Canonical Correlation

$\rightarrow r(^{1}\mathbf{y}, {}^{2}\mathbf{y}) \leftarrow$

$^{1}\mathbf{X}_{IJ}$ $^{1}\mathbf{w}_{J}$ $=$ $\approx$ $=$

$^{2}\mathbf{w}_{K}$ Canonical weights for K

$^{2}\mathbf{w}_{J}$

Canonical weights for J (side 2)

$^{1}\mathbf{X}_{IJ}\mathbf{w}_{J}$ $^{1}\mathbf{y}_{I}$ $^{2}\mathbf{y}_{I}$ $^{2}\mathbf{X}_{IJK}\, {}^{2}\mathbf{w}_{J}\, {}^{2}\mathbf{w}_{K}$

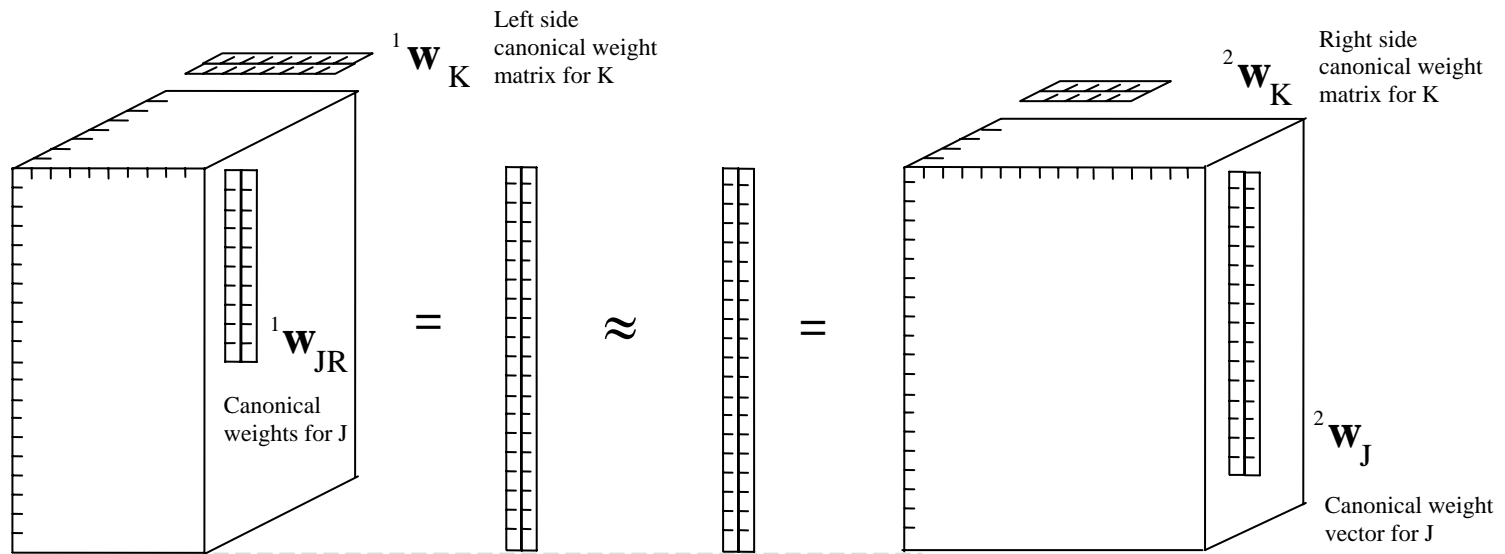| Left side source-data matrix | Left side canonical weights | Left canonical variate | Right canonical variate | Right side source-data *array* | Right side canonical weights for J | Right side canonical weights for K |

Multilinear Canonical Correlation (PARACCON /TUCCON): Level 1a.
Multilinear canonical-weights applied to 3-way data array (here on just one side)

$^1\mathbf{W}_K$ Left side canonical weight matrix for K

$^2\mathbf{W}_K$ Right side canonical weight matrix for K

$^1\mathbf{W}_{JR}$

Canonical weights for J

$=$ $\approx$ $=$

$^2\mathbf{W}_J$

Canonical weight vector for J

$$^1\mathbf{X}_{IJK} \quad * \quad {}^1\mathbf{W}_{JR} \quad = \quad {}^1\mathbf{Y}_{IR} \qquad {}^2\mathbf{Y}_{IR} \qquad {}^2\mathbf{X}_{IJK} \quad * \quad {}^2\mathbf{W}_{JR} \quad * \quad {}^2\mathbf{W}_{KR}$$

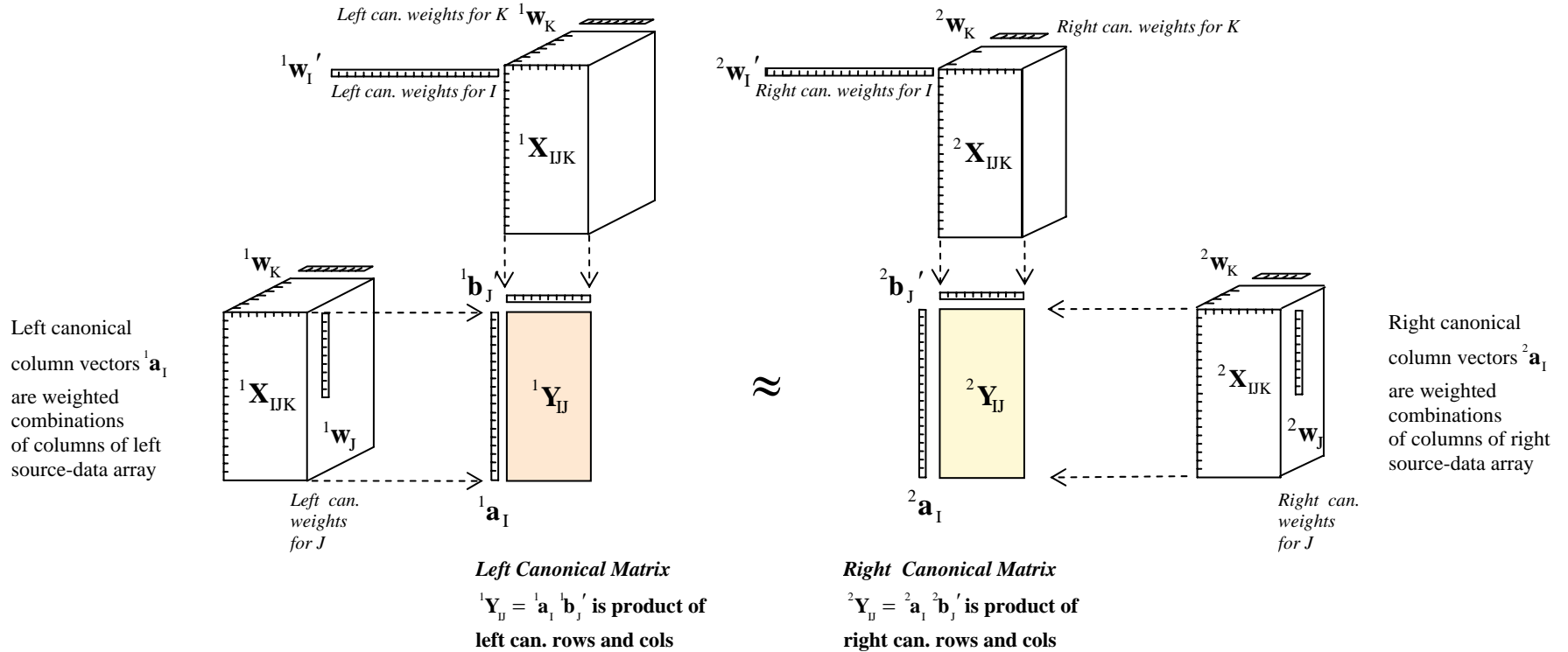| Left side source data array | Left side canonical weights for J | Left side canonical weights for K | Left canonical variate | Right canonical variate | Right side Source data array | Right side canonical weights for J | Right side canonical weights for K |

Multilinear CC: Level 1b. Multilinear Canonical weights and ML data source on both sides (1CC)

Left canonical row vectors
$^1\mathbf{b}_J{}'$ are weighted
combinations of rows of
left source-data array

Right canonical row vectors
$^2\mathbf{b}_J{}'$ are weighted
combinations of rows of
right source-data array

Left can. weights for K $^1\mathbf{w}_K$

$^1\mathbf{w}_I{}'$
Left can. weights for I

$^1\mathbf{X}_{IJK}$

$^2\mathbf{w}_K$ *Right can. weights for K*

$^2\mathbf{w}_I{}'$ *Right can. weights for I*

$^2\mathbf{X}_{IJK}$

$^1\mathbf{w}_K$

$^1\mathbf{b}_J$

Left canonical
column vectors $^1\mathbf{a}_I$
are weighted
combinations
of columns of left
source-data array

$^1\mathbf{X}_{IJK}$

$^1\mathbf{w}_J$

$^1\mathbf{Y}_{IJ}$

$\approx$

$^2\mathbf{b}_J{}'$

$^2\mathbf{Y}_{IJ}$

$^2\mathbf{X}_{IJK}$

$^2\mathbf{w}_K$

$^2\mathbf{w}_J$

Right canonical
column vectors $^2\mathbf{a}_I$
are weighted
combinations
of columns of right
source-data array

*Left can.
weights
for J*

$^1\mathbf{a}_I$

$^2\mathbf{a}_I$

*Right can.
weights
for J*

Left Canonical Matrix
$^1\mathbf{Y}_{IJ} = {}^1\mathbf{a}_I\,{}^1\mathbf{b}_J{}'$ **is product of**
left can. rows and cols

Right Canonical Matrix
$^2\mathbf{Y}_{IJ} = {}^2\mathbf{a}_I\,{}^2\mathbf{b}_J{}'$ **is product of**
right can. rows and cols

$$(^1\mathbf{X}_{IJK}{}^1\mathbf{w}_J{}^1\mathbf{w}_K)\otimes(^1\mathbf{X}_{IJK}{}^1\mathbf{w}_I{}^1\mathbf{w}_K) \;=\; {}^1\mathbf{a}\,^1\mathbf{b}' \;=\; {}^1\mathbf{Y}_{IJ} \;\approx\; {}^2\mathbf{Y}_{IJ} \;=\; {}^2\mathbf{a}\,^2\mathbf{b}' \;=\; (^2\mathbf{X}_{IJK}{}^2\mathbf{w}_J{}^2\mathbf{w}_K)\otimes(^2\mathbf{X}_{IJK}{}^2\mathbf{w}_I{}^2\mathbf{w}_K)$$

ML-CC Level 2b: ML canonical objects, ML canonical weights, and ML source data (1 CC)