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## **Algorithms Lab**

## Exercise – Aliens

Aliens have invaded earth! Let us say that n aliens are facing off against m humans in the final battle for the planet. During this battle, some humans and aliens are wounded. Let us write  $a \to b$  if alien/human a has given a wound to human/alien b. We assume that no human has wounded another human, an similarly that no alien has wounded another alien. Moreover, we assume for simplicity that for every pair human/alien, either the human has wounded the alien or vice-versa. (but not both!)

After the battle (which they won of course), the important thing for the aliens is that they can boast of their battle prowess. Alien a feels that it is superior to alien/human b if there exists a 'path' of wounding ending in b

$$a \to x_1 \to x_2 \to \ldots \to x_k \to b.$$

However, aliens only know how to count to three, so they disregard all paths with more than three arrows (i.e., paths in which k > 2). Note that it can happen that two aliens feel superior to each other.

Of course, an alien has extra bragging rights if it can feel both superior to every human as well as to every other alien. You have been forced to write a program which counts the number of aliens for which this is true.

To simplify things, we assume that the humans are ordered  $1, \ldots, m$ .

**Input** The first line contains the number t ( $t \le 15$ ) of test cases.

Each test case is structured as follows.

It starts with two numbers n and m, such that  $1 \le n, m \le 500000$ , the numbers of aliens and humans respectively.

The n following lines specify for each alien i which humans have been wounded by this alien. The specification consists of two numbers  $p_i$  and  $q_i$ , where  $p_i \leq q_i$ , which are either both 0 or are both between 1 and m. If  $p_i = q_i = 0$ , then this means that this alien has not wounded any humans. If  $1 \leq p_i, q_i \leq m$ , then it managed to wound the humans in the interval  $[p_i, q_i]$ . Assume that all humans not wounded by the alien have been able to wound it in their turn.

**Output** For every test-case the output should consist of a single line containing the number of aliens who feel superior to every other alien and every human.

Sample Input	0 0
1	
3 3	Sample Output
1 1	
2 3	2

*Hint:* If there are two aliens i and j such that  $[p_i,q_i]\subseteq [p_j,q_j]$ , then alien i cannot feel superior to alien j. Why? Try some examples. (\* *Points*)100