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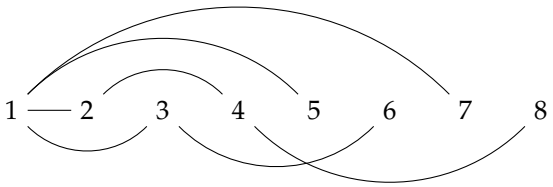
Algorithms Lab

Exercise – *Divisor Distance*

Let n be some positive integer. Let G_n be a graph with the numbers from 1 to n as vertices. Further in G_n there is an (undirected) edge between vertices i and j with $i > j$ if and only if j is the largest proper divisor of i. A proper divisor of a positive integer i is any other positive integer j less than i and such that j divides i without a remainder.

What is the minimal distance of two given vertices in this graph?

As an example here is the graph G_8



Input The first line of the input contains the number of test cases $t \leq 100$. Each of the t test cases is described as follows.

- It starts with a line that contains two integers n and c ($1 \leq n \leq 10'000'000$ and $1 \leq c \leq 100$). The first integer (n) denotes that we consider the graph G_n as defined above. The second integer indicates the number of pairs of vertices for which we wish to know their minimum distance in G_n.
- Each of the following c lines contain two integers v_1 and v_2 ($1 \leq v_1, v_2 \leq n$).

Output For each test case you have to print c lines (with c corresponding to a particular test case): for each of the c pairs of vertices v_1 and v_2 one line containing the minimal distance of v_1 and v_2 in the graph G_n.

Points There are four groups of test sets. The individual points are specified below; the total number of points is 100.

1. For the first group of test sets, worth 20 points, you may assume that $n \leq 1'000$.
2. For the second group of test sets, worth 40 points, you may assume that $n \leq 10'000$.
3. For the third group of test sets, worth 20 points, you may assume that $n \leq 100'000$.
4. For the fourth group of test sets, worth 20 points, there are no additional assumptions.

Sample input

```
1
8 3
1 2
1 8
1 6
```

Sample output

```
1
3
2
```