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Recurrence Quantification Analysis (RQA) and Cross-Recurrence Quantification Analysis (CRQA) are nonlinear methods for the analysis of nonstationary time series; such as EEG signals. The offer quantification of the recurring patterns in phase space trajectories [19, 20]. Introduced by Trulla et al.[19] and expanded by Webber and Zbilut[20], RQA measures metrics like recurrence rate, determinism, and laminarity to capture dynamic system behavior. Thomasson et al.[21] demonstrated RQA's applicability to EEG, highlighting its robustness to noise and nonstationarity. Marwan et al.[22] further advanced recurrence plot techniques, emphasizing their utility in detecting dynamic transitions in neuroimaging. These foundational works underpin the application of RQA and CRQA to EEG studies in epilepsy, cognitive disorders, and beyond, as explored in this review.

Frolov et al.[1] (2020) proposed an approach to analyze frequency based multiplex brain networks using recurrence quantification analysis (RQA) on EEG data, where they have demonstrated how recurrence-based synchronization indices can effectively capture both within-frequency (intralayer) and cross-frequency (interlayer) functional connectivity during cognitive tasks. Their work showed that RQA is particularly suitable for analyzing non-stationary EEG signals and revealed important insights about the evolution of functional connectivity patterns during prolonged cognitive tasks. In addition the dataset used in this research are openly available in a Figshare repository.

Lombardi et al.[5] have investigated the nonlinear properties in fMRI BOLD signals during a working memory task in schizophrenic patients and healthy controls. They have attempted by using RQA, to analyze recurrence plots for quantifying determinism, trapping time, and maximal vertical line length in functionally relevant brain clusters. Outcome revealed differences in the dynamics between the two groups, and more specific in working memory and DMN areas. While their work have focused on fMRI, the methodology can be adapted also into EEG signals, which can offer a higher resolution for capturing rapid neural dynamics.

Kang et al. (2023)[2], in their study explore the dynamics and functional connectivity of the Default Mode Network (DMN) in schizophrenia, applying RQA-CRQA on resting-state fMRI data. Findings include decreased *determinism* between specific DMN regions (vMPFC-posterios cingulate and vMPFC-precuneus) in first-episode schizophrenia patients, as a signal of disturbed predictability of functional interactions. Moreover, their results achieve to correctly classify using SVM(support vector machine) schizophrenia patients from healthy controls with 77% classification accuracy.

Núñez et al. [16] in their work, have analyzed resting-state EEG recordings from subjects with mild cognitive impairment(MCI), Alzheimer's disease(AD), and healthy ground truth controls in order to detect frequency based changes into their brain dynamics. By blending wavelet based Kullback–Leibler divergence (KLD) for capturing non-stationarity, and two recurrence quantification analysis (RQA) metrics(*entropy of the recurrence point density* and the *median* *of the recurrence point density*) insights have been extracted related to neurodegeneration presence. Research's findings show that MCI and AD are presenting notable changes in the recurrence structure and non-stationarity of EEG signals, and more specifics on the theta and beta frequency bands. Therefore, recurrence based dynamics show a capability as potential biomarkers for monitoring and detecting early Alzheimer's disease and its progression.

Fan and Chou [15] have also proposed an approach for real-time epileptic seizure detection using as a method the analysis of temporal synchronization patterns of EEG signals with recurrence networks and spectral graph theory. Recurrence plots are used for the modeling of the EEG dynamics, extracting graph theory's features for quantifying the synchronization. Results showed high sensitivity of 98.48% and low latency (6 seconds) for detecting seizure on the CHB-MIT dataset, performing better than other RQA measures.

Researchers in [3], have applied RQA on resting-state fMRI data from TgF344-AD rats(a transgenic rat model which will eventually develop Alzheimer's disease) and their healthy-control counterparts wild-type rats(WT), in order to detect early stage biomarkers for the disease. By analyzing Default Mode-Like Network (DMLN) using RQA metrics(*entropy, recurrence rate, determinism and average diagonal line length*) changes have been detected in regions of the basal forebrain, hippocampal fields (CA1, CA3), and visual cortices (V1, V2). Also on the study's findings include reduced predictability in WT rats with aging, while AD rats exhibited less decline in predictability, suggesting some unknown yet countereacting mechanisms. This study highlights RQA's sensitivity for nonlinear dynamics in preclinical AD and the code used is also publicly available.

Author in [6](2025), investigated changes related to aging in brain sensorimotor systems using RQA and theta-band functional connectivity in EEG signals. In the study a VR experimental paradigm was utilized with auditory stimulus across different age groups(young and elder subjects). Key findings revealed that elder subjects present decreased EEG complexity during motor preparation stages as measured by RQA metrics (ΔRR *and* ΔRTE), and had increased theta band functional connectivity highlighting the potential of RQA in detecting age related biomarkers that were not detectable using standalone signal spectral analysis.

Guglielmo et al. (2022)[7] utilized RQA features extracted by EEG signals for the purpose of classification of cognitive performance during mental arithmetic tasks. They used frontal and parietal EEG signals and analyzed them, from 36 participants by extracting six RQA metrics (*recurrence rate, determinism, laminarity, entropy, maximum diagonal line length and average diagonal line length*) from four electrodes (F7, Pz, P4, Fp1). Afterwards they applied machine learning(ML) classifiers (SVM, Random Forest, and Gradient Boosting) and they reached accuracy of classification above 0.85, showing the potential that RQA holds for generalizing on nonlinear dynamics.

Yang and co-authors [17], employed stereo-electroencephalography (sEEG) recordings from 10 patients with refractory focal epilepsy to analyze dynamical differences among discreet epileptic states (inter-ictal, pre-ictal, and ictal) and regions. Using recurrence plots and CRQA, they have identified epileptogenic channels with longer diagonal structures in RPs, which is a sign of more deterministic and recurrent dynamics. Their findings point out that synchronization between epileptogenic channels strengthened while seizures events occur, suggesting these regions dominate the epileptic network's dynamics.

Lopes et al. (2020)[8] have proposed a combinatorial framework by mixing RQA with dynamic functional network (dFN) analysis, applying it to both MEG and stereo EEG data. The methodology they described is split into five steps: data segmentation, functional network inference, distance computation alongside networks, recurrence plot construction and finally RQA. The study demonstrated that functional networks in epilepsy patients recur more quickly than in healthy controls, suggesting RQA on dFNs could serve as a potential biomarker. For the EEG dataset investigation, they have showed that the pre-ictal networks shown higher recurrence rates than post-ictal periods, with the τ-recurrence rate (RR_τ) proving particularly effective for seizure detection.

Rangaprakash [18] have proposed an application of RQA for the study of brain connectivity using multichannel EEG signals. In its work, a new CRQA-based feature was proposed (Correlation between Probabilities of Recurrence (CPR)), a nonlinear and non-parametric phase synchronization technique. Afterwards it was utilized for the analysis of functional connectivity in epilepsy subjects during eyes-open/eyes-closed conditions. The results demonstrated that CPR outperformed other known traditional linear methods on distinguishing seizure and pre-seizure states, identifying epileptic foci, and differentiating alongside eyes-open and eyes-closed conditions.

In their research, Pentari et al.[9] have applied CRQA to resting-state fMRI data for examining the dynamic functional connectivity on patients with neuropsychiatric systemic lupus erythematosus (NPSLE). Results contain the fact that CRQA metrics, such as determinism, appear more sensitive than conventional static functional connectivity methods in order to identify aberrant connectivity patterns that correlated with visuomotor performance. The study focused on 16 frontoparietal regions and found that CRQA could detect both increased and decreased connectivity in NPSLE patients compared against the healthy controls. Building on these findings, Pentari et al.[10] subsequently expanded the investigation to whole brain network analysis in a larger cohort. In this study they demonstrate the capability of CRQA to integrate multiple recurrence metrics for revealing both hyperconnectivity in parietal regions (angular gyrus and superior parietal lobule) and hypoconnectivity in medial temporal structures (hippocampus and amygdala). Notably, the dynamic connectivity measures showed stronger associations with cognitive performance than structural measures, particularly for verbal episodic memory.

Recent studies have demonstrated the effectiveness of RQA in analyzing EEG signals for epilepsy detection. Gruszczyńska et al.[11] have applied RQA in order to distinguish epileptic from healthy patients using EEG recordings from frontal and temporal lobe electrodes (Fp1, Fp2, T3, T4). In their findings they have showed that the epileptic signals present more periodic dynamics in comparison to healthy controls, by as evidenced by higher values of RQA parameters such as determinism, laminarity, and longest diagonal line. The study combined RQA with Principal Component Analysis for dimensionality reduction and visualization, achieving 86.8% classification accuracy with SVM. This work is particularly relevant as it demonstrates RQA's capability to identify pathological patterns in resting-state EEG without requiring seizure events during recording.

Another study utilizing advanced nonlinear analysis techniques for neural correlation investigation to cognitive functions [12] used *stereoelectroencephalography (sEEG)* combined alongside RQA for the examination of the relationship of the DMN and empathy. Correlations have been detected relating specific RQA metrics (mean diagonal line length, entropy of diagonal line lengths, trapping time) and empathy scores, particularly within DMN subsystems.

Regarding epilepsy diagnosis, authors in [13] proposed a new framework utilizing the combintation of RQA with genetic algorithms and Bayesian classifiers for identifying corresponding biomarkers for seizure detection. They utilized five distance norms (e.g., Euclidean, Mahalanobis) and multiple thresholds for extracting recurrence features from EEG signals, achieving 100% classification accuracy. More specific, the *transitivity* feature has shown capability of a highly discriminative biomarker, performing better compared to traditional linear methods.

Ngamga et al.[14] studied the performance achieved of RQA and Recurrence Network (RN) measures in identifying pre-seizure states from multi-day, multi-channel intracranial EEG (iEEG) recordings of epilepsy patients. Results highlighted the correlation among RQA measures (determinism, laminarity, and mean recurrence time) in detecting seizure precursors, while RN measures (average shortest path length and network transitivity) provided complementary but not so consistent insights than using the application of RQA measures alone.

In addition there have been works where simulated data have been used in conjunction with RQA. Lameu et al.[4], investigated burst phase synchronization in neural networks using RQA. They analyzed two network types; a small-world network and a network of networks (to mimic better the real human brain), using coupled Rulkov maps to model bursting neurons. By applying RQA, they identified synchronized neuron groups and quantified their sizes during synchronization transitions. The study showed that RQA measures (*recurrence rate, laminarity inspired*(custom feature)*, and average structure size*) complement traditional order parameters by revealing localized synchronization patterns, such as the formation and growth of synchronized clusters.

Heunis and co-authors[23] have utilized resting state EEG and RQA in order to distinguish individuals of ages 0-18 of two categories; ASD(autism spectrum disorder) and typically developing. They have extracted RQA features and tested various linear and nonlinear classifiers achieving 92.9% classification accuracy with nonlinear SVM classifier.

Table 1: Comparison among the retrieved studies using recurrence analysis

#	Reference	Modality	Analysis Methods	Network Type
1	Frolov et al. (2020)	EEG	RQA, CRQA	Multiplex functional networks
2	Kang el al. (2023)	fMRI	RQA, CRQA	DMN, schizophrenia
3	Rezaei el al. (2023)	fMRI	RQA	Default model-like network, AD
4	Lameu et al. (2018)	—	RQA	Small-world & cluster network
5	Lombardi et al. (2014)	fMRI	RQA	schizophrenia,working memory
6	Pitsik E. (2025)	EEG	RQA	aging
7	Guglielmo et al. (2022)	EEG	RQA	cognitive tasks
8	Lopes et al. (2020)	sEEG, MEG	RQA	epilepsy
9	Pentari et al. (2022)	fMRI	RQA, CRQA	NPSLE
10	Pentari et al. (2023)	fMRI	CRQA	NPSLE
11	Gruszczyńska et al. (2019)	EEG	RQA	epilepsy
12	Mo et al. (2022)	sEEG	RQA	DMN, epilepsy
13	Palanisamy et al. (2024)	EEG	RQA	epilepsy
14	Ngamga et al. (2016)	EEG	RQA,RN	epilepsy
15	Fan and Chou (2019)	EEG	RQA,RN	epilepsy, seizure detection
16	Nunez et al. (2020)	EEG	RQA	AD
17	Yang et al. (2019)	sEEG	RQA,CRQA	epilepsy
18	Rangaprakash (2014)	EEG	CPR(CRQA-based)	epilepsy
19	Heunis et al. (2018)	rsEEG	RQA	autism spectrum disorder

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