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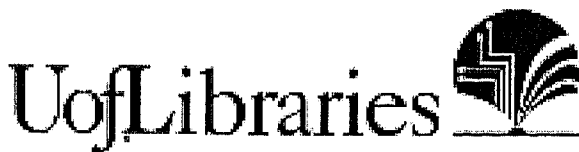
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**Please briefly describe the problem and initial:**

# Application of Recurrence Quantification Analysis to EEG Signals

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## Abstract

Numerous techniques have been applied to analyze electroencephalogram (EEG) dynamics. However, the nonstationarity of EEG signals makes it difficult to obtain reliable results. Most often, these techniques include Fast Fourier Transforms (FFTs) and wavelets, which are based on linear systems theory. An alternative method, not constrained by stationarity, namely, recurrence quantification analysis (RQA), was thus examined in its ability to reveal dynamical change in epileptic EEG signals.

**Key Words:** Recurrence quantification analysis, EEG, nonstationarity, signal processing.

## 1 Introduction

EEG signals can be considered as time series of wave-like voltage fluctuations generated by a large number of neurons. In this way, they have been analyzed using a wide range of time series analysis techniques [16]. Generally speaking, these techniques fall within two categories: linear and nonlinear methods.

Linear methods, such as power spectrum or frequency coherence, have been the traditional signal processing methods used in EEG analysis to search for brain rhythms [4, 11, 17]. Among them, the most widely used method for frequency analysis of EEG signals is the Fast Fourier Transform (FFT). The method itself transforms a time series by decomposing it into a series of sin/cosin functions [5]. Moreover, the FFT cannot properly separate periodic components of a signal when frequency bands are overlapping [6, 14]. The main disadvantage when applying linear methods to EEG signals is that they are insensitive to the nonlinear structure contained in the signals.

As a result, in the last decade attention has been focused on the development of nonlinear tools, since several researchers have provided evidence that the EEG is a nonlinear signal with deterministic properties [8, 11, 13]. Chaos-related tools such

as the correlation dimension, Lyapunov exponent or Kolmogorov entropy, have thus been used to characterize EEG dynamics, but with contradictory results. Indeed, while some studies reported chaotic properties in EEG segments [2-4; 10; 15; 22], others claimed an absence of the signature of low-dimensional chaos in EEG signals [20-21; 23-25]. The import of these studies suggests it is questionable to consider the brain as a low dimensional nonlinear dynamical system. In fact, Gribkov and Gribkova [9] recently reported evidence of high dimensional nonlinear dynamics in quasi-stationary EEG segments.

A major point is that real world dynamical processes and EEG signals, in particular, are nonstationary, and this restricts the possibility of dynamics extraction from the corresponding time series. In others words, in absence of stationary signals dynamical invariants (correlation dimension, Lyapunov exponents, entropies) and stationary statistical characteristics of the process (mean and standard deviation) are not very meaningful.

Consequently, analysis of EEG signals using a method that is not constrained by stationarity requirements, could bring insight into EEG dynamics. This paper presents such a method, Recurrence Quantification Analysis (RQA) as applied to EEG signals.

## 2 Recurrence Quantification Analysis

In 1987, Eckman et al., introduced the Recurrence Plots (RP) technique to visually analyze nonstationarity in complex systems [7]. Indeed, an RP allows one to visualize the dependence in time of orbits  $\vec{x}_i$  in phase space. Knowing that the existence of a state recurrence is a fundamental property of deterministic systems [1, 19], an RP thus represents recurrences of phase space trajectory for specific states.

The construction of an RP relies upon the calculation of a  $N \times N$  matrix:

$$R_{i,j} = \theta(r - \|\vec{x}_i - \vec{x}_j\|) \quad \text{for } i,j = 1 \dots N$$

where  $r$  is a predetermined radius;  $\|\vec{x}_i - \vec{x}_j\|$  the Euclidean norm of vector  $x_i$  and  $x_j$ ;  $\theta$  the Heaviside function. Then, a

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black pixel is drawn in the plot only if  $\|\bar{x}_i - \bar{x}_j\| < r$ , i.e., if they are recurrent points.

The main advantage of such a graph is that it reveals the nonstationarity of a system as well as structures characterizing specific dynamical behaviors. Stationary systems will give homogenous RPs (Figure 1), while nonstationary ones will depict brightened heterogeneous areas, revealing changes in the distribution of recurrent points (Figure 2). Furthermore, line segments parallel to the main diagonal indicate points that

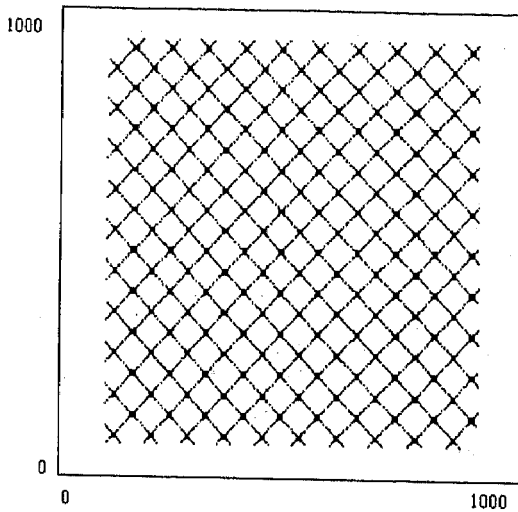


Figure 1: Illustration of a RP for a 1-Hz sine wave. The main diagonal goes from the lower left corner to the upper right one. Note the repetition of the same structure through the plot (revealing the periodicity of the signal), and its homogeneity, characteristic of stationary signals.

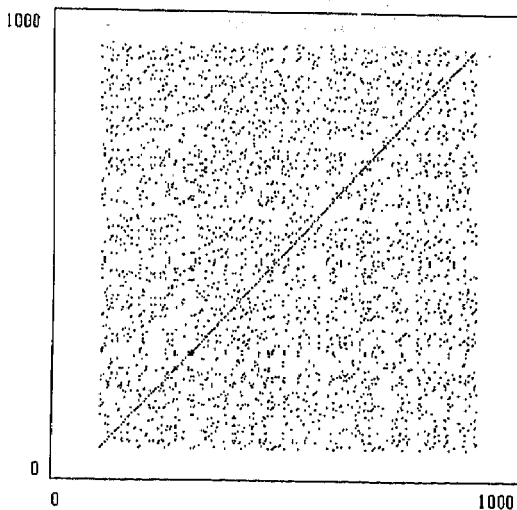


Figure 2: Illustration of a RP for the logistic equation in the chaotic regime. The main diagonal is drawn. Note the non-homogeneous aspect of the plot. Diagonal line segments of varying length are indicative of deterministic dynamics.

are close to each other successively forward in time (Figure 2). This phenomenon would not occur in random processes and points out deterministic ones. In addition, parallel diagonals formed by recurrent points at a regular distance between them depict periodic processes (Figure 1). Finally, a paling of the RP away from the main diagonal to the corners reveals a drift in the system.

However, all these graphical features remain qualitative. In order to obtain quantitative measures, RQA was developed [27, 29] and applied to a wide range of physical and biological research [12, 18, 30]. Furthermore, RQA has also proved its ability to deal with noisy or short data sets [32-33] and to detect bifurcation sequences [27].

Quantitative measures of RP are defined from recurrent points density (percent recurrence); from diagonal segments parallel to the main diagonal (percent determinism); and from the length of the line and paling in the RP (maxline, entropy and trend). Moreover, in order to follow the changes of these variables in time, their computation is done for small windows moving along the time series. Then, for an  $N$  point long series  $(s_1, s_2, \dots, s_N)$ , the windowed version of RQA will provide

$$E_1 = (s_1, s_2, \dots, s_N)$$

$$E_2 = (s_{1+w}, s_{2+w}, \dots, s_{N+w})$$

$$E_3 = (s_{1+2w}, s_{2+2w}, \dots, s_{N+2w})$$

$$\dots$$

$$E_p = (s_{1+(p-1)w}, s_{2+(p-1)w}, \dots, s_{N+(p-1)w})$$

where  $w$  is the offset and  $E_p$  the number of epochs (windows) that satisfies the relation,  $N + (p-1)w \leq n$ .

Hence, quantification of recurrences [29-31] leads to the generation of five variables including: percent of recurrences (percent of plot filled with recurrent points, %REC), percent determinism (percent of recurrent points forming diagonal lines, with a minimum of two adjacent points away from the central diagonal, %DET); entropy (Shannon information entropy of the line length distribution, ENT); maxline the length of the largest line segment (measure of the system divergence, the reciprocal of which is an approximation of the largest positive Lyapunov exponent, MAXLINE); and trend (measure of the paling of recurrent points away from the central diagonal, TREND).

### 3 Application to Epileptic EEG Segments

#### 1) EEG acquisition

Epileptic EEG time series were recorded on 20 AgCl EEG electrodes positioned according to the international 10-20 system. The data were band-pass filtered (1-70 Hz) and digitally stored with a sampling frequency of 200 Hz.

#### 2) RQA

Prior to analysis, a careful visual inspection of the 20 recording electrodes indicated the events that occurred during 90 seconds (18000 digitized points) of epileptic EEG data (Figure 3). Namely, (a) a change in the EEG

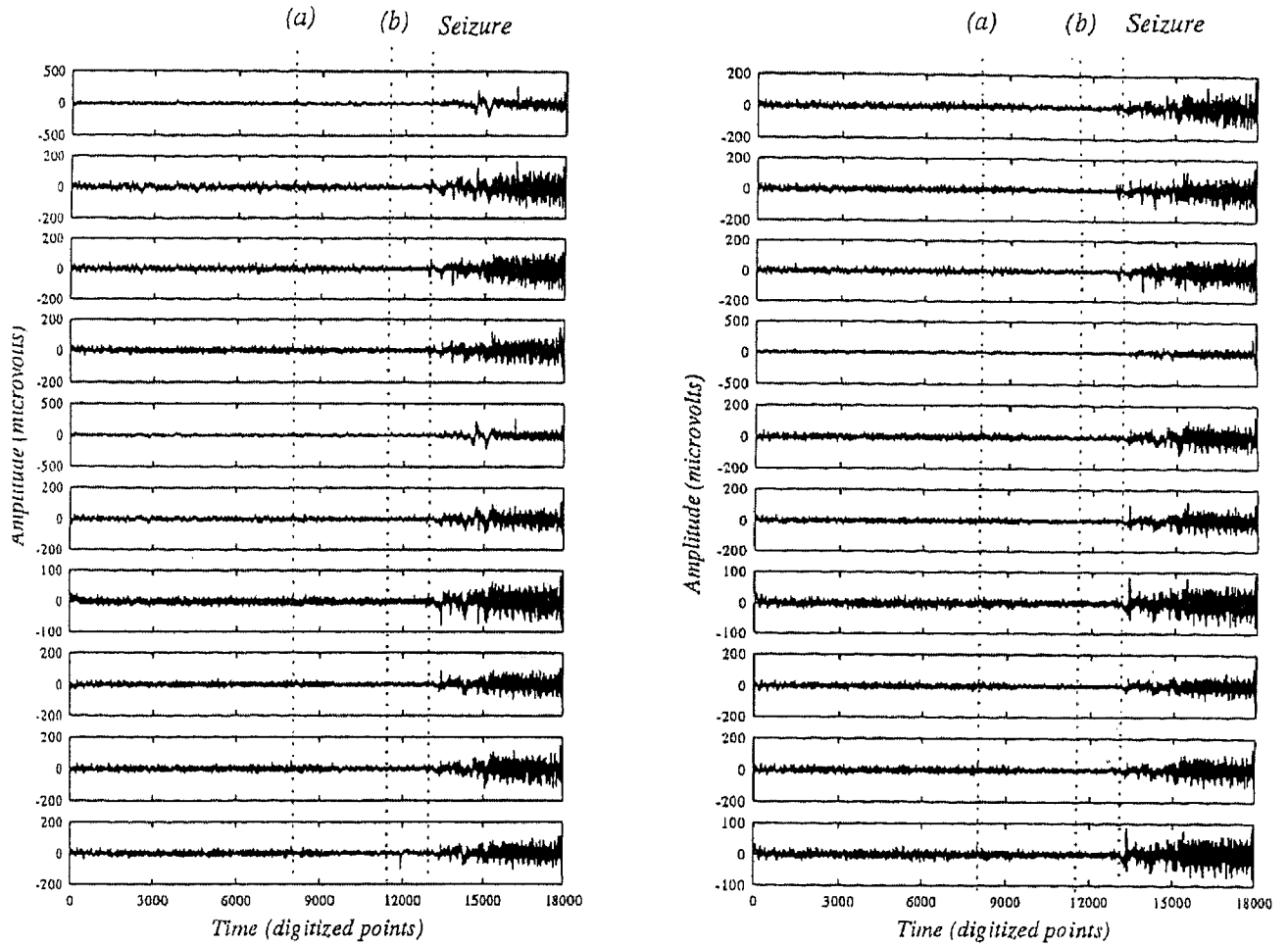


Figure 3: Twenty-lead EEG signals. Time is indicated in the x-axis (second); amplitude in y-axis (microvolt). Chronological events occurring with times are indicated: (a) change in background EEG activity, (b) widespread electromyogram, seizure onset.

background activity due to eye-blinking is noted at 40 sec, and (b) a widespread electromyogram activity probably contaminates brain electrical recording at 57 sec. Finally, the onset of the seizure occurs at around 65 sec. Such events may provoke changes in EEG dynamics that are depicted in an RP and quantitatively detected by RQA.

RP and RQA were performed on the time series obtained from the Euclidean norm of the 20 electrodes (Figure 4), defined by  $\sqrt{\sum_{i=1}^{20} |x_i|^2}$  where  $\bar{x}_i$  represents a vector of the successive EEG value for each lead.

The corresponding RP is represented in Figure 5. The plot is symmetrical with the main diagonal going from the left lower corner to the right upper corner. The nonstationarity of the data is salient; the plot is non-homogeneous, the repetition of a same structure through the plot does not occur, as opposed to a stationary process (cf. Figure 1). Moreover, the RP reveals specifics features. For example, the black part to the right of

the x-axis corresponds to the ictal period (Figure 5 *Seizure*). But changes in dynamics also seem to occur at approximately the center of the x-axis (Figure 5a), as well as later, where vertical black lines are becoming closer to each other (Figure 5b).

As previously pointed out, an RP allows a visual and a qualitative inspection of the changes in EEG dynamics. To confirm the observations, RQA was performed using an Euclidean norm for distance calculation (re-scaled on unit interval); a radius  $r$  of 5 to define the recurrences; and line segments parallel to the main diagonal were counted if composed of two or more points.

Results in the computation of RQA variables are illustrated in Figure 6. The first 65 seconds constitute the preictal period, followed by the seizure. The evolution of the five RQA variables (%REC, %DET, ENT, MAXLINE, TREND) with time exhibits major transients. Significance of these changes was determined using a running 500 ms window overlapping one point at a time to calculate a 99 percent confidence interval. This precaution thus singled out major transients in the dynamics. Such changes are found to be simultaneous in

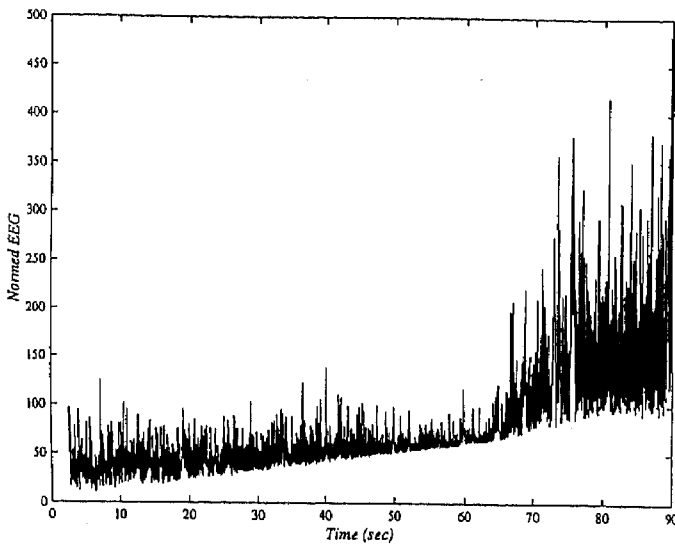


Figure 4: Sample of a normed epileptic EEG segment obtained by taking the Euclidean norm of the 20 recording electrodes. Time is given in the  $x$ -axis (sec); amplitude in  $y$ -axis (microvolt).

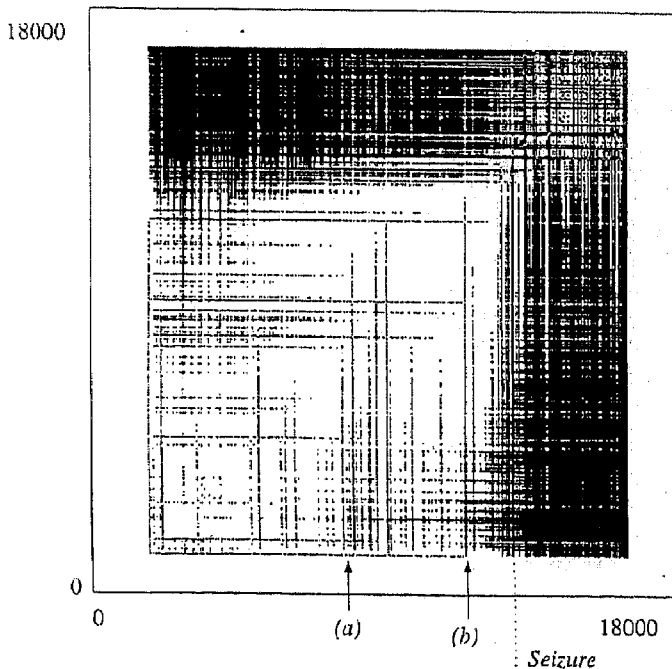


Figure 5: Representation of the RP of a 90 seconds normed epileptic EEG segment. Seizure onset is indicated by the dotted line; it clearly marks a change in dynamics. Other transients are observed in (a) where a few vertical black lines appear and in (b) where they become more numerous and close to each other as the seizure onset approaches.

the different RQA variables, and are concordant with the features observed in the RP. Indeed, a change in dynamics characterized by a drop in %REC, %DET, ENT and MAXLINE occurs at 40 seconds (Figure 6(a)). This change is

associated with a modification of EEG background activity due to eye-blinking. Moreover, a transient towards high values of RQA variables (except in the case of TREND where the relationship is reciprocal) is observed at 57 seconds, followed by others at 63 and 64 seconds (Figure 6(b)). This unstable state may reflect a transitional dynamic related to widespread electromyogram activity; a progressive "route" toward the seizure where abnormally discharging neurons act as pacemakers to recruit and entrain other normal neurons. Finally, the significant decrease in the variables observed at 65 seconds indicates the seizure onset (Figure 6). This study shows that RQA may be a sensitive diagnostic measure of pathologic changes in EEGs.

#### 4 Conclusion

The aim of this paper was to present the ability of RQA to reveal dynamical changes in EEG signals. Without requiring the signal to be stationary, RQA appears to be an appropriate tool in the analysis of brain electrical activity. Developed from the recurrence plot technique [7], RQA provides quantitative measures of system dynamics and is able to reveal transients in the data set [27-29]. Applied to epileptic EEGs it demonstrated transients occurring prior to seizures [26]. RQA may thus be a promising approach in the characterization of brain states, such as neuropathology or cognitive activity. Nevertheless, this study is preliminary and additional trials are necessary to determine the method's utility.

#### Acknowledgment

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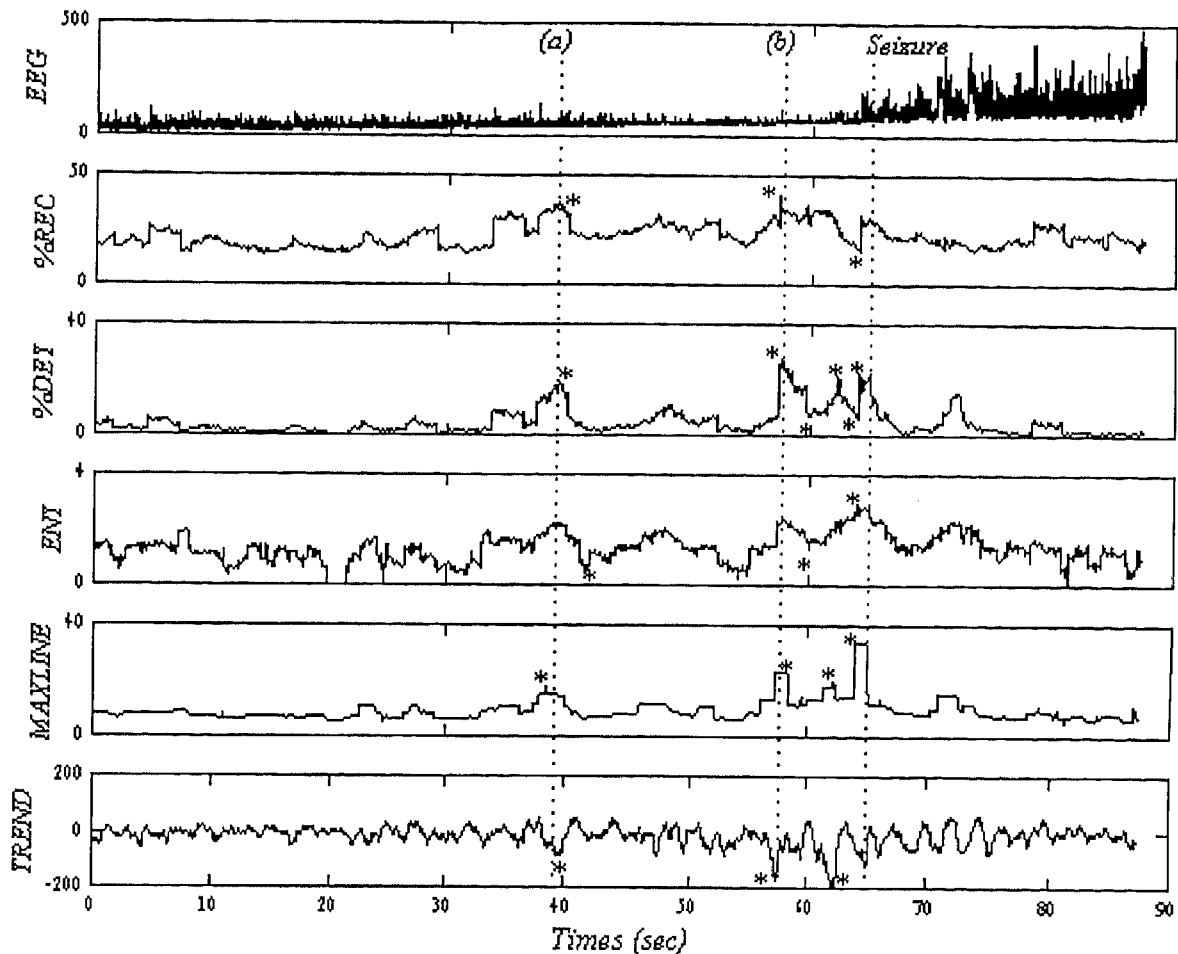


Figure 6: Results of the windowed RQA for Figure 4. Variations of the five RQA variables are depicted: %REC, %DET, ENT, MAXLINE, TREND. Dotted lines indicate main changes in dynamics related to RP features and electrophysiological event descriptions. (a) A drop in values of RQA variables (except TREND where it is reciprocal) is observed. This change is related to background EEG activity resulting from eye-blinking. (b) A general increase in RQA values is observed with many significant transients (denoted by stars). The ictal period is indicated by "Seizure", and corresponds to a decrease toward small values of RQA variables.

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