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Algorithms Lab

Exercise – *Return of the Jedi*

During the Star Wars, the Rebels showed an incredible amount of courage and diligence, but they lacked an efficient organization of their intelligence. Recently the Jedi developed a technology to build a direct information transmission channel between two planets. Such a channel can then be used to transmit information between the two planets in both directions. Using this technology Luke Skywalker wants to construct a network that allows to transmit—either directly or indirectly via other planets—information from every planet in the galaxy to every other planet. A network with this property we call a *warp network*. For a particular pair of planets the cost of building a direct channel between them depends on several parameters, such as distance, the number of army units of the Galactic Empire between the planets, cosmic dust, and so on. As the rebels are short on resources to spend, Luke asks a fellow rebel, Princess Leia, to come up with a cost effective warp network.

Princess Leia proposes the following method: enumerate all planets with numbers $1, \ldots, n$, sorted by increasing importance. (We assume that no two planets have the same importance.) In this way each planet receives a unique number. The method proceeds in n steps. In the first step, the network consists of the planet Tatooine only (the home planet of Luke and Darth Vader). Then repeat the following process $n - 1$ times: Pick a planet that is not yet part of the network and has the cheapest possible channel connecting to one of the planets already in the network. Build this channel and include the chosen planet into the network. All ties are broken in favor of the more important planet.[1] In each step the set of planets in the network increases by one, so that after n steps all planets will be included in the network.

Everybody likes this method, except for R2-D2. Although a droid, R2-D2 is very cautious. He thinks that the approach is too obvious and so Emperor Palpatine will expect Luke to build the network in exactly this way. Knowing their plan he would be able to infiltrate the network and eventually take it over. In order to prevent this from happening, R2-D2 wants to calculate the costs for a cheapest possible warp network that is *different* from the network created by Princess Leia's method, that is, there is at least one channel that is in one network but not in the other. The *cost* of a network is the sum of the costs of all constructed transmission channels. Compute this value and help the rebels to win the war.

Input The first line of the input contains the number $t \leq 15$ of test cases. Each of the t test cases is described as follows.

- It starts with a line that contains two integers `n i`, separated by a space and such that $3 \leq n \leq 10^3$ and $1 \leq i \leq n$. Here n denotes the number of planets and i denotes the index of Tatooine.

[1] If two or more planets are eligible to be added to the network because the cost of doing so is the same, then select the most important planet among them. If the selected planet has two or more channels of lowest cost that would connect it to the existing network, build the channel among those that leads to the most important planet.

- The following $n-1$ lines describe the costs for building transmission channels. The j-th line, for $j \in \{1, \ldots, n-1\}$, contains $n-j$ integer numbers between 1 and 2^{20}, where the k-th number, for $k \in \{1, \ldots, n-j\}$, denotes the cost for building a transmission channel between planets j and $j+k$.

Output For each test case output a line with one integer that is the minimum cost of a warp network that is different from the network constructed by Princess Leia's method.

Points There are three groups of test sets, worth 100 points in total.

1. For the first group of test sets, worth 40 points, you may assume that $n \leq 100$.
2. For the second group of test sets, worth 40 points, you may assume that the network created by Princess Leia's method is a star (that is, there is one planet that has channels to all other planets).
3. For the third group of test sets, worth 20 points, there are no additional assumptions.

Corresponding sample test sets are contained in `test`i`.in/out`, for $i \in \{1, 2, 3\}$.

Sample Input

```
3
3 1
1 1
1
3 2
1 2
1
4 3
2 1 1
2 1
2
```

Sample Output

```
2
3
4
```