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Algorithms Lab

Exercise – *The Phantom Menace*

On orders of the evil Sith Lord Darth Sidious the *Trade Federation* has invaded the remote planet Naboo, located in the Chommel Sector, with an army of battle droids. However, they have failed to capture *Queen Amidala* who has been saved by the Jedis Qui-Gon Jinn and Obi-Wan Kenobi. Now it has come to the attention of the Trade Federation that the Queen and the Jedis want to escape the planet on her royal starship. You have been forced to help the Trade Federation to block their getaway plan.

The royal starship can travel between *astronomical objects* such as planets and moons. Due to asteroid belts and solar flares, travel between certain pairs of astronomical objects may be possible in one direction only or not at all. Both Queen Amidala and the Trade Federation are aware of all usable links between astronomical objects. It is not known which astronomical object in the Naboo system the royal starship will launch its flight from nor where exactly in the rest of the Chommel Sector it will head for. But the Trade Federation's intelligence agency provides you with a set of conjectured starting locations and a set of imaginable destinations.

Your task is to block all possible escape routes (from any given starting location to any given destination) with battleships. A battleship can be placed on any astronomical object and blocks all escape routes that use this astronomical object. Furthermore, the Trade Federation wants to use as few battleships as possible. Since you have no choice but to help the Trade Federation, let us hope that the starship's hyperdrive will enable Queen Amidala, Qui-Gon Jinn and Obi-Wan Kenobi to break through the blockade...

Input The first line of the input contains the number $t \leq 30$ of test cases. Each of the t test cases is described as follows.

- It starts with a line that contains four integers `n m s d`, separated by a space and such that $4 \leq n \leq 10^3$, $m \leq 2 \cdot 10^4$ and $2 \leq s, d \leq n$. Here n denotes the number of astronomical objects, m the number of links between astronomical objects, s the number of given starting locations and d the number of given destinations.
- The following m lines define the links between astronomical objects (0-based). Each link is described by two integers `i j`, separated by a space and such that $0 \leq i, j \leq n-1$, meaning that the starship can travel from object i to object j (but not necessarily vice versa).
- Two lines follow. The first contains a list of s space separated integers i, $0 \leq i \leq n-1$, denoting the given set of starting locations S. The second contains a list of d space separated integers j, $0 \leq j \leq n-1$, denoting the given set of destinations D. You may assume that all starting locations and all destinations are pairwise distinct and that $S \cap D = \varnothing$.

Output For each test case output a line with one integer k that is the minimum number of battleships needed to block all escape routes, as defined below.

An escape route from S to D is a sequence $v_0, \ldots, v_\ell$ with $v_0 \in S$ and $v_\ell \in D$ such that for all $0 \leq i \leq \ell - 1$ there is a link from the astronomical object v_i to the astronomical object v_{i+1}.
A group of $k \geq 0$ battleships can *block* all possible escape routes from S to D, if there is a set $\{p_0, \ldots, p_{k-1}\}$ of k astronomical objects such that for every escape route $v_0, \ldots, v_\ell$ there exist $i, j \in \mathbb{N}$ with $v_i = p_j$ (that is, a battleship is positioned at the astronomical object v_i). Note that it is possible to place battleships on starting locations and destinations.

Points There are three groups of test sets, worth 100 points in total.

1. For the first group of test sets, worth 20 points, you may assume that $s = 2$ or $d = 2$.
2. For the second group of test sets, worth 20 points, you may assume that $n \leq 100$, that $m \leq 2 \cdot 10^3$, and that $s \leq 3$ or $d \leq 3$.
3. For the third group of test sets, worth 40 points, there are no additional assumptions.
4. For the fourth (hidden) group of test sets, worth 20 points, there are no additional assumptions.

Corresponding sample test sets are contained in `test`*i*`.in/out`, for $i \in \{1, 2, 3\}$.

Sample Input

```
2
7 9 2 2
0 2
0 3
1 2
1 4
2 3
3 4
4 2
3 5
4 6
0 1
5 6
7 5 3 3
0 4
1 3
2 3
3 5
3 6
0 1 2
4 5 6
```

Sample Output

```
2
2
```