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Αριθμητική Γραμμική Άλγεβρα

3η εργαστηριακή άσκηση
(παράδοση έως 15-04 00^{00})

Άσκηση 1

$$A=\begin{bmatrix} 0 & 2 & 1 \\ 1 & -2 & 4 \\ 4 & 1 & 2 \end{bmatrix}$$

$$\det(A)\begin{vmatrix} 0 & 2 & 1 \\ 1 & -2 & 4 \\ 4 & 1 & 2 \end{vmatrix} \xrightarrow{\Sigma_2=\Sigma_2-2\Sigma_3} \begin{vmatrix} 0 & 0 & 1 \\ 1 & -10 & 4 \\ 4 & -3 & 2 \end{vmatrix} = \cancel{0\cdot\begin{vmatrix} -10 & 4 \\ -3 & 2 \end{vmatrix}}^{0} + \cancel{0\begin{vmatrix} 1 & 4 \\ 4 & 2 \end{vmatrix}}^{0} + 1\begin{vmatrix} 1 & -10 \\ 4 & -3 \end{vmatrix}$$

$$= 1\cdot(3)-(-10)\cdot 4 \Rightarrow \boxed{\det(A)=37\neq 0}$$

$$\begin{vmatrix} 0 & 2 & 1 \\ 1 & -2 & 4 \\ 4 & 1 & 2 \end{vmatrix} \xrightarrow{r_1 \leftrightarrow r_2} \begin{vmatrix} 1 & -2 & 4 \\ 0 & 2 & 1 \\ 4 & 1 & 2 \end{vmatrix} \xrightarrow{r_3=r_3-4r_1} \begin{vmatrix} 1 & -2 & 4 \\ 0 & 2 & 1 \\ 0 & 9 & -14 \end{vmatrix}$$

$$\xrightarrow{r_3=r_3-\frac{9}{2}r_2} \begin{vmatrix} 1 & -2 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & -\frac{37}{4} \end{vmatrix}$$

Επομένως, $U=\begin{vmatrix} 1 & -2 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & -37/4 \end{vmatrix}$

και ο L έχει προκύψει από τις γραμμοπράξεις $L=\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 9/2 & 1 \end{vmatrix}$

Συνεπώς $P.A=L\cdot U \Rightarrow$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\cdot\begin{bmatrix} 0 & 2 & 1 \\ 1 & -2 & 4 \\ 4 & 1 & 2 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 4 & 9/2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & 4 \\ 0 & 2 & 1 \\ 0 & 0 & -37/4 \end{bmatrix}$$

Ασκηση 3

$$E = \begin{vmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\det(E) = 1\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + \cancelto{0}{0\begin{vmatrix} 6 & 0 \\ 0 & 1 \end{vmatrix}} + \cancelto{0}{0\begin{vmatrix} 6 & 1 \\ 0 & 0 \end{vmatrix}} = 1 - 0$$

$$\boxed{\det(E) = 1}$$

$$E^2 = \begin{vmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow \boxed{E^2 = \begin{vmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}}$$

$$E^4 = E^2 \cdot E^2 = \begin{vmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 12 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow E^4 = \begin{vmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$E^8 = E^4 \cdot E^4 = \begin{vmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 24 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow E^8 = \begin{vmatrix} 1 & 0 & 6 \\ 48 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$E^9 = E^8 \cdot E = \begin{vmatrix} 1 & 0 & 0 \\ 48 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 6 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \Rightarrow \boxed{E^9 = \begin{vmatrix} 1 & 0 & 0 \\ 54 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}}$$

Για την αντίστροφο έχουμε:

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 6 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\Sigma_2 = \Sigma_2 - 6\Sigma_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -6 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

Επομένως $E^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -6 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Άσκηση 4

$$A = \begin{bmatrix} 4 & 2 & 0 & 2 \\ 2 & 4 & -1 & -1 \\ 0 & -1 & 4 & 1 \\ 2 & -1 & 1 & 4 \end{bmatrix}$$

$A = A^T$

$A = L \cdot U \Rightarrow \quad U = L^T$

$A = L \cdot L^T$

$$\begin{vmatrix} 4 & 2 & 0 & 2 \\ 2 & 4 & -1 & -1 \\ 0 & -1 & 4 & 1 \\ 2 & -1 & 1 & 4 \end{vmatrix} = \begin{vmatrix} l_{11} & 0 & 0 & 0 \\ l_{21} & l_{22} & 0 & 0 \\ l_{31} & l_{32} & l_{33} & 0 \\ l_{41} & l_{42} & l_{43} & l_{44} \end{vmatrix} \cdot \begin{vmatrix} l_{11} & l_{21} & l_{31} & l_{41} \\ 0 & l_{22} & l_{32} & l_{42} \\ 0 & 0 & l_{33} & l_{43} \\ 0 & 0 & 0 & l_{44} \end{vmatrix} =$$

$$= \begin{vmatrix} l_{11}^2 & l_{11} l_{21} & l_{11} l_{31} & l_{11} l_{41} \\ l_{11} l_{21} & l_{21}^2 \; l_{22}^2 & l_{21} l_{31} + l_{22} l_{32} & l_{21} l_{41} + l_{22} l_{42} \\ l_{11} l_{31} & l_{21} l_{31} + l_{22} l_{32} & l_{31}^2 + l_{32}^2 + l_{33}^2 & l_{31} l_{41} + l_{32} l_{42} + l_{32} l_{43} \\ l_{11} l_{41} & l_{21} l_{41} + l_{22} l_{42} & l_{31} l_{41} + l_{32} l_{42} + l_{33} l_{43} & l_{41}^2 + l_{42}^2 + l_{43}^2 + l_{44}^2 \end{vmatrix}$$

$l_{11}^2 = 4 \Rightarrow l_{11} = \sqrt{4} = 2$

$l_{11} l_{21} = 2 \Rightarrow l_{21} = \frac{2}{l_{11}} = 1$

$l_{11} l_{31} = 0 \Rightarrow l_{31} = 0$

$l_{11} l_{41} = 2 \Rightarrow l_{41} = 1$

$l_{21}^2 + l_{22}^2 = 4 \Rightarrow l_{22} = \sqrt{4 - l_{21}^2} = \sqrt{3}$

$l_{21} l_{31} + l_{22} l_{32} = -1 \Rightarrow l_{32} = \frac{-1 - l_{31} l_{21}}{l_{22}} = \frac{-\sqrt{3}}{3}$

$l_{21} l_{41} + l_{22} l_{42} = -1 \rightarrow l_{42} = \frac{-1 - l_{41} l_{21}}{l_{22}} = \frac{-2\sqrt{3}}{3}$

$l_{31}^2 + l_{32}^2 + l_{33}^2 = 4 \Rightarrow l_{33} = \sqrt{4 - (l_{31}^2 + l_{32}^2} = \sqrt{\frac{11}{3}} = \sqrt{\frac{33}{3}}$

$l_{31} l_{41} + l_{32} l_{42} + l_{33} l_{43} = 1 \Rightarrow l_{43} = \frac{1 - (l_{41} l_{31} + l_{42} l_{32}}{l_{33}} = \frac{\sqrt{33}}{33}$

$l_{41}^2 + l_{42}^2 + l_{43}^2 + l_{44}^2 = 4 \Rightarrow l_{44} = \sqrt{4 (l_{41}^2 + l_{42}^2 + l_{43}^2} = \sqrt{\frac{18}{11}} = \frac{3\sqrt{22}}{11}$

Επομένως έχουμε:

$$L = \begin{vmatrix} 2 & 0 & 0 & 0 \\ 1 & \sqrt{3} & 0 & 0 \\ 0 & -\sqrt{3}/3 & \sqrt{33}/3 & 0 \\ 1 & -\frac{2\sqrt{3}}{3} & \sqrt{33}/33 & \frac{3\sqrt{22}}{11} \end{vmatrix}$$

$$L_T = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & \sqrt{3} & \sqrt{3}/3 & -\frac{2\sqrt{3}}{3} \\ 0 & 0 & \sqrt{33}/3 & \sqrt{33}/33 \\ 0 & 0 & 0 & \frac{3\sqrt{22}}{11} \end{bmatrix} = U$$

Ο πίνακας Δ είναι συμμετρικός ως προς τη διαγώνιο του και επίσης ισχύει ότι $L = \frac{1}{U}$ δηλ L και U αντίστροφοι.