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ΤΠ4726

ΘΕΜΑ 1

α - Σ
β - Σ
γ - Σ
δ - Λ

ΘΕΜΑ 2

$$A = \begin{pmatrix} 1 & 1-i \\ 1+i & -1 \end{pmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 1-i \\ 1+i & -1-\lambda \end{vmatrix}$$

$$= (1-\lambda)(-1-\lambda) - (1-i)(1+1)$$
$$= -1 - \lambda + \lambda + \lambda^2 - 1 - i + i + 1 + i^2$$
$$= \lambda^2 - 1 = P(\lambda)$$

Εφαρμογή του Θ Cayley - Hamilton

$$A^2 - 1 = 0 \Rightarrow A^2 = 1$$

$$A^4 = A^2 \Rightarrow A^4 = 1$$
$$A^3 = A \Rightarrow A^5 = A^3 = A^2 A = A$$

Με επαγωγή προκύπτει ότι

$A^{\alpha} = 1$, όπου α άρτιος
$A^{\beta} = A$, όπου β περιττός.

Για ΤΠ: 4726 έχουμε

$$A^{4726} = A$$

ΘΕΜΑ 3

$$A = \begin{vmatrix} 1 & \sqrt{2}-i \\ \sqrt{2}+i & -1 \end{vmatrix}$$

$$A^{-1} = \frac{1}{\det A} \begin{vmatrix} -1 & -\sqrt{2}+i \\ \sqrt{2}-i & 1 \end{vmatrix} \quad A^{-1} = -\frac{1}{2} \begin{vmatrix} -1 & -\sqrt{2}+i \\ -\sqrt{2}-i & 1 \end{vmatrix} = \begin{vmatrix} 1/4 & \frac{\sqrt{2}-i}{4} \\ \frac{\sqrt{2}+i}{4} & -\frac{1}{4} \end{vmatrix}$$

$$\det A = \begin{vmatrix} 1 & \sqrt{2}-i \\ \sqrt{2}+i & -1 \end{vmatrix} = -1-(\sqrt{2}-i)(\sqrt{2}+i)$$
$$-1-(2+(\sqrt{2}-(\sqrt{2}+i)))$$
$$= -1-2-1 = -4 \neq 0$$

άρα ~~det~~ A αντιστρέψιμος

$$\det(A-\lambda I) = \begin{vmatrix} 1-\lambda & \sqrt{2}-i \\ \sqrt{2}+i & -1-\lambda \end{vmatrix} = (-1-\lambda)(1-\lambda)-(\sqrt{2}-i)(\sqrt{2}+i)$$

$$= -1+\lambda-\lambda+\lambda^2-3 = \lambda^2-4$$
$$= (\lambda-2)(\lambda+2)$$

Άρα $\lambda=2$ ή $\lambda=-2$

$$\begin{vmatrix} -1 & \sqrt{2}-i \\ \sqrt{2}+i & -3 \end{vmatrix} \quad \text{ή} \quad \begin{vmatrix} 3 & \sqrt{2}-i \\ \sqrt{2}+i & 1 \end{vmatrix}$$

ΘΕΜΑ 5

$$x + 2y + 3z = 1$$
$$2x + 3y + z = 3$$
$$3x + y + 2z = 4$$

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{vmatrix}$$

$$\det A = 1\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 3\begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix}$$

$$= 1\cdot(6-1) - 2(4-3) + 3(2-9) = -18 \neq 0$$

Άρα A αντιστρέψιμος.

$A_{11} = (3\cdot2) - (1\cdot1) = 5$ $\quad A_{21} = -(2\cdot2) - (3\cdot1) = -1$

$A_{12} = -(2\cdot2) - (3\cdot1) = -1$ $\quad A_{22} = (1\cdot2) - (3\cdot3) = 7$

$A_{13} = (2\cdot1) - (3\cdot3) = -7$ $\quad A_{23} = -(1\cdot1) - (2\cdot3) = 5$

$A_{31} = (2\cdot1) - (3\cdot3) = -7$

$A_{32} = -(1\cdot1) - (2\cdot3) = 5$

$A_{33} = (1\cdot3) - (2\cdot2) = -1$

$$A^{-1} = \frac{1}{\det A}\begin{vmatrix} 5 & -1 & -7 \\ -1 & -7 & 5 \\ -7 & 5 & 1 \end{vmatrix} = \begin{vmatrix} -5/18 & 1/18 & 7/18 \\ 1/18 & 7/18 & -5/18 \\ 7/18 & -5/18 & 1/18 \end{vmatrix}$$

$$\begin{vmatrix} x \\ y \\ z \end{vmatrix} = \begin{vmatrix} -5/18 & 1/18 & 7/18 \\ 1/18 & 7/18 & -5/18 \\ 7/18 & -5/18 & 1/18 \end{vmatrix} \begin{vmatrix} -1 \\ 3 \\ 4 \end{vmatrix}$$

$$\begin{vmatrix} 5/18 & 3/18 & 28/18 \\ 1/18 & 21/18 & -20/18 \\ -7/18 & -15/18 & 4/18 \end{vmatrix} = \begin{vmatrix} 2 \\ 0 \\ -1 \end{vmatrix}$$