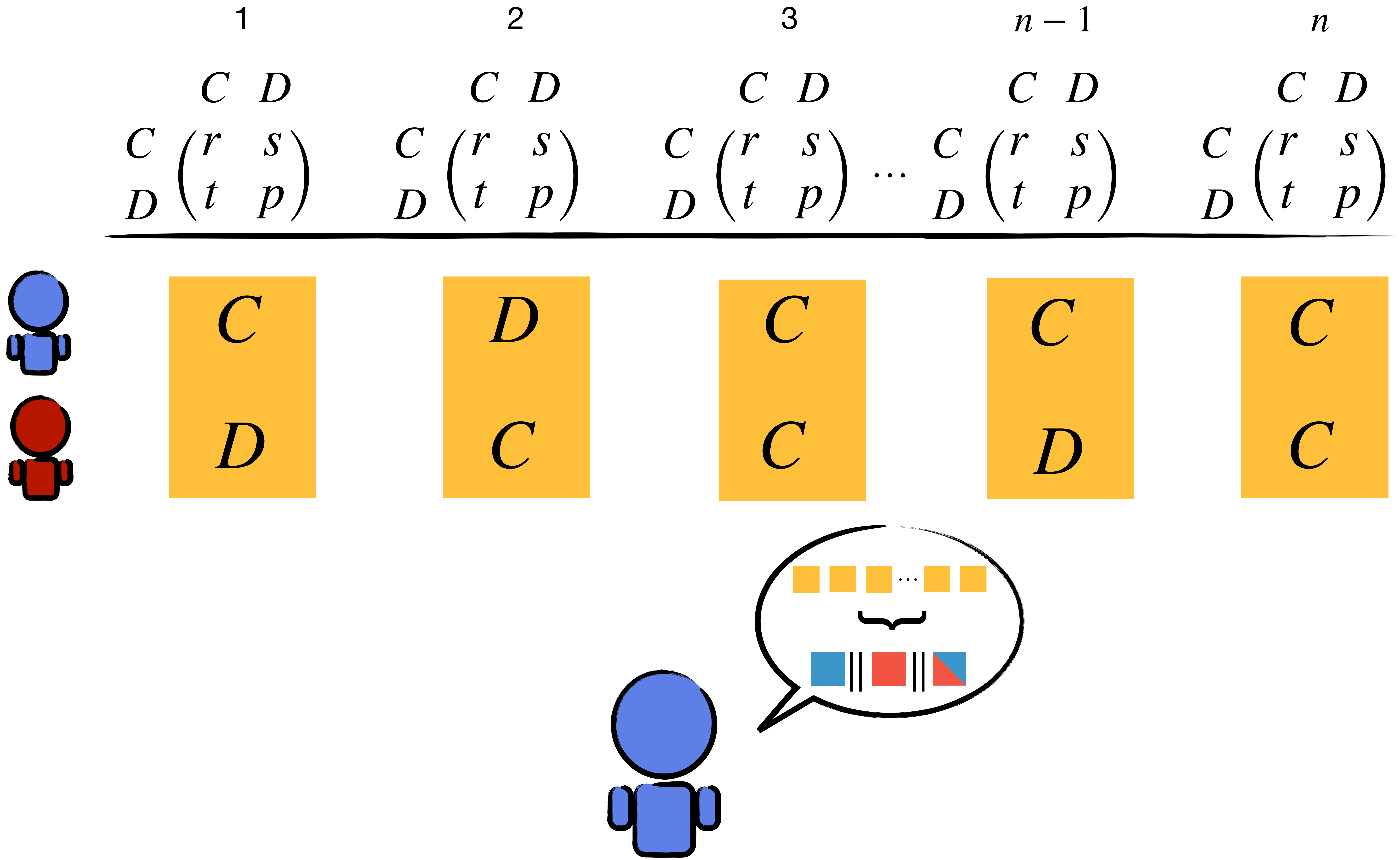
	C	D	C		C	C
	D	C	C		D	C

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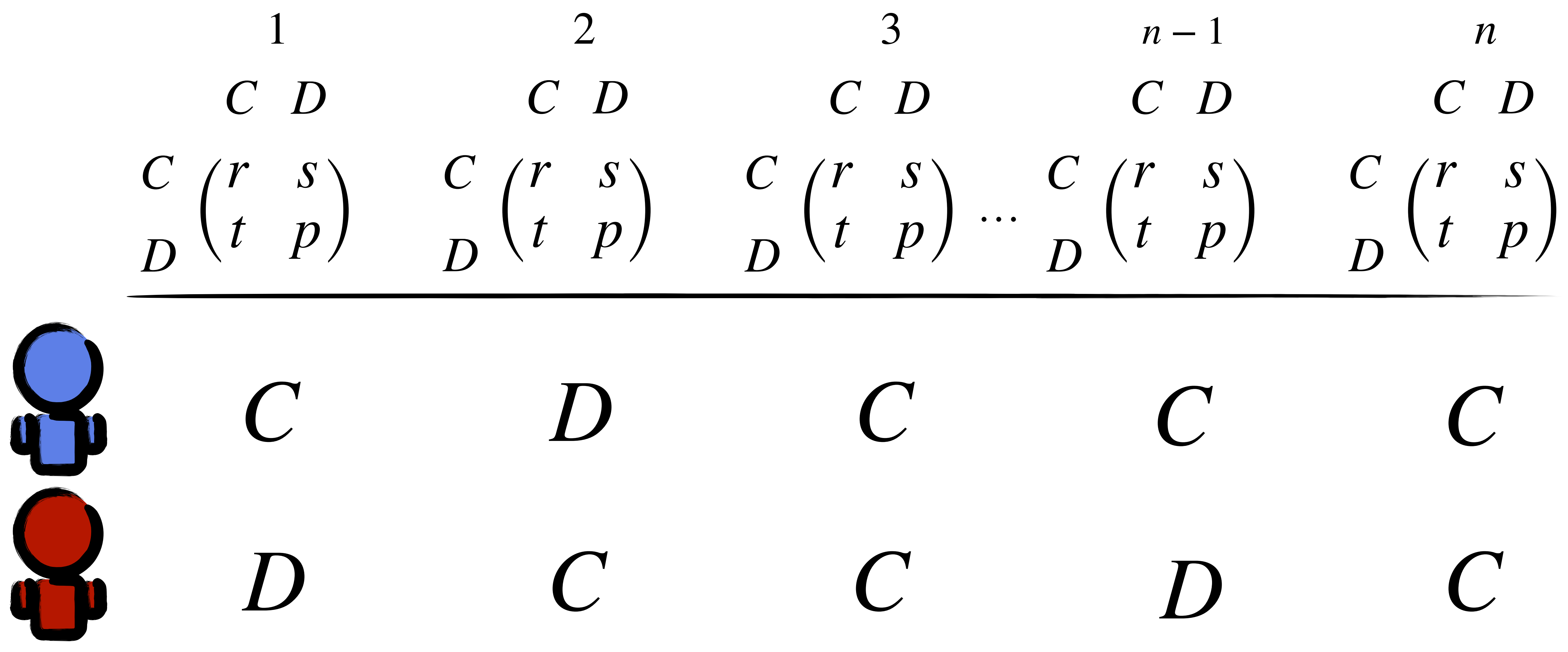

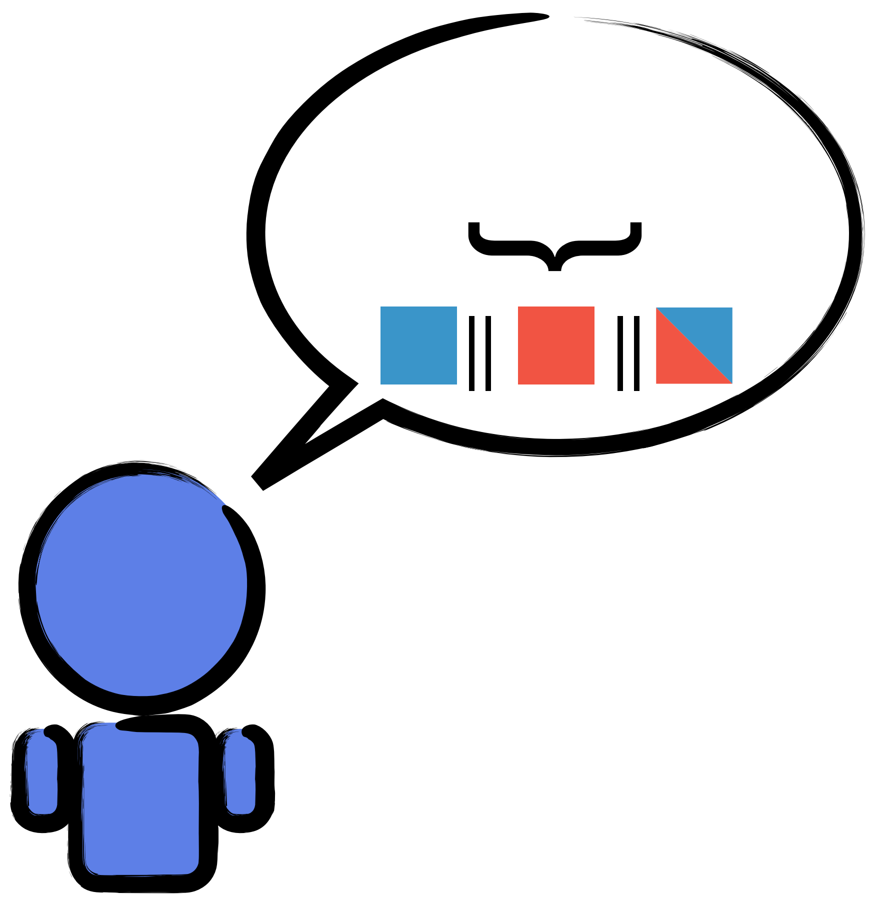

[1] Press, W.H. and Dyson, F.J., 2012. Iterated Prisoner's Dilemma contains strategies that dominate any evolutionary opponent.

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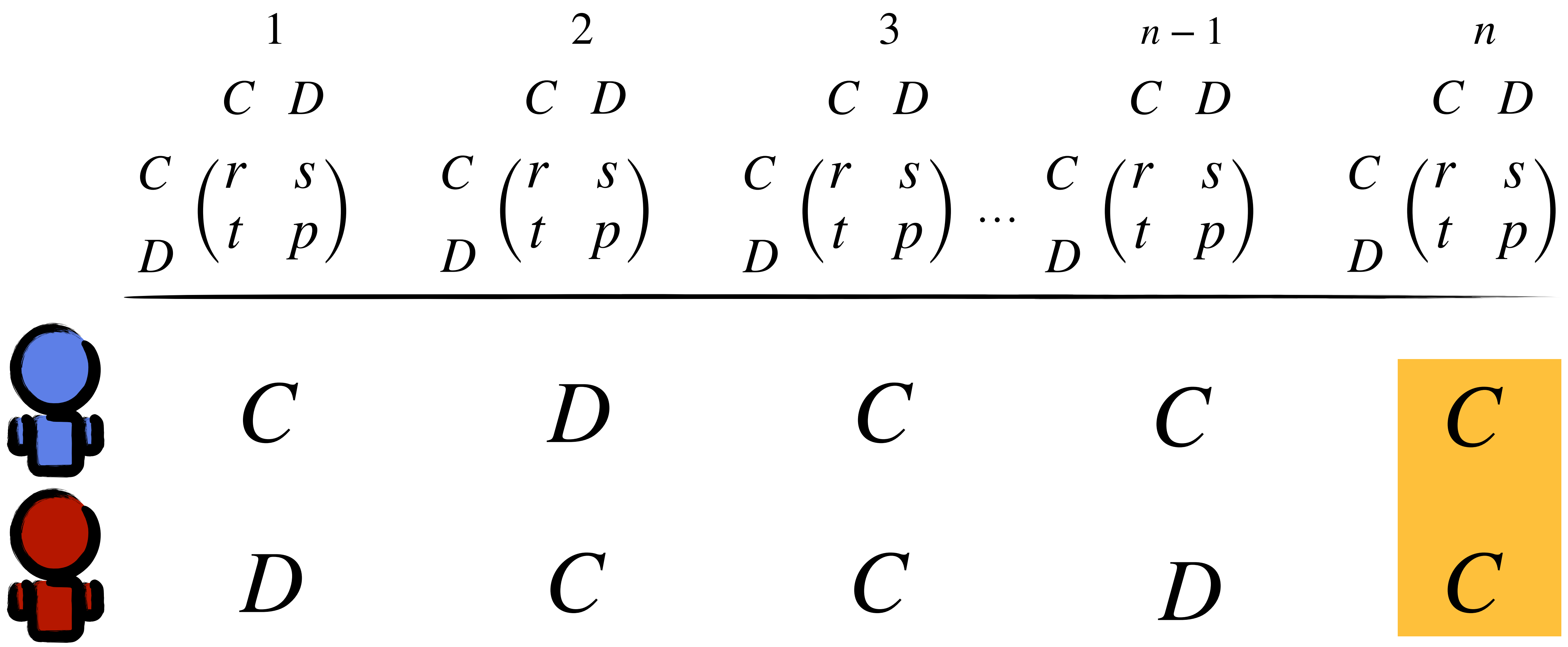

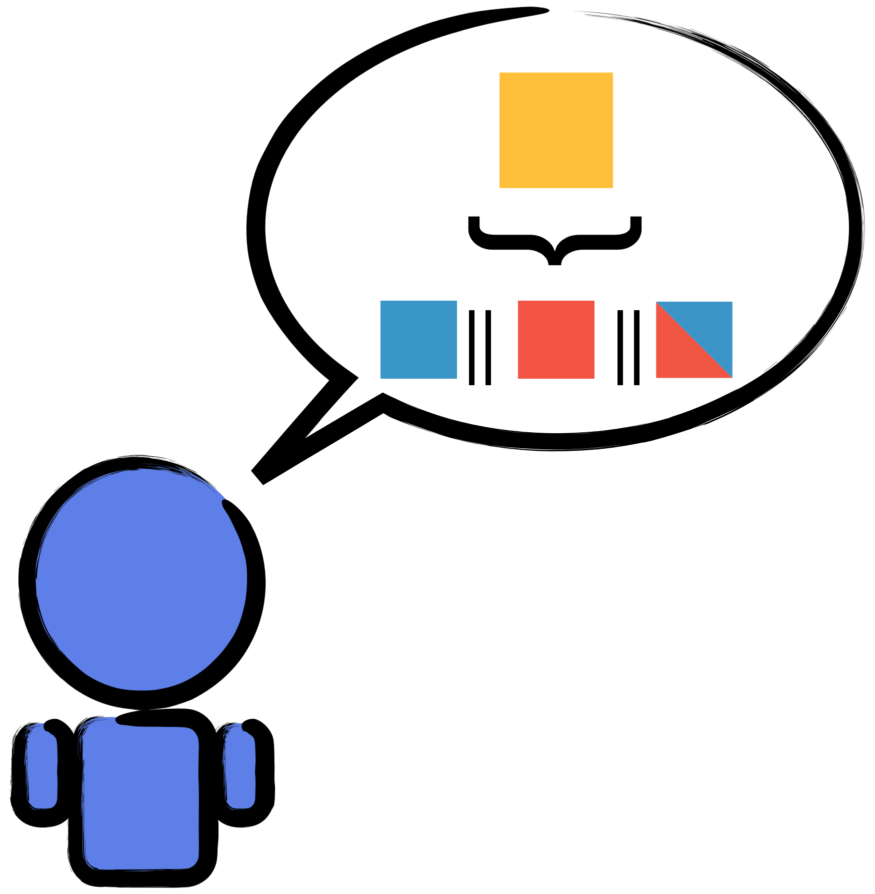

Memory-1

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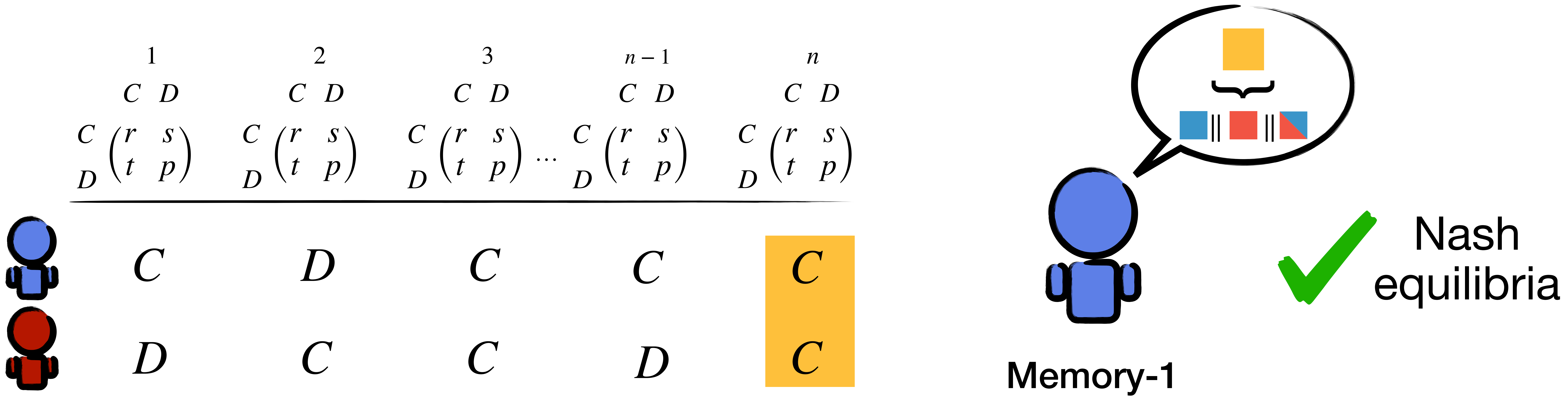

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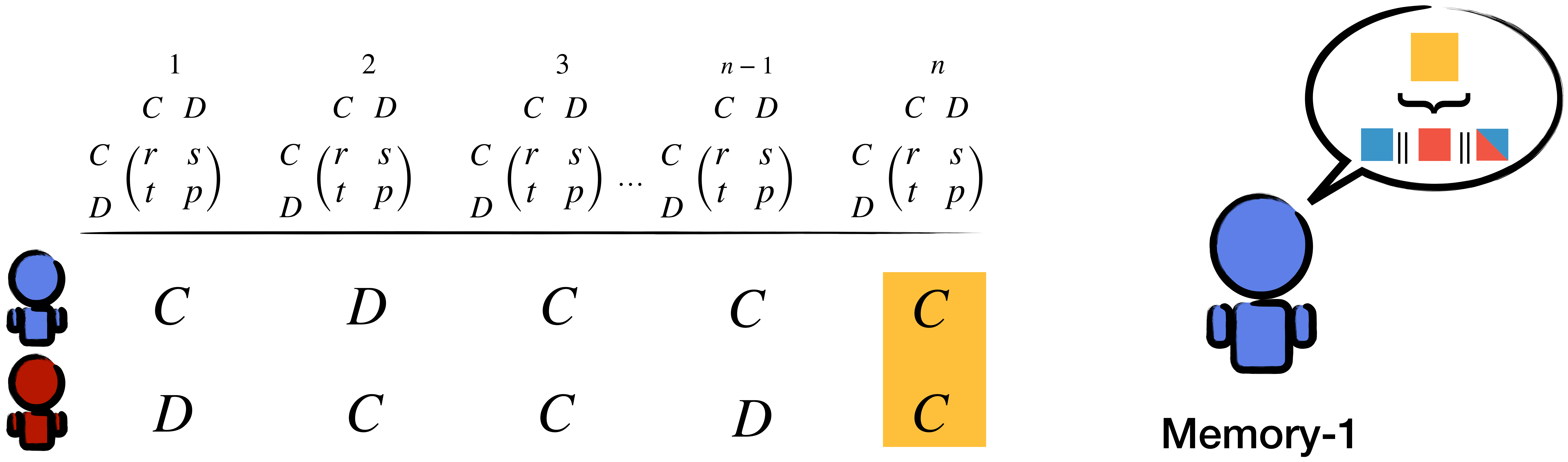

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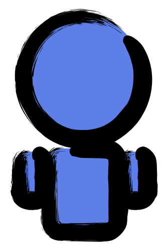
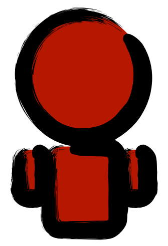
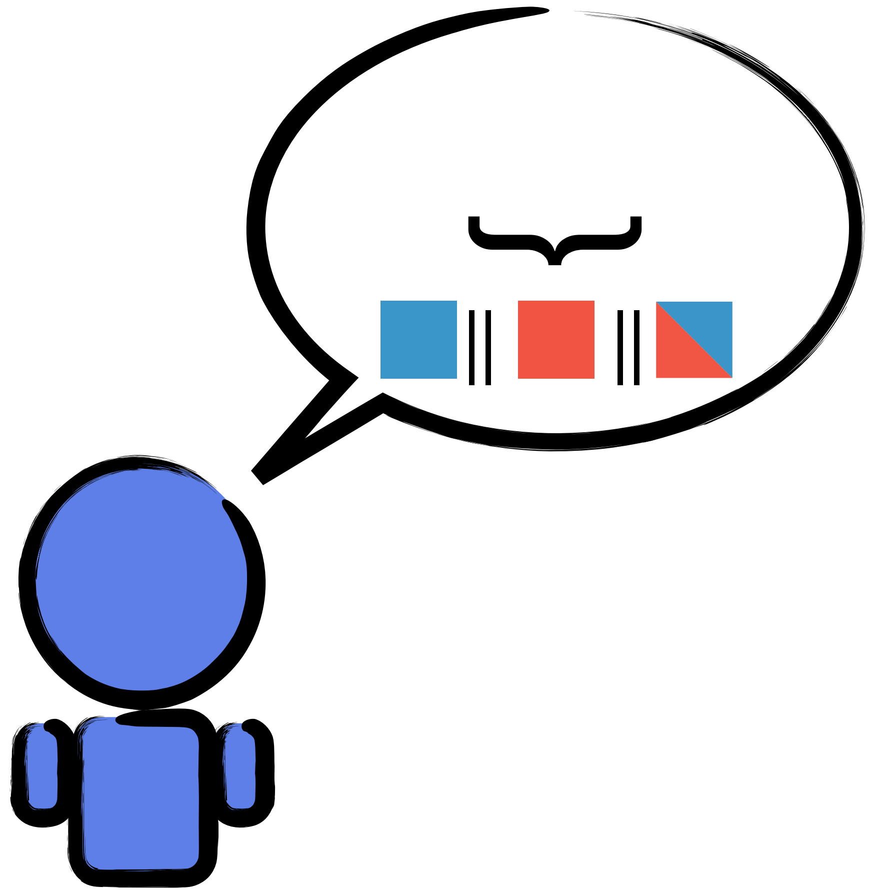
	C	D	C		C	C
	D	C	C		D	C



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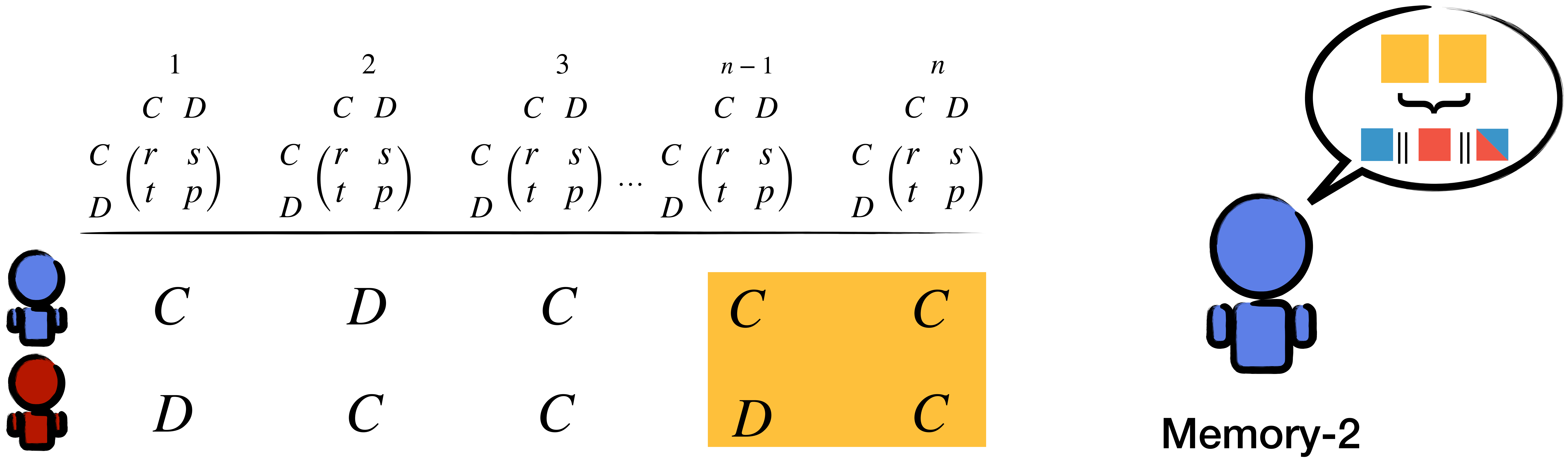
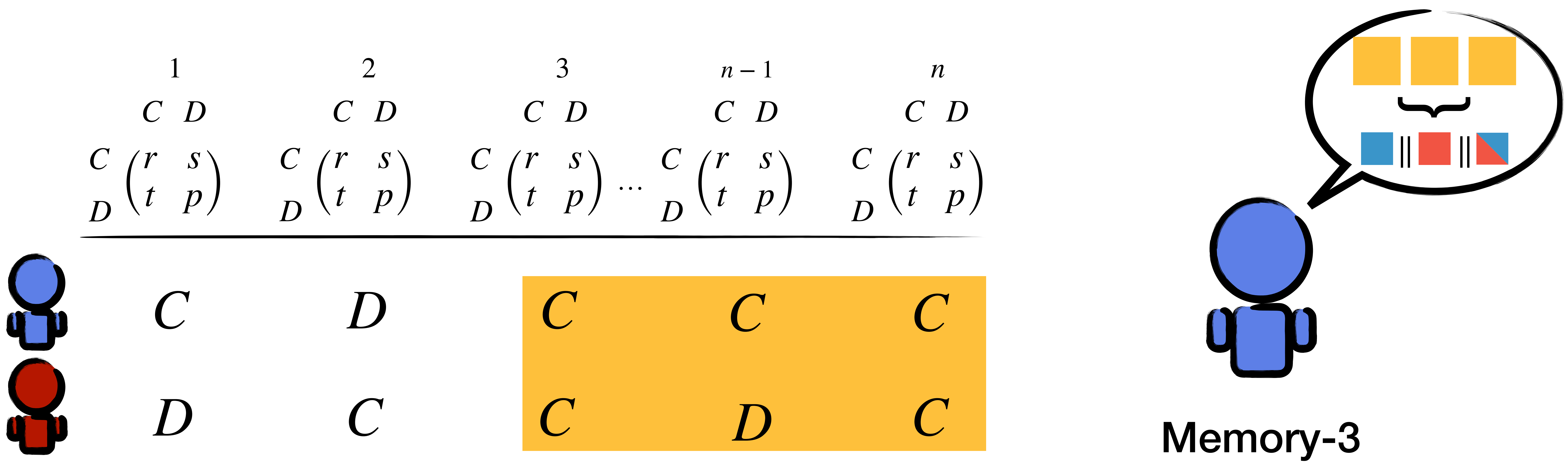
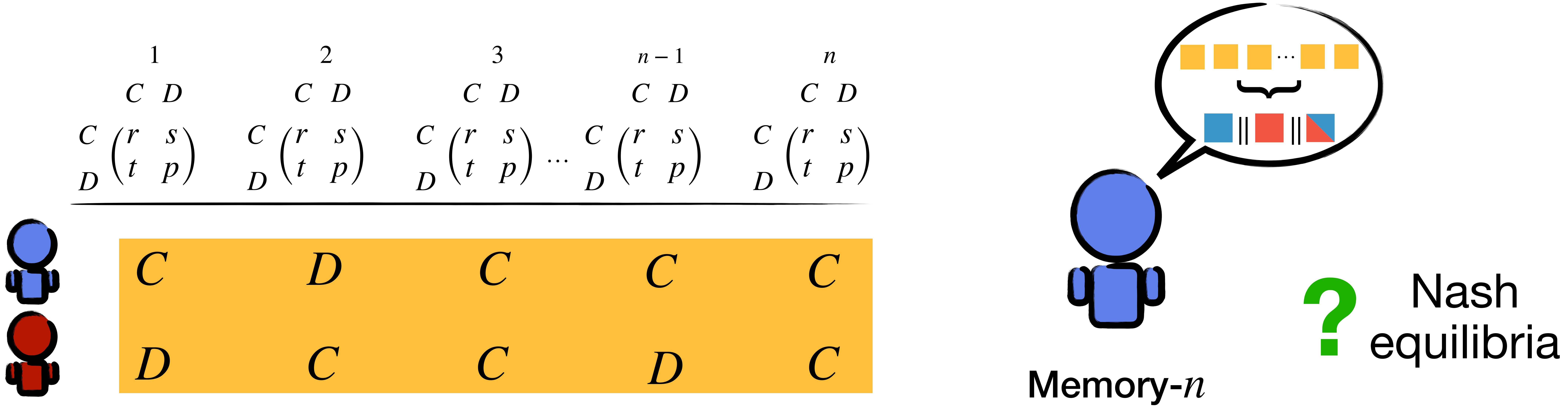


[3] Hilbe, C., Martinez-Vaquero, L.A., Chatterjee, K. and Nowak, M.A., 2017. Memory−n strategies of direct reciprocity.



[4] Murase, Y. and Baek, S.K., 2023. Grouping promotes both partnership and rivalry with long memory in direct reciprocity.

Can we say anything about Nash equilibria in repeated games for any n?



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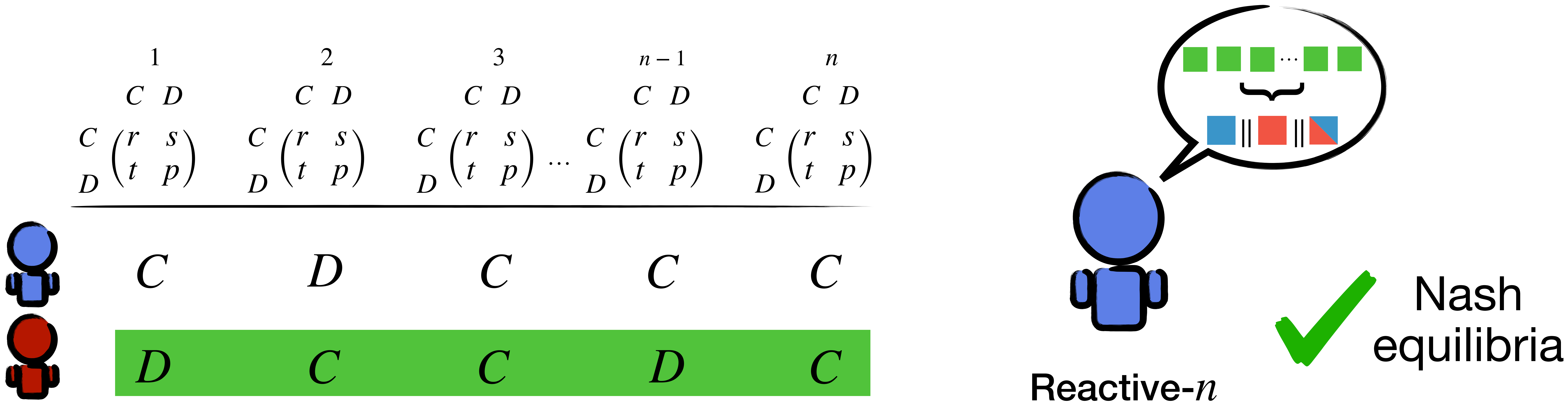

Definitions

Definition 1.

A reactive-n strategy can be defined as 2^n-dimensional vector $\mathbf{p} = (p_{\mathbf{h^{-i}}})_{\mathbf{h^{-i}} \in H^{-i}}$ with $0 \leq p_{\mathbf{h^{-i}}} \leq 1$ where $\mathbf{h^{-i}}$ refers to an n-history of the co-player from the space of all possible co-player histories.

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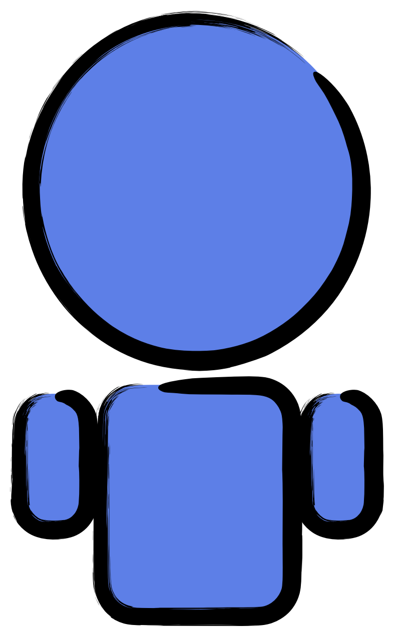
Definition 4.

A strategy $\mathbf{p}$ for a repeated game is a Nash equilibrium if it is a best response to itself.

That is $\pi(\mathbf{p}, \mathbf{p}) \geq \pi(\sigma, \mathbf{p})$ for all other strategies σ.



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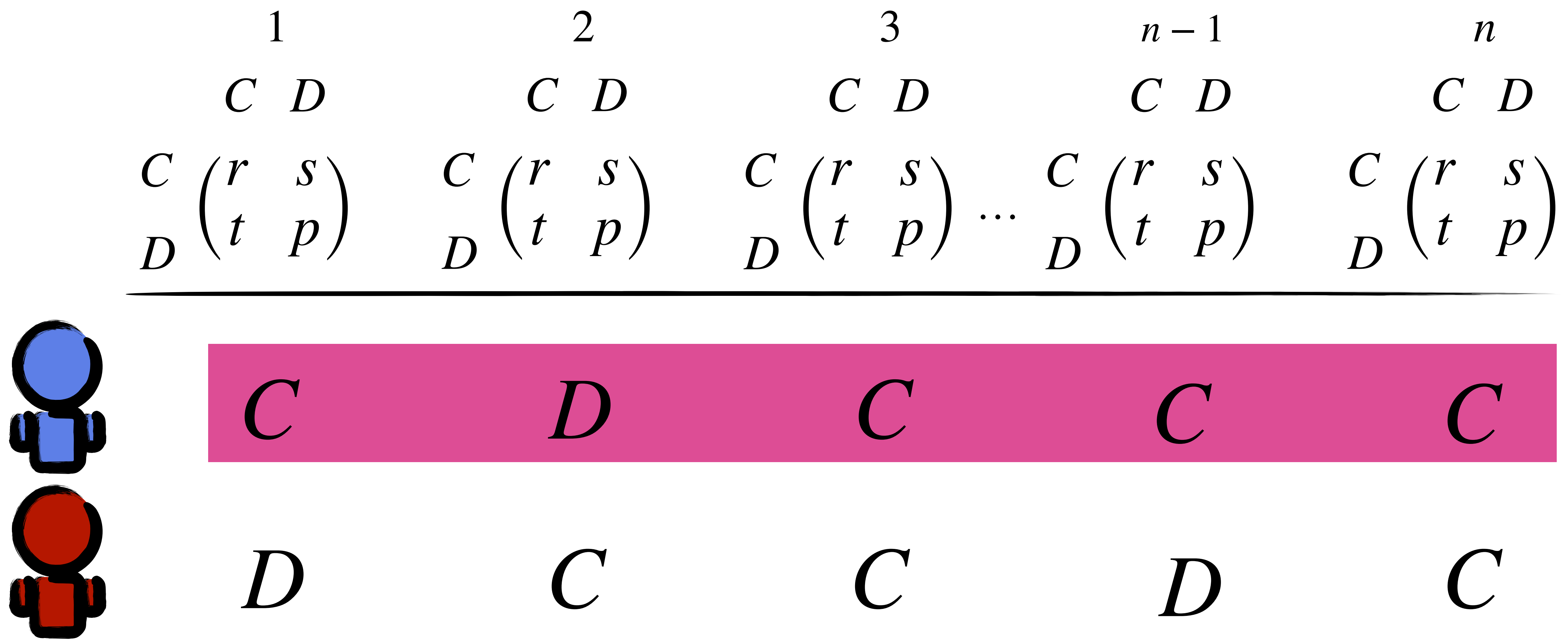

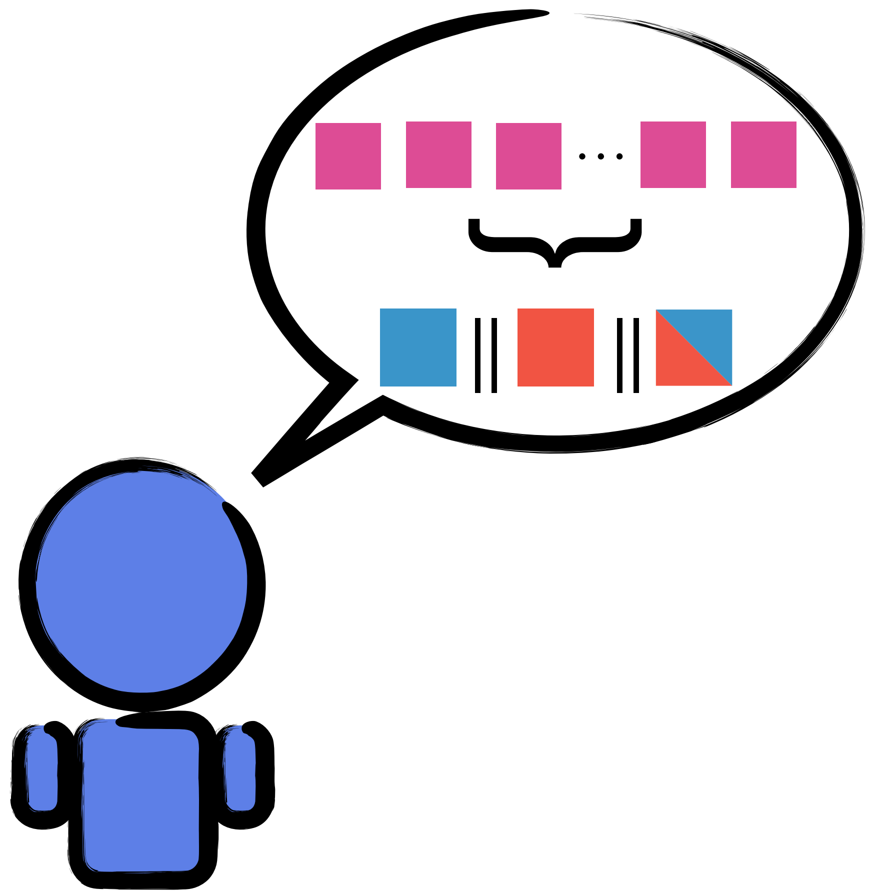

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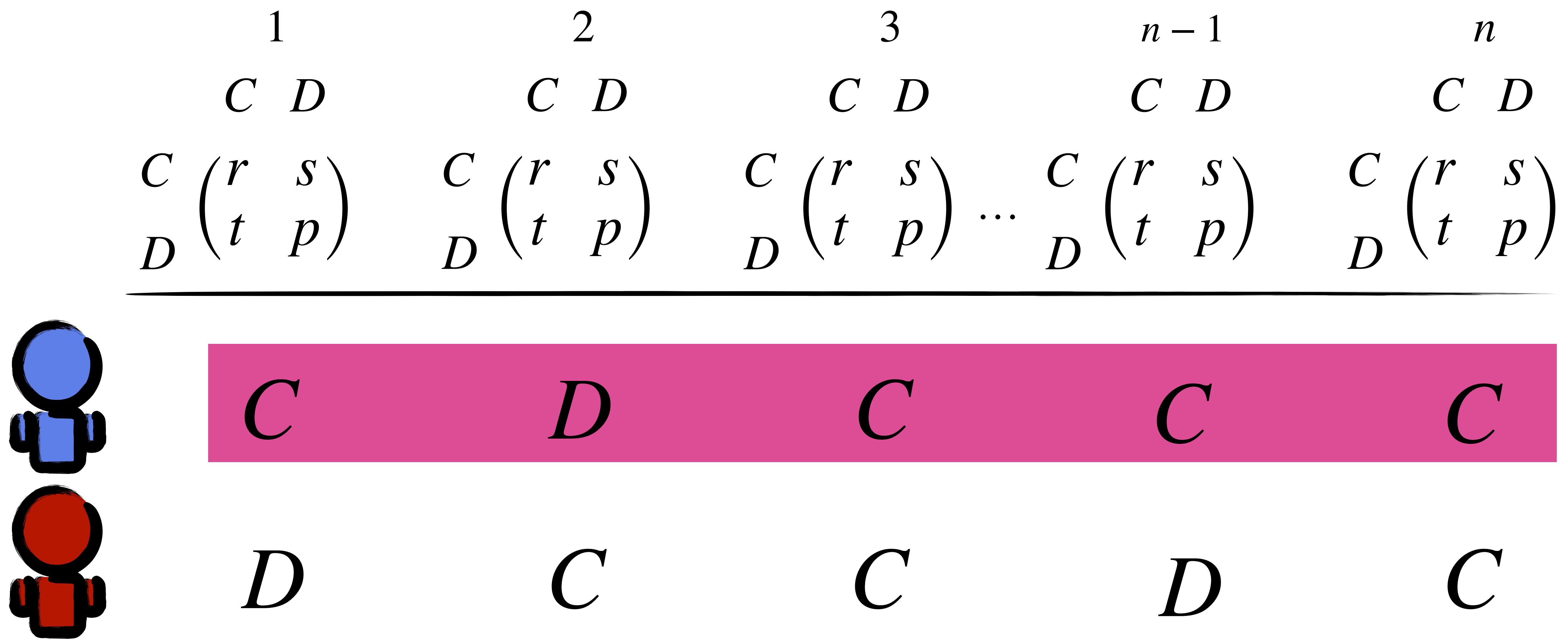

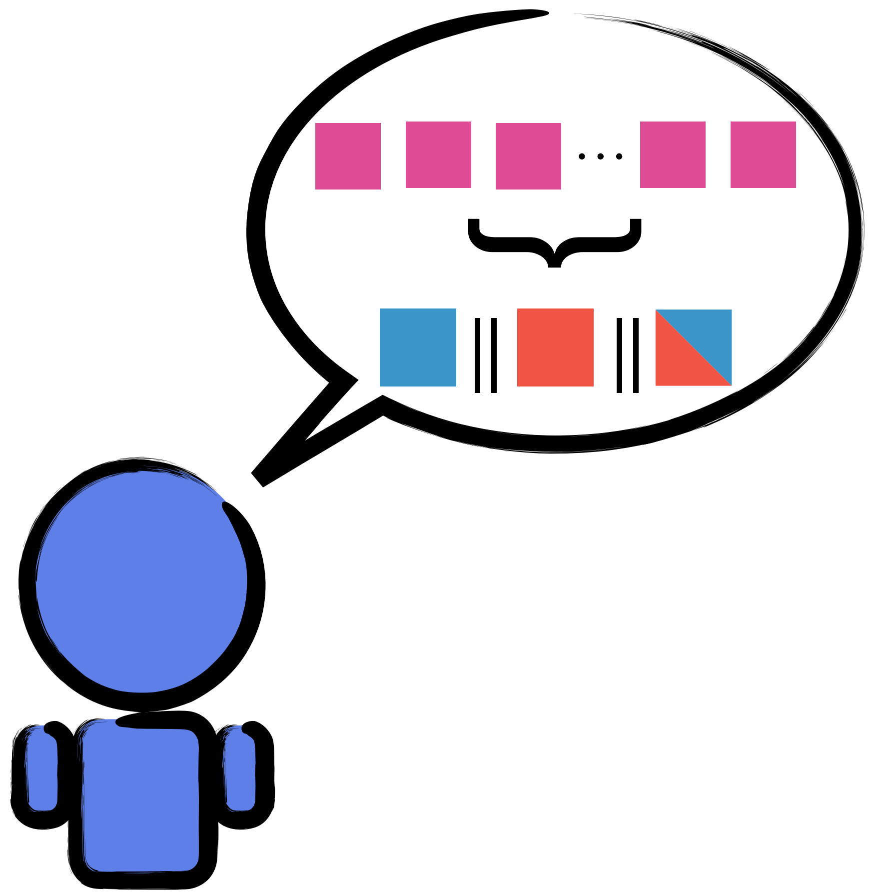

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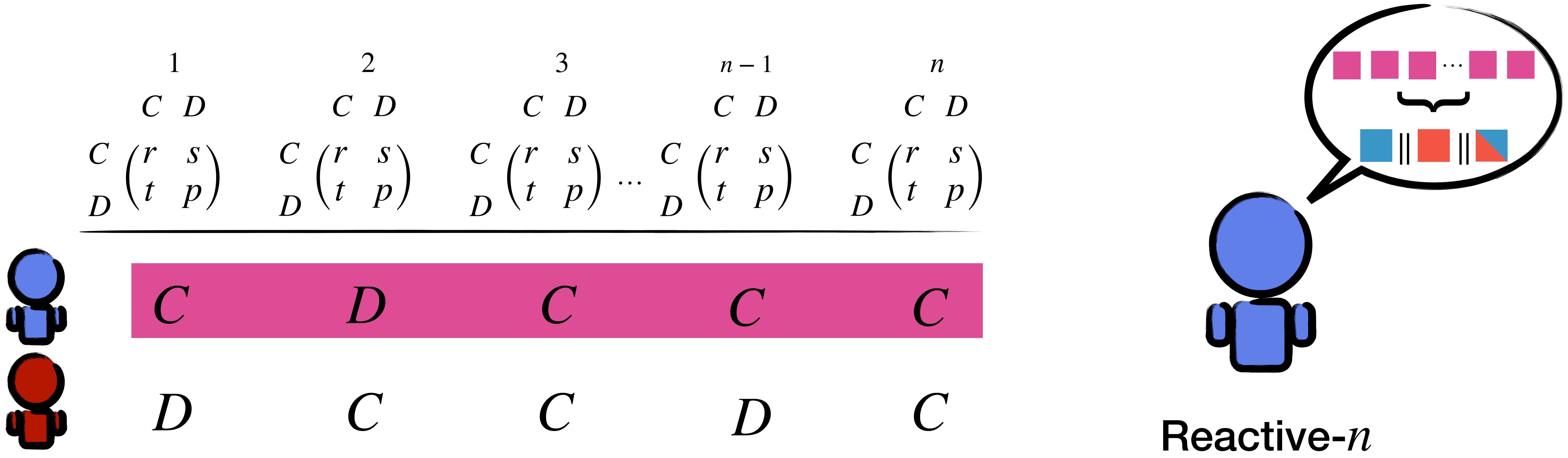
We established the following technical results:

1. Against reactive strategies, any feasible payoff can be generated with self-reactive strategies.

2. To any reactive strategy, there is a best response among the pure self-reactive strategies.



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Theorem. A reactive strategy $\mathbf{p} \in \mathscr{R}_n$ is a Nash equilibrium if and only if $\pi(\mathbf{p}, \mathbf{p}) \geq \pi(\tilde{\mathbf{p}}, \mathbf{p})$ for all pure self-reactive strategies $\tilde{\mathbf{p}}$.

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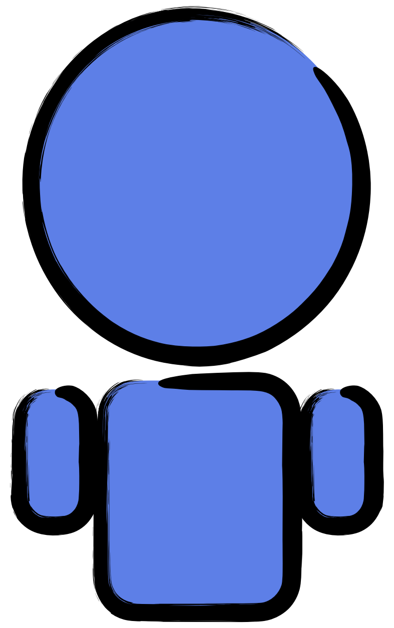
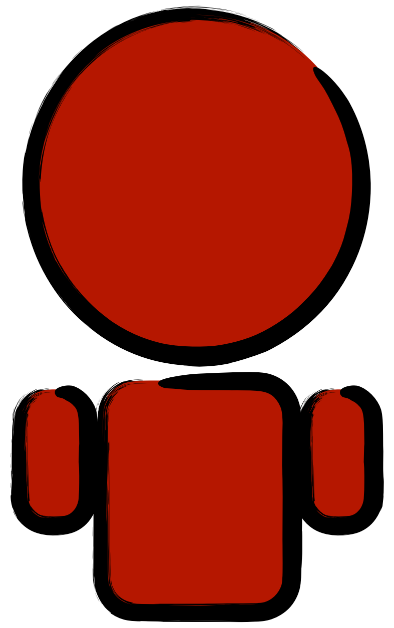


$\mathbf{p} = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$



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Nash equilibria in higher n

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$\mathbf{p} = (p_{CCC}, p_{CCD}, p_{CDC}, p_{CDD}, p_{DCC}, p_{DCD}, p_{DDC}, p_{DDD})$ 256

Donation game for $n = 2$ & $n = 3$

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} b-c & -c \\ b & 0 \end{pmatrix} \end{array}$$

$b > c > 0$

Cooperative Nash

Cooperative Nash

Theorem. A reactive-2 strategy $\mathbf{p}$ is a cooperative Nash equilibrium if and only if its entries satisfy the conditions,

$$p_{CC} = 1, \quad \frac{p_{CD} + p_{DC}}{2} < 1 - \frac{1}{2} \cdot \frac{c}{b}, \quad p_{DD} \leq 1 - \frac{c}{b}.$$

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Theorem. A reactive-3 strategy $\mathbf{p}$ is a cooperative Nash equilibrium if and only if its entries satisfy the conditions,

$$p_{CCC} = 1 \qquad \frac{p_{CDC} + p_{DCD}}{2} \leq 1 - \frac{1}{2} \cdot \frac{c}{b}$$

$$\frac{p_{CCD} + p_{CDC} + p_{DCC}}{3} \leq 1 - \frac{1}{3} \cdot \frac{c}{b} \qquad \frac{p_{CDD} + p_{DCD} + p_{DDC}}{3} \leq 1 - \frac{2}{3} \cdot \frac{c}{b}$$

$$\frac{p_{CCD} + p_{CDD} + p_{DCC} + p_{DDC}}{4} \leq 1 - \frac{1}{2} \cdot \frac{c}{b} \qquad p_{DDD} \leq 1 - \frac{c}{b}$$

Defective Nash

Defective Nash

Theorem. A reactive-2 strategy $\mathbf{p}$ is a defective Nash equilibrium if and only if its entries satisfy the conditions,

$$p_{CC} \leq \frac{c}{b} \quad \frac{p_{CD}+p_{DC}}{2} \leq \frac{c}{2b}, \quad p_{DD} = 0.$$

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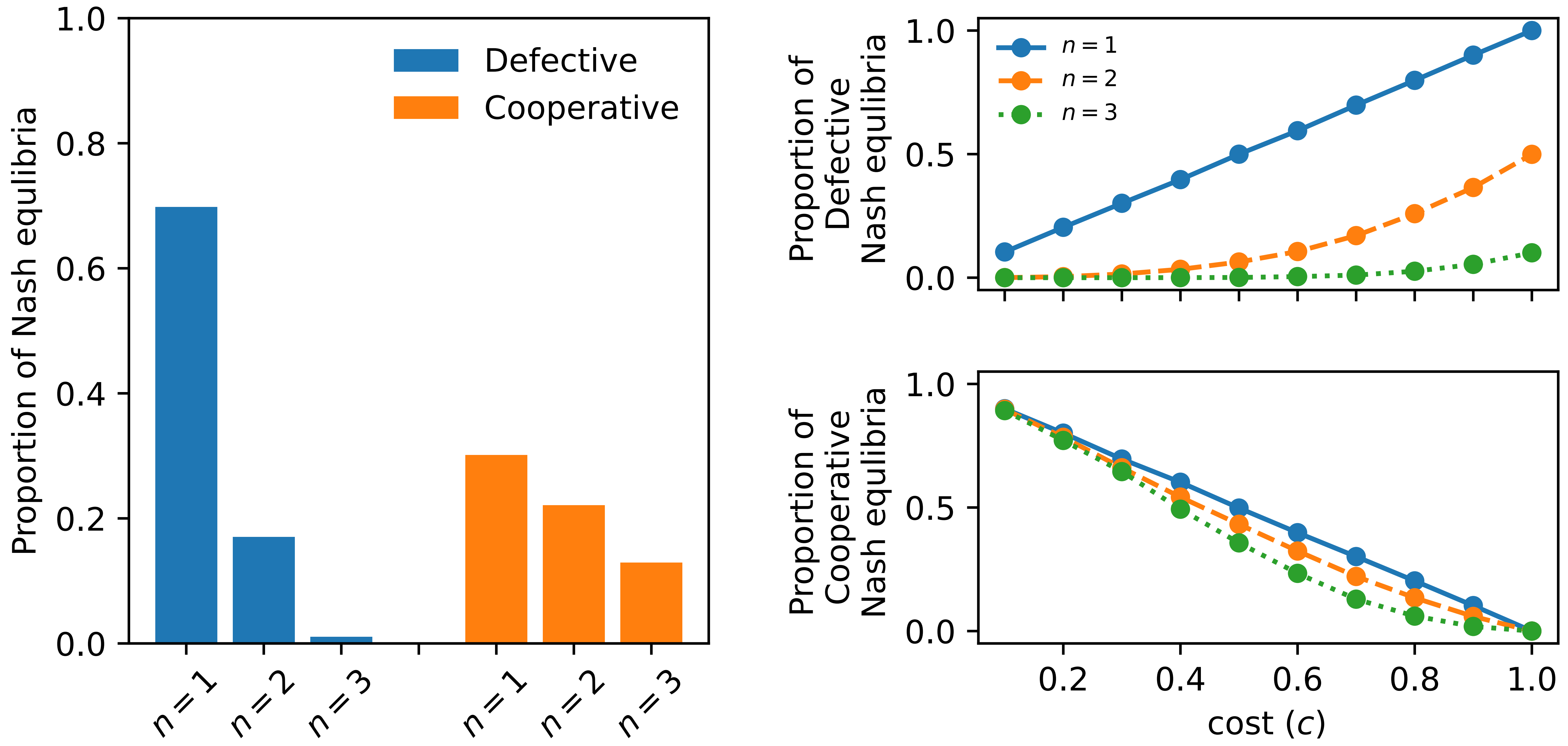
Theorem. A reactive-3 strategy $\mathbf{p}$ is a defecting Nash strategy if and only if its entries satisfy the conditions,

$$p_{CCC} \leq \frac{c}{b}, \qquad \frac{p_{CDC} + p_{DCD}}{2} \leq \frac{1}{2} \cdot \frac{c}{b}$$

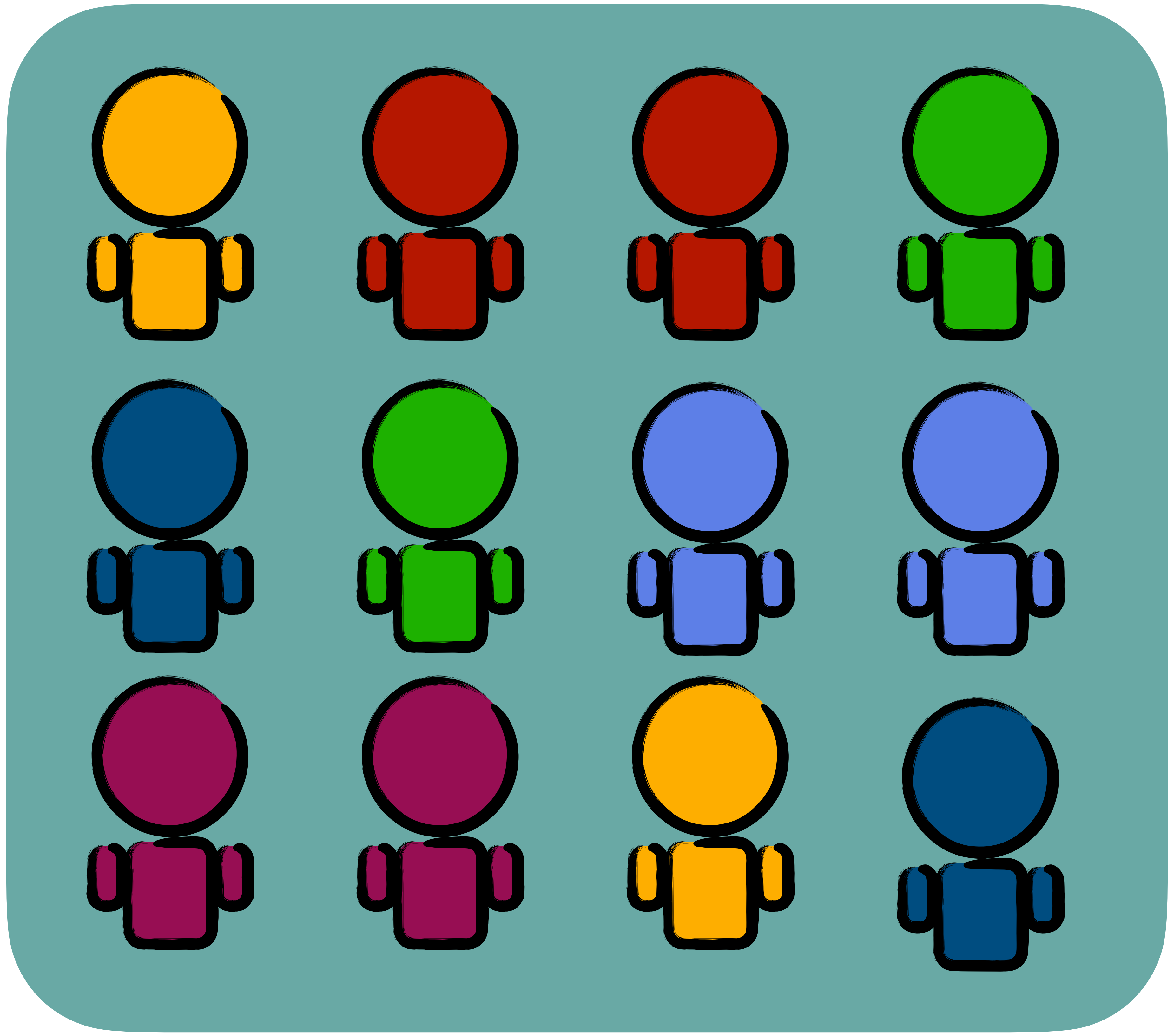
$$\frac{p_{CCD} + p_{CDC} + p_{DCC}}{3} \leq \frac{2}{3} \cdot \frac{c}{b}, \qquad \frac{p_{CDD} + p_{DCD} + p_{DDC}}{3} \leq \frac{1}{3} \cdot \frac{c}{b}$$

$$\frac{p_{CCD} + p_{CDD} + p_{DCC} + p_{DDC}}{4} \leq \frac{1}{2} \cdot \frac{c}{b}, \qquad p_{DDD} = 0.$$

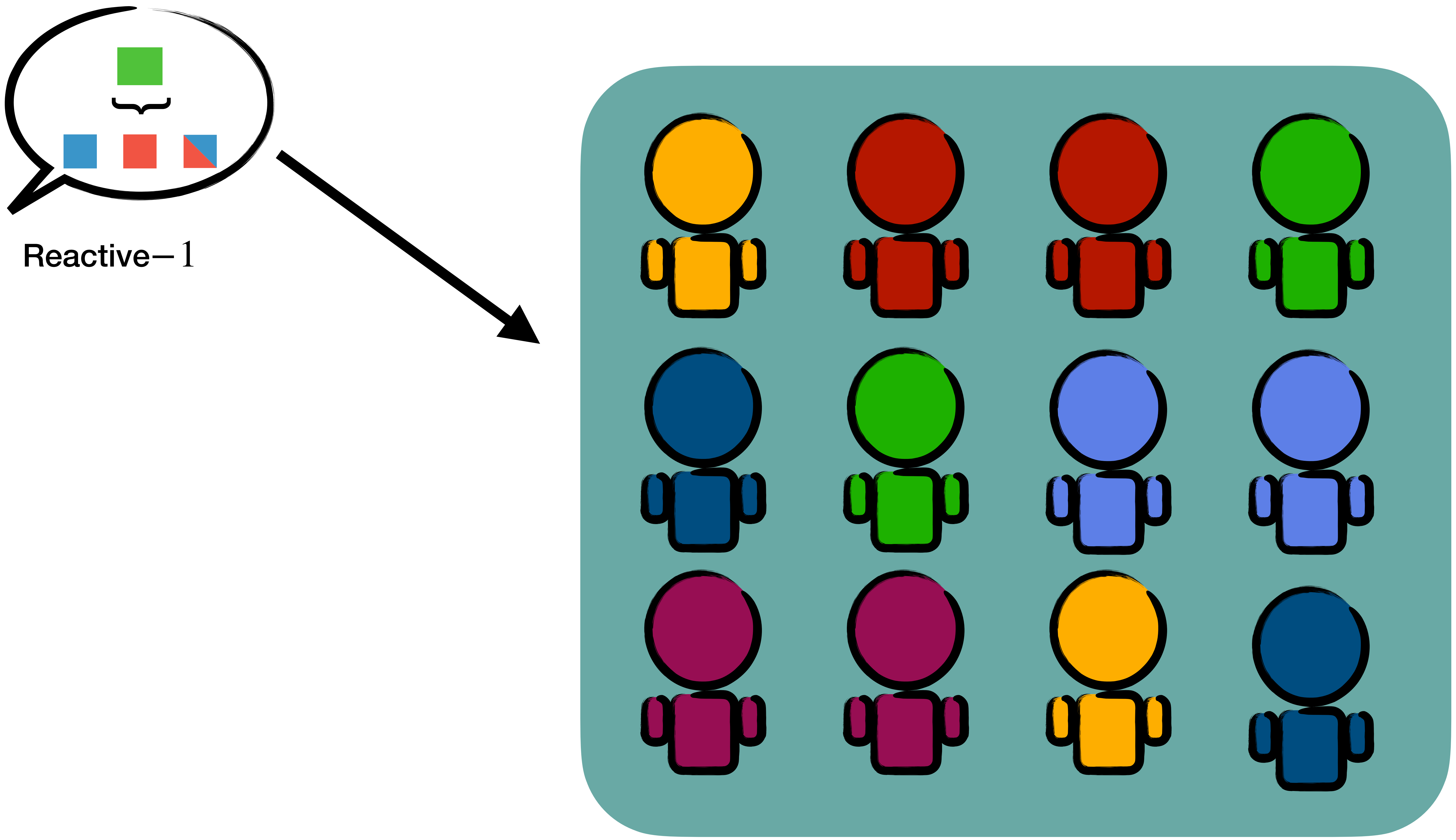
Cooperative & Defective Nash



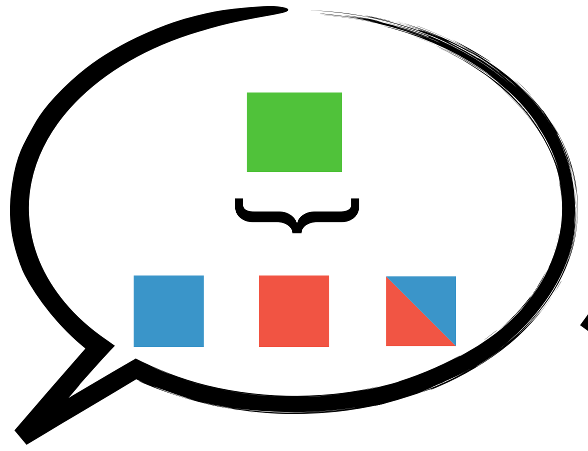
Evolutionary simulations



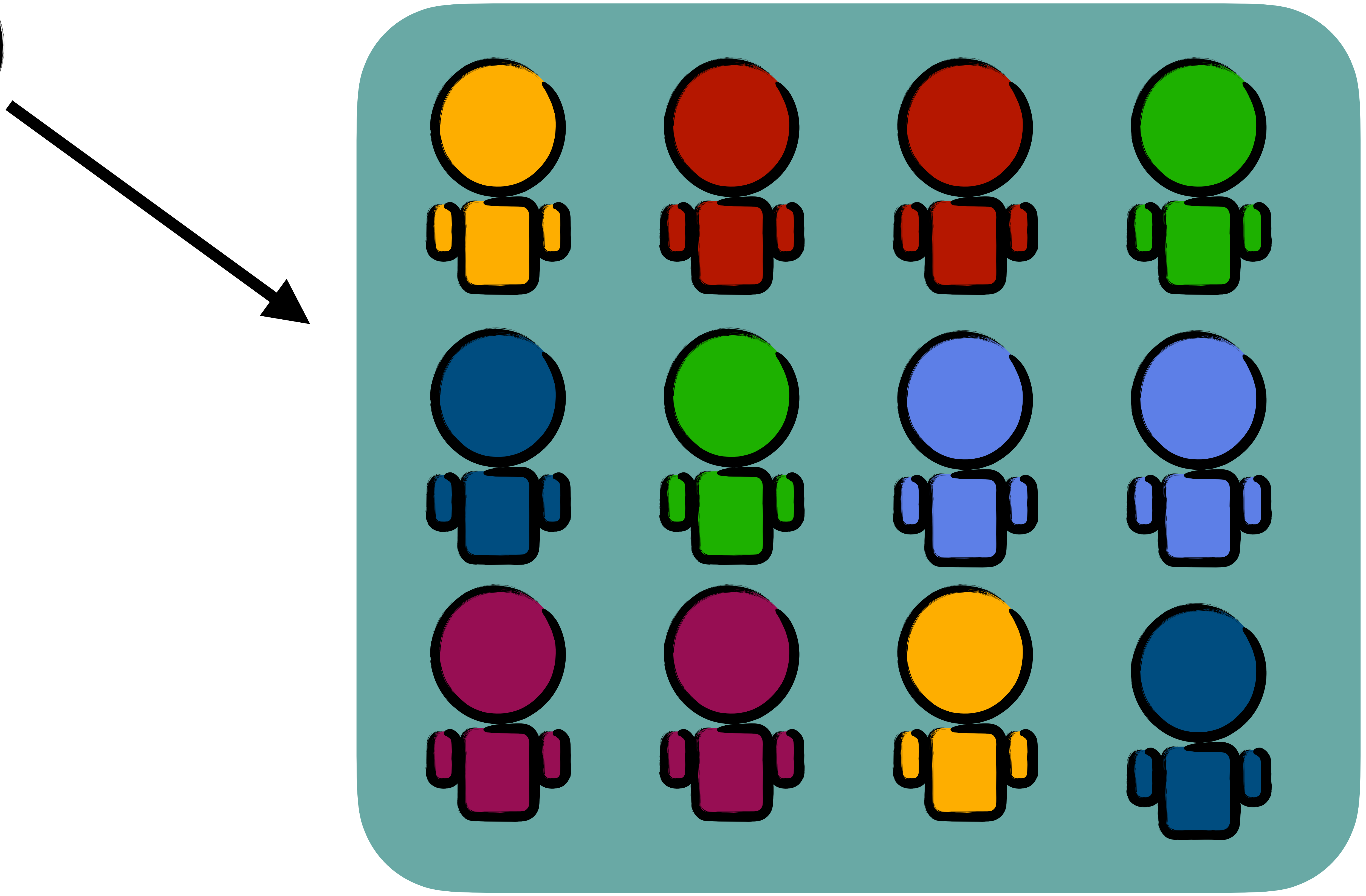
Evolutionary simulations



Evolutionary simulations

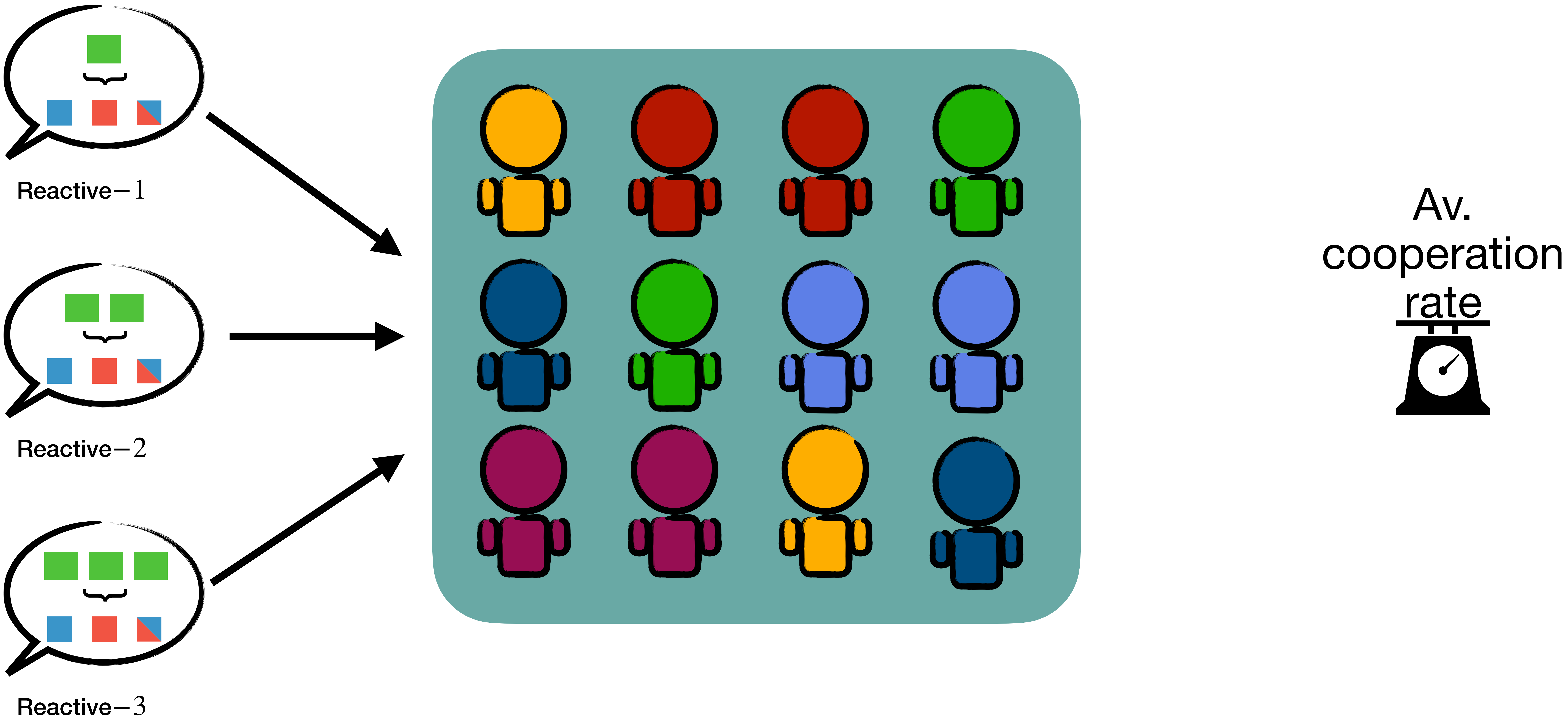


Reactive-1

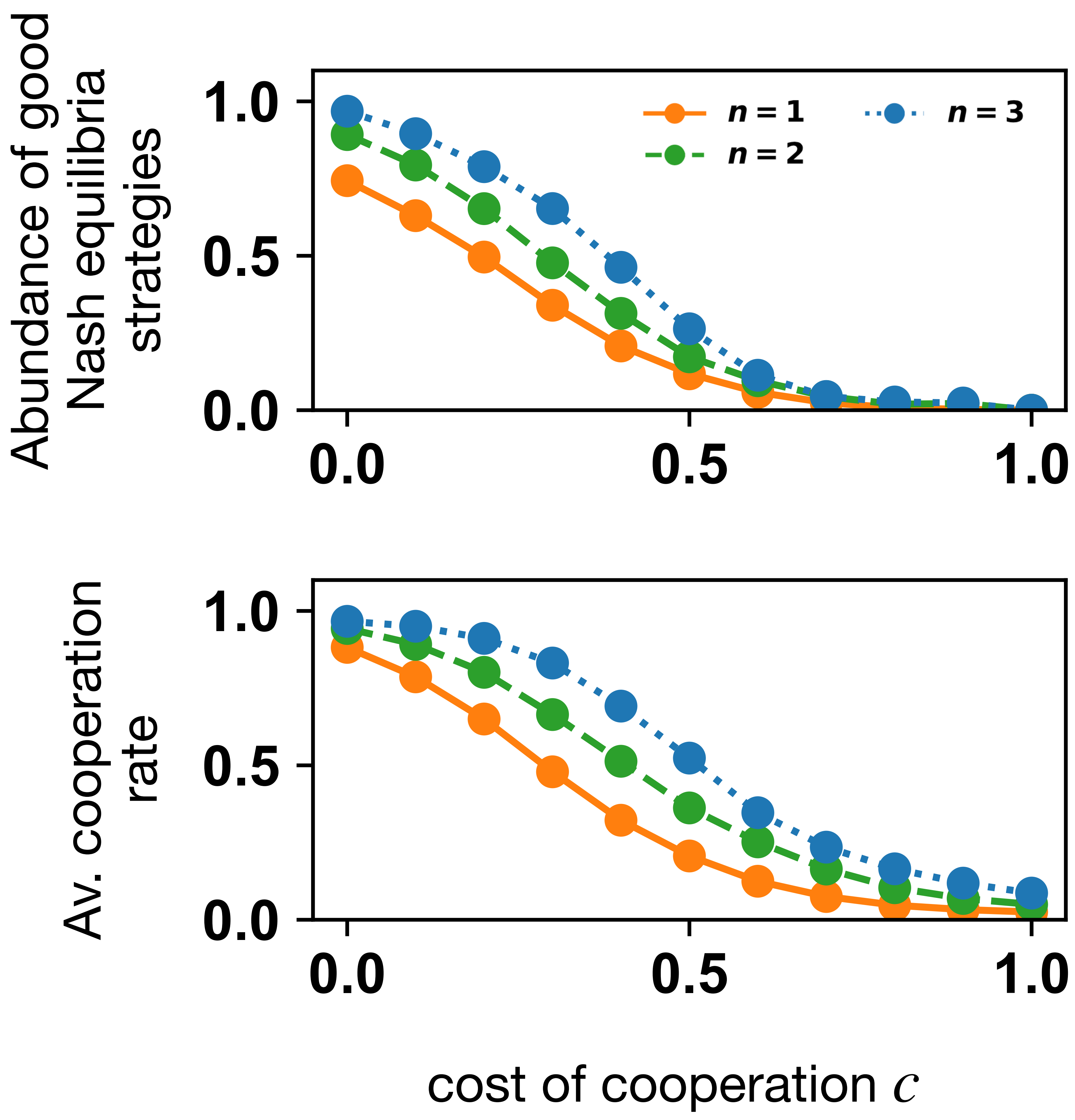


Av.
cooperation
rate

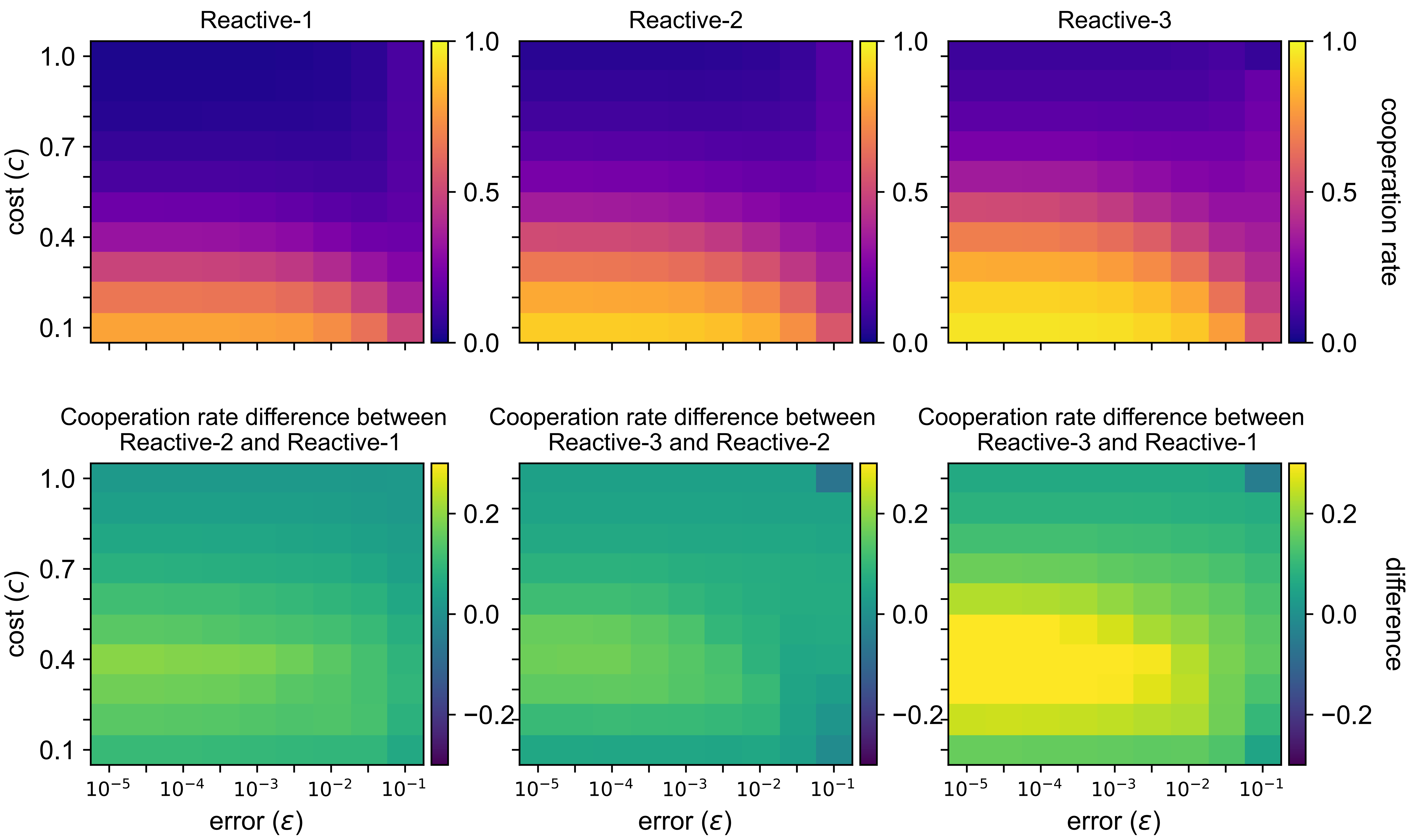
Evolutionary simulations



Evolutionary simulations



Simulations with error



1. Algorithm to verify whether a given reactive-n strategy is an equilibrium.

input: p, n

pure_self_reactive_strategies $\leftarrow \left\{\tilde{\mathbf{p}} \;\middle|\; \tilde{\mathbf{p}} \in \{0,1\}^{2^n}\right\}$;

isNash ←True;

for $\tilde{\mathbf{p}} \in$ *pure_self_reactive_strategies* **do**

    if $\mathbf{p}$ is not a best response $\tilde{\mathbf{p}}$ to **then**

        isNash ←False;

return ($\mathbf{p}$, isNash);

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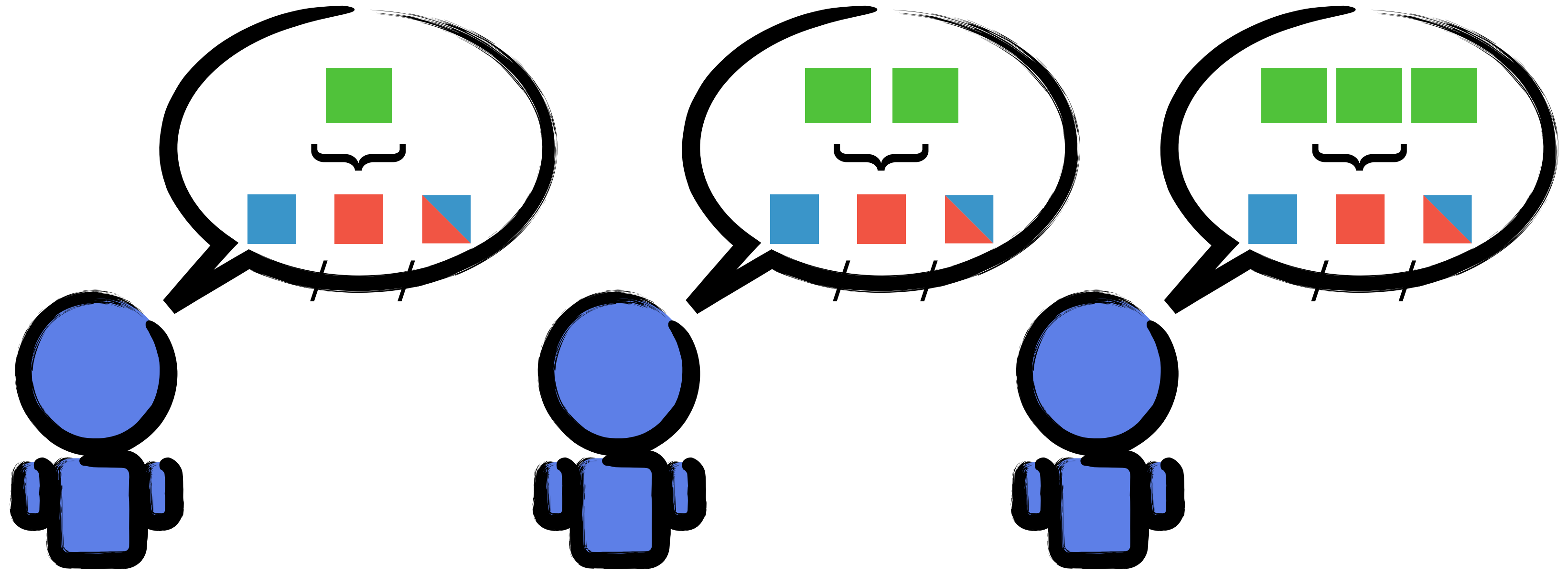
2. Fully characterize cooperative & defective equilibria for $n = 2$ and $n = 3$.



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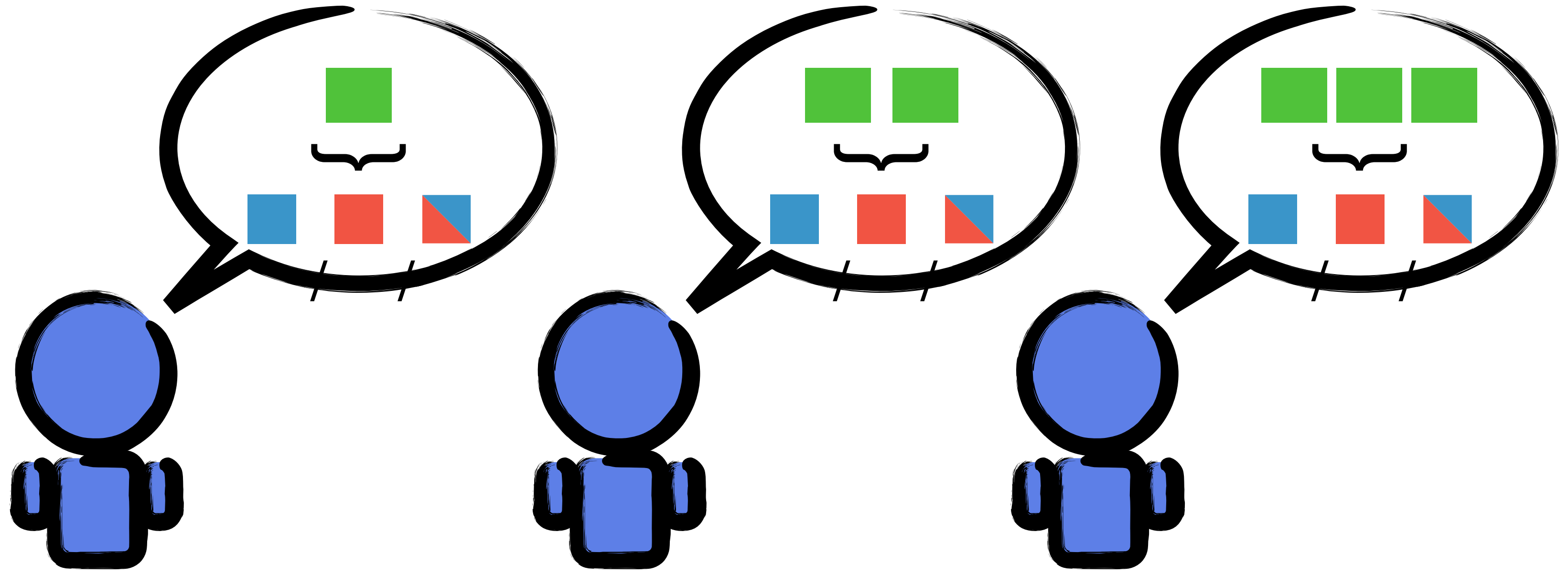
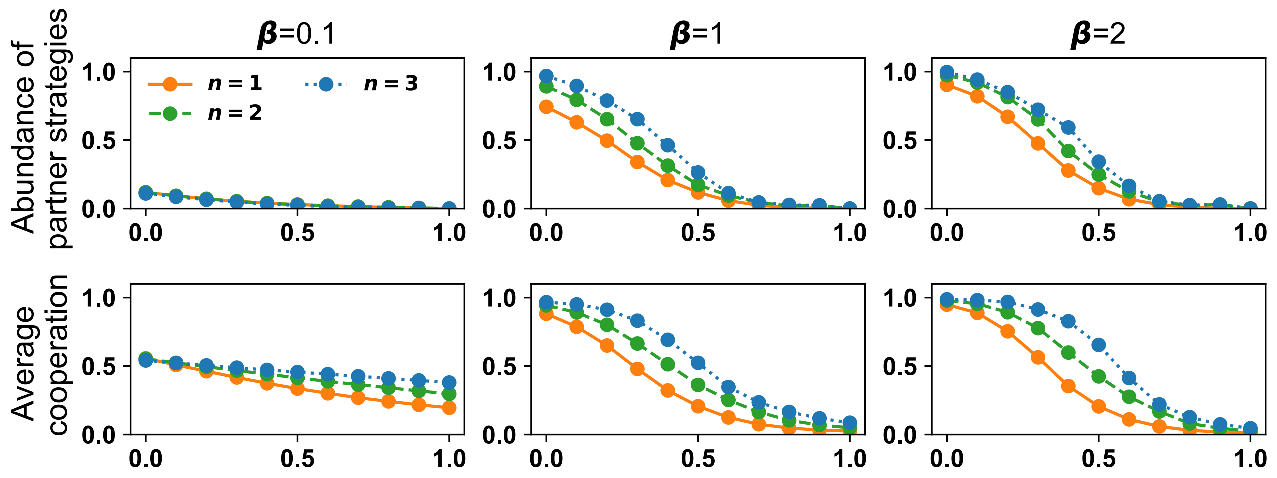


3. Longer memory helps sustain cooperation.



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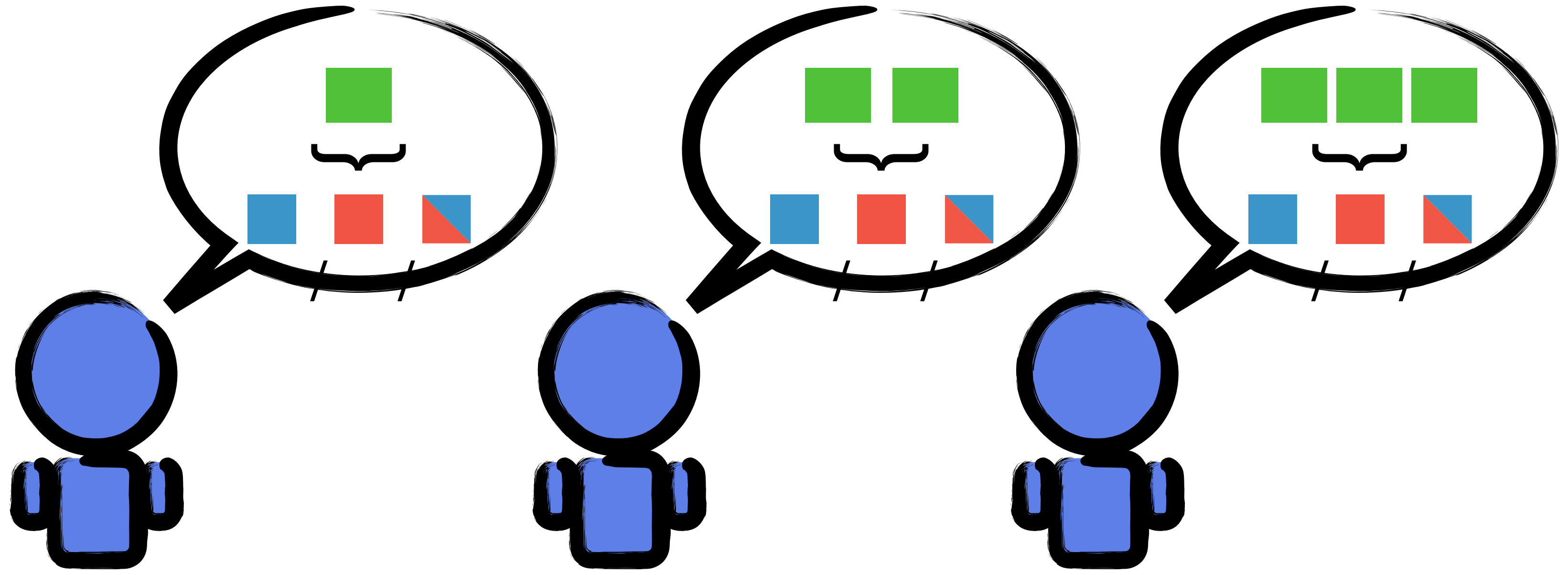
```
input: p, n
pure_self_reactive_strategies ← { p̃ | p̃ ∈ {0,1}^(2^n) };
isNash ←True;
for p̃ ∈ pure_self_reactive_strategies do
    if p is not a best response p̃ to then
        isNash ←False;
return (p, isNash);
```

2. Fully characterize cooperative & defective equilibria for $n = 2$ and $n = 3$.



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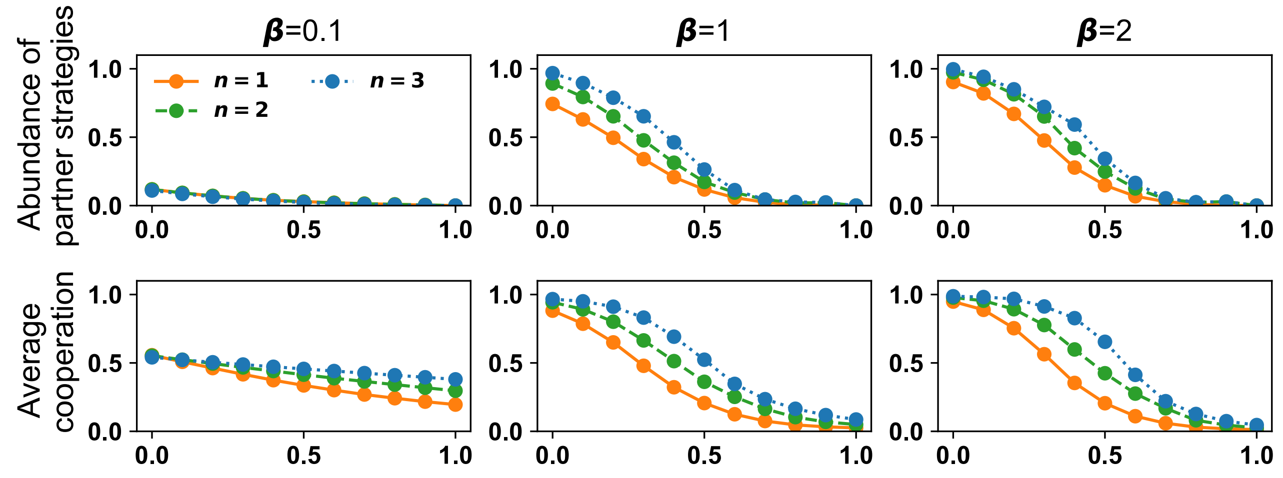

More information

Conditional cooperation with longer memory

https://arxiv.org/abs/2402.02437

@NikoletaGlyn

https://nikoleta-v3.github.io/

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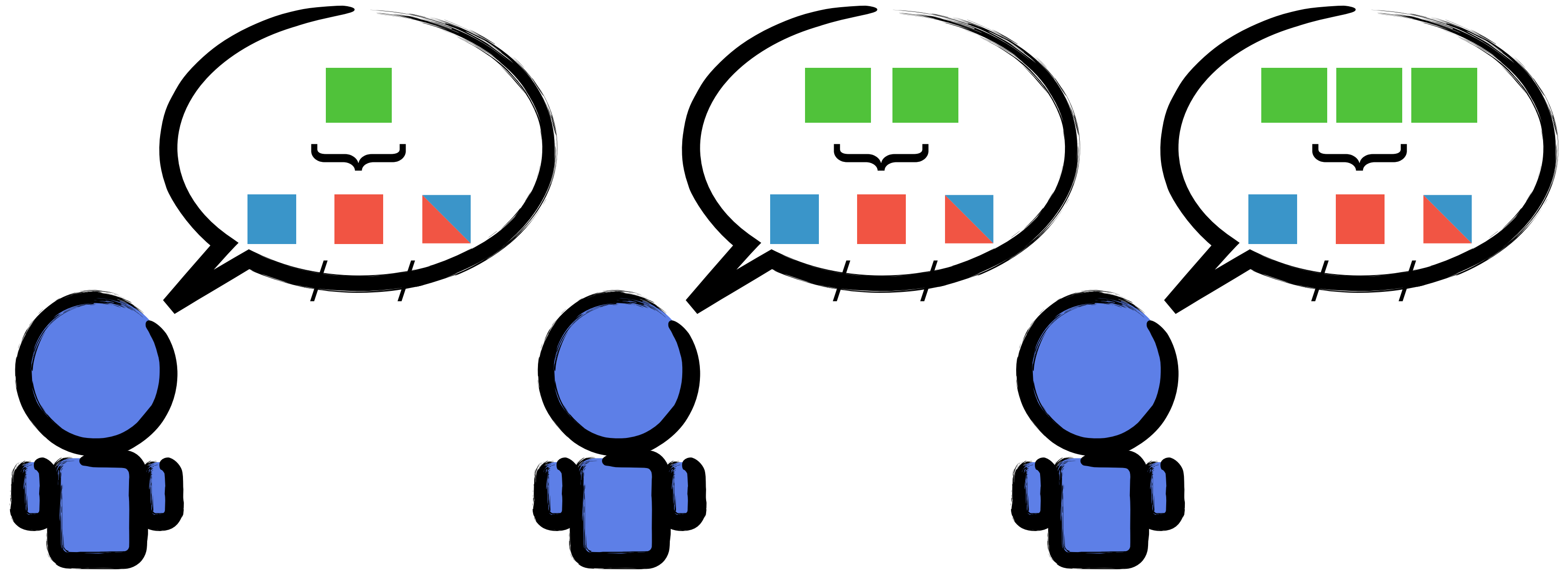

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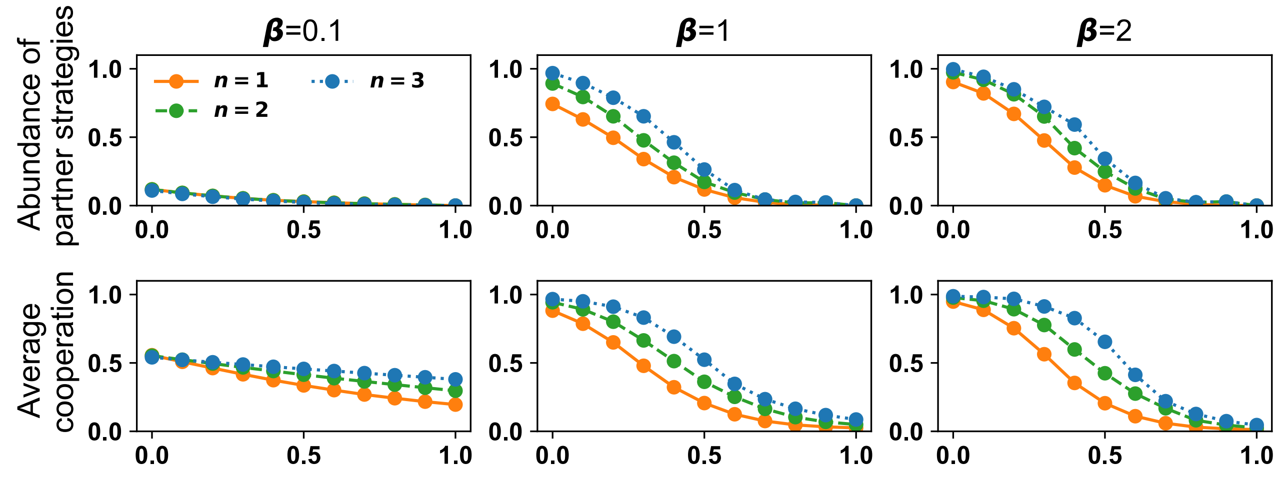

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