10. Magnetic Gears

Take several identical fidget spinners and attach neodymium magnets to their ends. If you place them side by side on a plane and rotate one of them, the remaining ones start to rotate only due to the magnetic field. Investigate and explain the phenomenon.

Rep: *Nikoloz Burduli*

Plan

- Explanation of the phenomenon
  - *Why does the spinner start spinning;*

- Theoretical Model
  - *Angular Momentum;*
  - *The force of repulsion of magnets;*

- Experimental Part
  - *Experimental setup;*
  - *Distance between the centres of spinners;*
  - *The number of fidgets;*
  - *The number of magnets;;*
  - *Angular velocity;*
  - *Friction;*
  - *Spinner radius;*

- Comparison of theoretical and experimental results
  - *Graphs;*

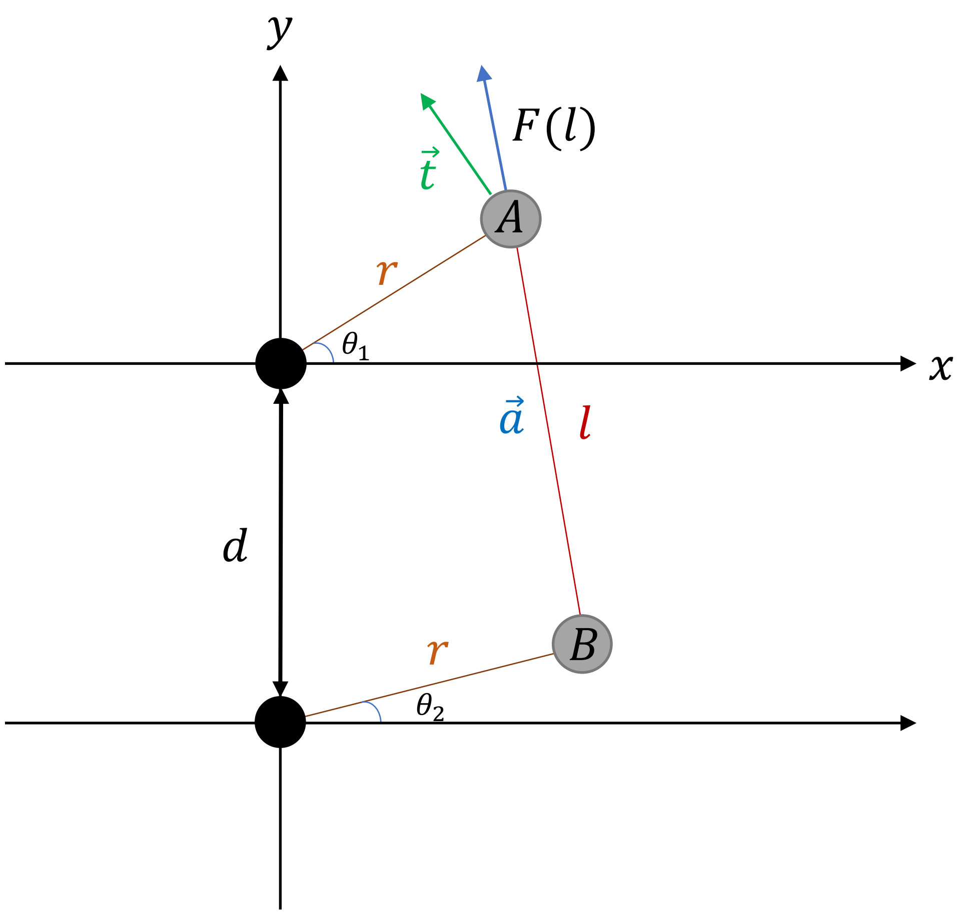
- Conclusion
  - *Important parameters;*
  - *Data analysis;*

Explanation of the phenomenon

F_t
$\vec{F}$
R
d

Theoretical Model

Theoretical Model



$$A_x = rcos\theta_1 \ A_y = rsin\theta_1$$

$$B_x = rcos\theta_2 \ B_y = rsin\theta_2 - d$$

$$a_x = r(cos\theta_1 - cos\theta_2)$$

$$a_y = r(sin\theta_1 - sin\theta_2) + d$$

$$l = \sqrt{{a_x}^2 + {a_y}^2}$$

$$\frac{a_x}{l}, \frac{a_y}{l} \ Unit\ vector$$

$$F_x = \frac{a_x}{l} F(l) \quad F_y = \frac{a_y}{l} F(l)$$

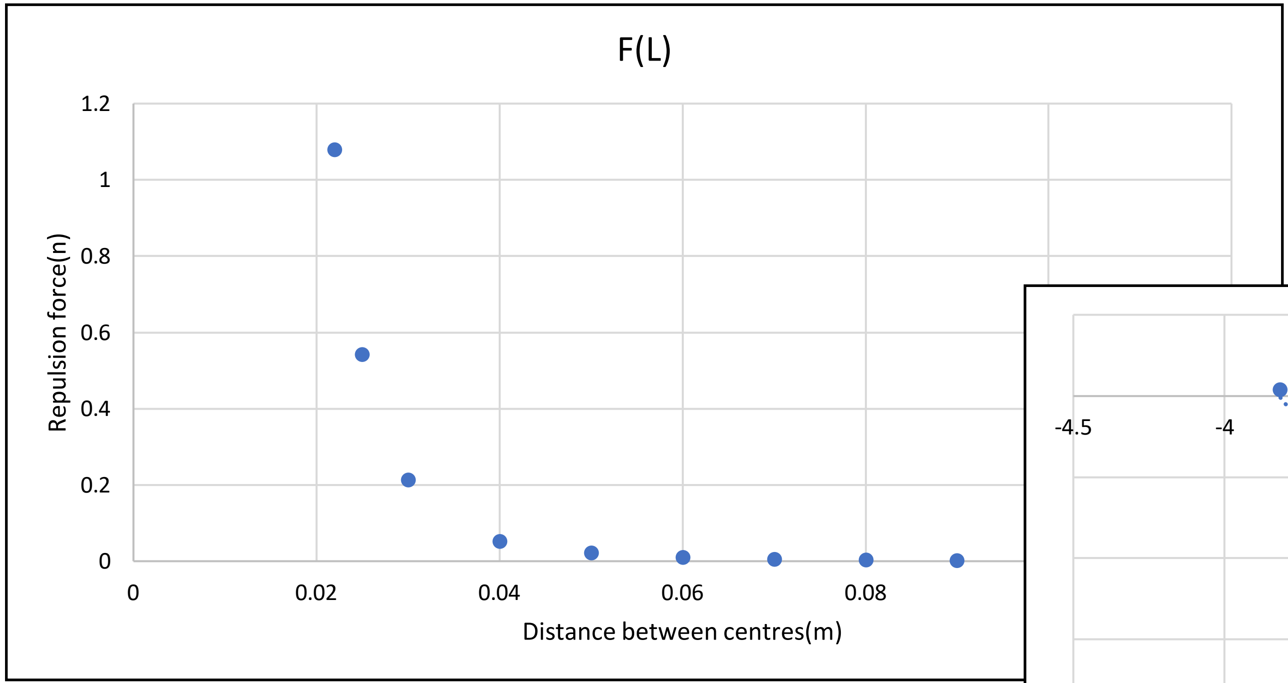
$$\vec{F} = (F_x, F_y)$$

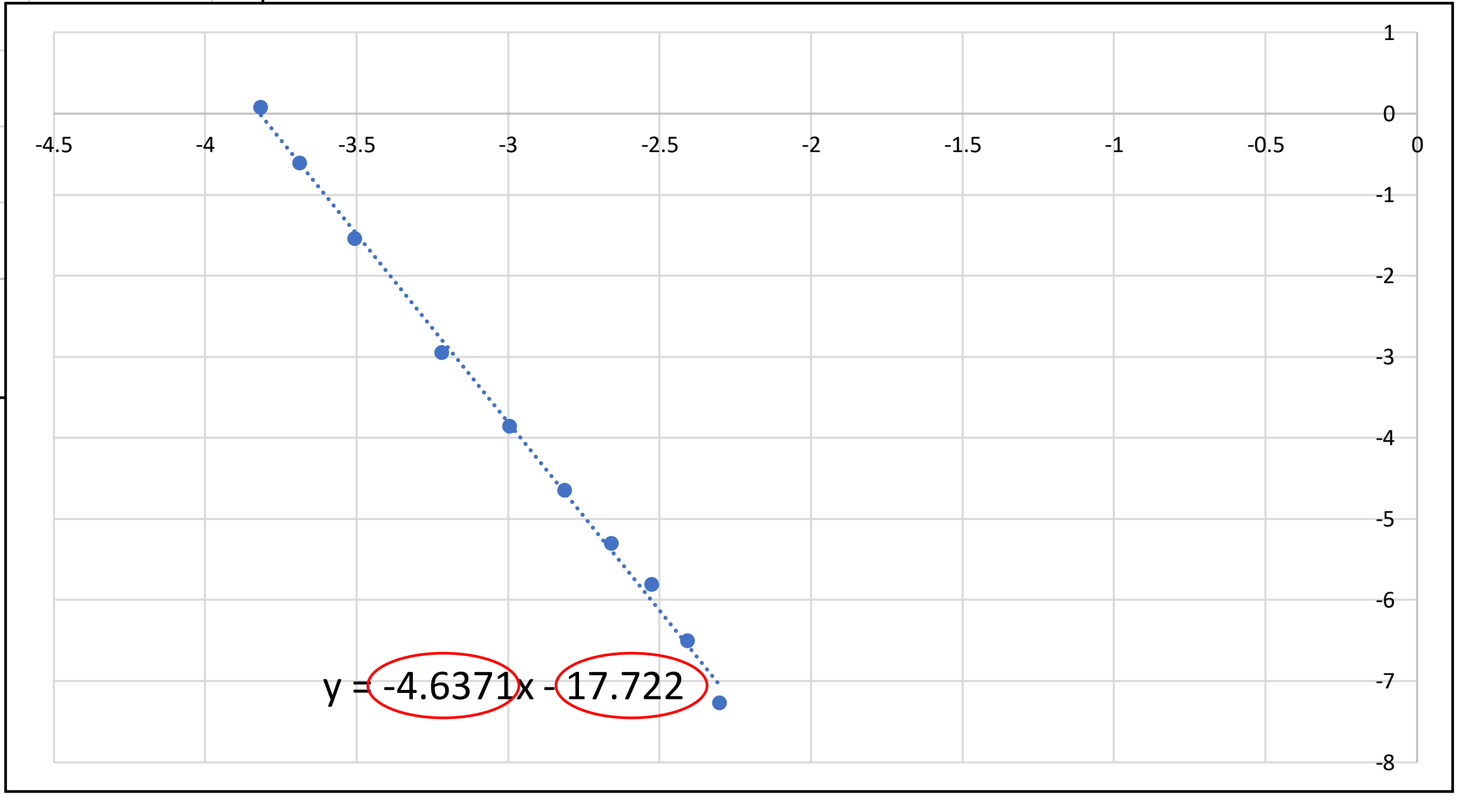
$$\vec{t} = (-sin\theta_1, cos\theta_1)$$

$$\vec{F} \cdot \vec{t} = F_t$$

$$I\varepsilon = I\ddot{\theta} = M = F_t r$$

$F(l) - Experimental\ determination$

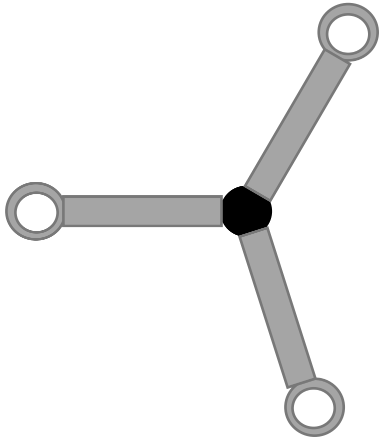


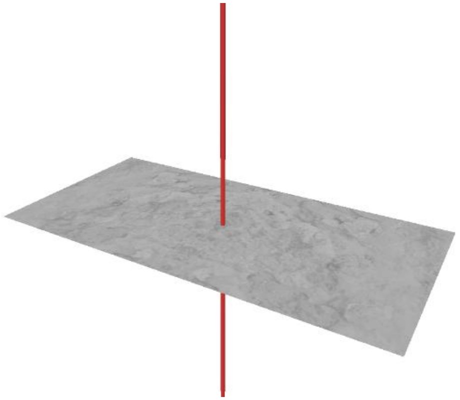


$$F = \frac{c}{l^n}$$

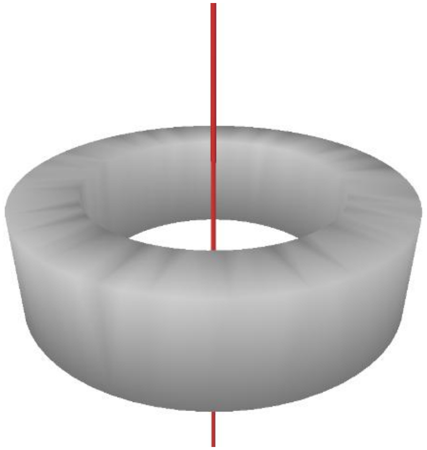
$$lnF = lgc - nlnl$$

Calculating the moment of inertia





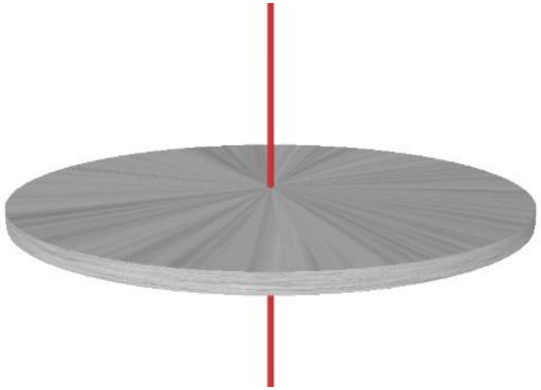
$$I = \frac{1}{12} m(A^2 + B^2)$$



$$I = \frac{1}{2} m(A^2 + B^2)$$

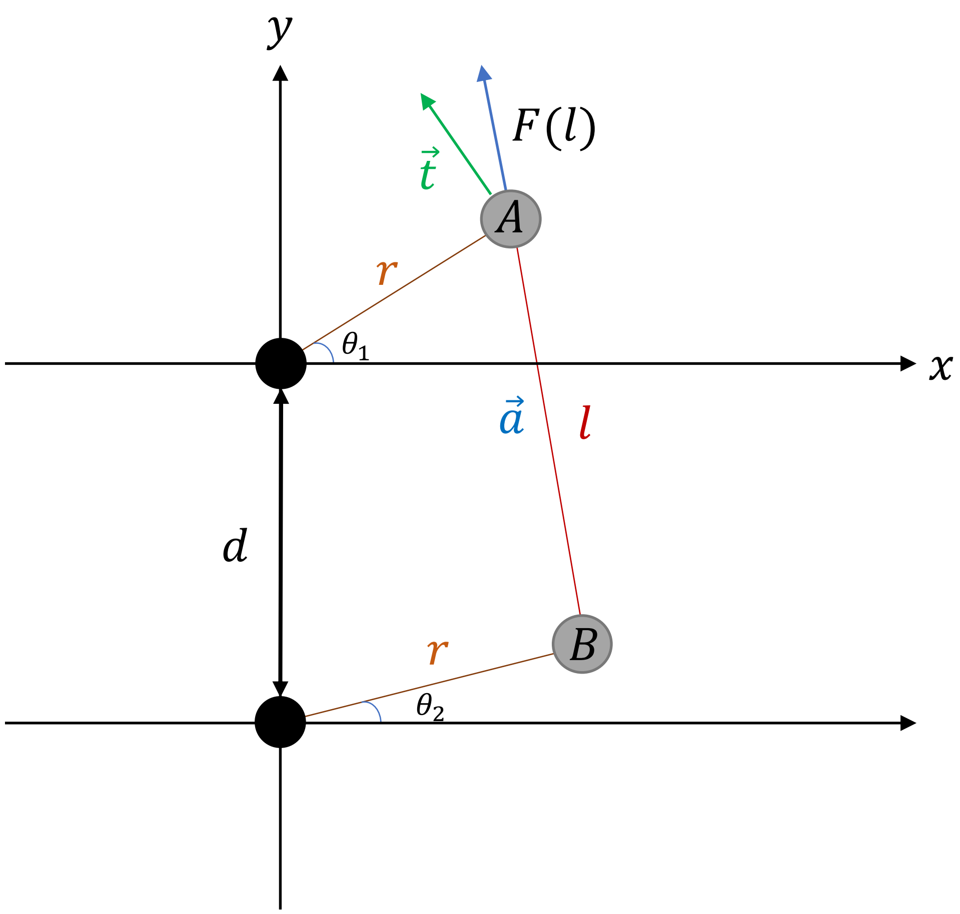
Parallel axis theorem

$$I = I_{cm} + md^2$$



$$I = \frac{1}{2} mR^2$$

Theoretical Model



$$A_x = rcos\theta_1 \; A_y = rsin\theta_1$$

$$B_x = rcos\theta_2 \; B_y = rsin\theta_2 - d$$

$$a_x = r(cos\theta_1 - cos\theta_2)$$

$$a_y = r(sin\theta_1 - sin\theta_2) + d$$

$$l = \sqrt{a_x{}^2 + a_y{}^2}$$

$$\frac{a_x}{l}, \frac{a_y}{l} \; Unit\ vector$$

$$F_x = \frac{a_x}{l}F(l) \quad F_y = \frac{a_y}{l}F(l)$$

$$\vec{F} = (F_x, F_y)$$

$$\vec{t} = (-sin\theta_1, cos\theta_1)$$

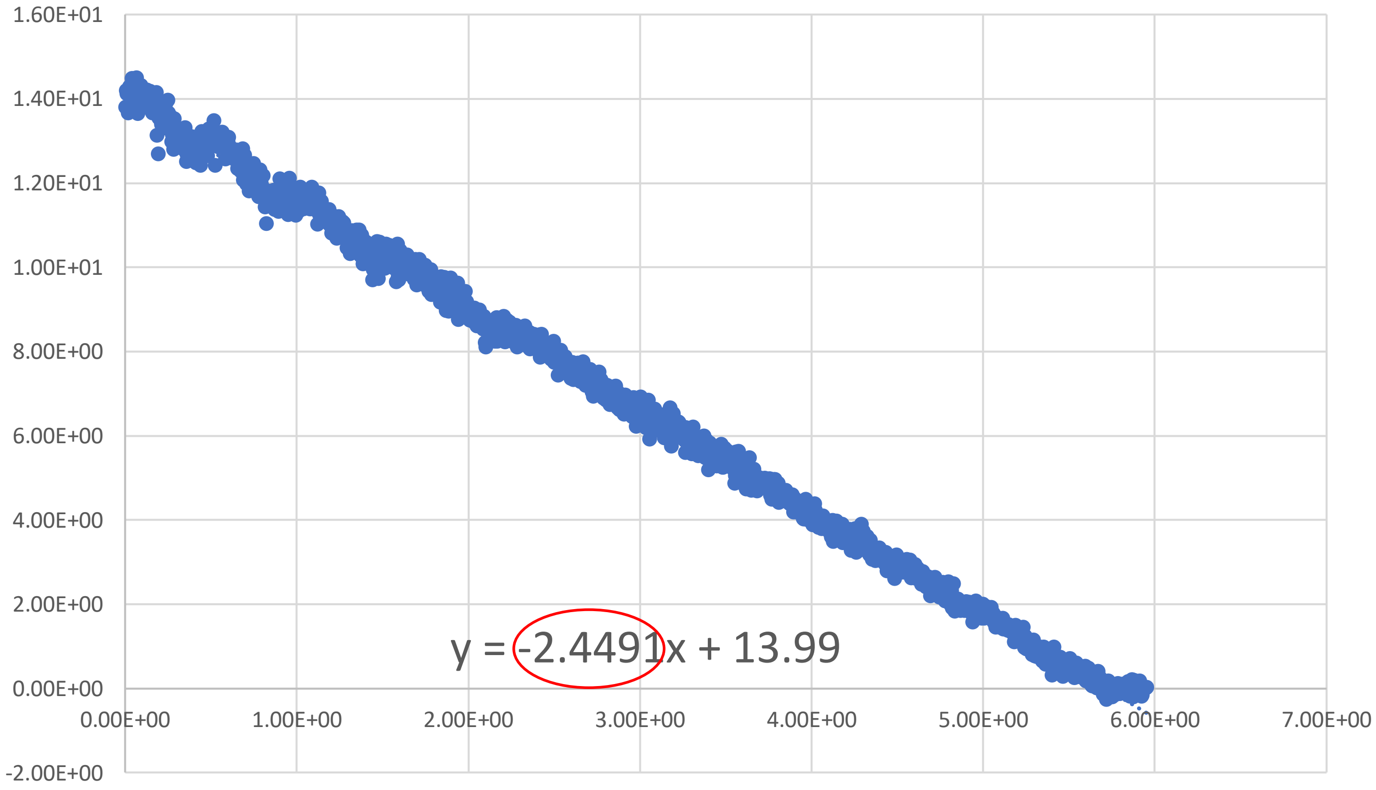
$$\vec{F} \cdot \vec{t} = F_{\text{ტ}}$$

$$I\varepsilon = I\ddot{\theta} = M = F_t r$$

$$\boxed{I\ddot{\theta} = M - M_{Friction}}$$

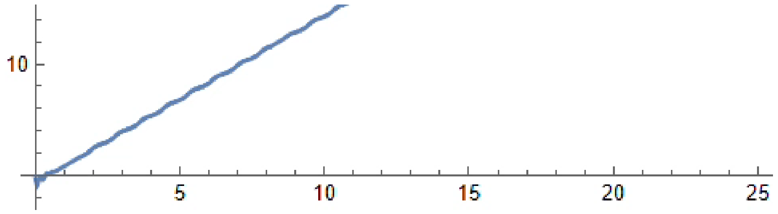
```
In[ ]:= i = 0.000103624;
w = 1.5;
d = 0.145;
r = 0.057;
tmax = 10;
number = 5;
motor = Table[w * t + i * 2 * Pi / number, {i, 0, number - 1}];
free = Table[th[t] + i * 2 * Pi / number, {i, 0, number - 1}];
connectingVector[th1_, th2_] := {r * (Cos[th1] - Cos[th2]), d + r * (Sin[th1] - Sin[th2]), 0};
distance[th1_, th2_] := Norm[connectingVector[th1, th2]];
forceMagnitude[l_] := 4 * Exp[-17.722] / l^4.6371;
forceVector[th1_, th2_] := forceMagnitude[distance[th1, th2]] * connectingVector[th1, th2] / distance[th1, th2];
torque[th1_, th2_] := Cross[{r * Cos[th1], r * Sin[th1], 0}, forceVector[th1, th2]][[3]];
M = Sum[torque[free[[n]], motor[[k]]], {n, 1, number}, {k, 1, number}];
sol = NDSolve[{th''[t] == M / i - 2.5 * Sign[th'[t]], th[0] == 0.3, th'[0] == 0.0}, th, {t, 0, tmax}, PrecisionGoal → 10, AccuracyGoal → 10];
Plot[-th[t] /. sol, {t, 0, tmax}, PlotRange → All]
Animate[
 Show[Join[Table[Graphics[Point[{r * Cos[free[[i]]], r * Sin[free[[i]]]}]], {i, 1, number}] /. sol[[1]],
    Table[Graphics[Point[{r * Cos[motor[[i]]], -d + r * Sin[motor[[i]]]}]], {i, 1, number}]], PlotRange → {{-1.1 * r, 1.1 * r}, {-4 * r, 1.1 * r}}] /. {t → k},
 {k, 0, tmax}, AnimationRate → 0.1]
```

Friction member



Theoretical model | experiment | conclusion

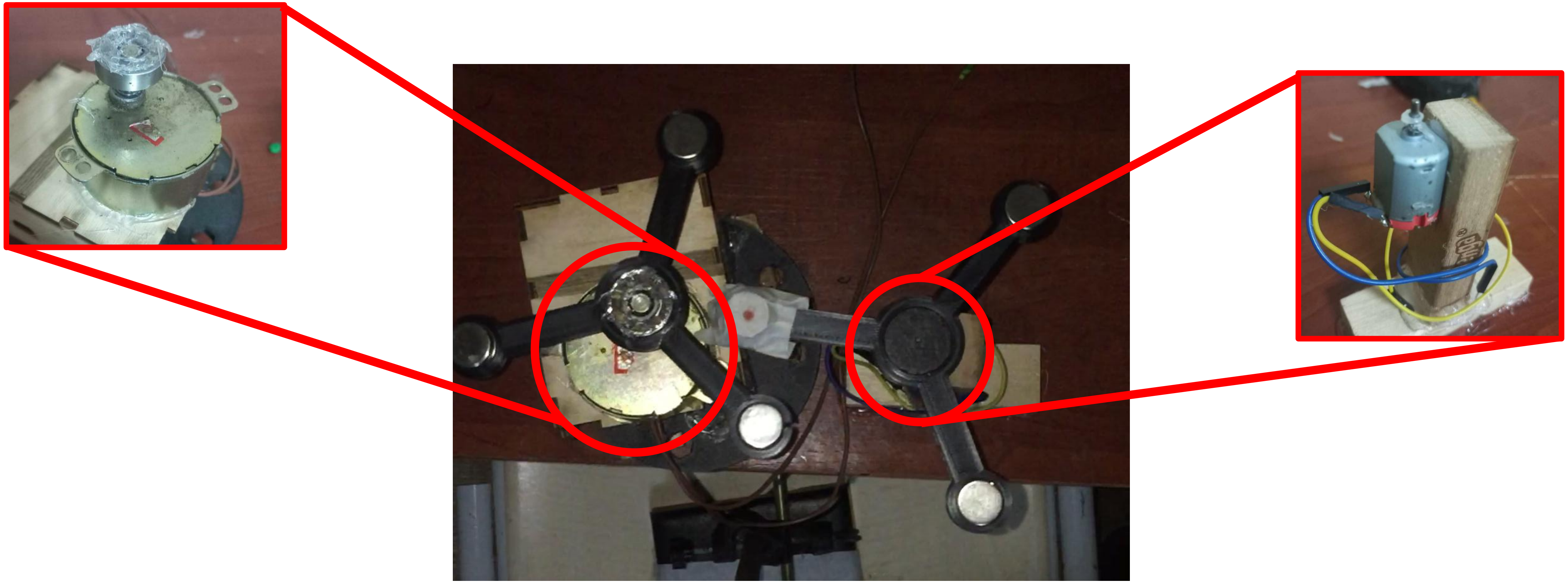
Simulation



Out[1287]=

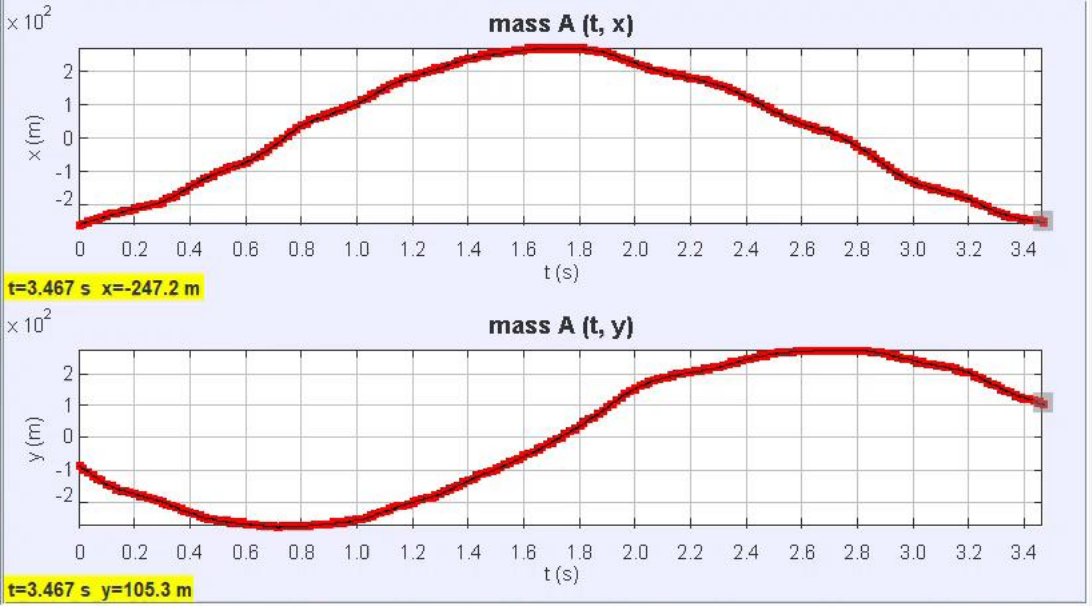
Experimental Part

Experimental Setup



mass A m 1.000 kg

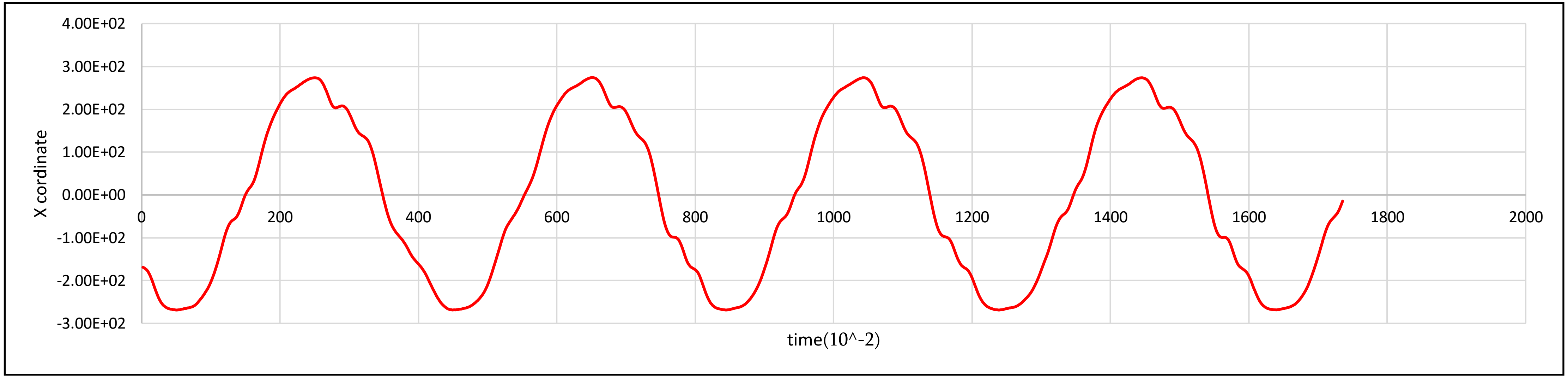
Plots mass A Sync

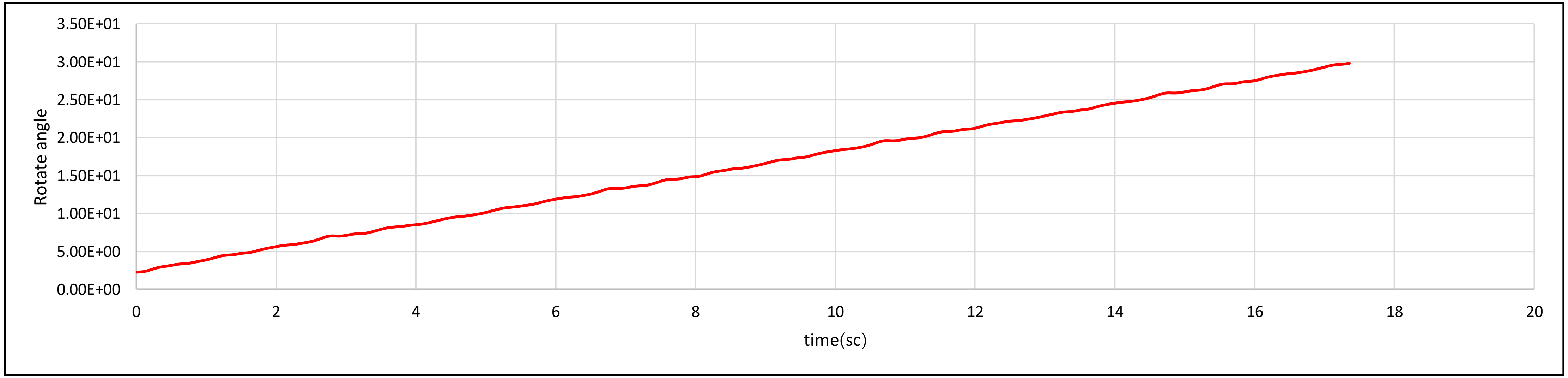


Columns mass A

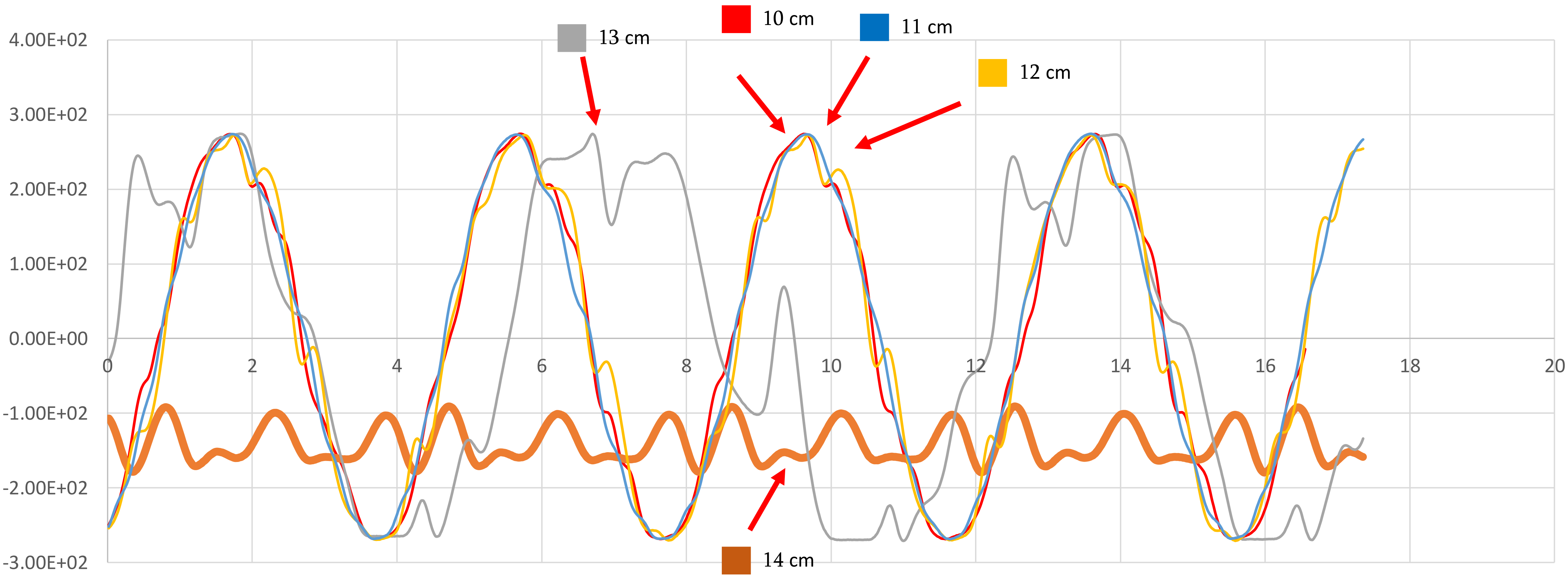
t (s)	x (m)	y (m)	θ	ω (°/s)
3.233	-192.5	188.5	135.6°	114.3
3.250	-198.7	181.8	137.5°	121.3
3.267	-205.0	174.2	139.6°	123.5
3.283	-211.1	166.9	141.7°	121.5
3.300	-216.7	159.2	143.7°	119.5
3.317	-221.8	151.6	145.6°	111.2
3.333	-226.4	144.8	147.4°	101.6
3.350	-230.7	138.4	149.0°	91.6
3.367	-233.8	132.5	150.4°	81.6
3.383	-236.6	127.1	151.7°	71.5
3.400	-239.1	122.7	152.8°	65.2
3.417	-241.6	118.2	153.9°	64.2
3.433	-243.7	113.8	155.0°	60.9

Experimental results



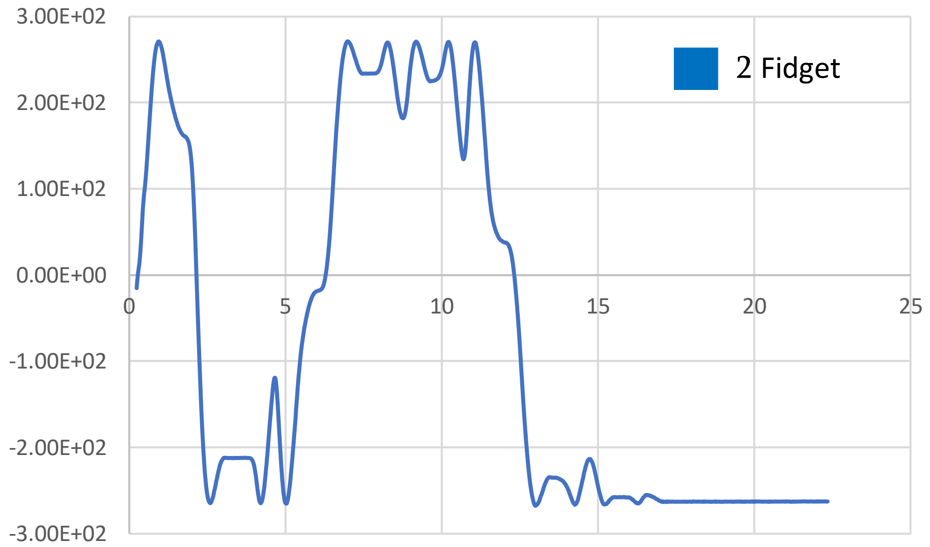


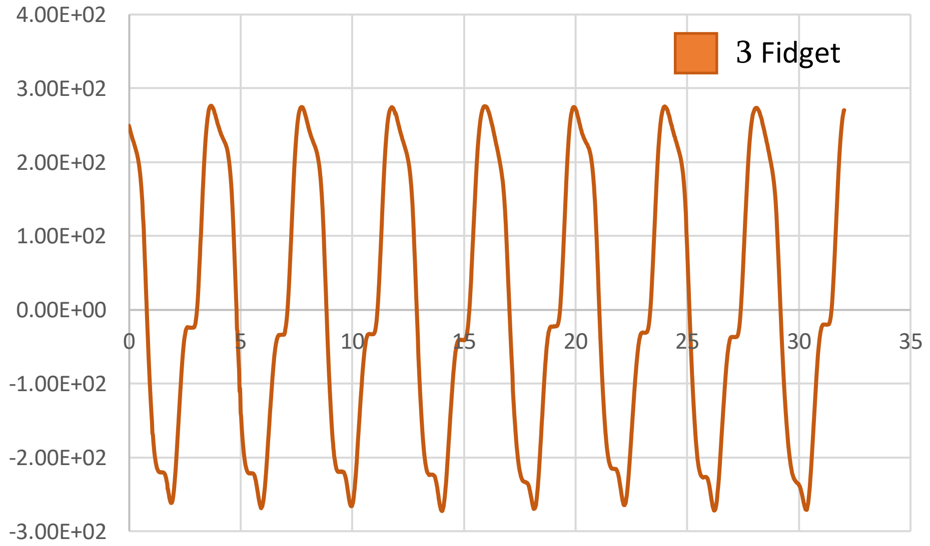
Distance between centres of spinners

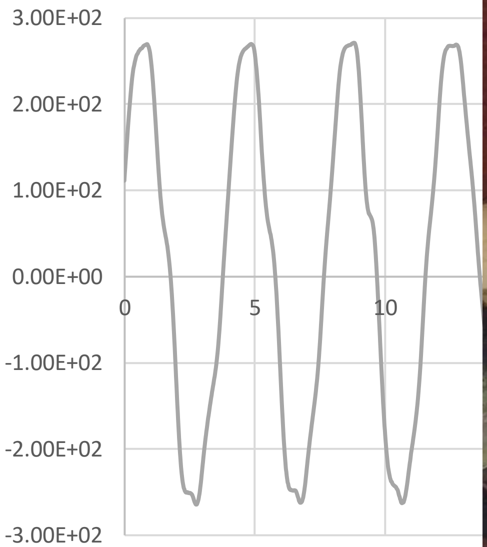


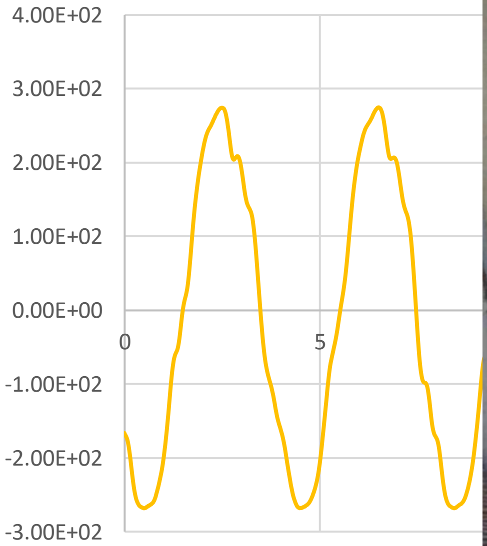
Distance – 10 cm

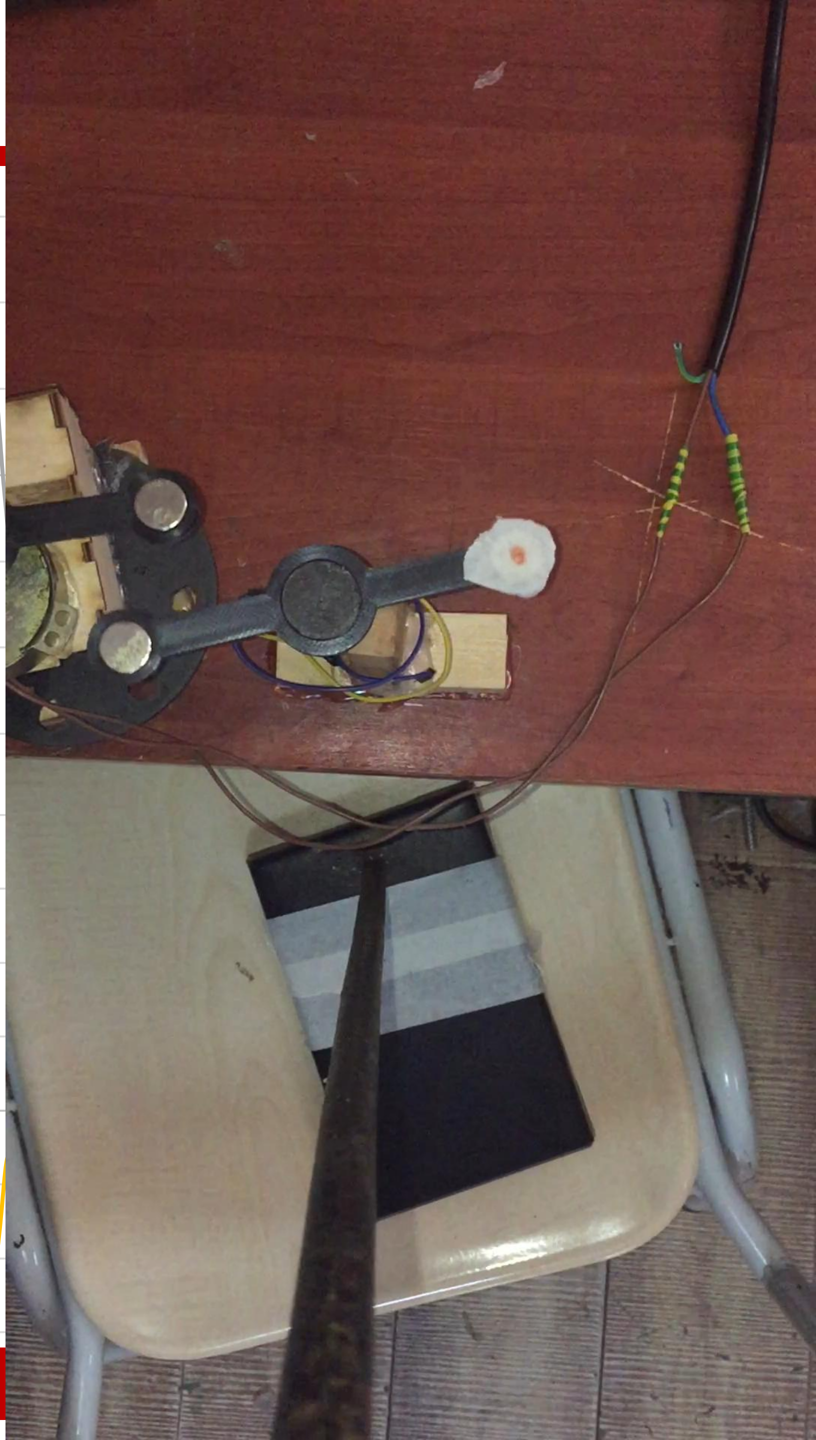
Fidgets number





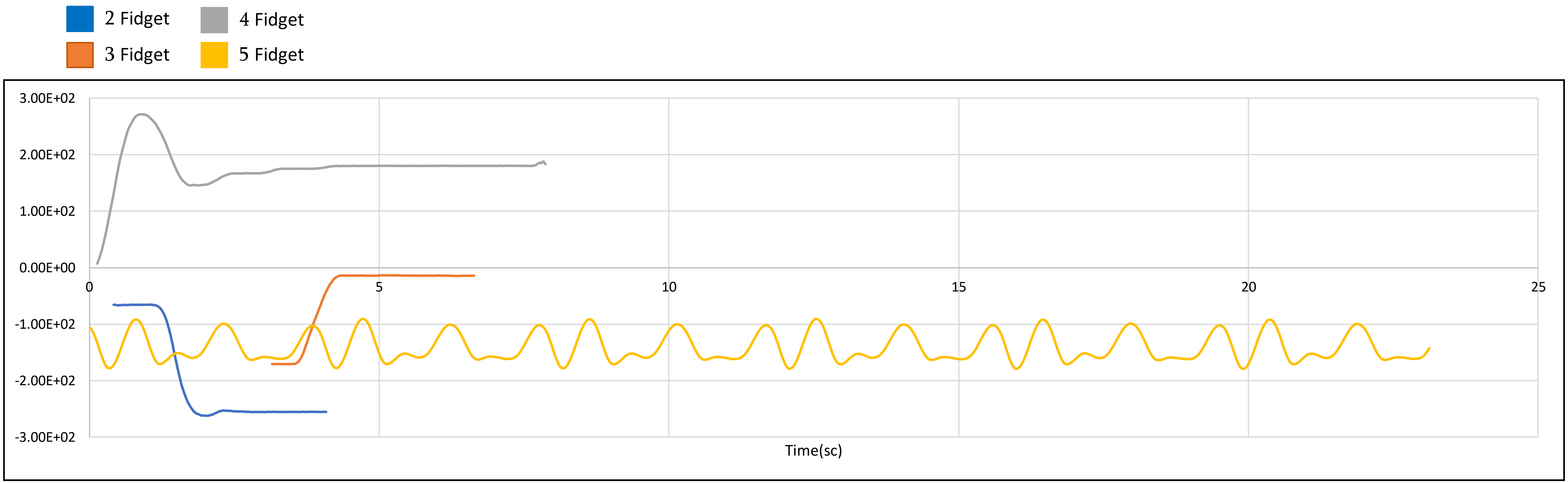






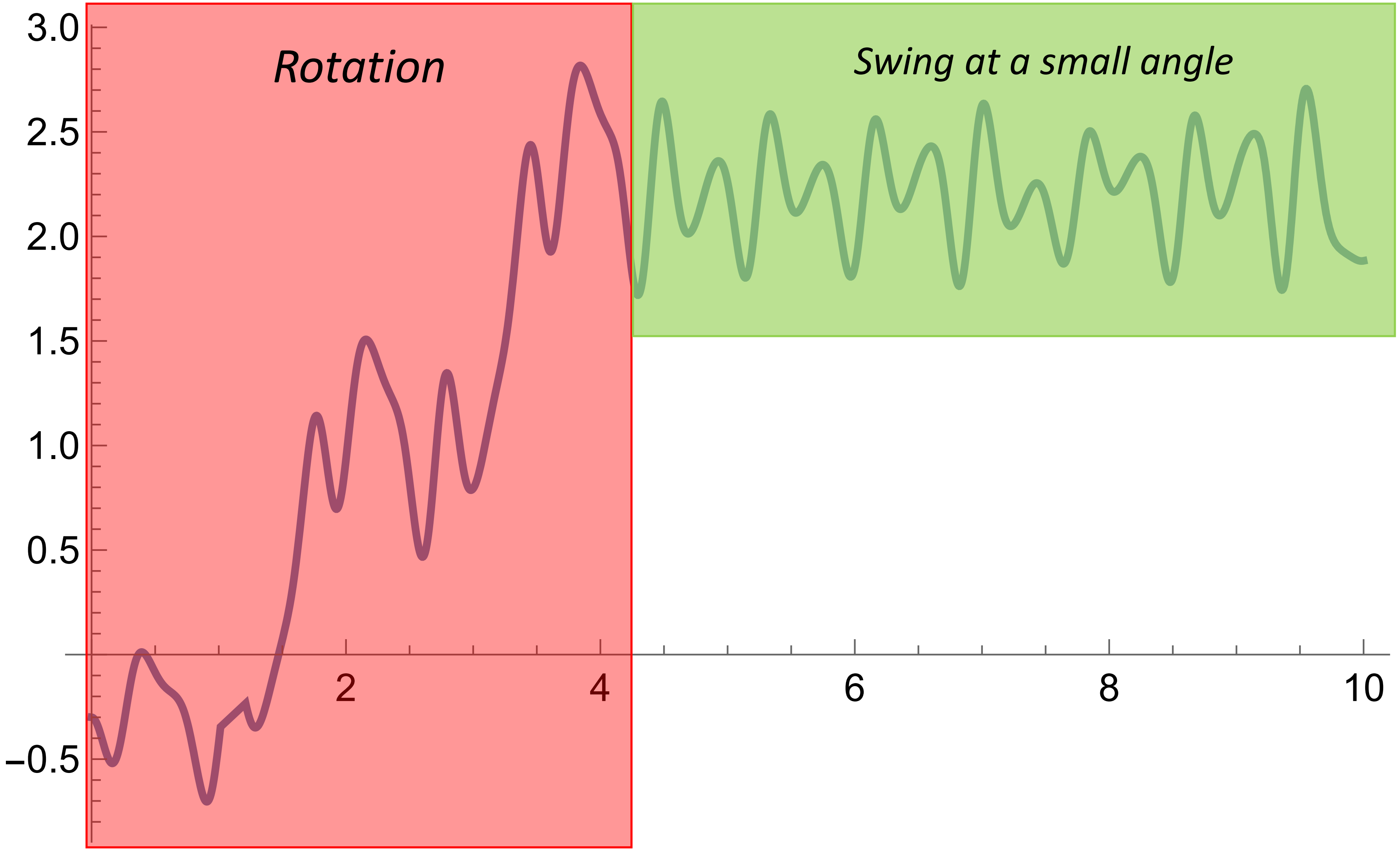
phenomenon | Theoretical model | experiment

Fidgets Number



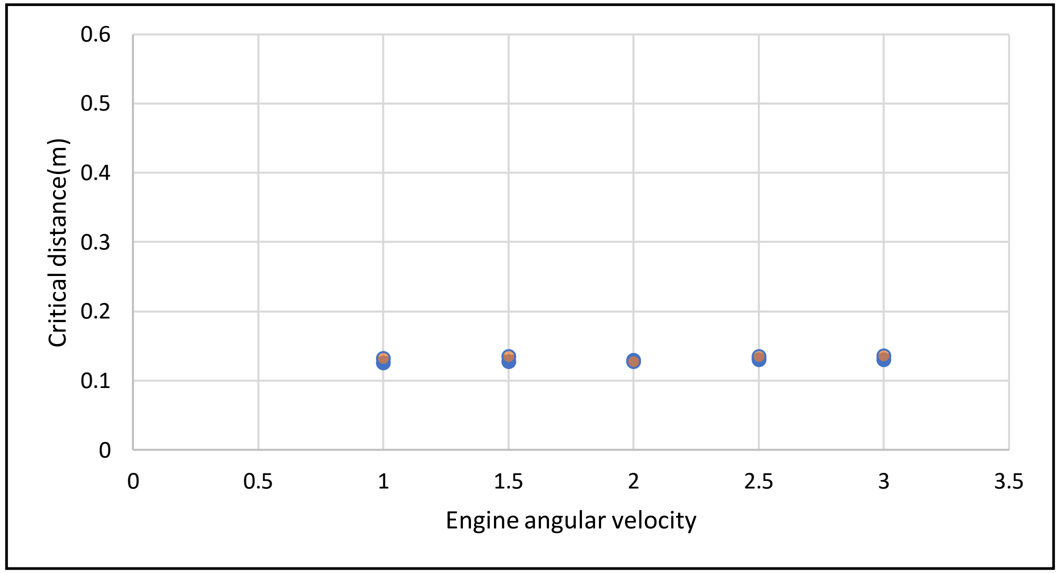
Distance – 14cm

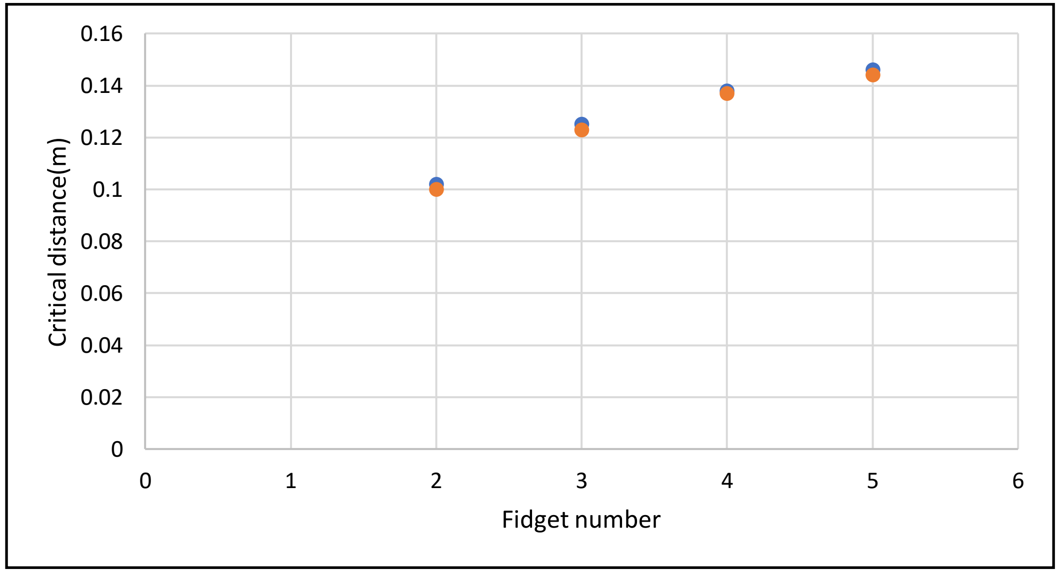
Critical Point

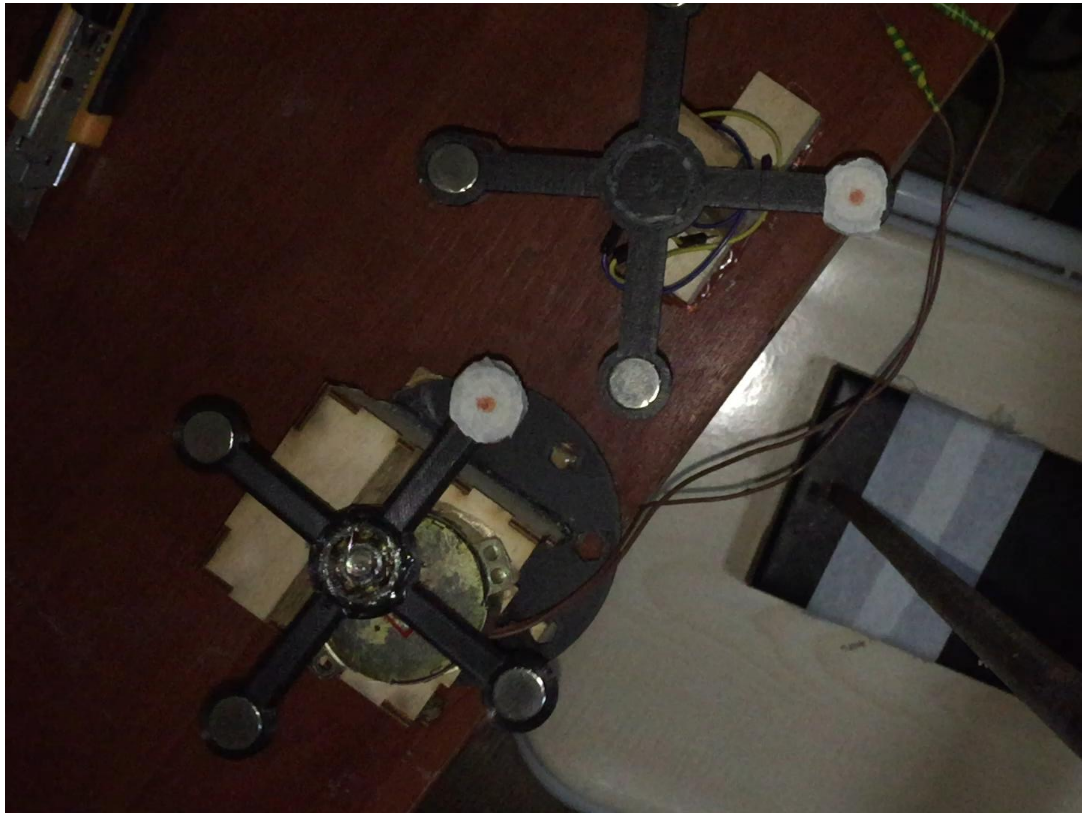


phenomenon | Theoretical model | **experiment** | conclusion

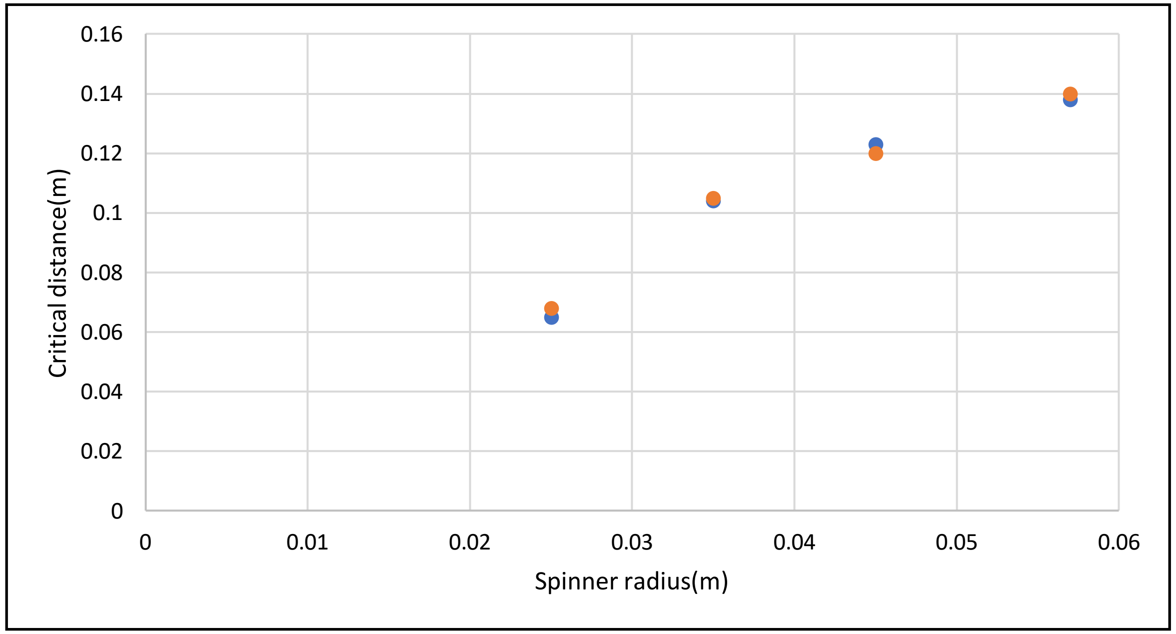
Critical Point

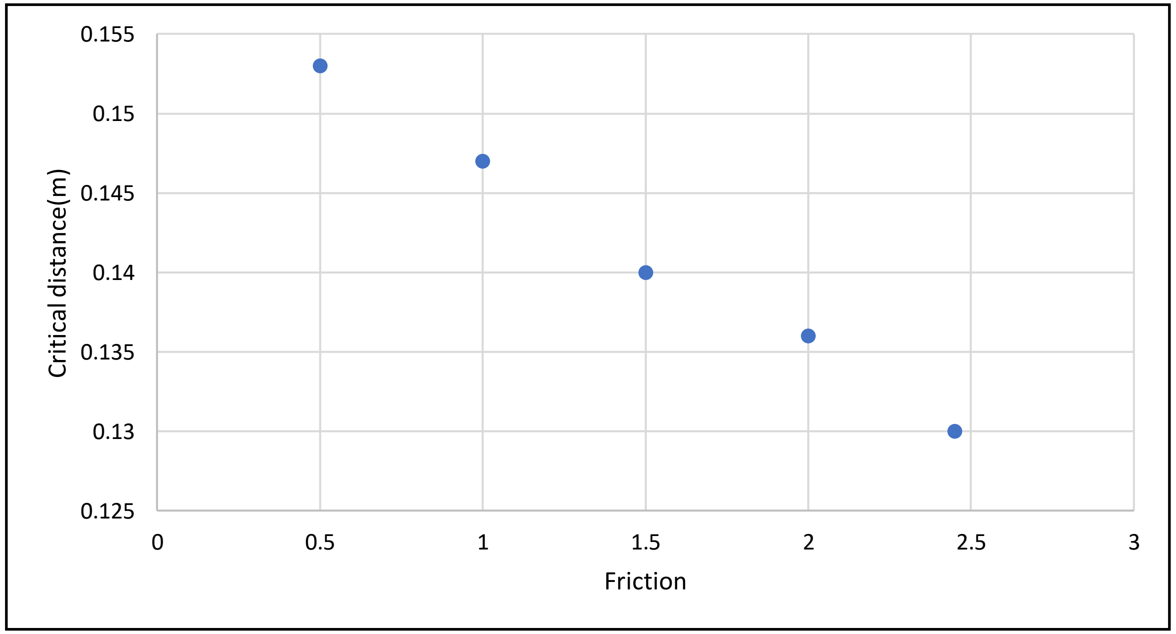






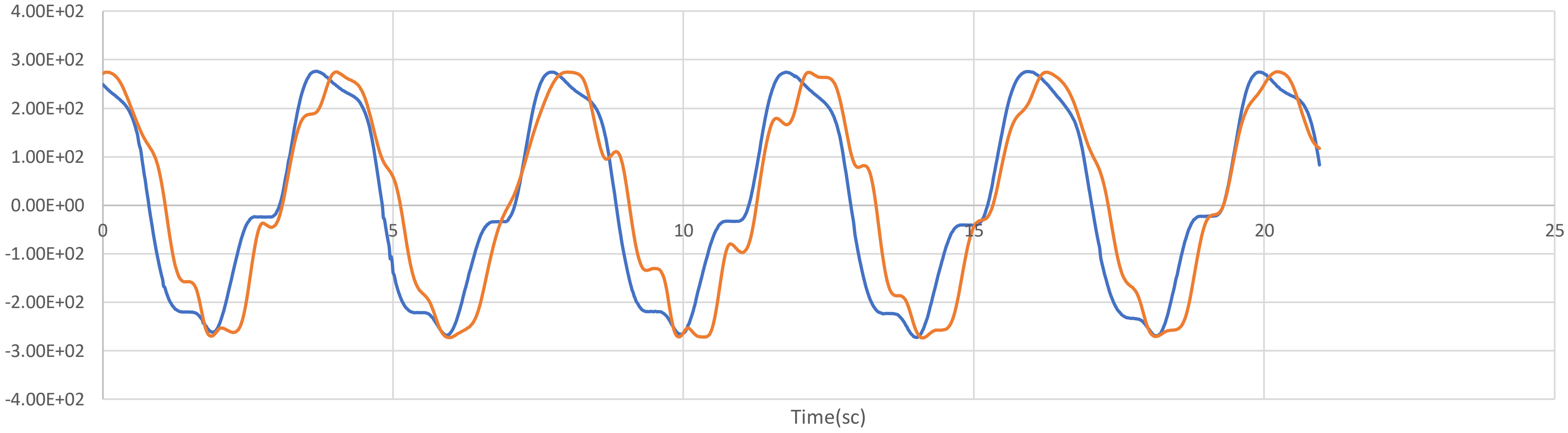
Critical Distance





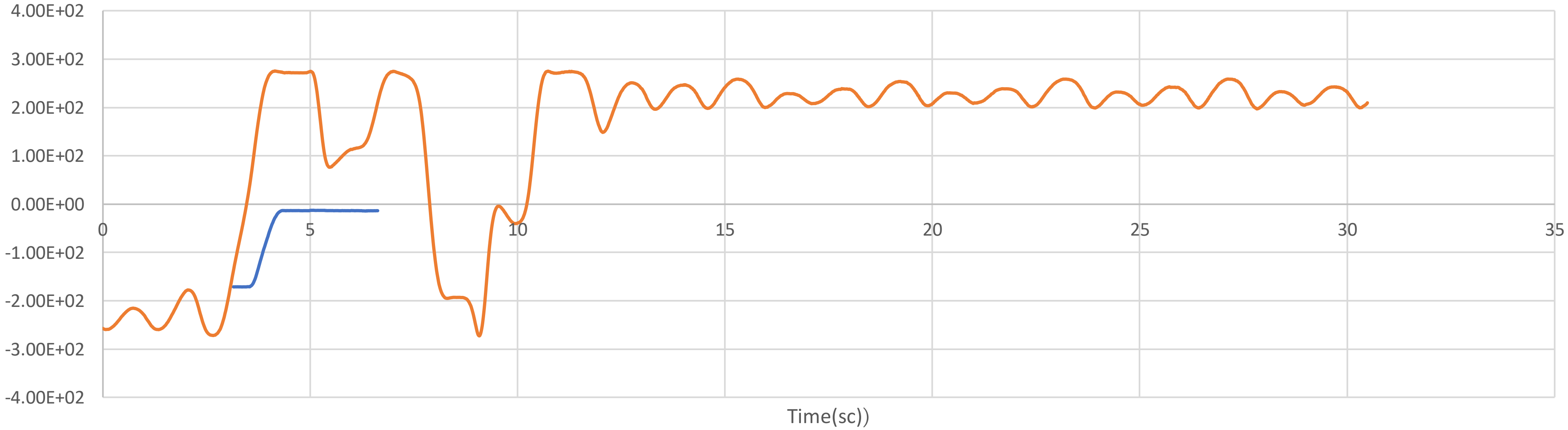
Distance – 10cm

Magnets Number



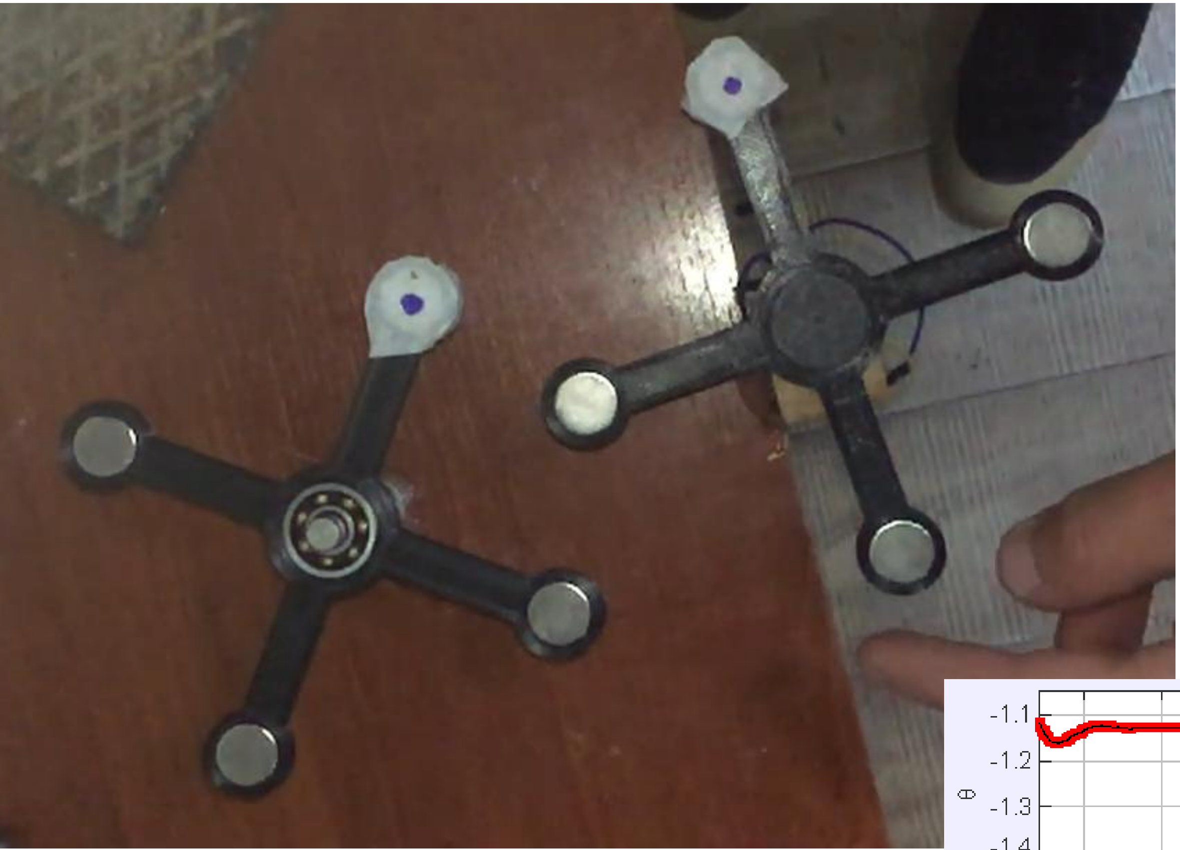
1 magnet

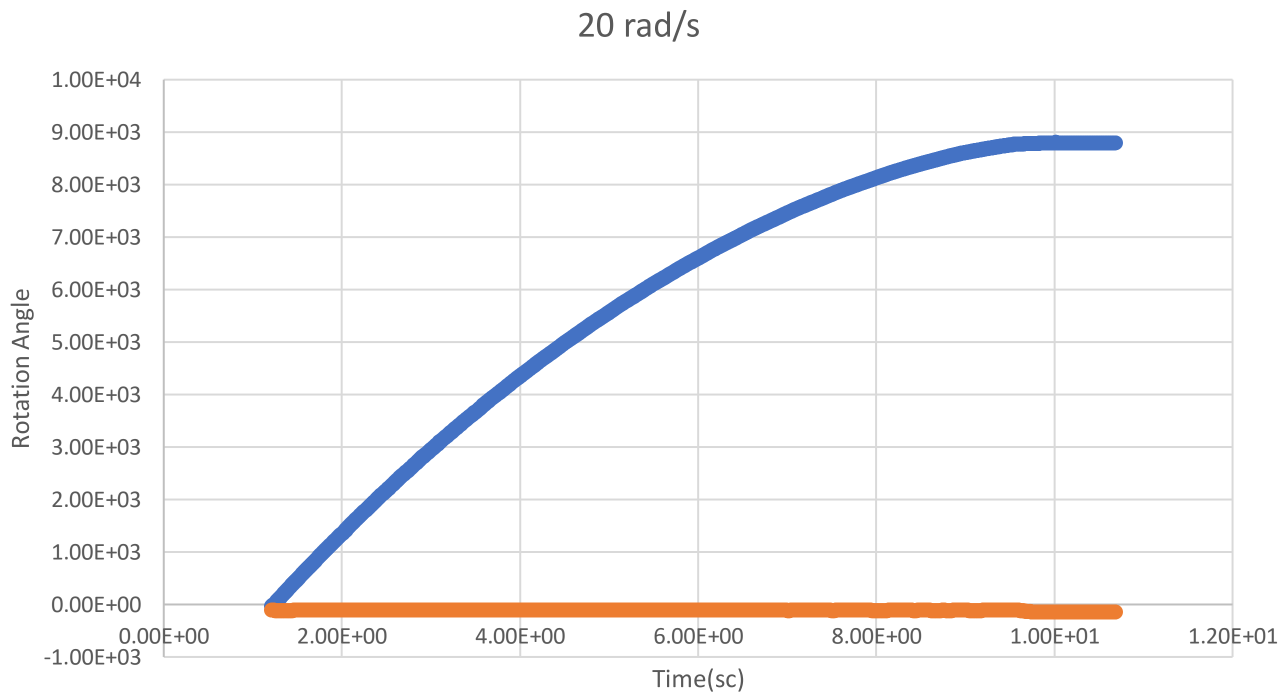
2 magnet

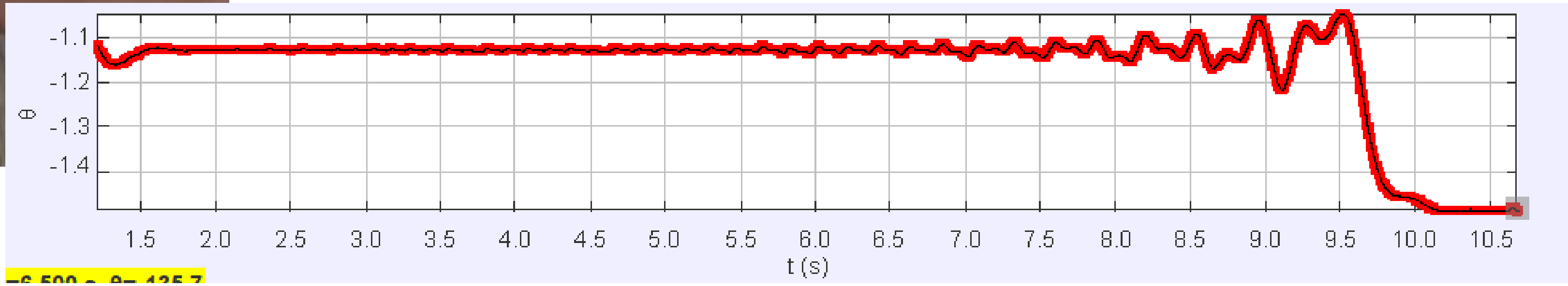


Distance – 14cm

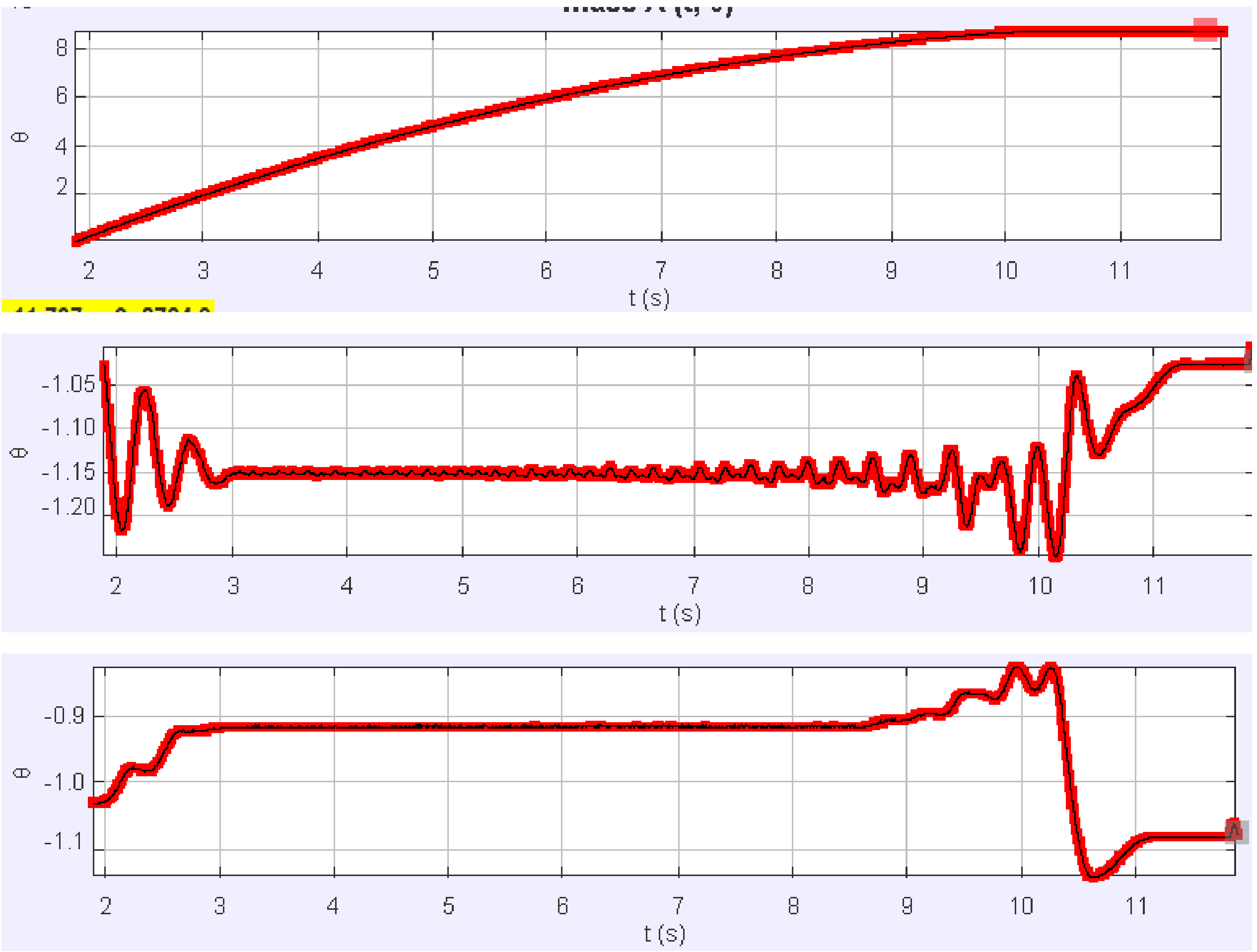
Without Motor





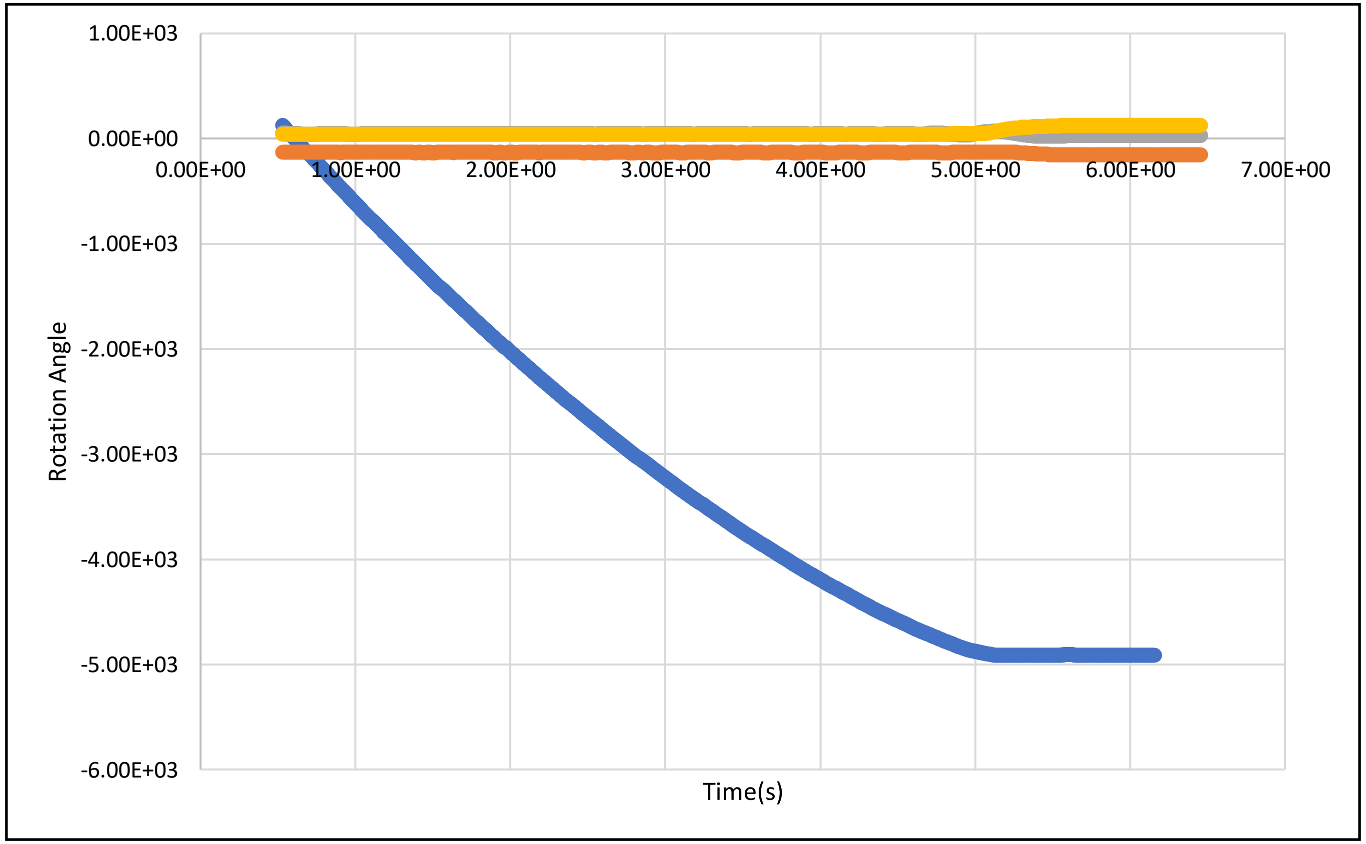


Number of Spinners



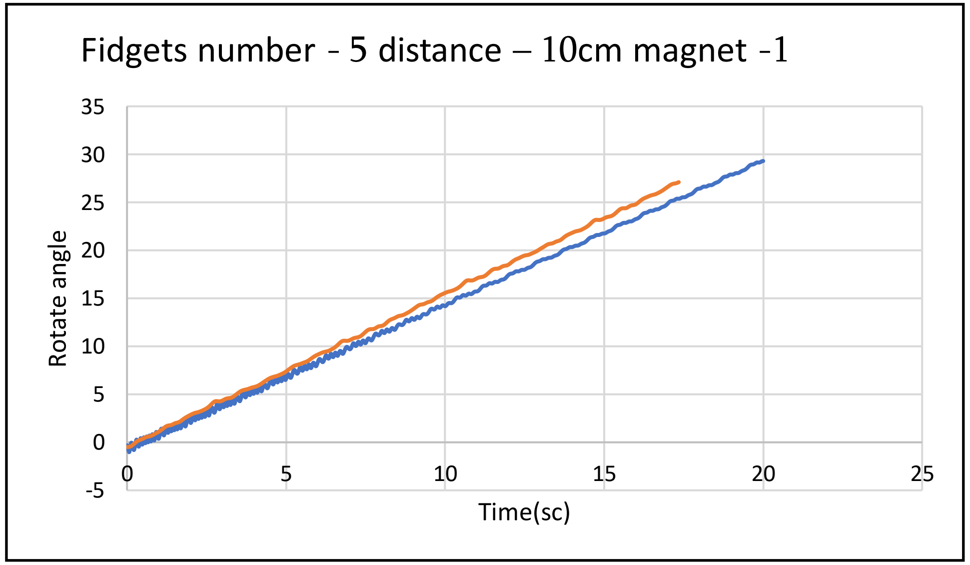
Theoretical model | **experiment** | conclusion

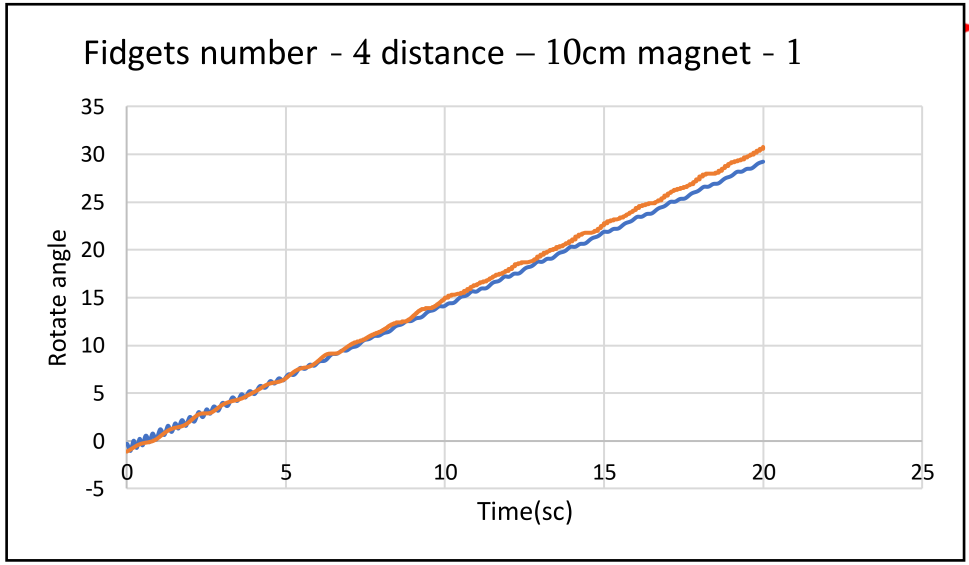
Number of Spinners

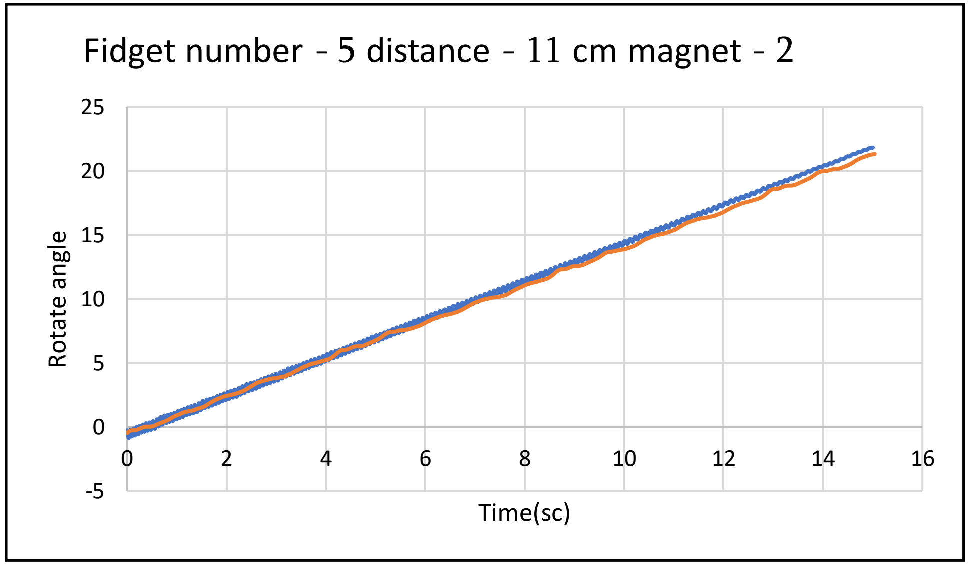


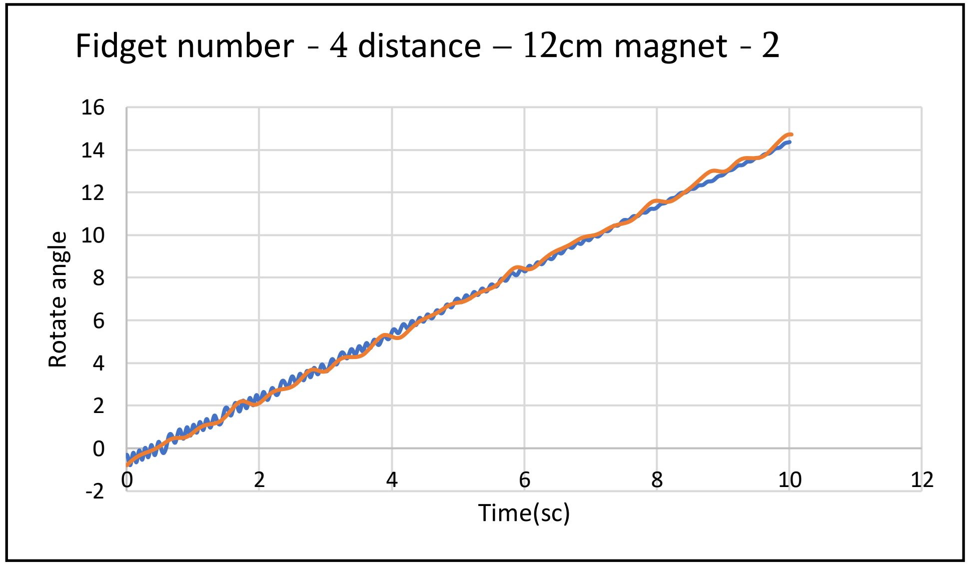
Conclusion

Theoretical and experimental graphs





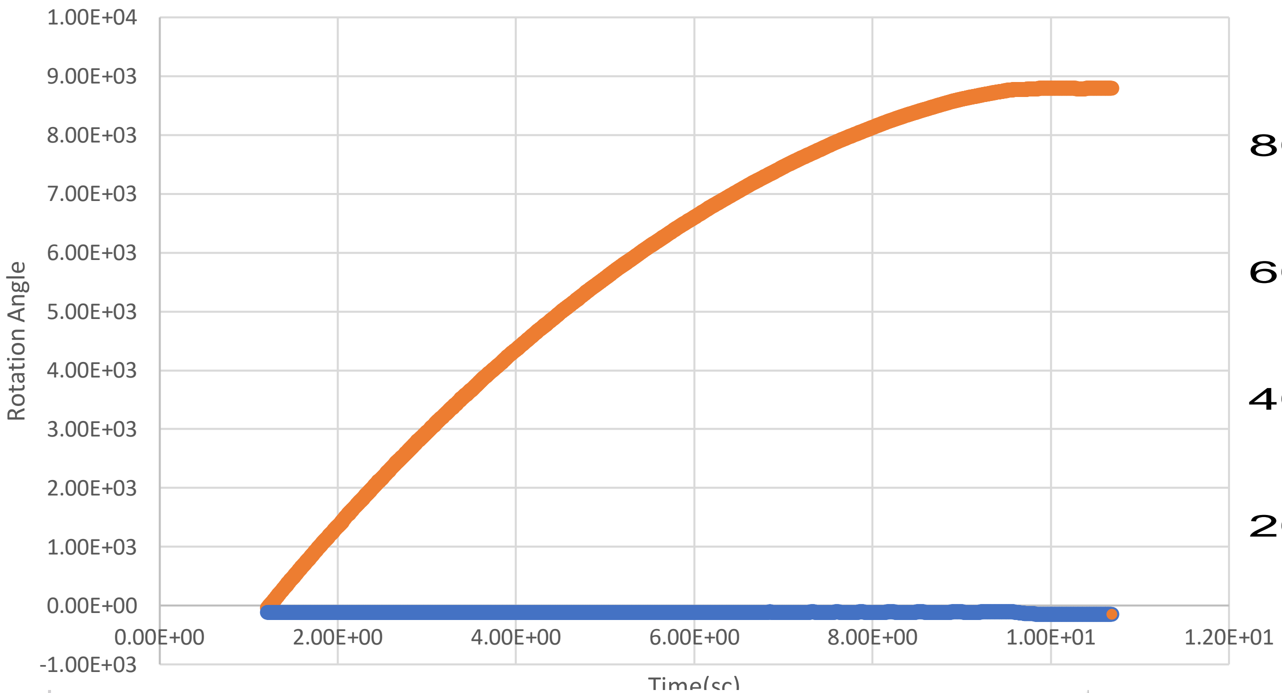


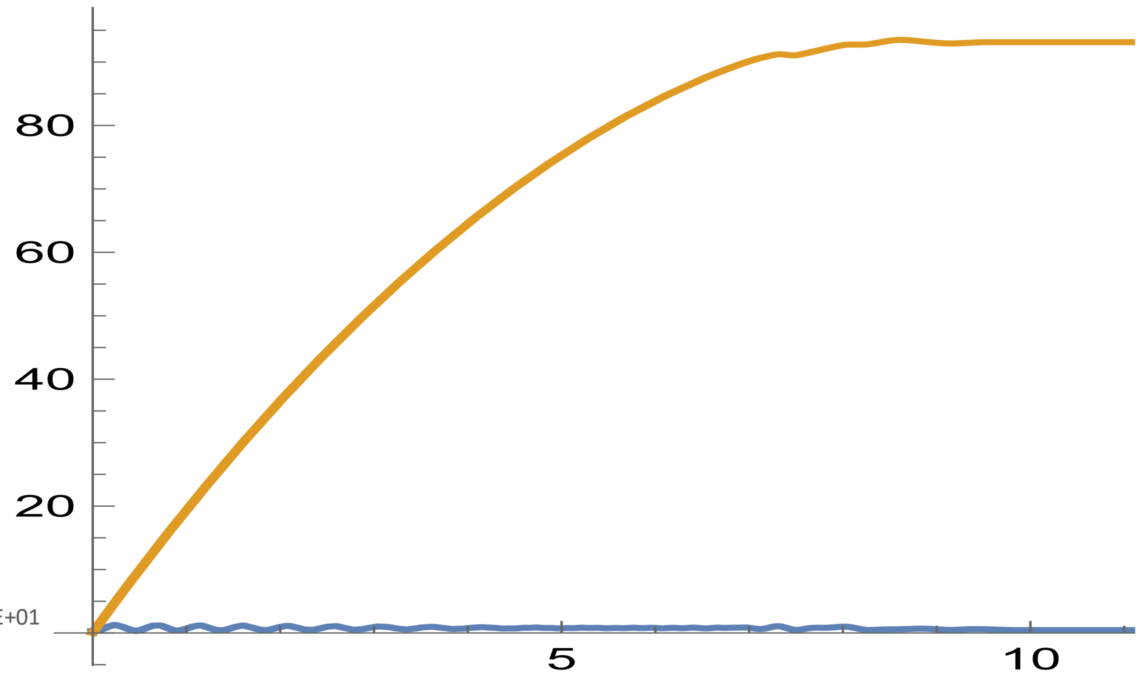


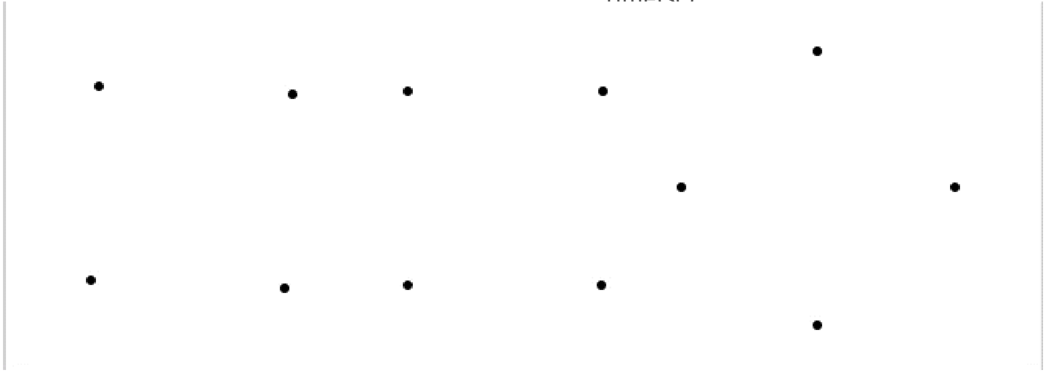
phenomenon | Theoretical model | experiment | conclusion

Theoretical and experimental graphs

Without Engine





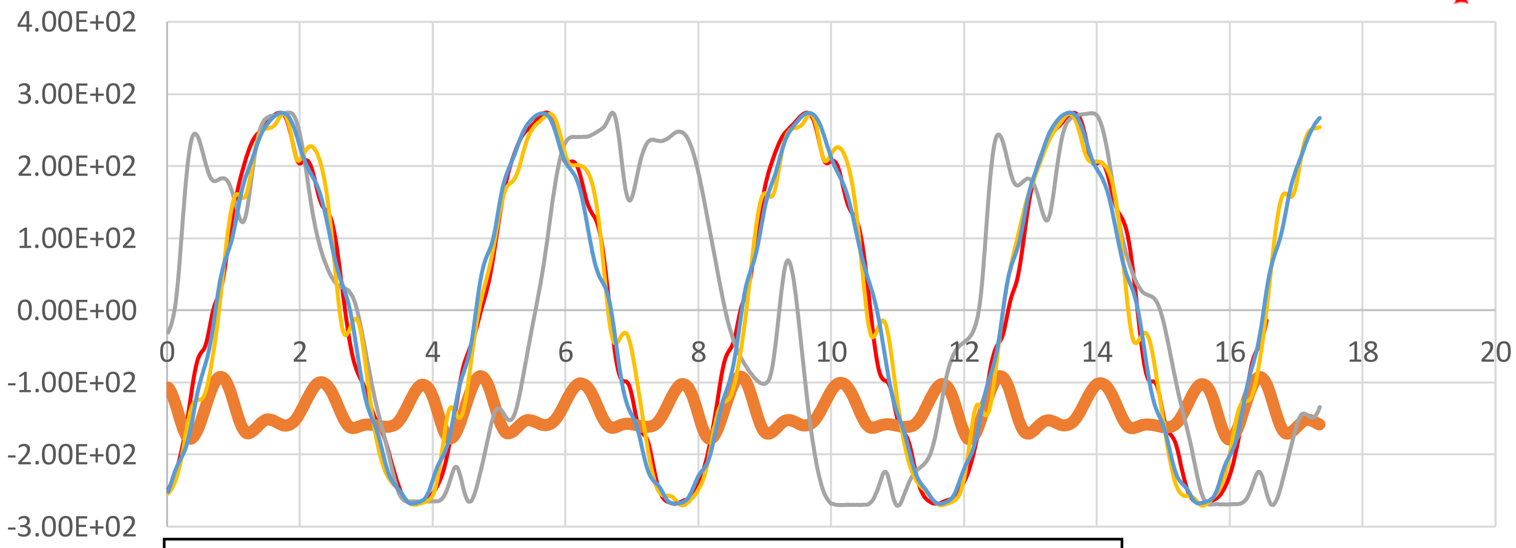


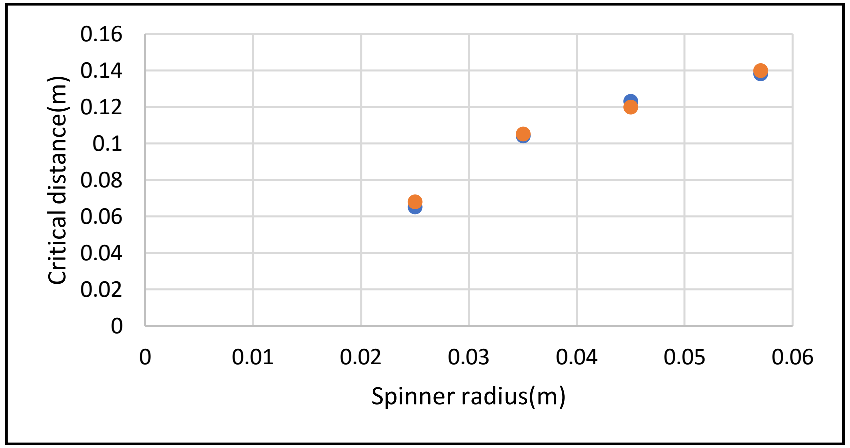
12.5 (rad/s)

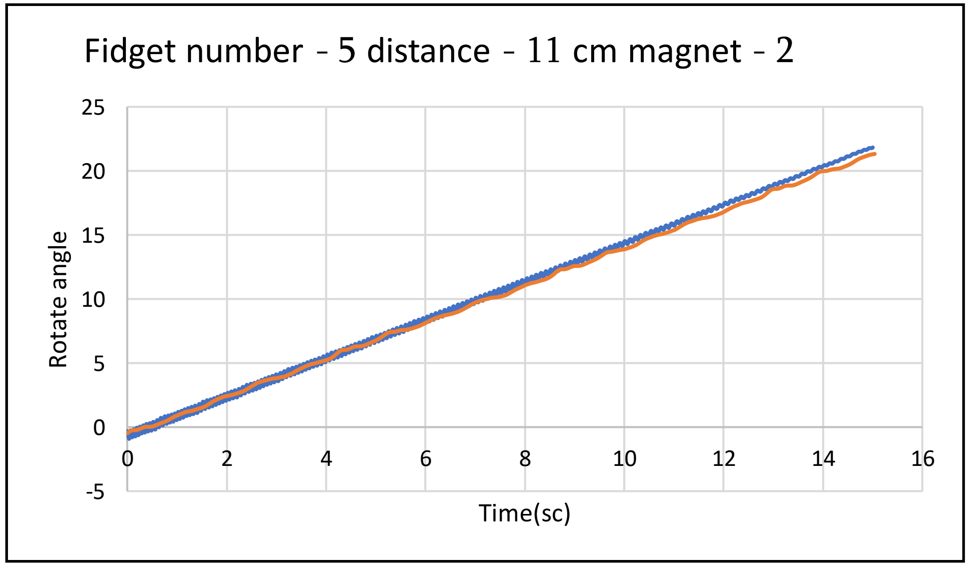
phenomenon | Theoretical model | experiment | conclusion

Important Parameters

- Distance between spinners;
- Fidgets Number;
- Magnets Number;
- Moment of Inertia;
- Angular velocity;
- Radius of the spinner;







$$\boxed{I\ddot{\theta} = M - M_{Friction}}$$

Thanks for your attention!

Whithout Engine