Constructor University Bremen

Natural Science Laboratory
Signals and Systems

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Lab Experiment 1 – RLC Circuits – Transient Response

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1. Introduction

The purpose of this experiment is to study the transient response of second-order systems, with a focus on the RLC circuit. These systems are called second order because the mathematical model that describes them contains a second-order differential equation. In electrical engineering, RLC circuits are a common example of such systems since they include two energy storage elements: the capacitor and the inductor.

RLC circuits are important because they can be used to control or filter signals. For example, they are applied in tuning radios to a specific frequency or in removing unwanted noise from power lines. The behavior of an RLC circuit depends on the values of resistance, inductance, and capacitance, which together determine how the circuit reacts when it is given an input signal.

In this experiment, the main objective is to observe and analyze the transient behavior of an RLC circuit. Both theoretical and practical approaches are considered. Using simulations, such as with MATLAB, the system's response is predicted and compared with the experimental results. The differences between the two are also discussed.

The study covers two main parts:

1. Understanding the mathematical background of second-order systems, including the differential equation that represents them and the possible types of transient response (overdamped, critically damped, and underdamped).
2. Applying this knowledge to a real RLC circuit, solving its governing equation, and comparing the transient and steady-state responses under different conditions.

2. Theory

An RLC circuit is an example of a second-order system because its behavior is described by a second-order differential equation. These circuits contain two energy storage elements: the inductor (L) and the capacitor (C). When a voltage or current is applied to such a system, the energy stored in these components interacts, which produces different types of responses depending on the resistance (R), inductance (L), and capacitance (C).

In general, a second-order system can be represented by the following equation:

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y(t) = x(t)$$

Here:

- y(t) is the system's response to the input x(t).
- a_0, a_1, a_2 are constants that depend on the circuit parameters.

For a series RLC circuit, this equation can be rewritten in a standard form:

$$\frac{d^2y(t)}{dt^2} + 2\varsigma\omega_n \frac{dy(t)}{dt} + \omega_n{}^2 y(t) = K\omega_n{}^2 x(t)$$

Where:

- $\omega_n = \sqrt{\frac{a_0}{a_2}} = \sqrt{\frac{1}{LC}}$ is the natural (undamped) frequency of the system.
- $\varsigma = \frac{a_1}{2\sqrt{a_0 a_2}} = \frac{R}{2}\sqrt{\frac{C}{L}}$ is the damping ratio.
- $K = \frac{1}{a_0}$ is the gain of the system.

A second-order non-homogeneous differential equation has two parts in its solution:

- The **homogeneous solution** (y_h)
- The **forced solution** (y_f)

The general form is:

$$y(t) = y_h(t) + y_f(t)$$

- The **homogeneous solution** represents the **transient response** of the system, which comes from the system's initial conditions when the input $x(t) = 0$.
- The **forced solution** represents the **steady-state response**, which is due to the external input when $x(t) \neq 0$.

The Homogeneous Solution

To find the homogeneous solution, we assume $x(t) = 0$. The equation then becomes:

$$\frac{d^2y(t)}{dt^2} + 2\varsigma\omega_n \frac{dy(t)}{dt} + \omega_n{}^2 y(t) = 0$$

We assume a trial solution of the form:

$$y(t) = Ce^{\lambda t}$$

Substituting this into the differential equation gives:

$$Ce^{\lambda t}(\lambda^2 + 2\varsigma\omega_n\lambda + \omega_n{}^2) = 0$$

This leads to the characteristic equation:

$$(\lambda^2 + 2\varsigma\omega_n\lambda + \omega_n{}^2) = 0$$

The solutions (roots) are:

$$\lambda_{1,2} = -\varsigma\omega_n \pm \omega_n\sqrt{\varsigma^2 - 1}$$

Thus, the homogeneous solution is:

$$y_h(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

Where C_1 and C_2 are constants determined by initial conditions.

Transient response Cases

The form of the transient response depends on the damping ratio (ς):

1. Underdamped ($0 < \varsigma < 1$)
    - Roots are complex.
    - The solution is oscillatory but with decaying amplitude:

$$y(t) = e^{-\varsigma\omega_n t}(C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t))$$

Where $\omega_d = \omega_n\sqrt{1 - \varsigma^2}$ is the damped natural frequency.

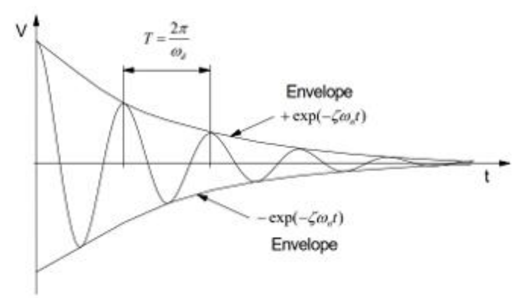

Figure 1.1: Under-damped 2nd order homogeneous D.E

2. Critically damped ($\varsigma = 1$)
    - Roots are real and equal.
    - The response is the fastest possible return to steady-state without oscillations:

$$y(t) = (C_1 + C_2 t)e^{-\varsigma\omega_n t}$$

3. Overdamped ($\varsigma > 1$)
    - Roots are real and distinct

- The response decays without oscillations but more slowly than the critically damped case:

$$y(t) = C_1 e^{(-\varsigma+\sqrt{\varsigma^2-1})\omega_n t} + C_2 e^{(-\varsigma-\sqrt{\varsigma^2-1})\omega_n t}$$

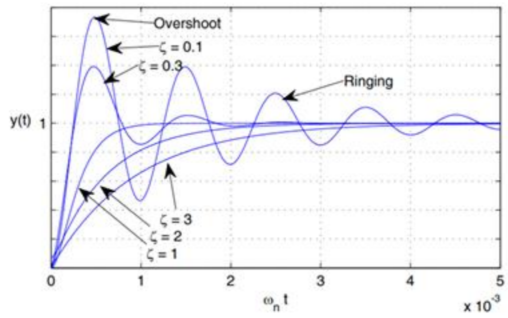

Figure 1.2: Solution for second-order homogeneous system with $\zeta = 0.1, 0.3, 1, 2, 3$

The Forced Solution

The **forced solution** comes from the external input x(t). Its form depends on the type of input:

- If the input is a constant (DC step), the forced response is constant.
- If the input is sinusoidal, the forced response will also be sinusoidal.
- If the input is linear, the forced response will be linear.

In other words, the forced response has the **same type of function** as the input but scaled and shifted depending on the system parameters.

Key definitions and Initial Conditions in Switched Circuits

- **Step Response**: The system's response when the input is a step function.

- **Steady-State Value**: The final value of voltage or current after the system has settled.

- **Ringing**: Oscillations that occur in underdamped systems. The amplitude decreases with time.

- **Overshoot**: When the response goes above the steady-state value before settling. It is measured as:

$$\%Overshoot = \frac{V_{max} - V_{steady-state}}{V_{steady-state}} * 100\%$$

Important properties of inductors and capacitors in switched circuits:

- Current through an inductor cannot change instantly, but its voltage can.
- Voltage across a capacitor cannot change instantly, but its current can.
- In steady-state DC conditions, an inductor acts like a short circuit.
- In steady-state DC conditions, a capacitor acts like an open circuit.

3. Experimental Set-Up and Results – Transient Response of RLC Circuits

Used tools and Instruments:

- 10 mH Inductor
- 6 n8F Capacitor
- Oscilloscope
- Function Generator
- Resistor Decade

Function Generator Settings:

- Waveform: Square
- Frequency: 100 Hz
- Peak-to-Peak Voltage (Vpp): 1V
- Offset: 0.5V

Execution

The experiment started by setting the function generator to produce a 100 Hz square wave with an amplitude of 0.5 V and an offset of 0.5 V, giving a signal between 0 V and 1 V as confirmed on the oscilloscope. The resistance decade was set to 100 Ω, and the oscilloscope was connected across the capacitor to observe the transient response. The damped frequency was then measured from the exponentially decaying sinusoidal signal, and the damped angular frequency ω_d was calculated while considering the 50 Ω internal resistance of the function generator. The calculated and measured values were compared and found to be consistent.

Next, the resistance needed for critical damping was calculated and applied to the circuit. The resulting waveform was displayed, and the response was checked to confirm whether it was critically damped by slightly adjusting the resistance. Finally, the resistance was increased to 30 kΩ to produce an over-damped response, and the transient voltage across the capacitor was recorded.

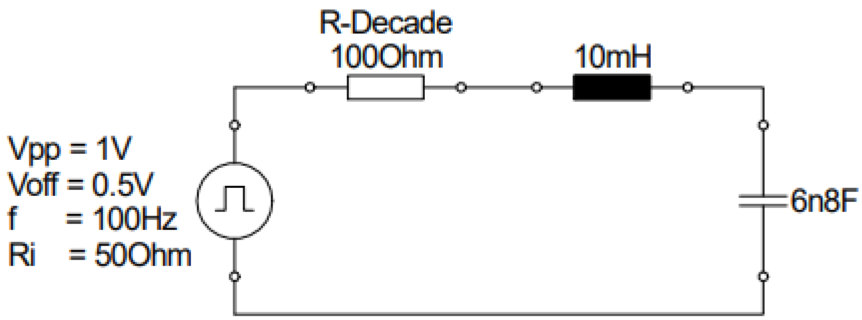

Figure 3: RLC Circuit for Experiment 1

After the circuit was built, the oscilloscope was connected across the capacitor to measure the damped frequency f_d. The frequency was determined in two ways: first, by using the period function to measure T directly, and second, by using the cursor function to measure the time difference between successive peaks. Both methods gave consistent results. The measured period was $T = 51.28\ \mu s$.

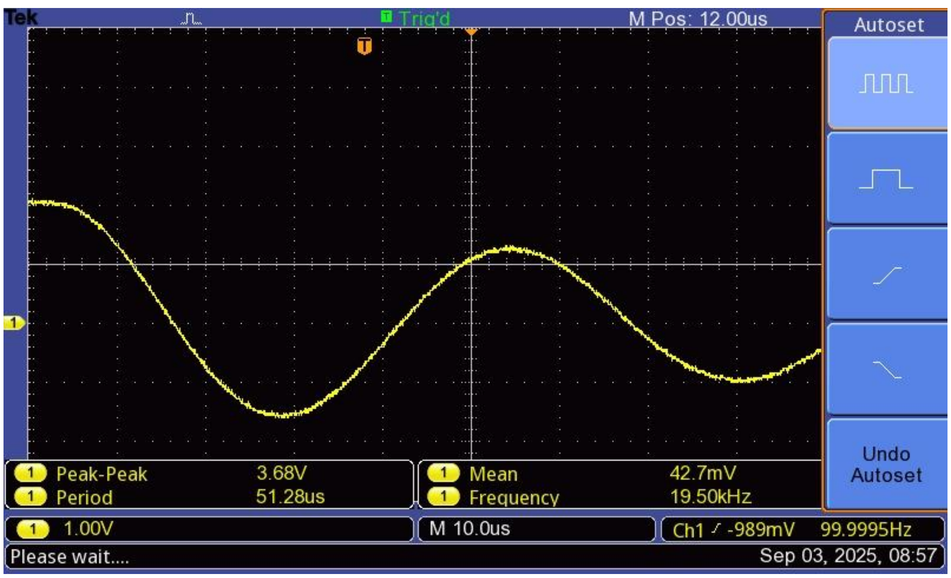

Figure 4: Period Measurement with Oscilloscope

From this value, the damped frequency was calculated as:

$$f_d = \frac{1d}{T} = \frac{1}{51.28 * 10^{-6}} = 19.5 * 10^3\ Hz$$

The corresponding damped angular frequency was then found to be:

$$\omega_d = 2\pi f_d = 2\pi * 19.5 * 10^3 = 1.23 * 10^5 rad/s$$

However, this calculation does not consider the internal resistance of the function generator, which is R=50 Ω. Taking this into account, the formula used was:

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

Substituting the circuit values gave:

$$\omega_d = \sqrt{\frac{1}{0.01 * 6.8 * 10^{-9}} - \left(\frac{150}{2 * 0.01}\right)^2} = 1.21 * 10^5 \ rad/s$$

To calculate the theoretical damped angular frequency ω_d, this requires knowing the natural frequency ω_n and the damping ratio ς. For a series RLC circuit, these are given as:

$$\omega_n = \sqrt{\frac{1}{LC}}$$

$$\varsigma = \frac{a_1}{2\sqrt{a_0 a_2}} = \frac{R}{2}\sqrt{\frac{C}{L}}$$

where L is the inductance, C is the capacitance, and R is the total resistance of the circuit (including the 50 Ω internal resistance of the function generator).

Using the measured values:

$$\omega_n = \frac{1}{\sqrt{10 * 10^{-3} * 6.8 * 10^{-9}}} = 121267.8 \, rad/s$$

$$\varsigma = \frac{100 + 50}{2}\sqrt{\frac{6.8 * 10^{-9}}{10 * 10^{-3}}} = 0.061846584$$

The damped angular frequency is then:

$$\omega_d = \omega_n\sqrt{1 - \varsigma^2} = 121267.8 * \sqrt{1 - 0.0618^2} = 121035.7 \, rad/s$$

This theoretical value agrees closely with the experimentally measured damped angular frequency of $1.23 * 10^5 \, rad/s$ The small difference is likely due to measurement errors from the oscilloscope.

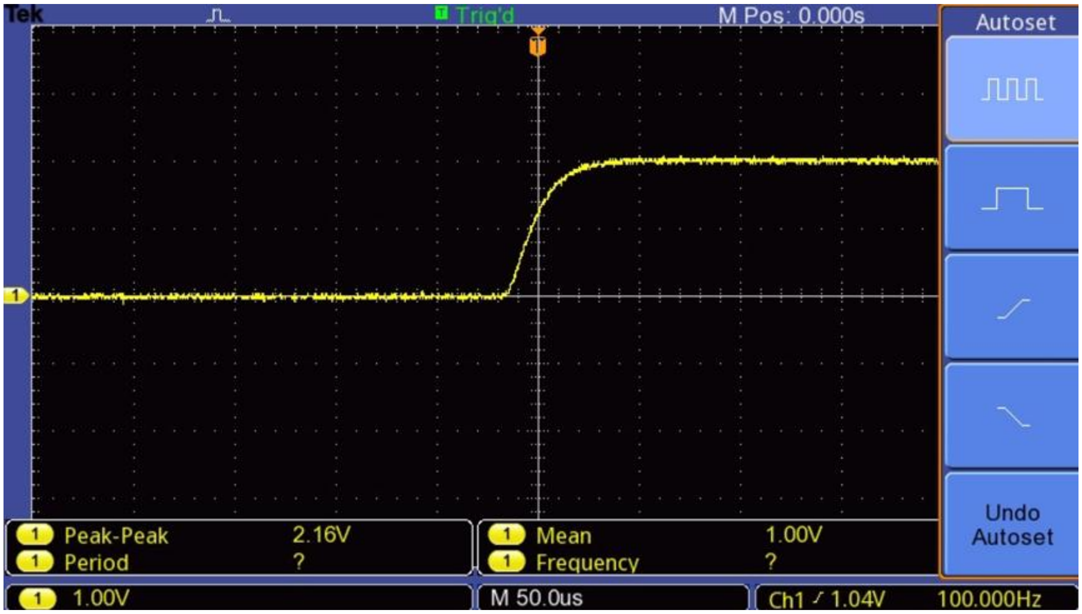

Figure 5: Critically damped signal R = 2375 Ohm

For a system to be critically damped, the damping ratio ζ must equal 1. The required resistance for critical damping was calculated as:

$$R_{total} = 2 * \varsigma * \frac{L}{C} = 2 * 1 * \frac{10 * 10^{-3}}{6.8 * 10^{-9}} = 2425.36\, Ohm$$

Since this includes the internal resistance of the function generator (50 Ω), the resistance set on the R-decade was:

$$R = 2425.36 - 50 = 2375.36\, Ohm$$

The oscilloscope displays at this resistance showed a critically damped response (see Fig. 6). However, it was observed that adjusting the R-decade value affected how close the response was to being truly critically damped. For example, increasing the resistance to R=4000 Ω resulted in a poorer critical damping effect compared to the calculated value.

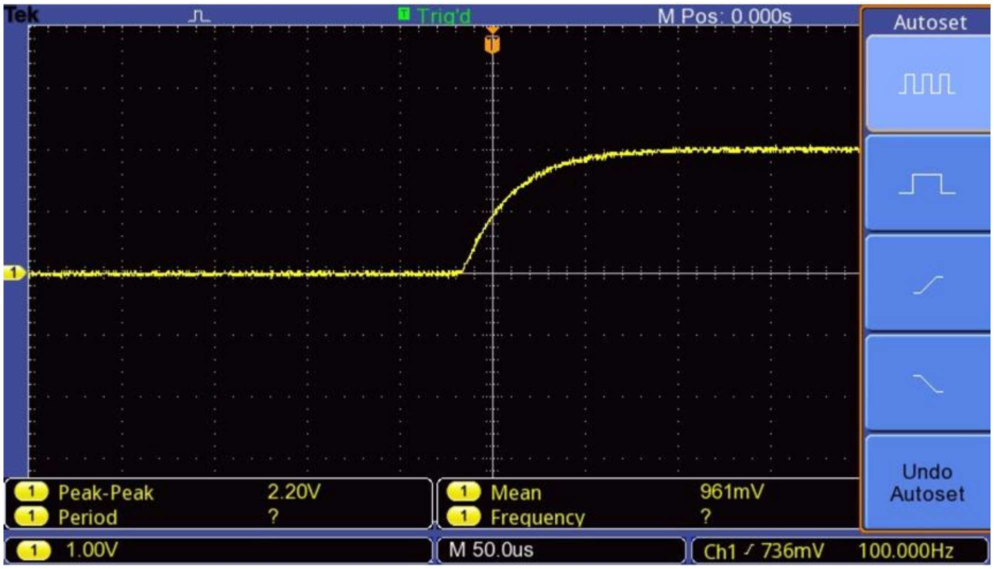

Figure 6: Critically damped signal R = 4000 Ohm

From the observations, it can be concluded that reducing the resistance slightly improves the critical damping of the circuit. When the R-decade was set to 1900 Ω, the damping improved noticeably, as shown in Figure 7. This confirms that small adjustments around the calculated resistance are sometimes necessary to achieve an ideal critically damped response in practice.

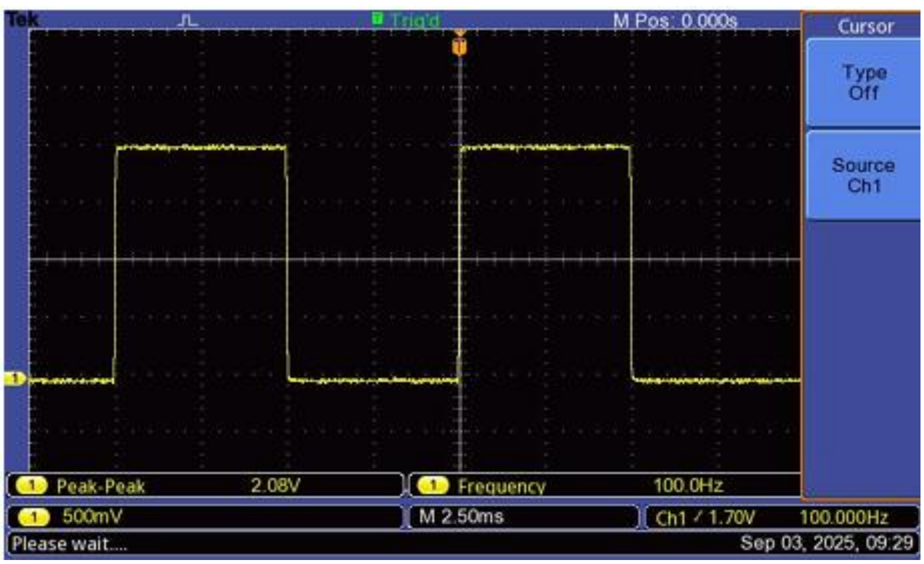

Figure 7: Critically damped signal R = 1900 Ohm

For the final part of the experiment, the R-decade was set to 30 kΩ to create an over-damped condition. The transient voltage across the capacitor was observed using the oscilloscope, as shown in Figure 8. In this case, the response decayed slowly without oscillations, demonstrating the typical behavior of an over-damped system.

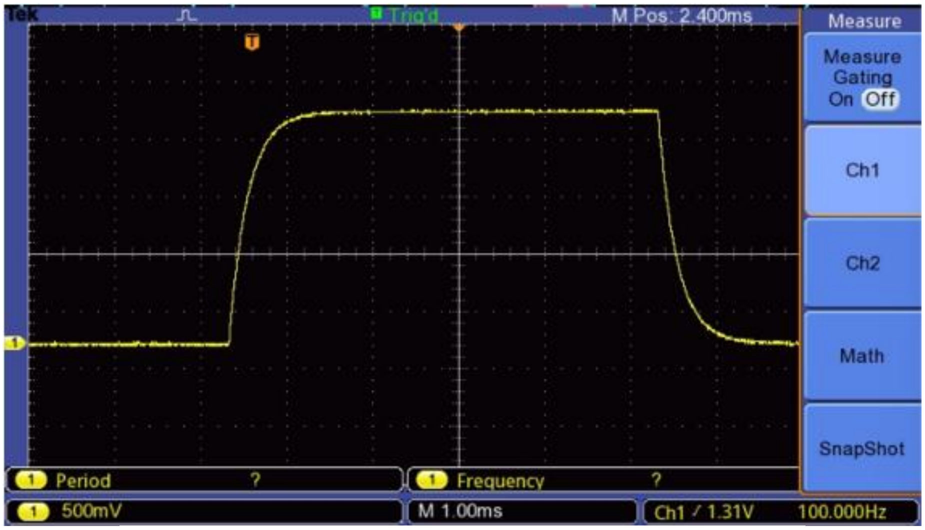

Figure 8: Over damped signal R = 30 KOhm

3. Evaluation

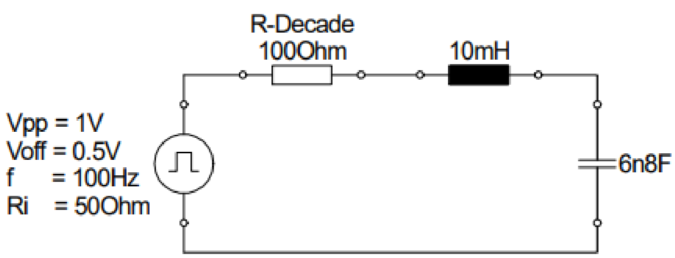

Figure 9: RLC Circuit for Experiment 1

From Figure 10, the differential equation describing the capacitor voltage $V_C(t)$ is given by:

$$\frac{d^2V_C}{dt^2}+\frac{RdV_C}{Ldt}+\frac{1}{LC}V_C(t)=V_S(t)\ -\ where\ R=150\ Ohm$$

To determine the system nature, we calculate the damping ratio using the following equation:

$$\varsigma=\frac{R}{2}\sqrt{\frac{C}{L}}$$

Substituting values:

$$\varsigma=\frac{150}{2}\sqrt{\frac{6.8*10^{-9}}{10*10^{-3}}}=0.0618$$

Since $\varsigma < 1$, the system is **underdamped**.

Determination of Constants

To solve for the constants C_1 and C_2 , we apply the initial conditions:

- The capacitor is initially uncharged, so

$$V_C(0) = 0$$

- The voltage across a capacitor cannot change instantly, and the initial current is zero, so

$$\left.\frac{dV_C}{dt}\right|_{t=0} = 0$$

The General Solution is:

$$V_C(t) = e^{-\varsigma\omega_n t}(C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t)) + 1$$

At $t = 0$:

$$V_C(0) = C_1 + 1 = 0 \longrightarrow C_1 = -1$$

Differentiating:

$$\frac{dV_C}{dt} = e^{-\varsigma\omega_n t}(-C_1\,\omega_d \sin(\omega_d t) + C_2\,\omega_d \cos(\omega_d t)) - \varsigma\omega_n e^{-\varsigma\omega_n}(C_1 \cos(\omega_d t) + C_2 \sin(\omega_d t))$$

At $t = 0$:

$$0 = C_2\omega_d - \varsigma\omega_n C_1$$

$$C_2 = \frac{\varsigma\omega_n C_1}{\omega_d} = -\frac{\varsigma\omega_n}{\omega_d}$$

Substituting values:

$$C_1 = -1, \quad C_2 = -0.062$$

The expression for $V_C(t)$ with the calculated constants was then plotted in MATLAB. The plot of capacitor voltage against time is shown in Figure 10. This plot clearly illustrates the underdamped response of the RLC circuit, where the oscillations gradually decrease in amplitude as the system settles toward its steady-state value.

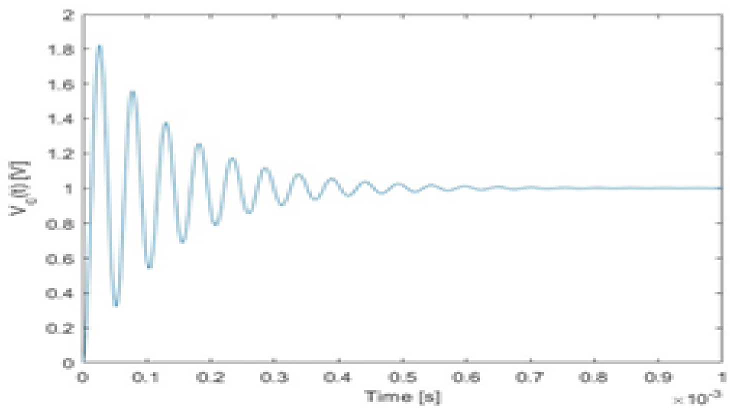

Figure 10: Underdamped case of an RLC circuit
(Voltage VS Time)

For the critically damped case, the damping ratio is $\varsigma = 1,$. The total resistance required for this condition was calculated as:

$$R_T = \frac{2L}{C} = 2425.356 \, Ohm$$

Considering the internal resistance of the function generator (50 Ω), the actual resistance to be set on the R-decade was:

$$R = R_T - 50 = 2375.356 \, Ohm$$

For a critically damped system, the voltage across the capacitor is given by:

$$V_C(t) = C_1 e^{-\omega_n t} + C_2 t e^{-\omega_n t} + 1$$

Using the initial conditions $V_C(0) = 0$ and $\left.\frac{dV_C}{dt}\right|_{t=0} = 0$, the constants were found as $C_1 = -1$ and $C_2 = -\omega_n$. Thus, the final solution becomes:

$$V_C(t) = -e^{-\omega_n t} - \omega_n t e^{-\omega_n t} + 1$$

where $\omega_n = 1.213 * 10^5 \, rad/s$. The plot of $V_C(t)$ against time was then generated in MATLAB, and the resulting graph is shown in Figure 11.

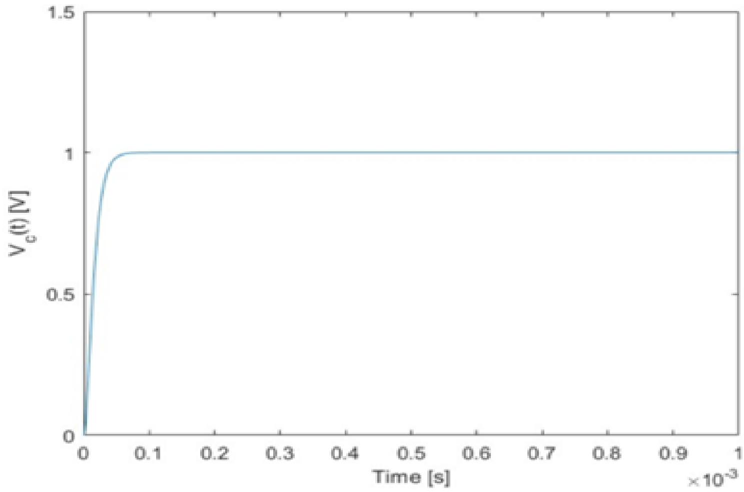

Figure 11: Critically damped case of an RLC circuit (Voltage VS Time)

The measured damped frequency was $f_d = 19.5\ kHz$, corresponding to $\omega_d = 1.23 * 10^5\ rad/s$. The calculated damped frequency was $f_d = 19.26\ kHz$, with$\omega_d = 1.21 * 10^5\ rad/s$. These values are very close, showing good agreement between theory and experiment.

For critical damping, the theoretical resistance was found to be R=2375.36 Ω (after considering the 50 Ω internal resistance of the generator). Experimentally, the best critical damping was observed at about R=1900 Ω. The small difference can be explained by practical effects such as component tolerances and measurement limitations.

The deviations between calculated and measured results can be attributed to factors like probe compensation errors, oscilloscope cursor accuracy, and parasitic resistances from the R-decade, breadboard, and connecting wires. Overall, the results confirm the theoretical expectations, with the observed differences falling within acceptable experimental limits.

4. Problem 5: RLC Circuit

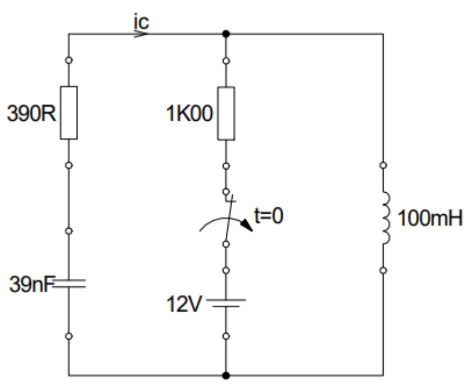

Figure 12: RLC Circuit 2

(a) Differential equation for $i_C(t)$

KCL at the node: $i_L - i_C = 0 \;\rightarrow\; i_L = i_C$

$$v(t) = V_R + V_C = Ri_c + V_C = V_L = L\frac{di_L}{dt} = L\frac{di_C}{dt}$$

We know that $i_C = C\frac{dV_C}{dt}$

$$L\frac{d^2 i_C}{dt^2} + R\frac{di_C}{dt} + \frac{1}{C}\,i_C = 0,$$

$$i_C'' + \frac{R}{L}\,i_C' + \frac{1}{LC}\,i_C = 0 \;=\; i_C'' + 3900\,i_C' + 2.5641\times 10^8\,i_C = 0$$

(b) Damping type and integration constants

$$\omega_n = \frac{1}{\sqrt{LC}} = 1.6013 * 10^4\;rad/s$$

$$\varsigma = \frac{R}{2}\sqrt{\frac{C}{L}} = 0.1218$$

The circuit is underdamped with:

$$\alpha = \varsigma\omega_n = 1.950 * 10^3 s^{-1},\;\; \omega_d = \omega_n\sqrt{1-\varsigma^2} = 1.589 * 10^4 rad/s$$

For $t < 0$ the inductor is a short and the capacitor is open (DC steady state)

$$i_L(0^-) = \frac{12\;V}{1\;kOhm} = 12\;mA, \quad V_c(0^-) = 0 \rightarrow i_L(0^+) = 12\;mA, \quad V_c(0^+) = 0$$

After opening $i_L = -i_C$

$$i_C(0^+) = C_1 = -12\;mA = \;-0.012\;A.$$

Computing C_2. Underdamped form after differentiating:

$$i_C(t) = e^{-\alpha t}(-\alpha(C_1 cos\omega_d t + C_2 sin\omega_d t) + \omega_d(-C_1 sin\omega_d t + C_2 cos\omega_d t))$$

When $t = 0, cos0 = 1\;and\;sin0 = 0$

$$i_C(0) = -\alpha C_1 + \omega_d C_2.$$

At $t = 0, V_C(0^+) = 0$:

$$-Li_C(0^+) = Ri_C(0^+) \rightarrow i_C(0^+) = -\frac{R}{L} i_C(0^+) = \frac{390}{0.1} * 0.012 = 46.8\ A/s$$

So

$$C_2 = \frac{i_C(0) + \alpha C_1}{\omega_d} = \frac{46.8 + (1950)(-0.012)}{1.589 * 10^4} = 1.47\ mA$$

Final expression for the current:

$$i_C(t) = e^{-1950t}(-0.012\cos(1.589 * 10^4 t) + 0.00147\sin(1.589 * 10^4 t))A$$

(C) Plot using MATLAB

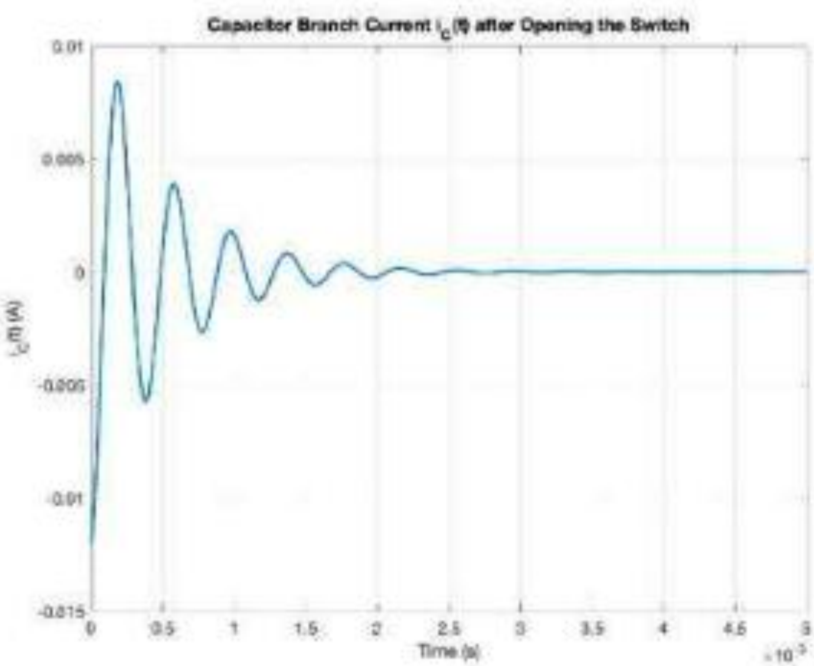

Figure 13: Plot Current dependence on the time.

5. Conclusion

The experiment successfully demonstrated the transient response of a second-order RLC circuit under different damping conditions. By varying the resistance, the three types of responses underdamped, critically damped, and overdamped—were clearly observed and compared with theoretical predictions.

In the underdamped case, the output showed oscillations that decayed exponentially over time. For the critically damped condition, the response returned to equilibrium without oscillation, while in the overdamped case the response was slower and non-oscillatory. The damped frequency measured with the oscilloscope was found to be close to the calculated value, with small deviations explained by probe compensation issues, cursor measurement limitations, and parasitic resistances from the circuit setup.

When determining the resistance required for critical damping, it was observed that the theoretical value did not perfectly match the practical result. This difference was due to internal resistances of the function generator, R-decade, breadboard, and component tolerances. Despite these small discrepancies, the results confirmed the theoretical models and highlighted the influence of practical limitations in real circuits.

Overall, the experiment provided a clear understanding of how the damping ratio affects the transient response of RLC circuits. It also emphasized the importance of considering measurement errors and component tolerances when comparing theoretical calculations with experimental results.

6. References

1. Alexander, C. K. and Sadiku, M. Fundamentals of Electric Circuits, 7th edition.
2. 20240829-co-520-b manual- U.Pagel, Page(11-22) [http://uwp-cu-lab.my-board.org/02.0.adveelab/02.1.signalsys/20240829-co-520-b manual.pdf](http://uwp-cu-lab.my-board.org/02.0.adveelab/02.1.signalsys/20240829-co-520-b%20manual.pdf)

7. Prelab: RLC Circuits – Frequency Response

1) Identify the filter type at different measurement nodes

Measuring the output across different elements of the RLC network yields the following classical filter behaviors:

- **Across the resistor:** Band–pass response
- **Across the capacitor:** Low–pass response
- **Across the inductor:** High–pass response
- **Across the capacitor and inductor together:** Band–stop (notch) response

2) Bode magnitude plots (100 Hz to 100 kHz

The magnitude responses for all four measurement cases were obtained using MATLAB. A 5 V input source was applied with a logarithmic frequency sweep ranging from 100 Hz to 100 kHz. The resulting magnitude characteristics illustrate the expected filter behaviors of the RLC circuit. Figure 14 shows the combined Bode magnitude plot for the resistor, capacitor, inductor, and the combined capacitor–inductor outputs.

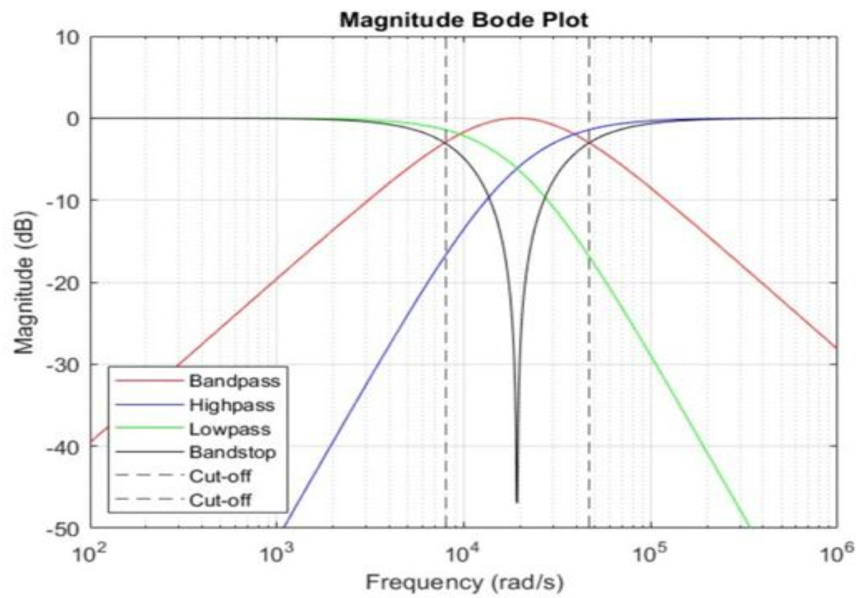

Figure 14: Magnitude Bode Plot for band-pass, high-pass and band-stop behaviors.

Code for the MATLAB:

```
% ----- Sweep and component values -----
w = logspace(2, 5, 1000);      % 10^2 to 10^5 rad/s (    100 Hz to 100 kHz)
R = 390;                       % ohm
C = 270e-9;                    % F
L = 10e-3;                     % H
s = 1j*w;

% ----- Transfer functions (node magnitudes) -----
H_R  = (s.*w.*R.*C)./(1 - (w).^2*L*C + s.*w.*C.*R);
H_L  = (-(w).^2*L*C)./(1 - (w).^2*L*C + s.*w.*C.*R);
H_C  = 1./(1 - (w).^2*L*C + s.*w.*C.*R);
H_LC = (-(w).^2*L*C + 1)./(1 - (w).^2*L*C + s.*w.*C.*R);

% ----- Plot magnitude (dB) on one axes -----
figure;
semilogx(w, 20*log10(abs(H_R)),  'r', 'LineWidth', 1.2); hold on;
semilogx(w, 20*log10(abs(H_L)),  'b', 'LineWidth', 1.2);
semilogx(w, 20*log10(abs(H_C)),  'g', 'LineWidth', 1.2);
semilogx(w, 20*log10(abs(H_LC)), 'k', 'LineWidth', 1.2);
grid on; ylim([-50 10]);
xlabel('Frequency␣(rad/s)'); ylabel('Magnitude␣(dB)');
title('Magnitude␣Bode␣Plot');
legend('Bandpass','Highpass','Lowpass','Bandstop','Location','southwest');

% ----- Bandwidth and Q of the band-pass (resistor measurement) -----
mag_HR = 20*log10(abs(H_R));
peak   = max(mag_HR);
th     = peak - 3;                          % -3 dB points

idx = find(mag_HR >= th);
w_low  = w(idx(1));
w_high = w(idx(end));

B       = w_high - w_low;                    % rad/s
B_Hz    = B/(2*pi);                          % Hz
w0      = sqrt(1/(L*C));                     % rad/s
Q       = w0 / B;

fprintf('w_low␣␣=␣%.2f␣rad/s\n', w_low);
fprintf('w_high␣=␣%.2f␣rad/s\n', w_high);
fprintf('Bandwidth␣␣B␣=␣%.2f␣rad/s␣␣(%.2f␣Hz)\n', B, B_Hz);
fprintf('Resonant␣w0␣=␣%.2f␣rad/s,␣␣␣Q␣=␣%.4f\n', w0, Q);

% Mark -3 dB frequencies
xline(w_low,  'k--');
xline(w_high, 'k--');
hold off;
```

3) Bandwidth and Quality Factor

For the band-pass case, where the output was measured across the resistor, both the theoretical bandwidth and quality factor were determined. The theoretical −3 dB bandwidth is given by:

$$B = \omega_2 - \omega_1 = \frac{R_S}{L} = \frac{390}{10 * 10^{-3}} = 39000\frac{rad}{s} = 6207.04\ H$$

and the quality factor by:

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{1}{B\sqrt{LC}} \quad where\ \omega_0 = \frac{1}{\sqrt{LC}} - resonant\ radian\ frequency$$

From MATLAB post-processing, using the −3 dB threshold, the estimated bandwidth was found to be approximately 6120 Hz. This result agrees well with the theoretical calculation. The small differences can be explained by factors such as the numerical resolution of the frequency sweep, the idealized assumptions in the theoretical model, and the discrete thresholding procedure used to locate the cutoff points.