Experiment 6 – Diffraction of Visible Light

Group 1

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We hereby confirm that this lab report is entirely our own. Where we have consulted the work of others this is always clearly stated. All statements taken literally from other writings or referred to by analogy are marked and the source is always given. This paper has not yet been submitted to another examination office, either in the same or similar form.

Abstract

In this study, the diffraction of electromagnetic waves through various objects, including single slits, wires, and gratings, was investigated using a coherent laser beam. The resulting diffraction patterns were analyzed to reveal the maxima and minima, which were utilized to determine the dimensions of the diffracting objects or the wavelength of the laser light. Furthermore, a diffraction grating was employed to analyze light emitted from different vaporized elements, allowing for their identification based on the observed spectral lines. Key concepts, such as electromagnetic waves, phase, amplitude, intensity, and coherence, were addressed, along with phenomena including diffraction and interference. The principles of Fraunhofer and Fresnel diffraction were examined, with results indicating the effectiveness of each method under different conditions. Additionally, Babinet's theorem was applied, demonstrating the relationship between the geometrical configuration of apertures and the resulting diffraction pattern. The implications of the Bohr atomic model were discussed in relation to light emission and electronic transitions, revealing how these concepts underpin the observed experimental phenomena.

Introduction

For the description of diffraction effects, the wave-like nature of light must be utilized. Light waves, which are electromagnetic in nature, possess a certain wavelength λ and frequency f that are related by the speed of light c through the equation:

$$c = \lambda f \qquad (Eq. 1)$$

The basic principles underlying diffraction are derived from Huygens-Fresnel's principles, which state that every point on a wave front acts as a source of secondary wavelets. When light passes through a narrow slit or around an obstacle, these wavelets interfere, resulting in a diffraction pattern that includes regions of constructive and destructive interference.

To quantitatively analyze diffraction patterns, the well-known equations governing the behavior of light through slits were referred to. For a double slit arrangement, the positions of maxima and minima in the interference pattern are expressed by:

$$sin\theta_k = \pm\frac{k\lambda}{d};\ \ (k = 0,1,2,3\ldots); for\ maxima \qquad (Eq. 2)$$

$$sin\theta_k = \pm\frac{(2k+1)\lambda}{2d};\ \ (k = 0,1,2,3\ldots); for\ minima \qquad (Eq. 3)$$

Where d represents the distance between the slits, and θ is the angle relative to the central axis. Additionally, Babinet's principle was explored, which states that identical diffraction patterns are produced by complementary diffracting objects. This principle allows for the application of the same equations for analyzing the diffraction patterns generated by thin slits and thin wires.

In addition to the observation of diffraction patterns, the spectral lines produced by gas lamps were analyzed to identify the unique wavelengths emitted by different elements. The relationship between the diffraction angle θ and the wavelength λ is given by:

$$\lambda = G sin\theta_1 \qquad (Eq.4)$$

where G is the grating constant. This relationship enables the determination of the wavelengths corresponding to various spectral lines based on their observed diffraction angles.

Through this experiment, a deeper understanding of the fundamental concepts of wave interference and diffraction is aimed to be elucidated, contributing to the knowledge of light's behavior as a wave and its applications in analytical techniques such as spectroscopy.

The Bohr model for the hydrogen atom was developed as one of the earliest successes of atomic physics. The most important properties of the hydrogen atom, particularly its spectral lines, are described accurately by this model. It is assumed that the electrons of an atom move around the nucleus on discrete orbits. The energies of these orbits are given discrete values indexed by an integer n, as expressed in the following equation:

$$E_n = -\frac{1}{8}\frac{e^8 m_e}{\in_0^2 h^2}\frac{1}{n^2},\ n = 1,2,3 \ldots \qquad (Eq.5)$$

where $\in_0 = 8.8542 \ * \ 10^{-12} As/Vm$ is the electric permittivity of free space, $e = 1.6021 * 10^{-19} c$ is the electric charge of the electron, and $m_e = 9.1091 * 10^{-31}$ is the rest mass of the electron.

When atoms are excited by absorbing energy, the electrons are "lifted" to higher orbits or energy levels, and subsequently decay back to lower energy levels. During the decay process, energy is released in the form of light with a frequency v that is related to the difference in energy levels $\Delta E = hv$. For instance, when an excited electron in the hydrogen atom transitions from an energy level E_m to a lower level E_n when m > n, a photon is emitted with energy $\Delta E = hv$,corresponding to the electron's change in energy, as represented in the equation:

$$\Delta E = hv = \frac{1}{8}\frac{e^8 m_e}{\in_0^2 h^2}\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \qquad (Eq.6)$$

The relationship between frequency and wavelength is given by $c = \lambda v$, allowing (Eq.6) to be expressed in terms of the wavelength of the emitted photon:

$$\frac{1}{\lambda} = R\left(\frac{1}{n^2} - \frac{1}{m^2}\right) \qquad (Eq.7)$$

where $R = \frac{1}{8}\frac{e^8 m_e}{\in_0^2 h^2} = 1.097 * 10^7 m^{-1}$ is Rydberg's constant, $c = 2.9979 * 10^8 m/s$ is the speed of light, and $h = 6.6256 \ * \ 1^{34} J$ is Planck's constant.

For n < m, the index n denotes a specific spectral series, while m varies within this series (see Fig. 6-6). For example, when n = 2, the series is referred to as the Balmer series, with spectral lines ranging from ultraviolet to red. The spectral lines in the Balmer series corresponding to m = 3, 4, 5, 6, and 7 are designated as the Hα, Hβ, Hγ, Hδ, and Hε lines, respectively. As illustrated in Fig. 6-6, the energy associated with m→∞ represents the ionization energy (or binding energy) for an electron in the n - th permitted state. The binding energy can be calculated using the following equation:

$$E_n = -Rhc\frac{1}{n^2} \qquad (Eq. 8)$$

Hydrogen is noted for its prominent spectral lines within the visible range (red, blue, and violet).

Light from various spectral lamps is observed through a grating, enabling the identification of the element in the atomic vapor by its spectral fingerprint, characterized by specific lines of different colors and intensities. This detection principle is analogous to that employed in modern spectrometers used for identifying different atoms or molecules based on their emitted light.

The intensity maxima of light with wavelength λ diffracted by a grating with constant G occur when the angle of diffraction satisfies the following condition:

$$\lambda * k = G sin\alpha; k = 0,1,2 \ldots \qquad (Eq. 9)$$

For this part of the experiment, a simple setup, as depicted in Fig. 6-7, can be utilized. The emission spectra are observed visually and will be perceived as an overlay on the meter scale. If light from a spectral line is diffracted at angle α, the position of the line on the meter scale is determined by the backward prolongation of the light beam in the direction of diffraction. For the diffraction of the k-th order, the following relation is derived from the geometrical scheme illustrated in Fig. 6-7:

$$k * \lambda = G * \frac{l}{\sqrt{d^2 + l^2}} \qquad (Eq. 10)$$

The experiment conducted with the hydrogen tube is employed to determine the spectral lines of the Balmer series for the calculation of Rydberg's constant.

Experimental methods and materials

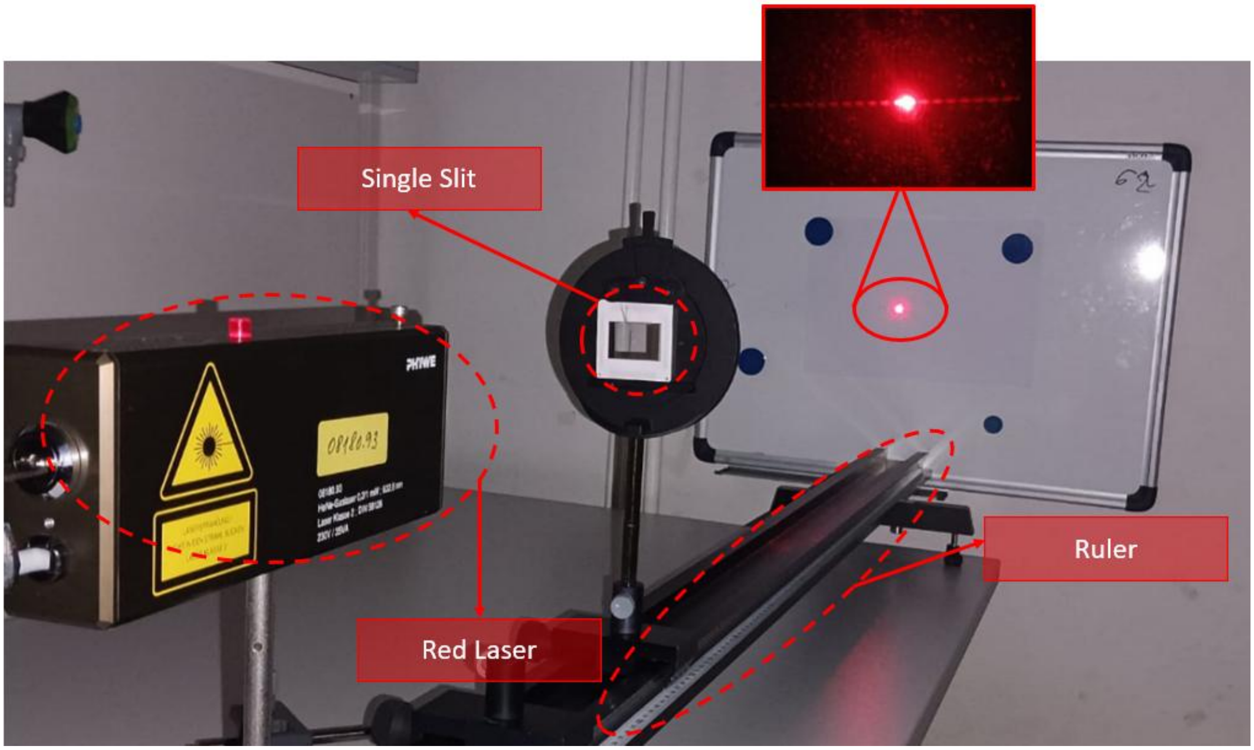

Figure 1. Experimental set up for single slit diffraction.

Single Slit Diffraction

A red laser was passed through a single slit, and the diffraction minima were marked on a screen at two different distances (D = 0.70 m to 1.50 m). Three diffraction orders (k) were measured to calculate the slit width (a). Statistical and propagated error analyses were performed.

Diameter of a Hair by Diffraction

The diffraction pattern of a human hair was measured at two different distances (D), with three diffraction orders used to calculate the hair's diameter (a). Statistical and propagated errors were evaluated.

Grating Constant

A grating was placed between the laser and screen, and maxima positions were recorded for two distances (D). The grating constant (G) was calculated, followed by error analysis.

Wavelength of a Green Laser

A green laser was used to repeat the diffraction experiment. The wavelength (λ) was calculated using previous grating data and analyzed for statistical and propagated errors.

Atomic Spectra

Spectral lines of gas discharge lamps were observed, compared to literature values, and analyzed using the Balmer series for the hydrogen spectrum. The Rydberg constant was determined.

In the final part of the experiment, Rydberg's constant was investigated. The spectral tube, serving as the source of radiation, was connected to the high-voltage power supply unit. The diffraction grating, with a spacing of d = (0.34 ± 0.0005) m, was set up at the same height as the center of the spectral tube. The power supply was switched on and adjusted to 1.8 kV. The light from the spectral tube was then observed through the grating.

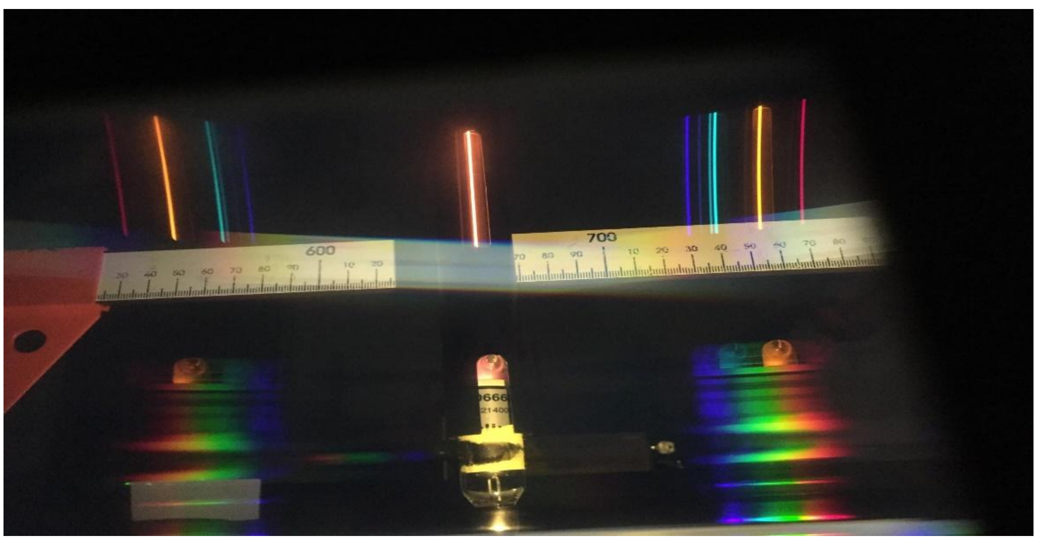

Figure 2. The visible lines of the Balmer series in the Helium.

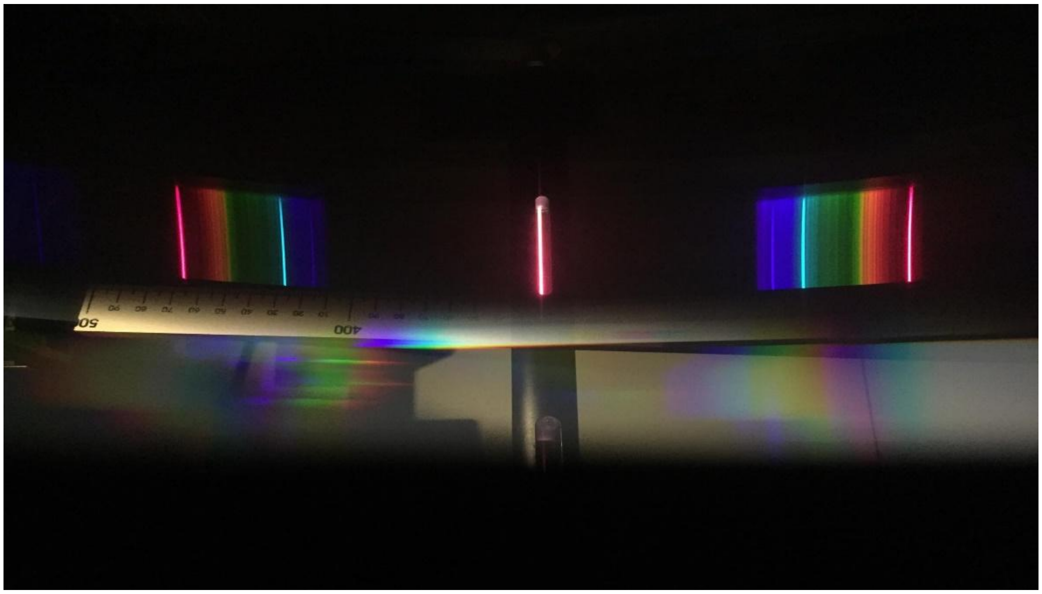

Figure 3. The visible lines of the Balmer series in the Hydrogen.

How 2l (the distance between two lines of the same color) can be determined?

Since the distance 2l is difficult to estimate by eye, it was decided to use the Tracker computer program, which would allow for a more accurate calculation of the distance. The calibration stick was used to indicate to the program that two centimeters on the ruler corresponded to the actual two centimeters visible in the photo. With this tool, the distance between two lines of the same color could be determined using proportion.

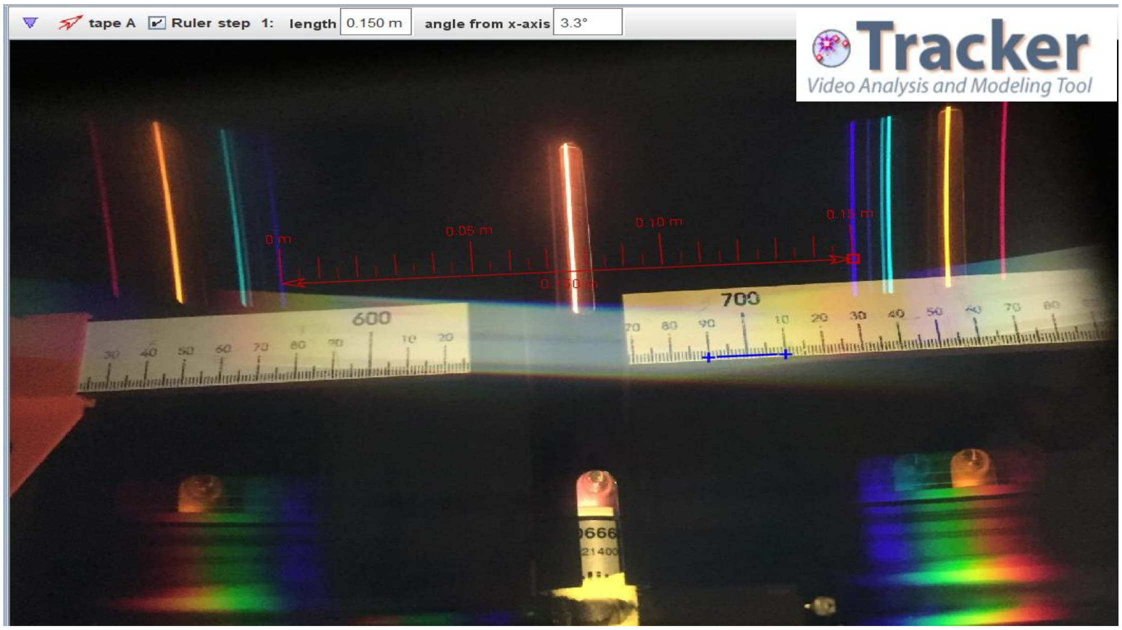

Figure 3. The method to determine the distance between two identical lines(2l)

To determine λ for different colors we used (eq10). And to calculate experimental value for R we used (eq7). R = (1.09908 ± 0.00015) m^{-1}.

Results and Data Analysis

The single slit diffraction experiment was conducted using a laser with a wavelength of 632.8 nm. Two different distances, D, between the screen and the diffraction object were used: D1 = (1.214 ± 0.0005) m and D2 = (1.055 ± 0.0005) m. For each distance, three different diffraction orders, k, were used to measure the distances, d, between minima of identical order. Based on this data, the sine of the angle θ was first calculated for each case. Subsequently, the slit width, ***a = (0.090102 ± 0.001402) mm*** (eq.), was determined. The width of the given slit was 0.1 mm. These results are presented for the two different distances.

D(1) 1.214m

k	d(m)	a(m)	sin(theta)
2	0.033	9.312E-05	0.013591
3	0.052	8.864E-05	0.021417
4	0.068	9.038E-05	0.028007

D(2) 1.055m

k	d(m)	a(m)	sin(theta)
2	0.031	8.61E-05	0.014692
3	0.047	8.52E-05	0.022275
4	0.055	9.71E-05	0.026066

Table 1. Calculating the slit width a for two different distances.

The average slit width, a = (0.090102±0.001402) mm, was then calculated.

The following formula was used to determine the standard deviation:

$$\sigma_a = \sqrt{\frac{1}{n(n-1)}\sum_{i=1}^{n}(a_i - \bar{a})^2} \qquad \sigma_a = 0.004468\, mm \qquad (Eq)$$

For the error of the mean:

$$\Delta\bar{a} = \frac{\sigma}{\sqrt{n}} \qquad \Delta\bar{a} = 0.001824mm \qquad (Eq)$$

And for the Propagation:

$$\frac{\Delta a}{a} = \left|\frac{\Delta D}{D}\right| + \left|\frac{\Delta d}{d}\right| \qquad (\mathrm{Eq}\ )$$

From this, the propagation error in a was obtained, resulting in Δa = 0.001402 mm.

When comparing the propagated and statistical errors, it can be stated that the propagated error is smaller than the statistical error and does not depend on k. Since k represents the number of minima, there is no associated error in it.

An identical experiment was conducted for a hair sample, and the average width of the hair was determined to be ***a = (0.104143±0.001903) mm***. These results are presented for the two different distances.

D(1) 1.214 m

k	*d(m)*	*a(m)*	*sin(theta)*
2	0.028	0.0001097	0.011532
3	0.044	0.0001048	0.018122
4	0.059	0.0001042	0.0243

D(2) 1.055 m

k	*d(m)*	*a(m)*	*sin(theta)*
2	0.026	0.000103	0.012322
3	0.039	0.000103	0.018483
4	0.053	0.000101	0.025118

Table 2. Calculating the hair width a for two different distances.

For the errors:

Standard Deviation: $\sigma_a = 0.003074\ mm$

Error of the mean: $\Delta\bar{a} = 0.001255\,mm$

Propagation Error: $\Delta a = 0.001903\ mm$

In the next experiment, the goal was to determine the lattice constant G. The theoretical value was G = 0.001666 mm, while the experimental value (averaged) was found to be ***G = (0.00172 ± 0.00002347) mm.*** Using equation (x), the sine of the angle θ was calculated, followed by the experimental determination of G.

D(1) 0.13 m

k	*s(m)*	*sin(theta)*	*G*
1	0.051	0.3652092	1.73E-06
2	0.141	0.7352031	1.72E-06

D(2) 0.06 m

k	*s(m)*	*sin(theta)*	*G*
1	0.024	0.371391	1.7E-06
2	0.065	0.734803	1.72E-06

Table 3. Calculating the lattice constant for two different distances.

For the errors:

Standard Deviation: $\sigma_G = 0.0000119629\,mm$

Error of the mean: $\Delta\bar{G} = 0.00005981\,mm$

Propagation Error: $\Delta G = 0.00002347\ mm$

The calculated Rydberg constant was used in the next experiment.

After that, the red laser was replaced with a green laser, and the goal was to determine the wavelength of this laser. To calculate it, the value of G obtained from the previous experiment was used. The same experiment was repeated for two different distances between the grating and the screen, and for each distance, two different orders of maxima were measured. The following table was created:

D(1) 0.13 m

k	*s(m)*	*sin(theta)*	*lambda*
1	0.043	0.314036	5.40171E-07
2	0.107	0.6354991	5.46558E-07

D(2) 0.06 6

k	*s(m)*	*sin(theta)*	*lambda*
1	0.019	0.301892	5.19281E-07
2	0.049	0.632535	5.44009E-07

Table 4. Calculating the wavelength of the green laser.

The averaged value of "λ" was calculated using the (eq). ***λ = (537.5047 ± 10.1865) nm***.

For the errors:

Standard Deviation: $\sigma_{\lambda} = 12.429\ nm$

Error of the mean: $\Delta\bar{\lambda} = 6.2146 nm$

The following equation was used to calculate the propagation error in λ:

$$\frac{\Delta\lambda}{\lambda} = \frac{\Delta G}{G} + \frac{\Delta s}{s} + \frac{\Delta D}{D} \qquad \textit{(Eq.)}$$

And the result for the propagation error was $\Delta\lambda = 10.1865\ \mathrm{nm}$.

In the next experiment, the goal was to determine the wavelengths of the visible lines in the Balmer series for Hydrogen and Helium. The distances between the maxima were calculated using Tracker, and a table of useful parameters was created.

Color	k	G	L(m)	D(m)	2l(m)	lambda(m)	lambda(nm)	Theo(nm)
red	1	1.72E-06	0.1165	0.29	0.233	6.41198E-07	***641.1976***	***668***
yellow	1	1.72E-06	0.101	0.29	0.202	5.65737E-07	***565.7372***	***566***
blue	1	1.72E-06	0.0855	0.29	0.171	4.8643E-07	***486.4298***	***492***
violet	1	1.72E-06	0.075	0.29	0.15	4.30681E-07	***430.6814***	***447***

Color	k	G	L(m)	D(m)	2l(m)	lambda(m)	lambda(nm)	Theo(nm)
red	1	1.72E-06	0.14	0.34	0.28	6.54924E-07	***654.9245***	***650***

blue	1	1.72E-06	0.1	0.34	0.2	4.85352E-07	***485.3519***	***485***
violet	1	1.72E-06	0.0885	0.34	0.177	4.33292E-07	***433.2918***	***434***

Table 4. Calculating the wavelength of the visible lines of the Balmer series in the Helium.

Table 5. Calculating the wavelength of the visible lines of the Balmer series in the Hydrogen.

To calculate λ for different colors the (eq) was used.

To calculate the Rydberg constant, we first determined the values of *n* and *m* in its formula. For hydrogen, *n* is always 2, while *m* is 3 for red, 4 for blue, and 5 for violet. After determining these values, the Rydberg constant *R* was calculated using the equation(eq), **R = (1.09908 ± 0.00015) m^{-1}**. The theoretical value for *R* is **1.097 m^{-1}**.

n	m	R	1/lambda	R m(-1)
2	3	10993633.6	1526893.56	1.099363
2	4	10988590.8	2060360.774	1.098859
2	5	10990063.7	2307913.383	1.099006

Table 5. Calculating the Rydberg’s constant.