Experiment 3 - Ideal Gas Laws

Group 1

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Abstract

In this study, the core relationship between pressure, volume, temperature, and the number of moles in an ideal gas was explored. Boyle's, Gay-Lussac's, and Charles's laws were examined and discussed. A glass enclosure system, a gas syringe, a thermometer, and a pressure sensor were used to conduct experiments to observe these gas laws in action. The objective was to confirm the Ideal Gas Law, while also calculating the Van der Waals constants, 'a' and 'b'. Graham's law was further examined, and the rate of air volume leakage was measured. Avogadro's Law was supported by the experimental findings, with the exception of a small discrepancy, likely due to air leakage under increased pressure. Gay-Lussac's Law was analyzed, yielding a thermal coefficient β = (0.0036 ± 0.0002) k^{-1}, which deviated from the expected value of 0.0036 k^{-1}, possibly due to systematic errors or deviations from ideal gas behavior. The uncertainty in β was determined by evaluating the error in the slope of the pressure-temperature graph. Based on the analysis of the temperature, pressure, and volume data, the Van der Waals constants were found to be a = 0.91 and b = 0.00005, illustrating the finite volume of gas molecules and intermolecular forces. The theoretical gas laws under the specified experimental conditions were confirmed by these results.

Introduction

In this experiment, the relationship between pressure, volume, temperature, and the number of moles in an ideal gas was examined. For a gas to be considered "ideal," four key assumptions are made: The gas particles occupy a negligible volume. The gas particles are uniform in size and experience no intermolecular forces (neither attraction nor repulsion) with other particles. The gas particles move randomly, adhering to Newton's Laws of Motion. The connection between these gas properties was formulated in the early 19th century and is known as the "Ideal Gas Law."

$$pV = nRT \qquad\qquad (Eq.1)$$

Here R = 8.3143 $JK^{-1}mol^{-1}$ represents the gas constant, T is the absolute temperature in Kelvin, and p is the absolute pressure, defined as $p = p_{atm} + p_{relative}$ in Pascals. n denotes the number of moles (where one mole contains (6.022 times 10^23 molecules), and V is the volume. From this equation, it can be observed that if two different gases have the same pressure and temperature, they will contain the same number of molecules.

Three major studies on gas laws, derived through experimentation, include Boyle's Law, Gay-Lussac's Law, and Jacques Charles's Law. Each of these laws was investigated in this experiment. Let's now discuss each one in detail.

Boyle's Law (The relationship between pressure and volume at constant temperature).

Boyle's Law states that the volume of a gas is inversely proportional to its pressure when the temperature remains constant. In other words, as the pressure on a gas increases, its volume decreases proportionally, provided that the temperature and the amount of gas remain unchanged.

$$p \propto \frac{1}{V} \quad or \quad pV = const. \quad at\ const.T \qquad (Eq.2)$$

Gay-Lussac's Law (The relationship between pressure and temperature at constant volume)

Gay-Lussac's Law states that the pressure of a gas is directly proportional to its absolute temperature, provided the volume remains constant. This means that as the temperature of the gas increases, its pressure increases proportionally, assuming no change in volume or the amount of gas.

$$p \propto T \qquad\qquad at\ const.V \qquad\qquad (Eq.3)$$

Charles's Law (The relationship between volume and temperature at constant pressure)

Charles's Law states that the volume of a fixed amount of gas at constant pressure is directly proportional to its absolute temperature. This implies that as the temperature of the gas increases, the volume increases proportionally, assuming the pressure and the quantity of gas remain unchanged.

$$V \propto T \qquad\qquad at\ const.p \qquad\qquad (Eq.4)$$

Unlike solids, where the increase in volume is proportional to the temperature change and the relative change in volume varies between different materials, for gases the thermal tension (β) remains constant across different gases. The thermal tension β is given by:

$$\beta_{T^\ominus} = \frac{1}{p_{T^\ominus}} \left(\frac{\partial p}{\partial T}\right)_{V,n} \qquad\qquad (Eq.5)$$

Here, $p_{T^\ominus}$ represents the pressure at standard temperature. The partial derivative of pressure with respect to temperature, while keeping volume V and the number of moles n constant, is given by:

$$\left(\frac{\partial p}{\partial T}\right)_{V,n} = \frac{nR}{V}$$

The pressure at the standard temperature $T^\ominus$ is: $\frac{nRT^\ominus}{V}$

Substituting these into the thermal tension equation:

$$\beta_{T^\ominus} = \frac{1}{p_{T^\ominus}} \left(\frac{\partial p}{\partial T}\right)_{V,n} = \frac{V}{nRT^\ominus} \cdot \frac{nR}{V} = \frac{1}{T^\ominus} = 3.661 \cdot 10^{-3} K^{-1} \qquad (Eq.6)$$

As there was some small gas leakage during the experiment, the volume flow rate in the syringe was measured and investigated. Generally, Graham's law of effusion describes the relationship between the rates of effusion (or diffusion) of two different gases. It is formulated as follows:

$$\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}} \qquad\qquad (Eq.7)$$

$R_1\ and\ R_2$ denote the rates of effusion or diffusion for gases 1 and 2, respectively, while M$_1$ and $_{M2}$ signify the molar masses of these gases.

In the final phase of the experiment, the Van der Waals constants 'a' and 'b' will be calculated. The Van der Waals equation is an enhancement of the ideal gas law that accounts for the non-ideal behavior of real gases, factoring in both molecular interactions and the finite volume of gas particles.

The ideal gas law is modified to reflect the volume occupied by gas molecules and the attractive forces present between them.

$$\left(P + a\frac{n^2}{V^2}\right)(V - nb) = nRT \qquad\qquad (Eq.8)$$

P represents the pressure of the gas, V is the volume of the gas, n indicates the number of moles of the gas, R is the universal gas constant R= 8.314, and T is the temperature of the gas measured in Kelvin. The constants a and b are specific Van der Waals constants for the particular gas. The term $a\frac{n^2}{V^2}$ accounts for the attractive forces between gas molecules, while (nb) represents the volume occupied by the gas molecules themselves.

Experimental Methods and Materials

A glass jacket system was utilized to regulate the temperature of the air within a gas syringe, which was equipped with a thermometer. To measure the relative pressure inside and outside the syringe, a pressure sensor connected to a Cassy interface was employed. Additionally, an immersion thermostat was placed in a water bath to control the water temperature, thereby heating the air inside the syringe. An external barometer was used to measure the initial pressure for a specific volume, helping to detect any potential leakage in the syringe.
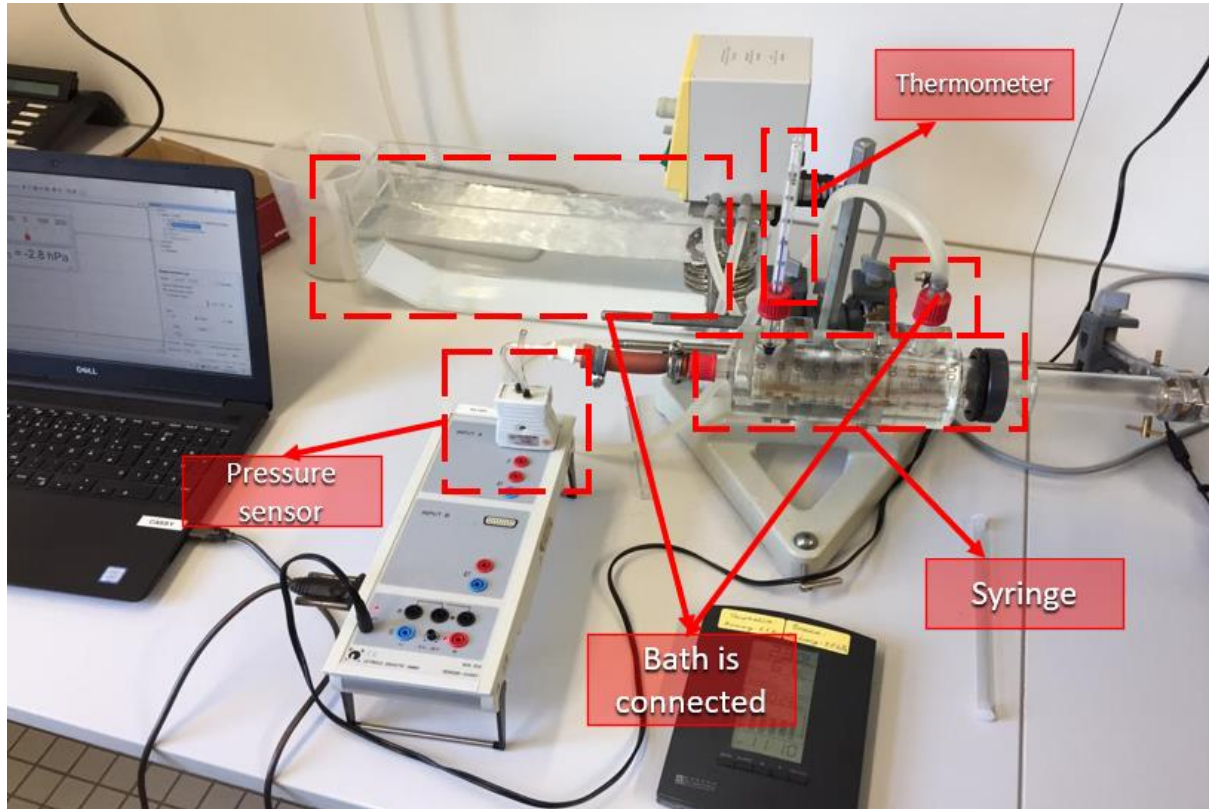
The pressure sensor and temperature bath were connected to the calibrated glass syringe. Using the data acquisition system, Cassy, the relative pressure between the inside and outside of the syringe was monitored. When the tube was connected to the syringe, it was ensured that the relative pressure was set to zero throughout all experiments. The system was not completely airtight, resulting in some air entering or exiting the syringe. To enhance its airtightness, a semi-solid, jelly-like substance was applied around the syringe. The air inside the syringe was heated using a glass jacket connected to a water bath, with the water temperature controlled by a thermostat. The tip of the syringe was linked to the pressure sensor. For the first experiment, a gas volume of $V = (4.33 \cdot 10^{-5} \pm 0.05 \cdot 10^{-5}) m^3$ was selected, and the water temperature was maintained at room temperature (constant). The system was closed, ensuring that n remained constant.

To measure leakage in the syringe, the initial pressure for the specified volume was recorded $p = (111 \pm 1) hPa$. Pressure loss was monitored over a period of 10 minutes, with measurements taken every 10 seconds during the first minute, every 20 seconds during the second minute, and subsequently every 30 seconds during the third and fourth minutes. Afterward, measurements were taken every minute until the 10 minutes were completed. The effusion rate, represented as the volume flow rate, (the slope was determined to be -0.1296) by multiplying the slope of the linear plot (pressure dependence over time) by the initial volume.

In the second part of the experiment (Boyle's Law), the temperature in the syringe was maintained at a constant 300.15 K, with the immersion thermostat kept at the same temperature for 4 minutes. The gas volume in the syringe was varied by pressing or pulling the plunger. Pressure ranged from $p = (0 \pm 1) \, hPa \, to \, p = (475 \pm 1) \, hPa$. The relationship between pressure and volume was observed, and the number of moles was calculated using (Eq.1).

The third part of the experiment focused on investigating the volume dependence on temperature at constant pressure (Charles's Law). Initially, the volume at which the relative pressure was zero was recorded as V=$50 \cdot 10^{-5} \, m^3$. The temperature of the water bath was adjusted, and volumes corresponding to a pressure of $p = (0 \pm 1) \, hPa$ were measured at ten different temperatures. This allowed us to study the volume dependence on temperature while keeping pressure constant.

In the final part of the experiment, Gay-Lussac's Law was investigated. The initial volume at which the relative pressure was zero was noted as V=$44.5 \cdot 10^{-5} \, m^3$. The plunger was securely fixed, preventing movement regardless of the pressure inside the syringe. Ice was added to the water bath, resulting in a system temperature of 5 °C, at which pressure was measured as p = 29hPa. The temperature was then gradually increased in 5 °C increments until reaching 30 °C (5, 10, 15, 20 °C, etc.), with the corresponding pressure recorded for each temperature. The temperature data was later converted to Kelvin for analysis.

Results and Data Analysis

During the leakage test, the room temperature was 21.5 °C. In this experiment, it was checked whether the experimental setup had any issues with gas leakage.

Initial volume of the air was V = (4*10^{-5} +- 0.05*10^{-5}) m^3. The pressure in the syringe was reduced to zero, and the tube of the Cassy sensor was connected. The volume of the gas was then varied. After waiting for 10 minutes, the pressure change was observed, and a graph showing the dependence of pressure on time was plotted. The slope of the graph was 0.129 and by multiplying the slope by the

initial volume flow rate was calculated R = 0.516 * $10^{-5}m^3$/s. As observed, the pressure remained constant, indicating that the experimental setup worked well and that there was not much leakage. To verify Graham's law of effusion experimentally, different gases were needed. However, since the experiment was conducted with only one gas, it was not possible to do so. Nonetheless, the volume flow rate in experiment 1 was calculated.
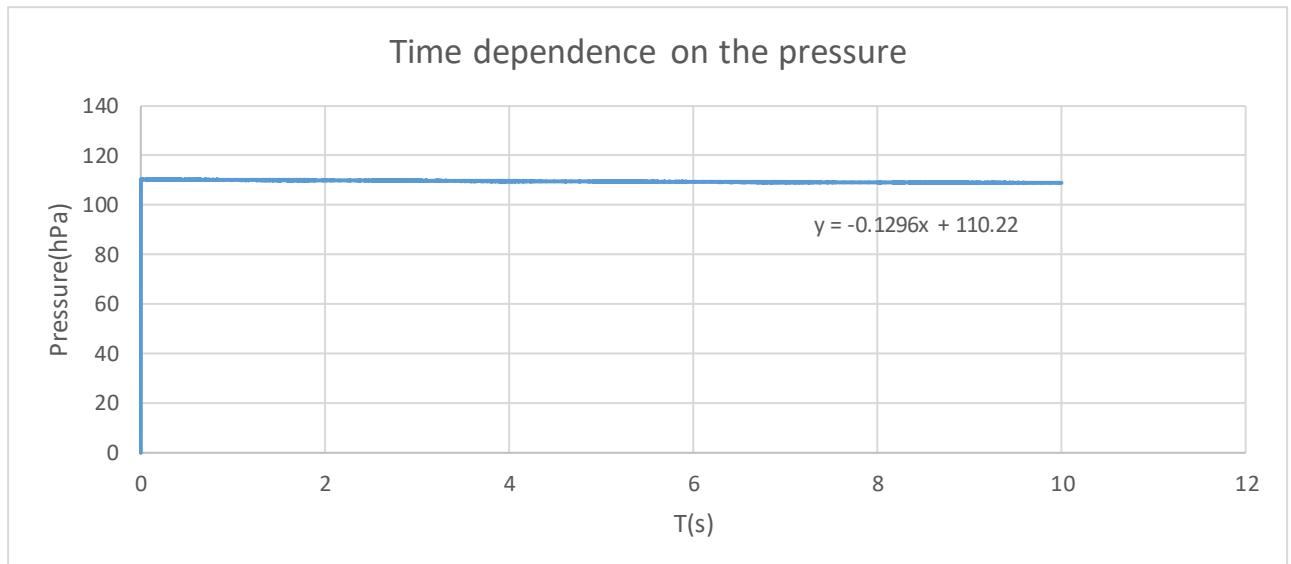
Plot 1 Pressure dependance on time (experiment 1).

In the next experiment, the dependence of pressure on volume was studied while keeping the temperature constant at (295.5 ± 0.5) K. The uncertainty arose from the thermometer, which has a precision of 1 degree Celsius, resulting in an error of 0.5 degrees Celsius, or 0.5 K in this case. Since relative pressure was measured during the experiment, the standard atmospheric pressure of 102,600 Pa was added to the results. The pressure uncertainty was ±1 hPa because the pressure sensor displayed values in hPa on the computer screen. In addition, there was also an instrumental error. The experiment demonstrated that pressure decreases as volume increases, provided that the temperature remains constant.
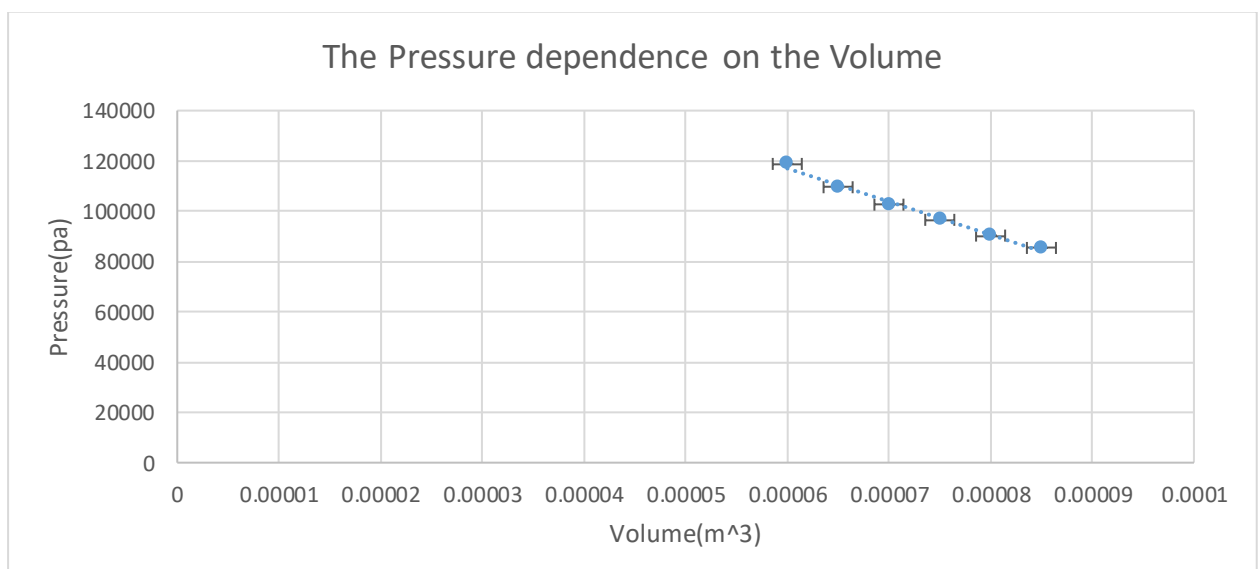
Plot 2 Pressure dependance on volume (Boyle's Law, experiment 2).

Then from the (Eq 1) we calculated number of moles which should be constant according to the Avogadro's law. The result was n = (0.002937 +- 0.000034) mol.
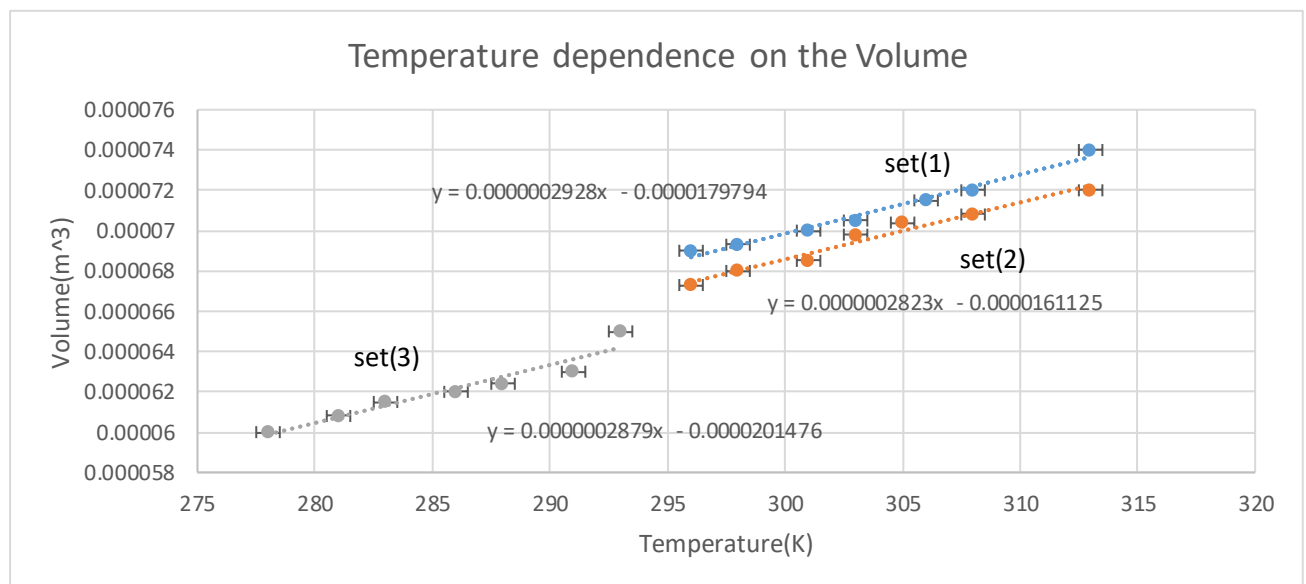
Volume in m^3	Pressure (Pa)	Moles (n)
0.00007	102600	0.002933151
0.000065	109520	0.00290734
0.00006	118800	0.002911097
0.000075	96560	0.002957655
0.00008	90300	0.002950304
0.000085	85380	0.002963904

Table 1. Calculating number of moles using volume and absolute pressure

To calculate the error, the following formula was used:

$$\frac{\Delta n}{n} = \left|\frac{\Delta p}{p}\right| + \left|\frac{\Delta T}{T}\right| + \left|\frac{\Delta V}{V}\right| \qquad \text{(Eq 9)}$$

Afterwards, when increasing the temperature, the volume of the air was measured at pressures of 102,600 Pa (set 1), 105,100 Pa (set 2), and 107,600 Pa (set 3), while keeping the constant in each case. This was done for seven different temperatures. Subsequently, a plot was created based on the results.

Temperature dependence on the Volume

set(1)
y = 0.0000002928x - 0.0000179794

set(2)
y = 0.0000002823x - 0.0000161125

set(3)
y = 0.0000002879x - 0.0000201476

Volume(m^3) / Temperature(K)

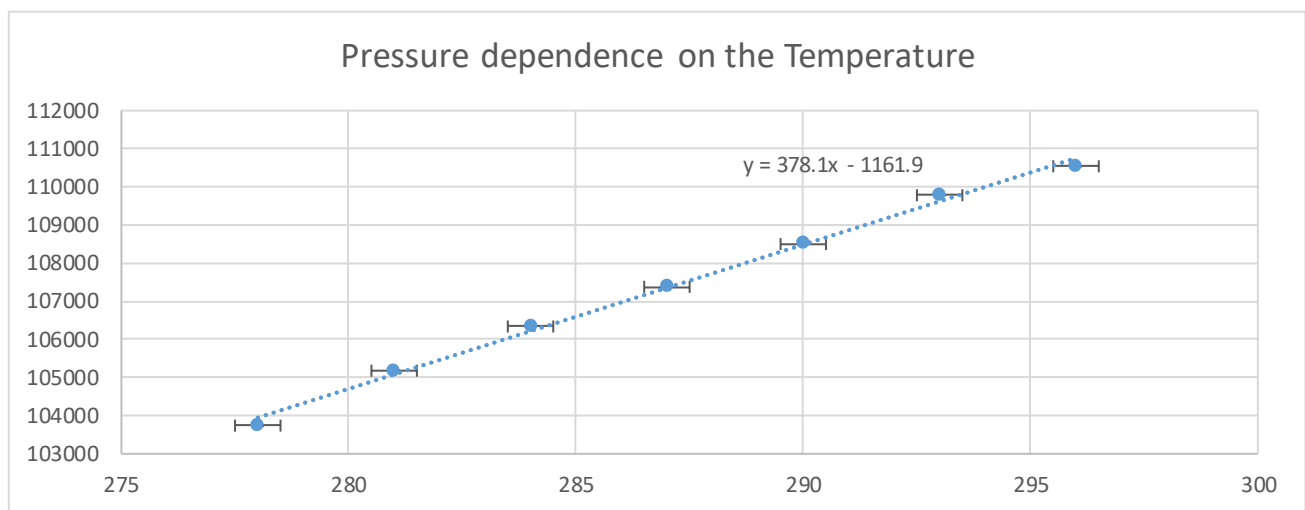**Plot 3 Volume dependance on temperature (Charles's Law, experiment 3).**

From this plot, it can be observed that as the temperature increases, the volume also increases, indicating a positive correlation between volume and temperature. Trendlines were then created, and the slopes for all the graphs were calculated. After that, the ratios of the pressures and slopes for all combinations of these sets were compared.

Ratios P	P1/P2	0.976213
	P1/P3	0.953532
	P2/P3	0.976766

ratios a	a1/a2	0.9641393
	a1/a3	0.983265
	a2/a3	1.0198371

As observed, the ratios of the pressures are very close to the ratios of the slopes. This serves as evidence for Charles's Law.

The next experiment focused on the investigation of Gay-Lussac's Law. In this case, the volume was kept constant, and a plot of pressure dependence on temperature was created.

Plot 4 Pressure dependance on temperature (Gay-Lussac's Law, experiment 4)

From the plot, it can be observed that as temperature increases, pressure also increases, indicating a positive correlation between pressure and temperature. The relationship appears to be linear. The slope and intercept of the function were found to be 378.1 and -1161.9, respectively. According to the equation, the derivative of pressure with respect to temperature would be the slope of that graph. The slope of this graph was divided by the pressure at standard temperature, p=102,600p = 102,600p=102,600 Pa, to calculate the thermal tension (plot 4), $\beta = (0.00368 \pm 0.00025)\ K^{-1}$. The theoretical value for thermal tension is 0.0036.

When measuring the error of the slope of the P(T) graph (plot 4), the following formula was used to calculate the error of the thermal tension:

$$\frac{\Delta\beta}{\beta} = \frac{\Delta m}{p_{T\ominus}} \qquad\qquad (Eq.\,10)$$

Where β is thermal tension, Δm is error of the slope of plot 4, $p_{T\ominus}$ is pressure at normal temperature.

To calculate Δm the following formulas were used:

$$R = \frac{\sum_i (T_i - \bar{T})(P_i - \bar{P})}{\sqrt{\sum_i (T_i - \bar{T})^2 \cdot \sum (P_i - \bar{P})^2}} \qquad \text{(Eq. 11)}$$

$$\Delta m = m\sqrt{\frac{1}{n-2} \cdot \frac{1-R^2}{R^2}} \qquad \text{(Eq. 12)}$$

Where m is slope of the function, n is number of measurements, T is temperature P is pressure.

The last step is to calculate the coefficients a and b in the Van der Waals equation experimentally. First of all, the Van der Waals equation will be simplified. The general formula is as follows:

$$\left(P + \frac{an^2}{v^2}\right)(v - nb) = nRT$$

Let's open the brackets.

$$Pv - Pnb + \frac{an^2}{v} - \frac{abn^3}{v^3} = nRT$$

A and b are very small numbers, so we can say that the product of them is equal to zero.

$$\frac{an^2}{v} - Pnb = nRT - PV$$

$$a\left(\frac{n^2}{v}\right) - b(Pn) = nRT - PV$$

These products are always constants, so let's consider $(\frac{n^2}{v})$ as β, (Pn) as δ and $(nRT - PV)$ as γ. After that we can make a system and solve them with respect to a and b.

$$\begin{cases} a\beta_1 - b\delta_1 = \gamma_1 \\ a\beta_2 - b\delta_2 = \gamma_2 \end{cases}$$

The calculations were performed, and the results are as follows: a=0.91 and b=0.00005.

Discussion and conclusion

In this experiment, the fundamental gas laws and the behavior of ideal gases were examined, specifically Boyle's, Gay-Lussac's, and Charles's laws. The main objective was to validate the Ideal Gas Law, calculate thermal tension, and determine the Van der Waals constants 'a' and 'b'. Using a glass jacket system, a gas syringe, a thermometer, and a pressure sensor, the relationships among pressure, volume, number of moles, and temperature were explored.

The analysis supported Avogadro's Law, demonstrating a consistent number of moles of gas in relation to volume and pressure, with n=(0.002937±0.000034)n = (0.002937 \pm 0.000034)n=(0.002937±0.000034) mol. However, a minor deviation was observed, likely due to air leakage within the experimental setup at higher pressures and the assumptions made regarding ideal gases.

The inverse relationship between pressure and volume, as described by Boyle's Law, was successfully illustrated, confirming the negative correlation between these two variables while maintaining a constant temperature. By varying the temperature while keeping pressure constant, a direct proportionality between volume and temperature was established, in line with Charles's Law. Additionally, the investigation of Gay-Lussac's Law affirmed the proportional relationship between pressure and temperature at a constant volume. The calculated thermal tension $\beta = (0.0036 \pm 0.0002)\ K^{-1}$ showed a slight deviation from the theoretical value, which may be attributed to experimental limitations and the assumptions made for ideal gases.

The Van der Waals constants we calculated were a = 0.91 and b = 0.00005, indicating the finite volume occupied by gas molecules and the presence of intermolecular forces.

One significant limitation of the experiment was the observed air leakage, particularly at higher pressures. Future experiments should aim to enhance the airtightness of the system to minimize this effect and achieve more accurate results. Improving the precision of measurements could be accomplished by utilizing more accurate equipment, thereby reducing the percentage uncertainty of each measurement (for example, by using a longer syringe with finer scale markings). Additionally, taking more measurements in each part could help decrease random errors, and experimenting with different gases would broaden our understanding of gas behavior.

References:

[1] CPL manual-F23 (BK)_3rd-Draft, Dr. Patrice Donfack.

[2] Errorbooklet_S2023_Version S23 - P01, Dr. Patrice Donfack.