Conjoint Analysis

Importing libraries, downloading data, and convert it to the tidy dataset style for online and offline cases

In [55]:
```
import numpy as np
import pandas as pd
import statsmodels.api as sm
import seaborn as sns
import matplotlib.pyplot as plt
from pylab import rcParams
rcParams['figure.figsize'] = 8, 5
import itertools as it
```

In [56]:
```
columns = ["exterior", "size", "price", "strap_pad", "pocket", "interior"]
cat_features = ["exterior", "size", "strap_pad", "pocket", "interior"]
df_online = pd.read_csv('responses_online_1.csv', header=None)
df_offline = pd.read_csv('responses_offline_1.csv', header=None)
df_design = pd.read_csv('design.csv', header=None, names=columns)
```

In [57]:
```
# The fnction to convert datasets to the more useful format

def melt(data: pd.DataFrame) -> pd.DataFrame:
    data['respondent'] = data.index
    return pd.melt(data,
                   var_name='bag',
                   value_name='rate',
                   value_vars=np.arange(20),
                   id_vars=['respondent'])
```

In [58]:
```
# Decoding price and redefining columns names

respondent_bag = melt(df_online)

price_dict = {
    1: 120,
    2: 140,
    3: 160,
    4: 180
}

names = {
    'exterior_1': 'exterior design: black',
    'exterior_2': 'exterior design: reflective',
    'exterior_3': 'exterior design: colorful',
    'exterior_4': 'exterior design: blue',
    'size_1': 'size: small',
    'size_2': 'size: large',
    'price': 'price',
```

```
    'strap_pad_1': 'strap pad: no',
    'strap_pad_2': 'strap pad: yes',
    'pocket_1': 'water bottle pocket: no',
    'pocket_2': 'water bottle pocket: yes',
    'interior_1': 'interior compartment: no dividers',
    'interior_2': 'interior compartment: divider for files',
    'interior_3': 'interior compartment: padded laptop'
}
```

In [59]:

```
# One-hot encoding

def get_dummies_and_price(design: pd.DataFrame) -> pd.DataFrame:
    one_hot_design = pd.get_dummies(design,
                                    drop_first=True,
                                    columns=cat_features, dtype=np.int64)
    one_hot_design["price"] = one_hot_design["price"].apply(lambda x: price_di
    return one_hot_design.rename(columns=names)
```

In [60]:

```
# Dummy coded attributes except price. Price is in $120-$180

bag_attributes_encoded = get_dummies_and_price(df_design)
bag_attributes_encoded['bag'] = bag_attributes_encoded.index
bag_attributes_encoded.head()
```

Out[60]:

	price	exterior design: reflective	exterior design: colorful	exterior design: blue	size: large	strap pad: yes	water bottle pocket: yes	interior compartment: divider for files	con
0	160	0	0	0	0	0	1	0	
1	140	0	0	1	0	1	1	1	
2	120	0	1	0	0	1	0	1	
3	160	1	0	0	0	1	0	0	
4	160	0	1	0	1	0	1	0	

In [61]:

```
# Merging our datasets and checking it for possible errors or NA values

online = respondent_bag.merge(bag_attributes_encoded, on='bag', how='inner')
online['bag'] = online['bag'].astype(int)

online.info()
online.head()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 2440 entries, 0 to 2439
Data columns (total 12 columns):
 #   Column                                 Non-Null Count  Dtype
---  ------                                 --------------  -----
 0   respondent                             2440 non-null   int64
 1   bag                                    2440 non-null   int32
 2   rate                                   2440 non-null   int64
 3   price                                  2440 non-null   int64
 4   exterior design: reflective            2440 non-null   int64
 5   exterior design: colorful              2440 non-null   int64
 6   exterior design: blue                  2440 non-null   int64
 7   size: large                            2440 non-null   int64
 8   strap pad: yes                         2440 non-null   int64
 9   water bottle pocket: yes               2440 non-null   int64
 10  interior compartment: divider for files  2440 non-null   int64
 11  interior compartment: padded laptop    2440 non-null   int64
dtypes: int32(1), int64(11)
memory usage: 219.3 KB
```

Out[61]:

	respondent	bag	rate	price	exterior design: reflective	exterior design: colorful	exterior design: blue	size: large	strap pad: yes	wa bo pocl
0	0	0	1	160	0	0	0	0	0	
1	1	0	1	160	0	0	0	0	0	
2	2	0	2	160	0	0	0	0	0	
3	3	0	1	160	0	0	0	0	0	
4	4	0	3	160	0	0	0	0	0	

In [62]:

```python
# Doing the same for offline case

respondent_bag_f = melt(df_offline)
offline = respondent_bag_f.merge(bag_attributes_encoded, on='bag', how='inner'
offline['bag'] = offline['bag'].astype(int)
offline['rate'] = offline['rate'].astype(np.int64)

offline.info()
offline.head()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 2440 entries, 0 to 2439
Data columns (total 12 columns):
 #   Column                                   Non-Null Count  Dtype
---  ------                                   --------------  -----
 0   respondent                               2440 non-null   int64
 1   bag                                      2440 non-null   int32
 2   rate                                     2440 non-null   int64
 3   price                                    2440 non-null   int64
 4   exterior design: reflective              2440 non-null   int64
 5   exterior design: colorful                2440 non-null   int64
 6   exterior design: blue                    2440 non-null   int64
 7   size: large                              2440 non-null   int64
 8   strap pad: yes                           2440 non-null   int64
 9   water bottle pocket: yes                 2440 non-null   int64
 10  interior compartment: divider for files  2440 non-null   int64
 11  interior compartment: padded laptop      2440 non-null   int64
dtypes: int32(1), int64(11)
memory usage: 219.3 KB
```

Out[62]:

	respondent	bag	rate	price	exterior design: reflective	exterior design: colorful	exterior design: blue	size: large	strap pad: yes	wa bo poc
0	0	0	1	160	0	0	0	0	0	
1	1	0	3	160	0	0	0	0	0	
2	2	0	2	160	0	0	0	0	0	
3	3	0	1	160	0	0	0	0	0	
4	4	0	3	160	0	0	0	0	0	

OLS Estimations online

1) For all users

In [63]:

```
# Run regression over all objects - estimation of the population average partw

X_online = online.drop(['rate', 'respondent', 'bag'], axis=1)
y_online = online['rate']

model = sm.OLS(y_online, sm.add_constant(X_online))
results = model.fit()
print(results.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   rate   R-squared:                       0.276
Model:                            OLS   Adj. R-squared:                  0.274
Method:                 Least Squares   F-statistic:                     103.0
Date:                Thu, 14 Nov 2024   Prob (F-statistic):         1.58e-163
Time:                        23:36:49   Log-Likelihood:                -3682.8
No. Observations:                2440   AIC:                             7386.
Df Residuals:                    2430   BIC:                             7444.
Df Model:                           9
Covariance Type:            nonrobust
===============================================================================
============================
                                             coef    std err          t
P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------
----------------------------
const                                      3.7232      0.172     21.701
0.000       3.387       4.060
price                                     -0.0111      0.001    -10.945
0.000      -0.013      -0.009
exterior design: reflective               -0.3135      0.070     -4.465
0.000      -0.451      -0.176
exterior design: colorful                 -1.0642      0.065    -16.468
0.000      -1.191      -0.937
exterior design: blue                     -0.2199      0.065     -3.403
0.001      -0.347      -0.093
size: large                                0.2680      0.044      6.035
0.000       0.181       0.355
strap pad: yes                             0.5100      0.045     11.296
0.000       0.421       0.599
water bottle pocket: yes                   0.4484      0.044     10.096
0.000       0.361       0.535
interior compartment: divider for files    0.4071      0.057      7.101
0.000       0.295       0.520
interior compartment: padded laptop        0.6192      0.054     11.413
0.000       0.513       0.726
==============================================================================
Omnibus:                       59.817   Durbin-Watson:                   1.994
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               41.908
Skew:                           0.210   Prob(JB):                     7.94e-10
Kurtosis:                       2.515   Cond. No.                     1.19e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctl
y specified.
[2] The condition number is large, 1.19e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

As we can see above, all the coefficients are significant (p-values are less than 0.05). For example, the coefficient on size: large is associated with the incresing of rating by 0.268 if a bag is large. We can highlight the properties of a bag which statistically positively affect the rating: large size, with strap pad, with water bottle pocket, with divider for files, and with padding for a laptop. It is not surprising because this features increase the comfort of using a bag. However, the price and some design elements (especially the color ones) lower the bag's rating. Consumers prefer black bags and are sensitive to price increases. It is also quite intuitive because people tend to wear things in neutral tones such as black, white or gray and not to spend a lot of money on not too important things.

Some conclusions: online consumers tend to prefer neutral colored not very expensive large bags with functional improvements that increase comfort and convenience (especially for laptops because nowadays a lot of people carry it all times for work, for example).

2) For individual users

In [64]:
```
# Run regressions for each individual -  estimations of the individual online

Individual_level_OLS = [] # list for OLS result for each respondent

respondents = online.respondent.unique()
Individual_level_OLS = {}

for respondent in respondents:
    X = X_online[online.respondent == respondent] # choose the needed responde
    y = y_online[online.respondent == respondent]
    model = sm.OLS(y, sm.add_constant(X))
    results = model.fit()
    Individual_level_OLS[respondent] = results
```

In [65]:
```
# Computing metrics to analyze

# Individual_level_params is a table with coefficients for each respondent.
# Individual_level_std is a table with standard errors for each respondent's c

Individual_level_params = pd.DataFrame(index = respondents,
                                       columns = ['const'] + X_online.columns.

Individual_level_std = pd.DataFrame(index = respondents,
                                    columns = ['std_const'] + (['std_'] + X

for respondent in respondents:
    Individual_level_params.loc[respondent] = Individual_level_OLS[respondent]
    Individual_level_std.loc[respondent] = Individual_level_OLS[respondent].bs
```

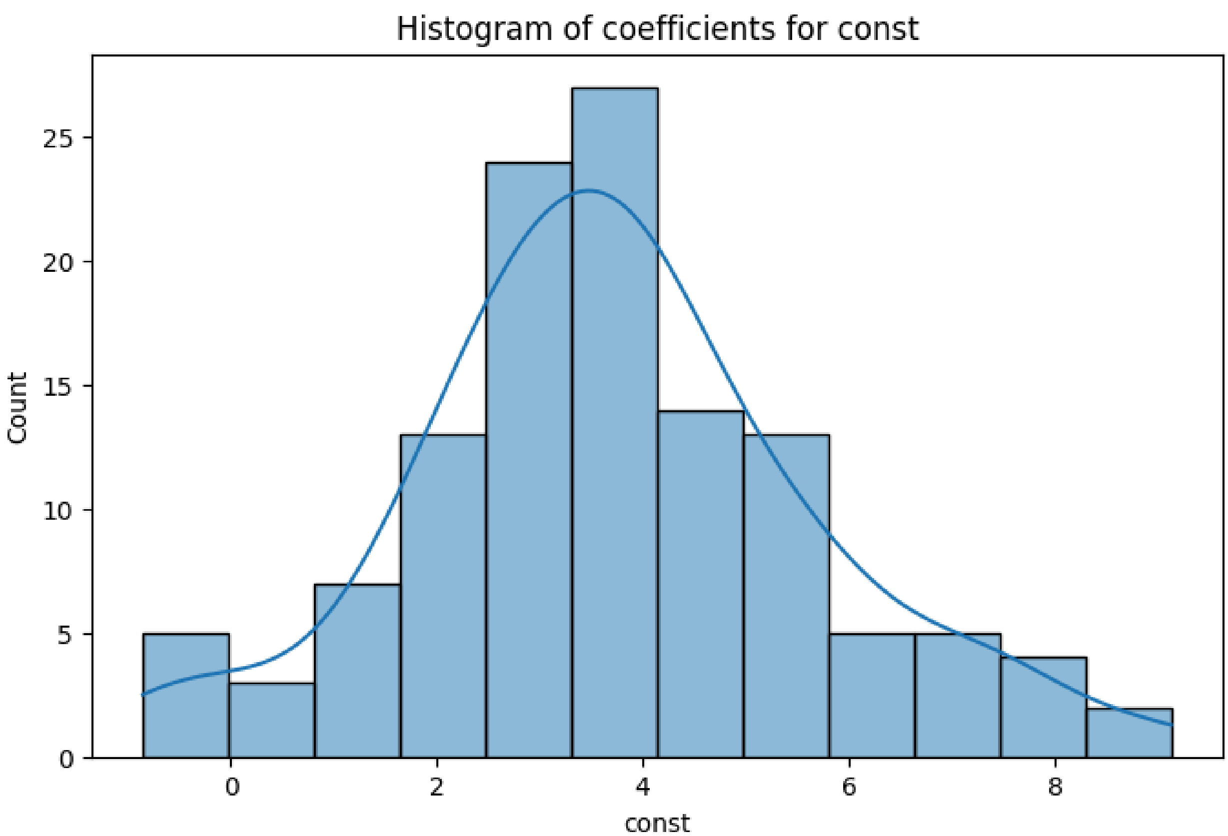
In [66]:
```
Individual_level_params.head()
```

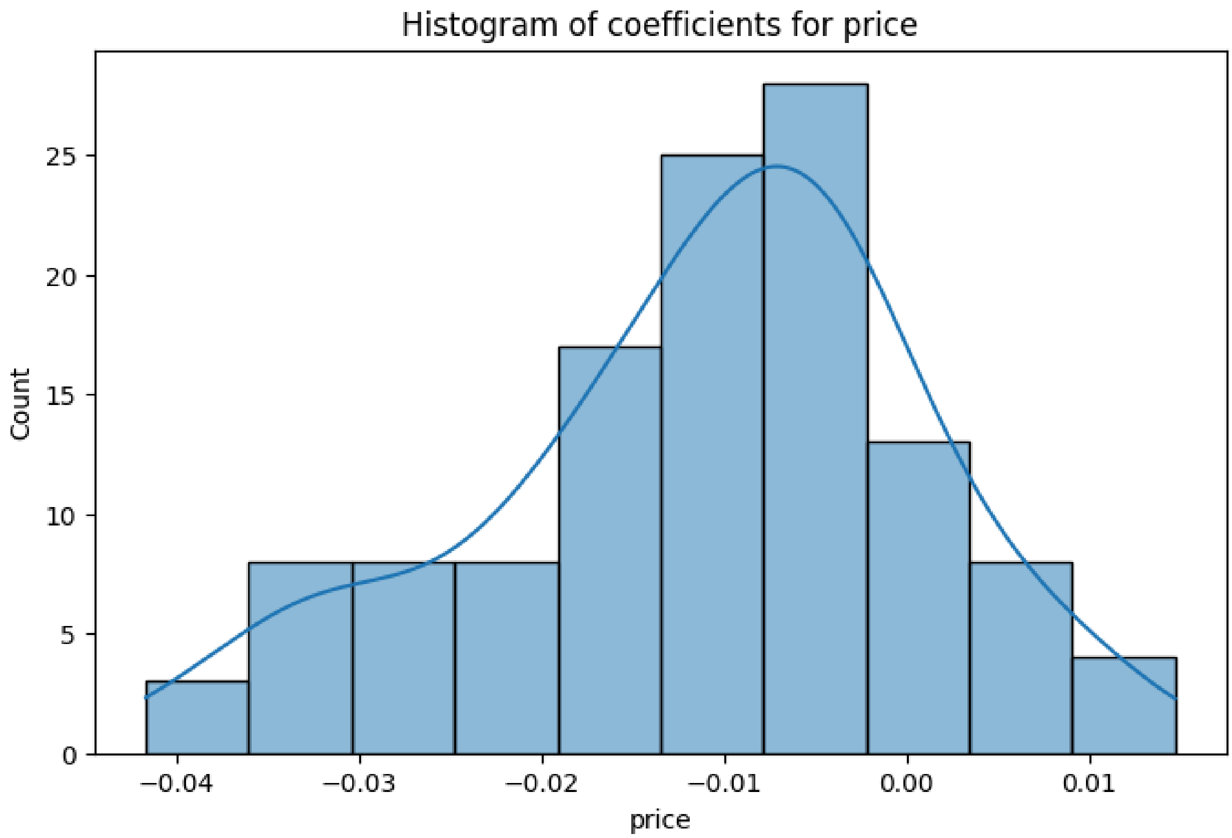
Out[66]:

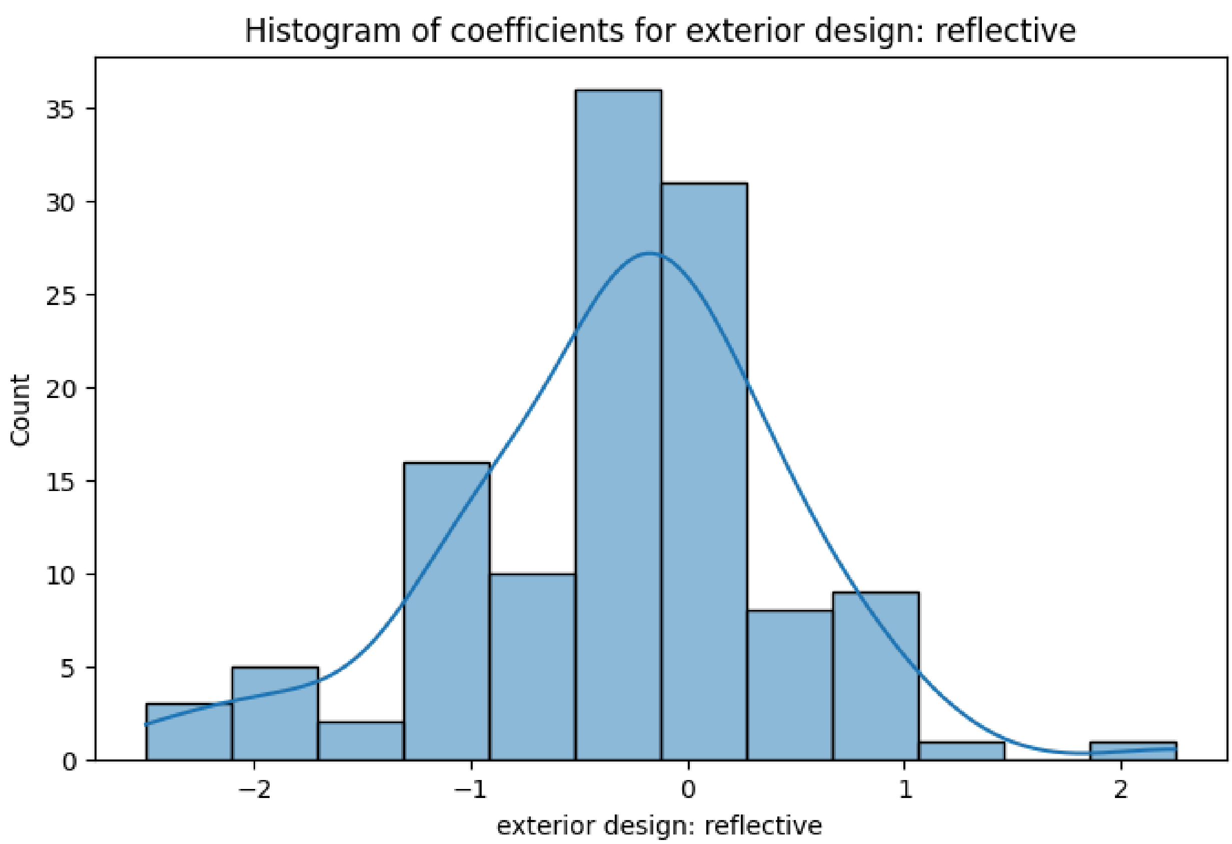
	const	price	exterior design: reflective	exterior design: colorful	exterior design: blue	size: large	strap pad: yes	water bottle pocket: yes
0	3.673077	-0.018269	0.0	-0.397436	-0.064103	0.6	0.653846	0.4
1	3.138862	-0.017007	0.25	0.064503	-0.26883	0.1	0.963942	0.7
2	7.333894	-0.030349	-0.25	-1.354567	-0.354567	0.1	0.257212	0.1
3	2.928846	-0.013462	-0.25	-0.580128	-0.246795	0.9	0.192308	0.3
4	2.747196	-0.004507	0.0	0.60617	0.439503	-0.3	0.463942	0.3

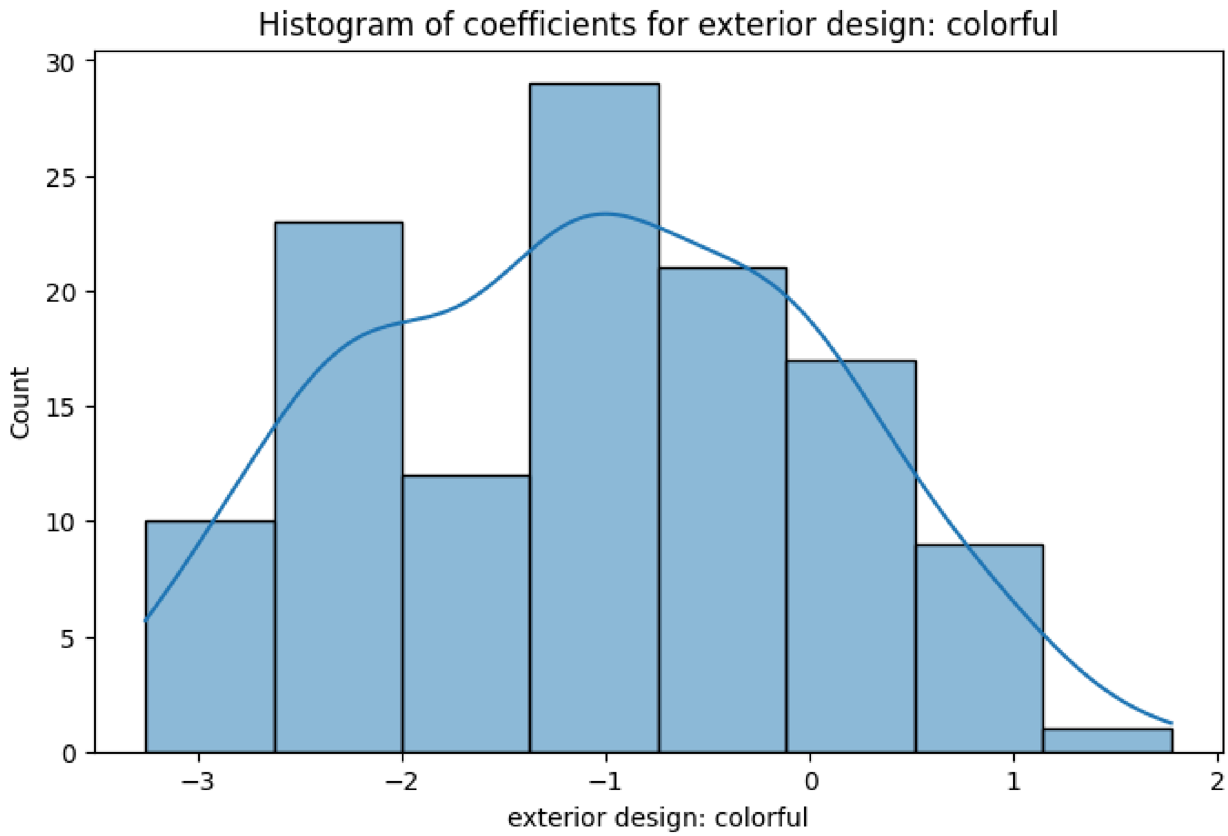
In [67]:

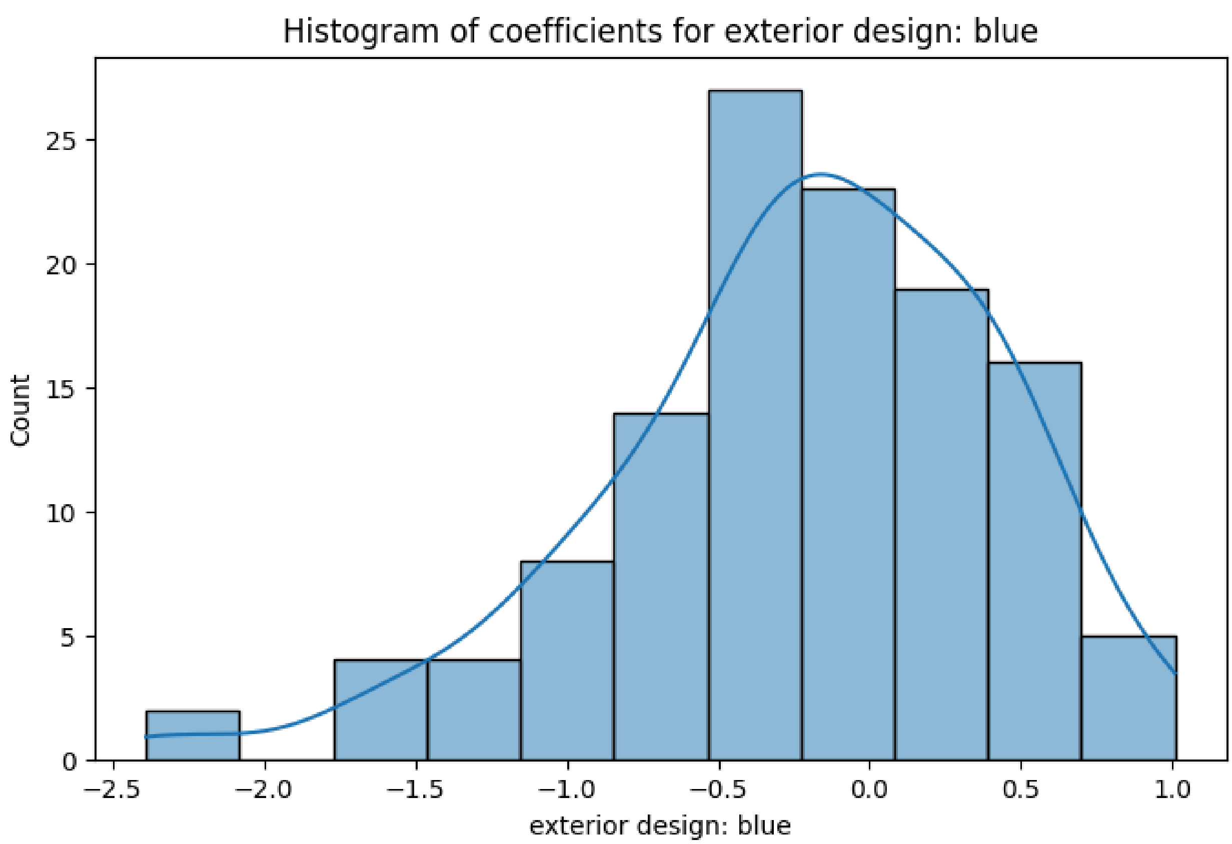
```
for col in Individual_level_params.columns:
    sns.histplot(Individual_level_params[col], kde=True)
    plt.title(f'Histogram of coefficients for {col}')
    plt.show()
```

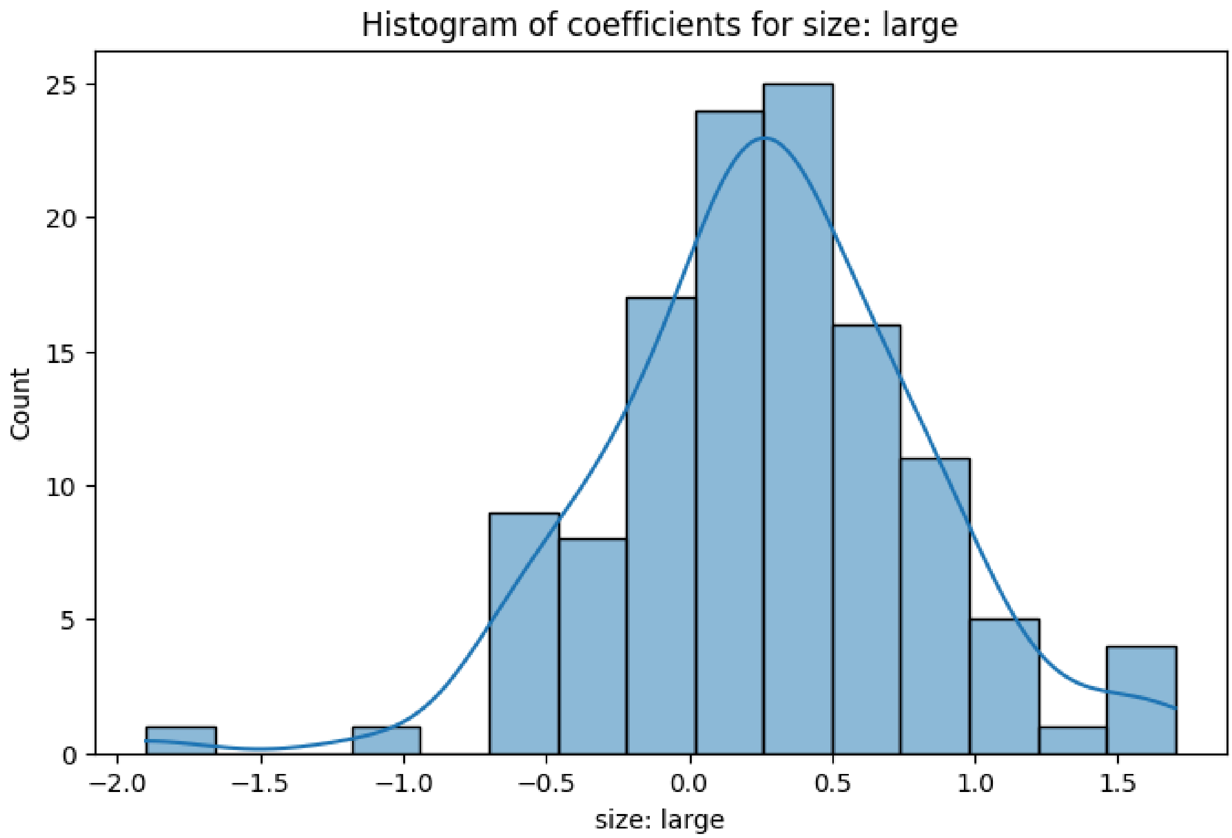


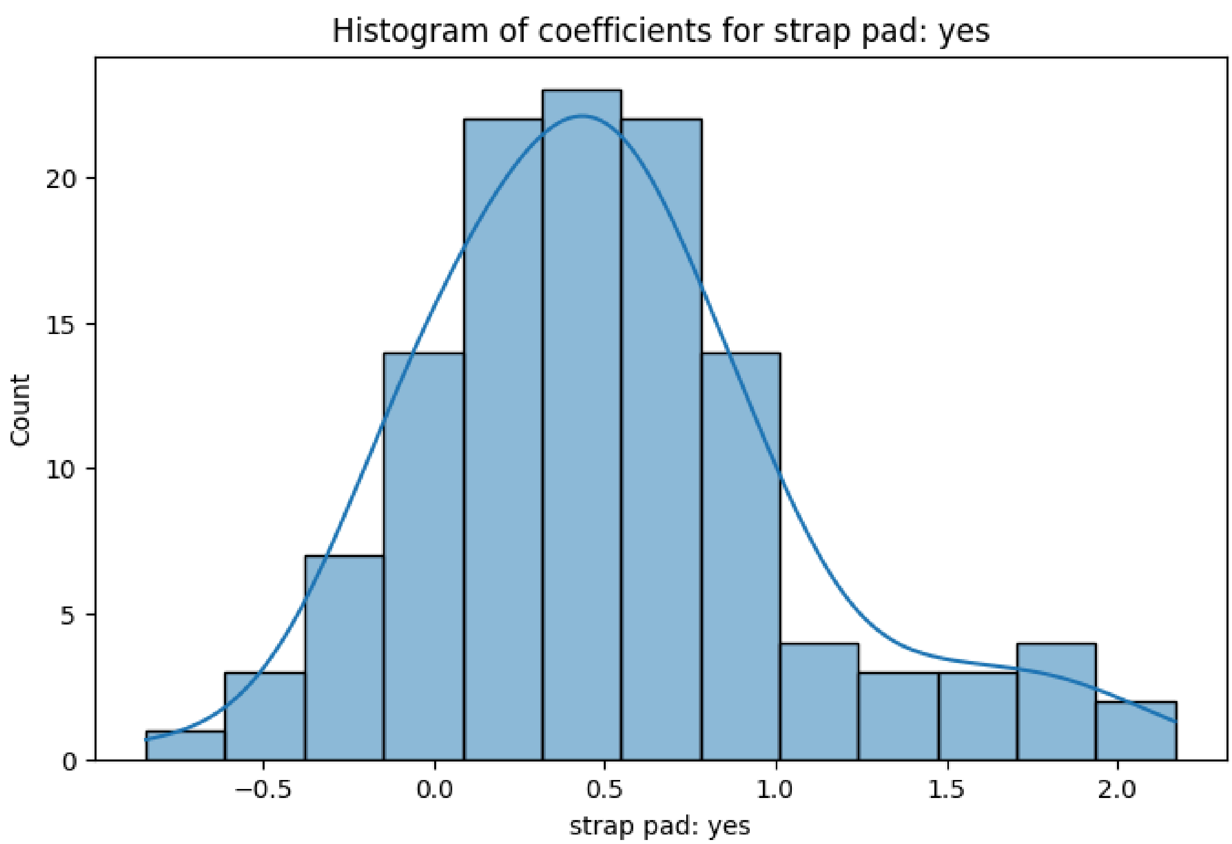

Histogram of coefficients for price
Count
25
20
15
10
5
0
−0.04
−0.03
−0.02
−0.01
0.00
0.01
price

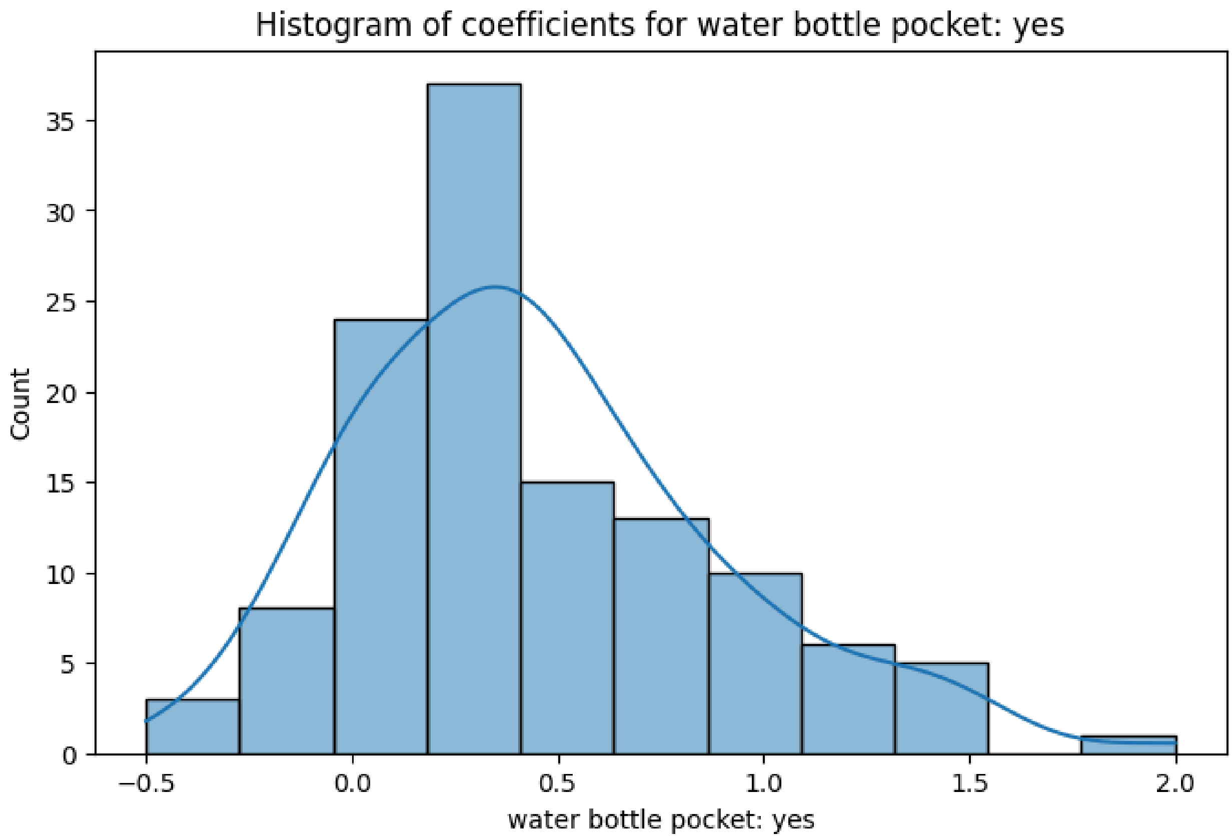

Histogram of coefficients for exterior design: reflective
Count
35
30
25
20
15
10
5
0
−2
−1
0
1
2
exterior design: reflective

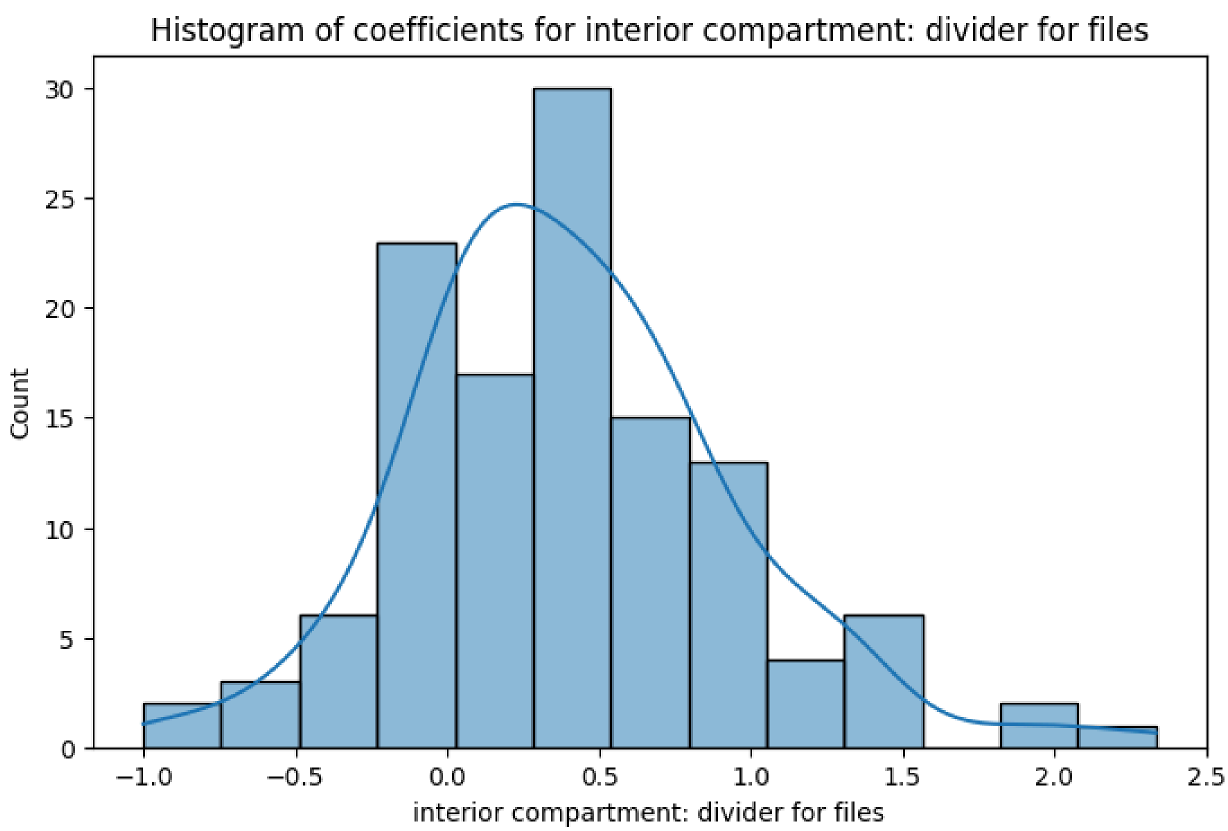

Histogram of coefficients for exterior design: colorful
Count
exterior design: colorful

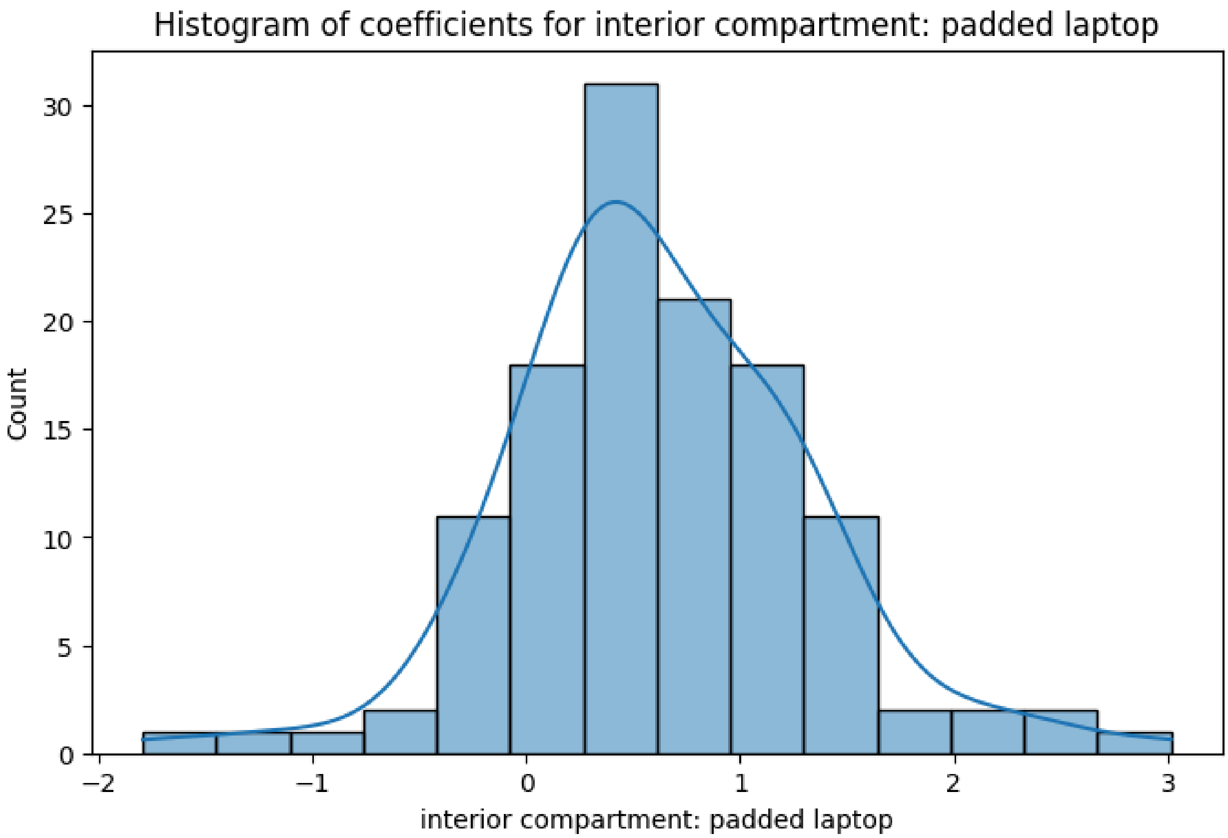

Histogram of coefficients for exterior design: blue
Count
exterior design: blue


Histogram of coefficients for size: large
Count
size: large
0
5
10
15
20
25
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

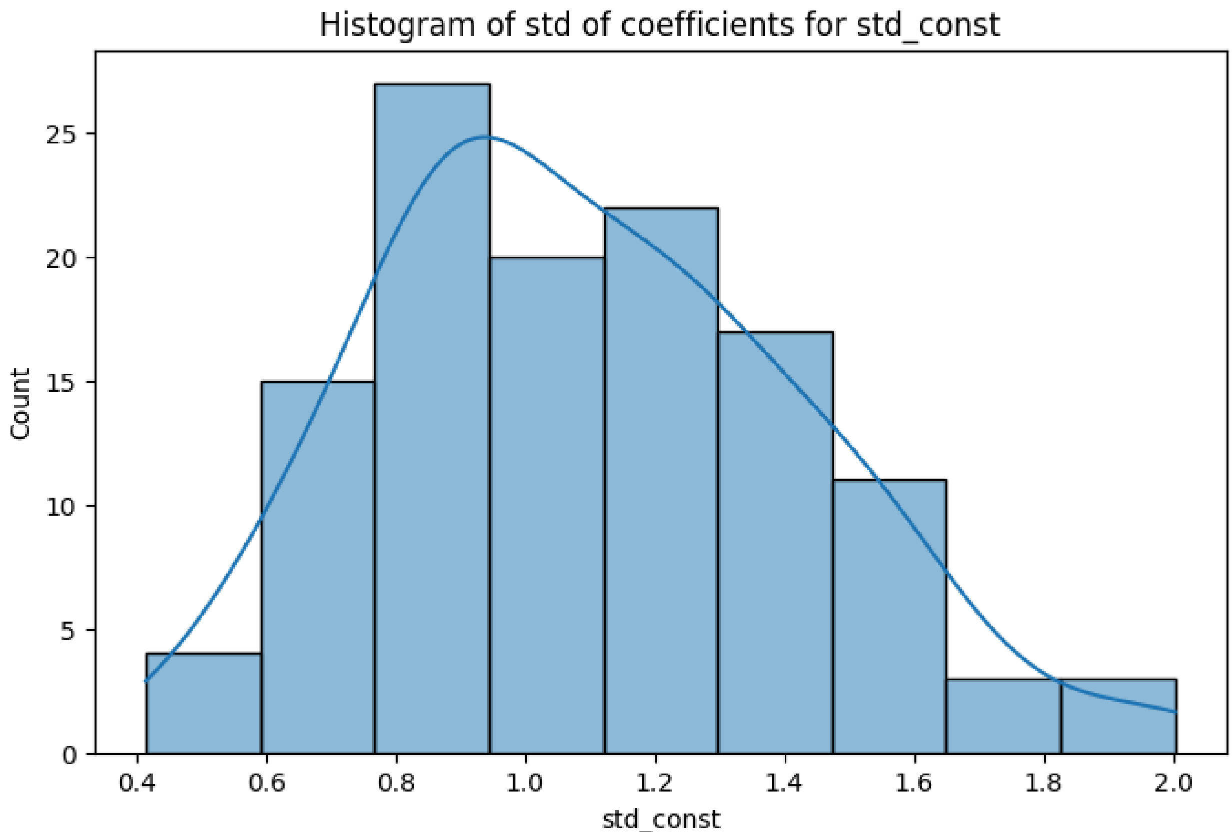

Histogram of coefficients for strap pad: yes
Count
strap pad: yes
0
5
10
15
20
−0.5
0.0
0.5
1.0
1.5
2.0

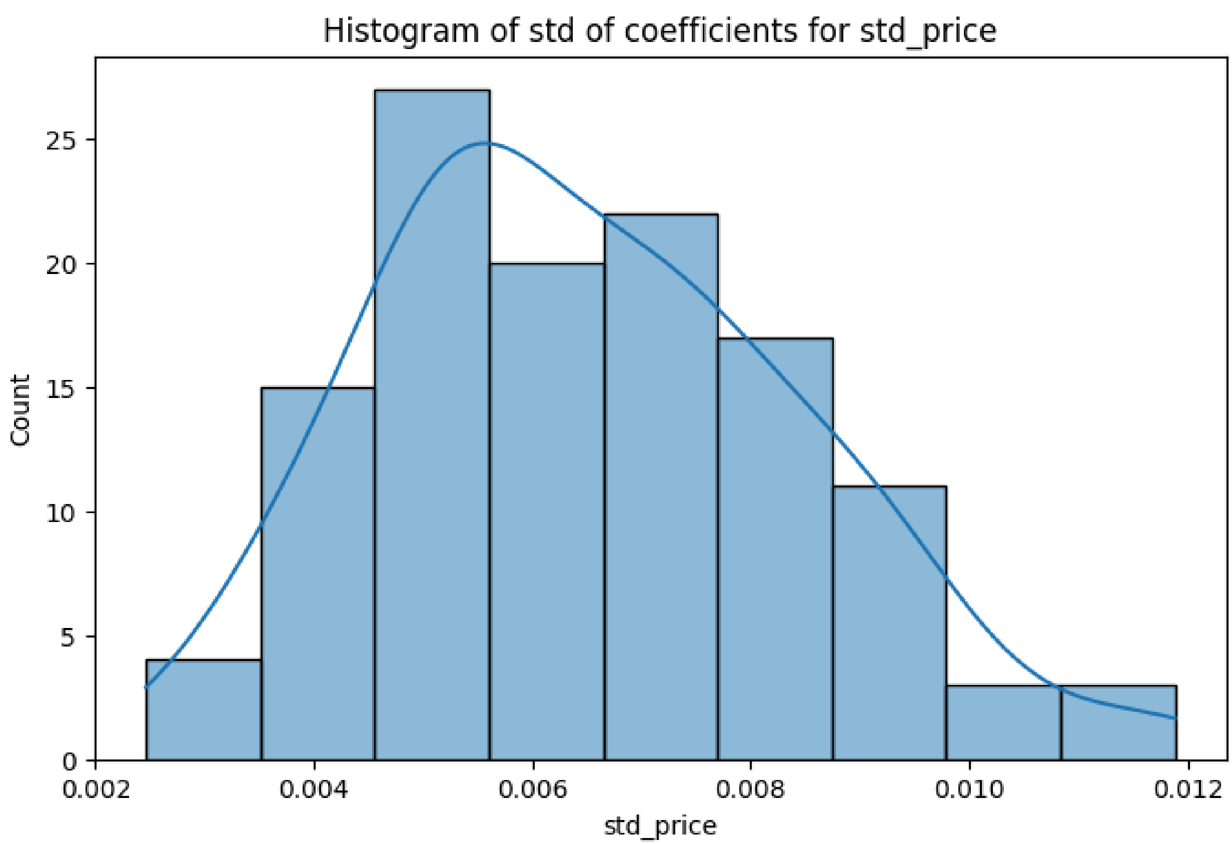

Histogram of coefficients for water bottle pocket: yes
Count
water bottle pocket: yes
0
5
10
15
20
25
30
35
−0.5
0.0
0.5
1.0
1.5
2.0

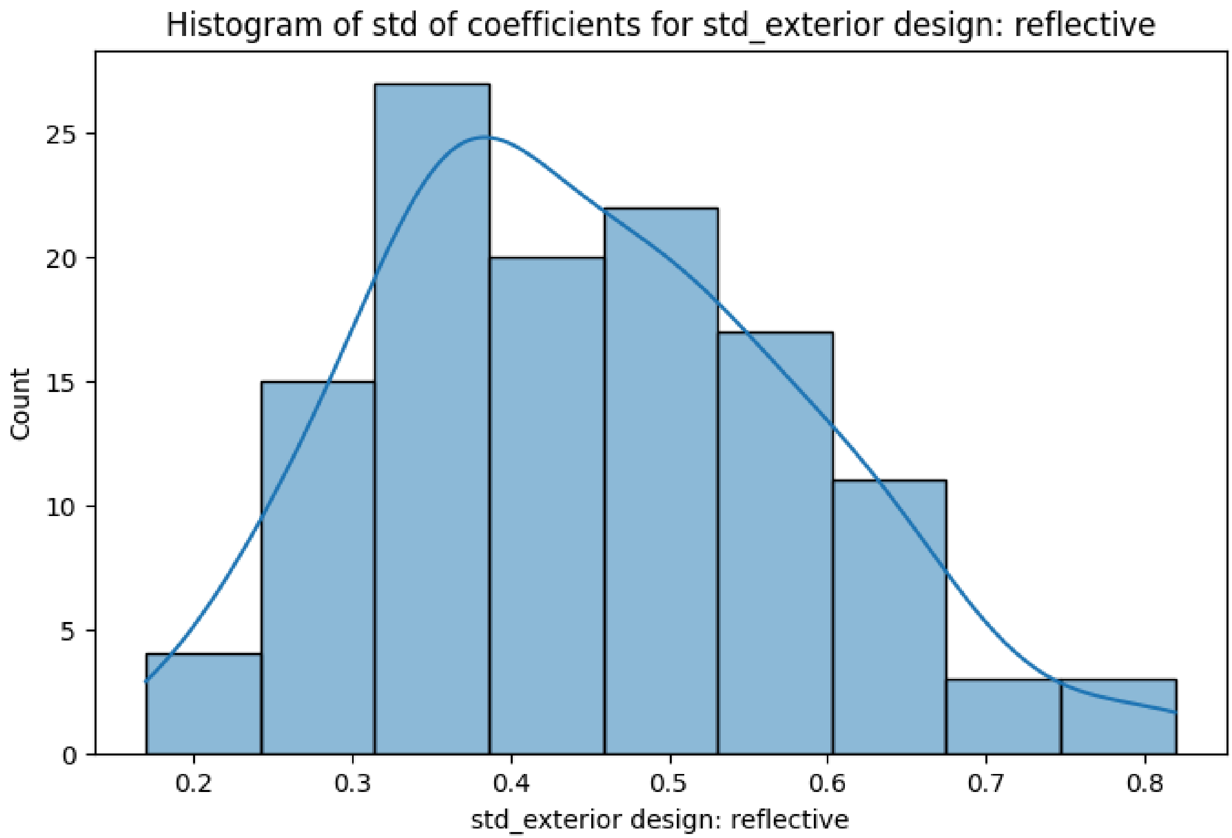

Histogram of coefficients for interior compartment: divider for files
Count
interior compartment: divider for files
0
5
10
15
20
25
30
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5



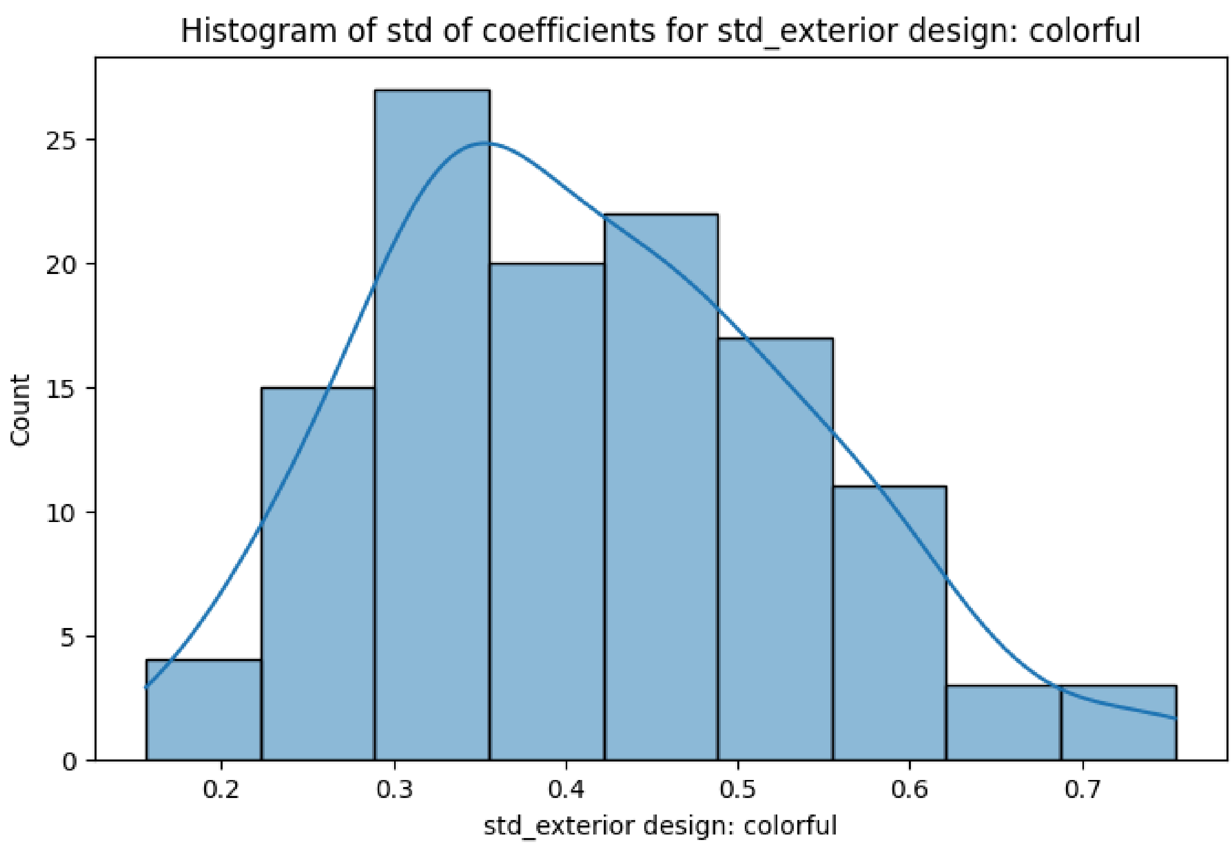
In [68]:
```
for col in Individual_level_std.columns:
    sns.histplot(Individual_level_std[col], kde=True)
    plt.title(f'Histogram of std of coefficients for {col}')
    plt.show()
```


Histogram of std of coefficients for std_const
Count
std_const
0
5
10
15
20
25
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0

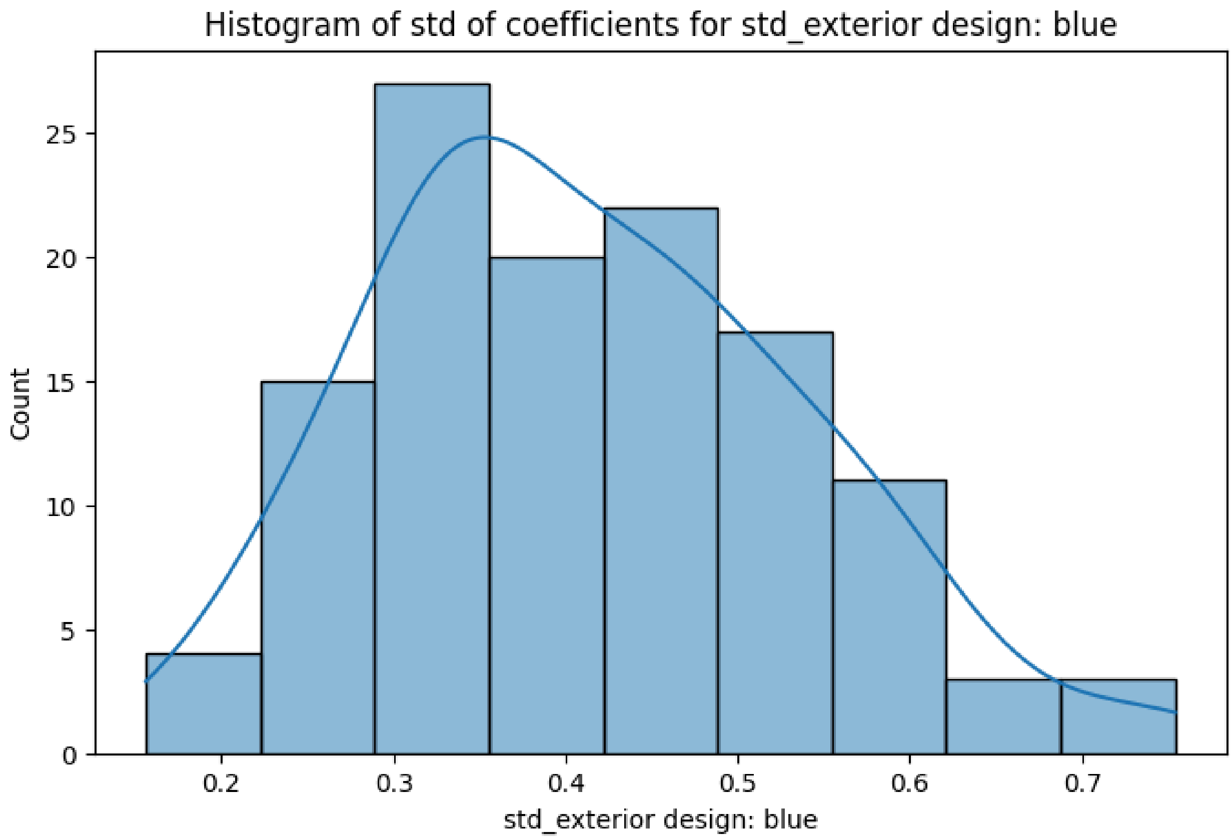

Histogram of std of coefficients for std_price
Count
std_price
0
5
10
15
20
25
0.002
0.004
0.006
0.008
0.010
0.012

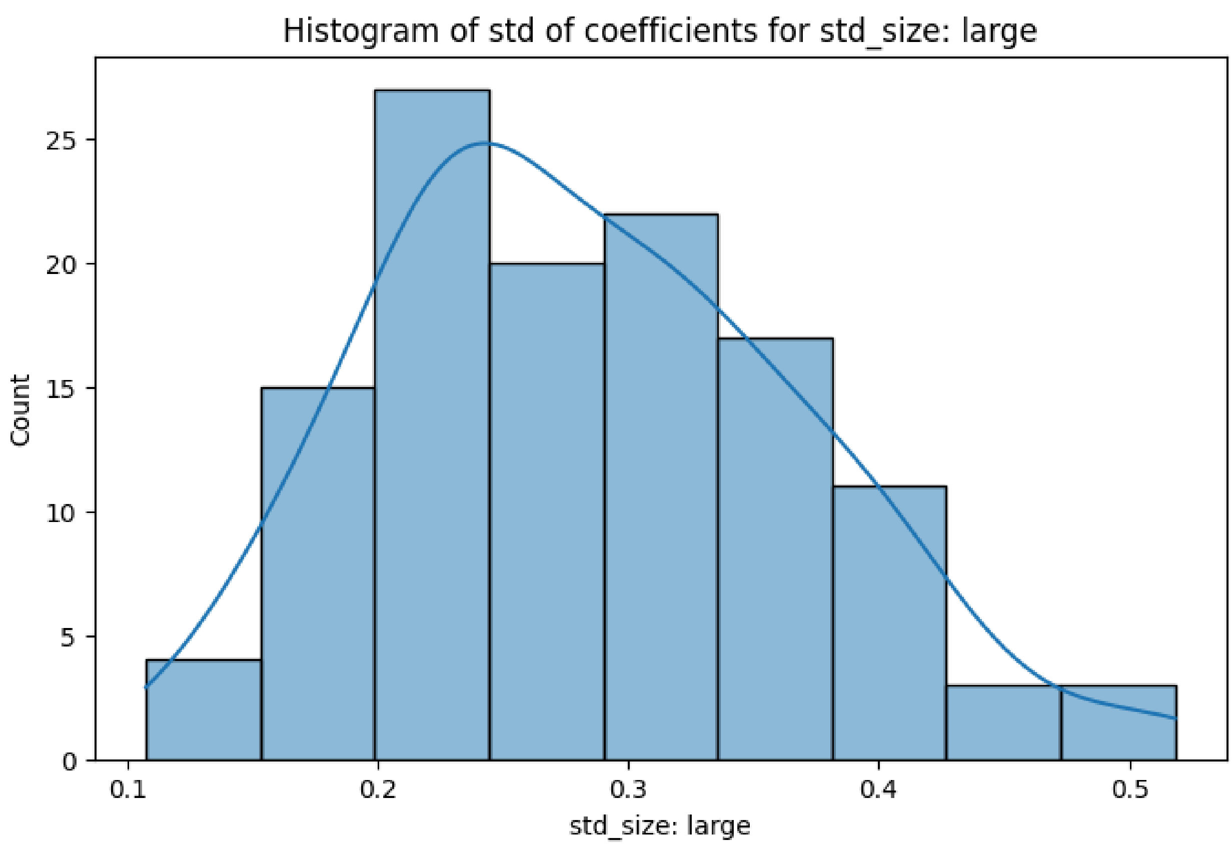


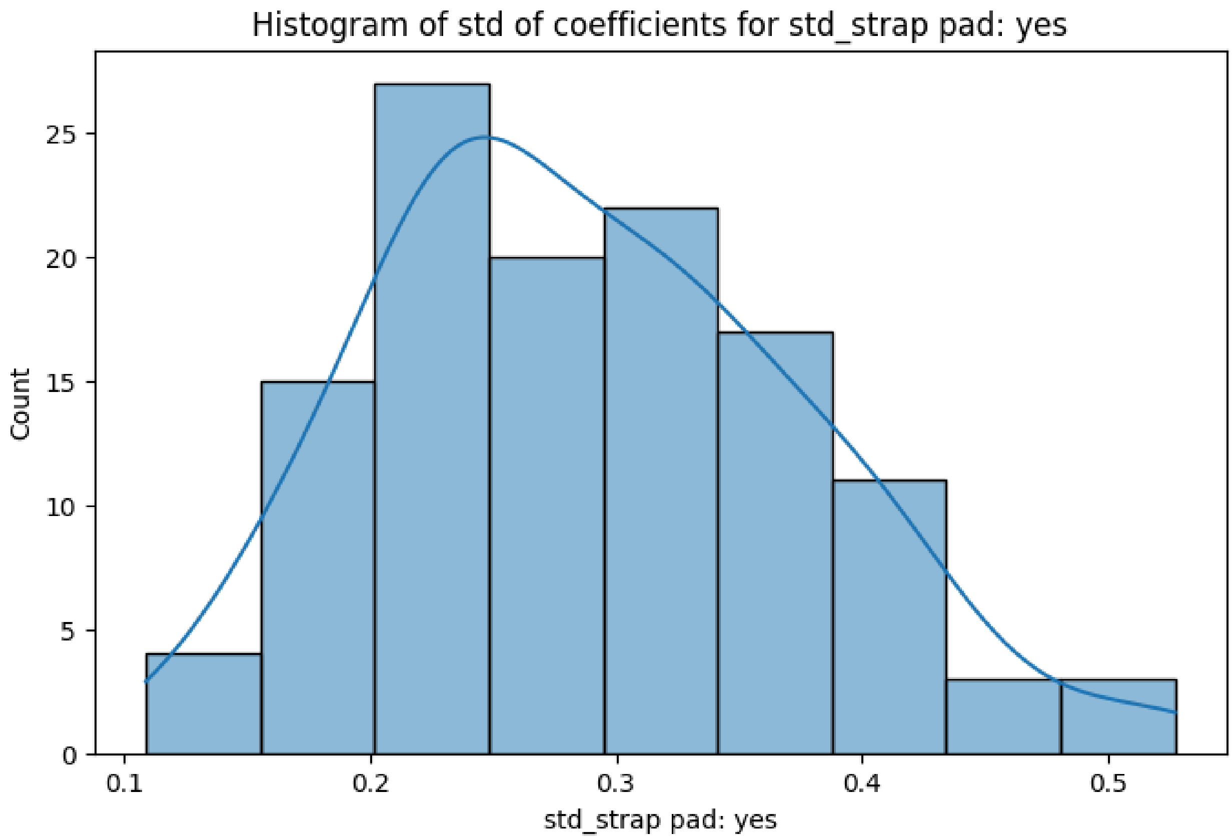
Histogram of std of coefficients for std_exterior design: reflective
Count
0
5
10
15
20
25
0.2
0.3
0.4
0.5
0.6
0.7
0.8
std_exterior design: reflective

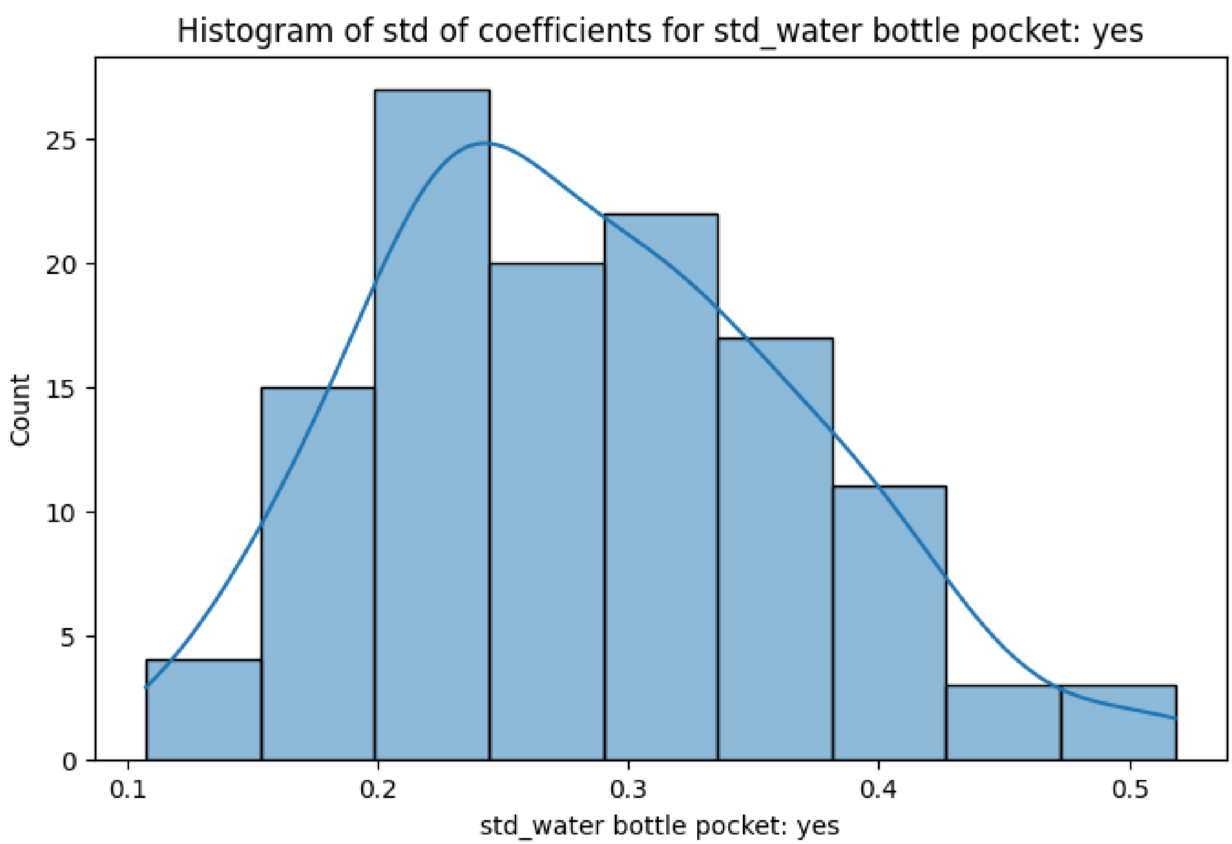


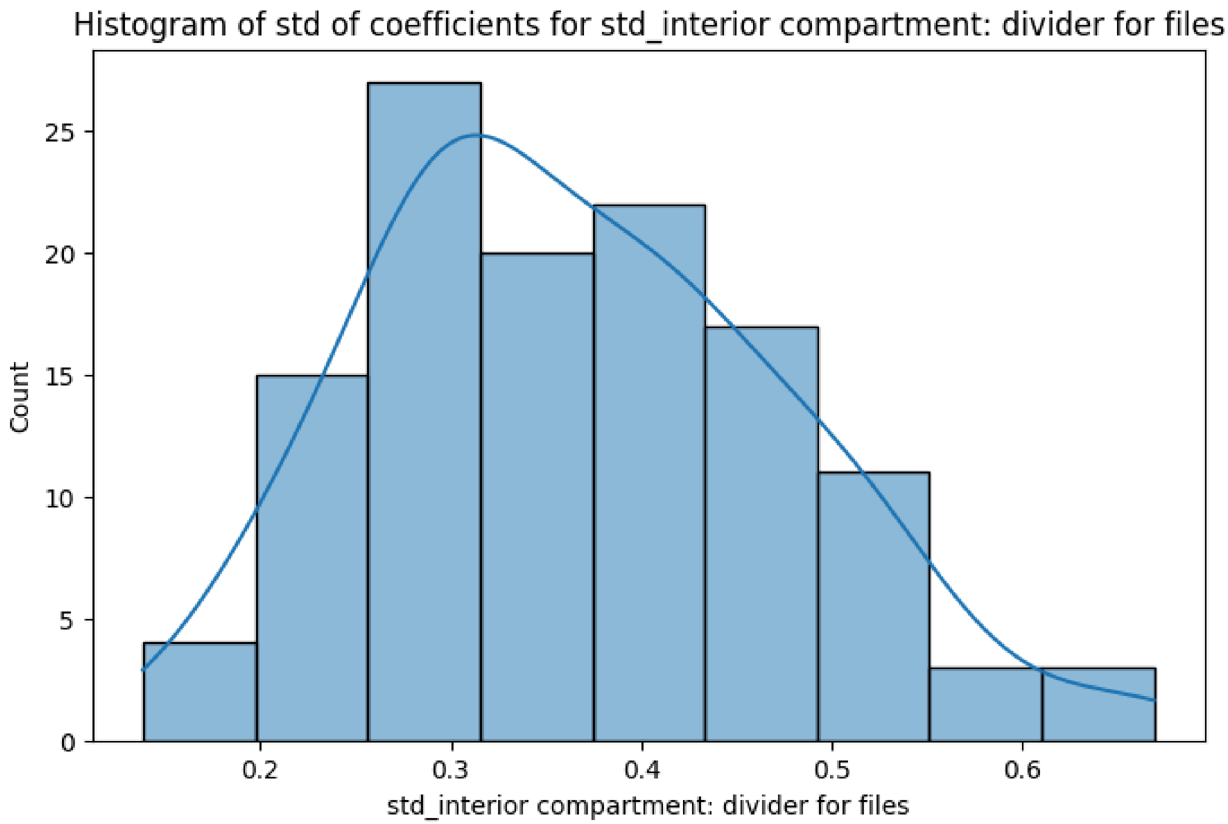
Histogram of std of coefficients for std_exterior design: colorful
Count
0
5
10
15
20
25
0.2
0.3
0.4
0.5
0.6
0.7
std_exterior design: colorful

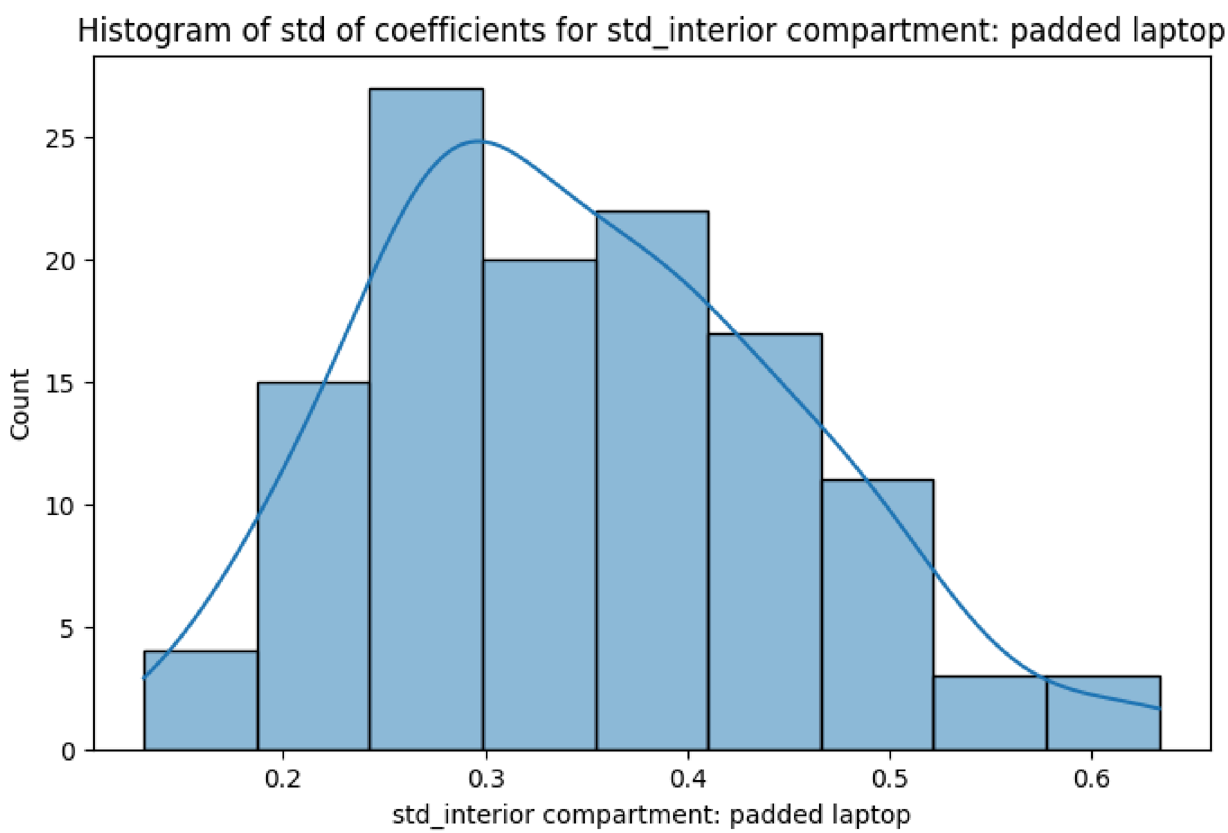

Histogram of std of coefficients for std_exterior design: blue
Count
0
5
10
15
20
25
0.2
0.3
0.4
0.5
0.6
0.7
std_exterior design: blue


Histogram of std of coefficients for std_size: large
Count
0
5
10
15
20
25
0.1
0.2
0.3
0.4
0.5
std_size: large


Histogram of std of coefficients for std_strap pad: yes
Count
std_strap pad: yes
0
5
10
15
20
25
0.1
0.2
0.3
0.4
0.5


Histogram of std of coefficients for std_water bottle pocket: yes
Count
std_water bottle pocket: yes
0
5
10
15
20
25
0.1
0.2
0.3
0.4
0.5





In [69]:
```
# Summarization and taking the mean (examples)

print("Average coef on price", Individual_level_params['price'].mean())
```

```
print("Average coef on prie", Individual_level_params['exterior design: colorf
```

```
Average coef on price -0.011146358764186619
Average coef on prie -1.064194514501884
```

A low standard deviation for any attribute indicates that most participants rate it about the same. A high standard deviation indicates significant differences in tastes. The histograms on std show well the average of std of coefficients (the peak of the distrubution line). In our case std is quite high for any feature, it shows us that among our sample the preferences are heterogeneous.

For each attribute level, I plotted the coefficient histograms for all respondents. This gave me how the preferences of the participants are distributed. If the distribution is highly stretched or has several peaks, this indicates heterogeneity in preferences. We can see that 'colorful', for example, has two peaks which are separated by some other data. Moreover, the histograms are quite stretched with two peaks in some cases. It allows us to suppose that the group (a sample of people) was at least quite heterogeneous.

it is easy to notice that the average of all the individuals coefficients by one feature is equal to the result form the regression on the all sample objects.

Revenue-maximizing strategies

Revenue-maximizing configuration

In this simple case we assume that $u_{ij} = \hat{\beta}^{OLS}_i x_j$ - utility and that user i will buy the bag j if $\hat{\beta}^{OLS}_i x_j >= 3$. 3 is the most logical constant in this case because it is the nearest number more than the mean of the possible rates.

In [70]:
```
# For each configuration of the bag we calculate the total revenue and find th

all_bags = list(it.product([1, 2, 3, 4], [1, 2], [1, 2, 3, 4],
               [1, 2], [1, 2], [1, 2, 3])) # all possible allocations
all_bags_df = pd.DataFrame(all_bags, columns=columns)
all_bags_full = sm.add_constant(get_dummies_and_price(all_bags_df))
```

In [71]:
```
def max_revenue_bag(coeffs: pd.DataFrame, bags: pd.DataFrame, threshold: float
    res = coeffs.to_numpy() @ bags.to_numpy().T # shape N * all bags combinati
    mask = res >= threshold  # dummy by threshold
    revenues = np.sum(mask, axis = 0) * bags['price'].to_numpy()
    index = revenues.argmax()
    max_revenue = revenues[index]
    return (index, max_revenue)
```

In [72]:
```
index_online, max_revenue_online = max_revenue_bag(Individual_level_params, al
print(f'Max revenue for online: {max_revenue_online}')
print(all_bags_full.loc[index_online])
```

```
Max revenue for online: 15840
const                                           1.0
price                                         160.0
exterior design: reflective                     0.0
exterior design: colorful                       0.0
exterior design: blue                           0.0
size: large                                     1.0
strap pad: yes                                  1.0
water bottle pocket: yes                        1.0
interior compartment: divider for files         0.0
interior compartment: padded laptop             1.0
Name: 83, dtype: float64
```

We use the approach which takes into account individual preferences rather than the one which uses the average coefficient for the entire market (the second strategy to determine the most beneficial configuration in the seminar). The advantages of the approach we realized are that it allows you to accurately model each user's preferences, which can lead to more personalized recommendations (which is better if we are talking about people with heterogeneous preferences). Therefore, if the market is diverse and includes different segments with different preferences, this approach will allow us to more accurately meet the needs of each group, and individual preferences will provide more accurate forecasts and help us choose the products that will bring the most income, which is better suited for the task of optimizing income. Therefore, as I've seen above that preferences of individuals are distributed quite extended with multiple peaks (two peaks - I reckon that if there isn't a highly prominent peak, then we can talk about multiple peaks in preferences), I've decided to use this approach to solve the given problem.

The most important features by this approach are the ones with 1 (which are the configuration points which maximize revenue given this approach). As expected, the color is black. Surprisingly, the price is not the lowest to attract more customers, however, it can be explained in terms of the reason that people prefer to buy things in the middle price range, becuase they often believe that this gives the maximum value for money. Additional features of a bag such as the presense of water bottle pocket and large size are also presented because they provide more comfort and convenience for customers. However, the divider for files is zero. It can be explained as insufficient increase for the invested sources and heterogeneous preferences on this feature (two peaks in the distribution).

Revenue-maximizing product line of two bags

$u_{ij} = \hat{\beta}^{OLS}_i x_j$, where i - respondent, j - bag.

Assumptions for product line of two bags:

1. $u_{i1} > 3$ and $u_{i1} > u_{i2}$. Then the i-th customer chooses the 1 bag.

2. $u_{i2} > 3$ and $u_{i2} > u_{i1}$. Then the i-th customer chooses the 2 bag.

3. Otherwise the customer chooses not to buy anything.

In [73]:

```
all_bags = list(it.product([1, 2, 3, 4], [1, 2], [1, 2, 3, 4], [1, 2], [1, 2],
all_bags_df = pd.DataFrame(all_bags, columns=columns)
all_bags_full = sm.add_constant(get_dummies_and_price(all_bags_df))

def max_revenue_product_line(coeffs: pd.DataFrame, bags: pd.DataFrame, thresho
    bag_combinations = list(it.combinations(range(len(bags)), n)) # all possib
    max_revenue = 0
    best_combination = None

    for item in bag_combinations:
        selected_bags = bags.iloc[list(item)].to_numpy()
        res = coeffs.to_numpy() @ selected_bags.T

        chosen_bags = np.max(res, axis=1) >= threshold  # res[i, j] contains t
        chosen_indices = np.argmax(res, axis=1) # find the bag indices that gi

        revenue = 0
        for i, mask in enumerate(chosen_bags):
            if mask:  # If buying
                chosen_bag_idx = chosen_indices[i]
                bag_price = selected_bags[chosen_bag_idx, bags.columns.get_loc
                revenue += bag_price

        if revenue > max_revenue: # check if this combination has the highest
            max_revenue = revenue
            best_combination = item

    return best_combination, max_revenue

best_comb, max_revenue = max_revenue_product_line(Individual_level_params, all

print(f"Max revenue for online: {max_revenue}\n")
for index in best_comb:
    print(all_bags_full.iloc[index], "\n")
```

```
Max revenue for online: 18020.0

const                                          1.0
price                                        180.0
exterior design: reflective                    0.0
exterior design: colorful                      0.0
exterior design: blue                          0.0
size: large                                    1.0
strap pad: yes                                 1.0
water bottle pocket: yes                       1.0
interior compartment: divider for files        0.0
interior compartment: padded laptop            1.0
Name: 95, dtype: float64

const                                          1.0
price                                        140.0
exterior design: reflective                    0.0
exterior design: colorful                      0.0
exterior design: blue                          1.0
size: large                                    1.0
strap pad: yes                                 0.0
water bottle pocket: yes                       1.0
interior compartment: divider for files        0.0
interior compartment: padded laptop            1.0
Name: 353, dtype: float64
```

Find the revenue-maximizing product line of two bags:

As we can see, these bags have many differences to satisfy as much demand as possible in terms of customer's preferences. As we've noticed before, the preferences are distributed widely, so in order to satisfy the demand for those who are not in the highest peak or near it, the bags are with different characterictics and different price level which provide maximum income in given conditions.

OLS Estimations offline

1) For all users

In [74]:
```
# Run regression over all objects - estimation of the population average partw

X_offline = offline.drop(['rate', 'respondent', 'bag'], axis=1)
y_offline = offline['rate']

model = sm.OLS(y_offline, sm.add_constant(X_offline))
results = model.fit()
print(results.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                   rate   R-squared:                       0.177
Model:                            OLS   Adj. R-squared:                  0.174
Method:                 Least Squares   F-statistic:                     57.96
Date:                Thu, 14 Nov 2024   Prob (F-statistic):           3.03e-96
Time:                        23:37:16   Log-Likelihood:                -3883.5
No. Observations:                2440   AIC:                             7787.
Df Residuals:                    2430   BIC:                             7845.
Df Model:                           9
Covariance Type:            nonrobust
===============================================================================
============================
                                              coef    std err          t
P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------
----------------------------
const                                       3.3905      0.186     18.201
0.000       3.025       3.756
price                                      -0.0076      0.001     -6.836
0.000      -0.010      -0.005
exterior design: reflective                -0.5963      0.076     -7.822
0.000      -0.746      -0.447
exterior design: colorful                  -0.7057      0.070    -10.058
0.000      -0.843      -0.568
exterior design: blue                      -0.1101      0.070     -1.569
0.117      -0.248       0.028
size: large                                -0.3074      0.048     -6.375
0.000      -0.402      -0.213
strap pad: yes                              0.2535      0.049      5.170
0.000       0.157       0.350
water bottle pocket: yes                    0.1697      0.048      3.519
0.000       0.075       0.264
interior compartment: divider for files     0.5191      0.062      8.339
0.000       0.397       0.641
interior compartment: padded laptop         0.8775      0.059     14.896
0.000       0.762       0.993
==============================================================================
Omnibus:                      168.933   Durbin-Watson:                   2.092
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               86.919
Skew:                           0.294   Prob(JB):                     1.34e-19
Kurtosis:                       2.287   Cond. No.                     1.19e+03
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctl
y specified.
[2] The condition number is large, 1.19e+03. This might indicate that there are
strong multicollinearity or other numerical problems.
```

As we can see above, all the coefficients are significant except for exterior design: blue (p-values are less than 0.05). We can suppose that offline customers have more smoothed color preferences. We can highlight the properties of a bag which statistically positively affect the rating: with strap pad, with water bottle pocket, with divider for files, and with padding for a laptop. It is not surprising because this features increase the comfort of using a bag. Consumers prefer black bags and are sensitive to price increases. It is also quite intuitive because people tend to wear things in neutral tones such as black, white or gray and not to spend a lot of money on not too important thongs.

Comparison with the online case:

Online: The price has a stronger negative impact on the rating (-0.0111) than with offline purchases (-0.0076). It can be explained that online shoppers seem to be more sensitive to price changes, perhaps due to easier access to price comparisons and discounts which may increase price awareness. In contrast, offline buyers may be more willing to pay higher prices due to the possibility of physical inspection of goods, which increases the perceived value of a good.

Online: Larger bags increase the rating (0.2680), while the offline rating decreases (-0.3074). The convenience of larger sizes is more appreciated online, where shoppers may assume they need more space when they buy discreetly. However, when offline, a large bag may seem bulky, and consumers can directly find out the size, preferring compact options if the bag seems too bulky to them. This highlights the importance of visual effect and practical considerations.

Offline: Functional features offline have smaller coefficient because offline customers value more the texture, the material (by tactile features), and real looking of a product which can be different (better looking) online. However, offline customers value more padding for a laptop, probably because they can physically test the product and check if this will be suitable for their laptop.

2) For individual

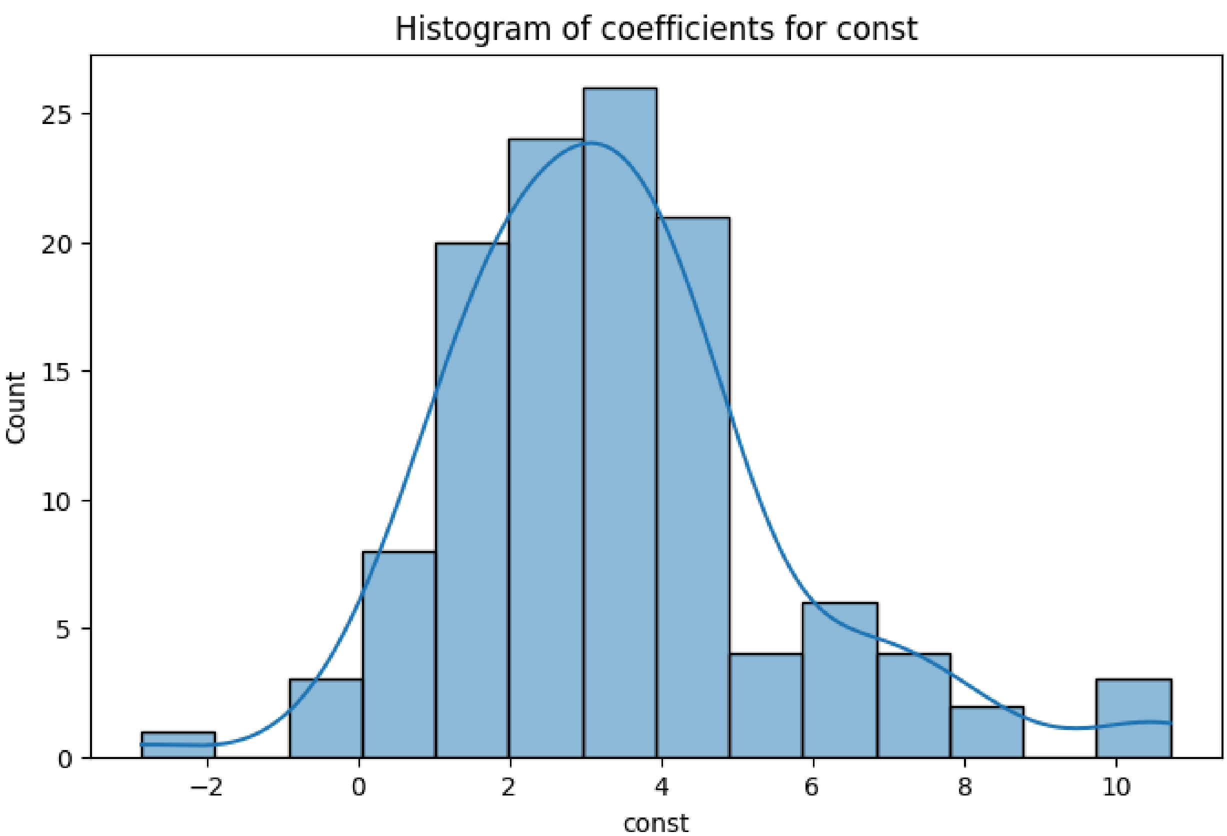
In [75]:
```
# Run regressions for each individual -  estimations of the individual online

Individual_level_OLS_f = [] # list for OLS result for each respondent

respondents_f = offline.respondent.unique()
Individual_level_OLS_f = {}

for respondent in respondents_f:
    X = X_offline[offline.respondent == respondent] # choose the needed respon
    y = y_offline[offline.respondent == respondent]
    model = sm.OLS(y, sm.add_constant(X))
    results = model.fit()
    Individual_level_OLS_f[respondent] = results
```

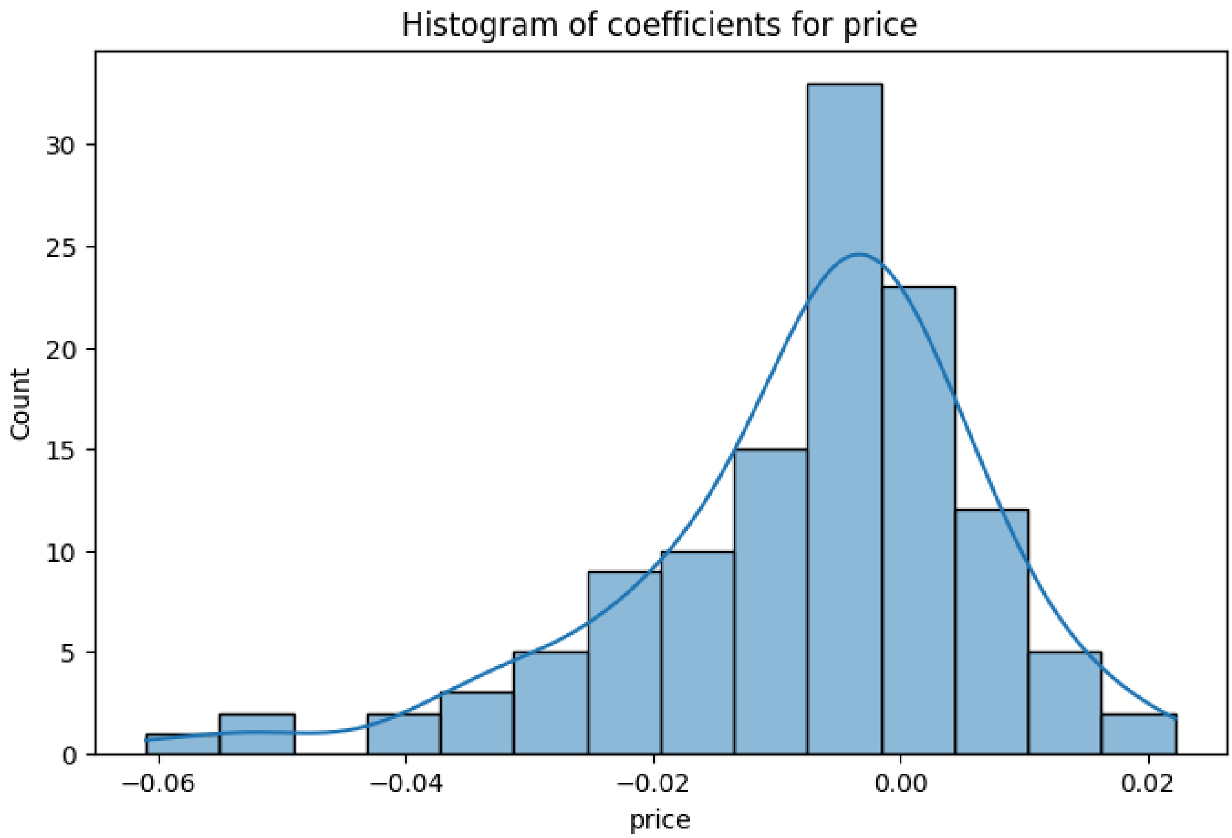
In [76]:
```
# Computing metrics to analyze
```

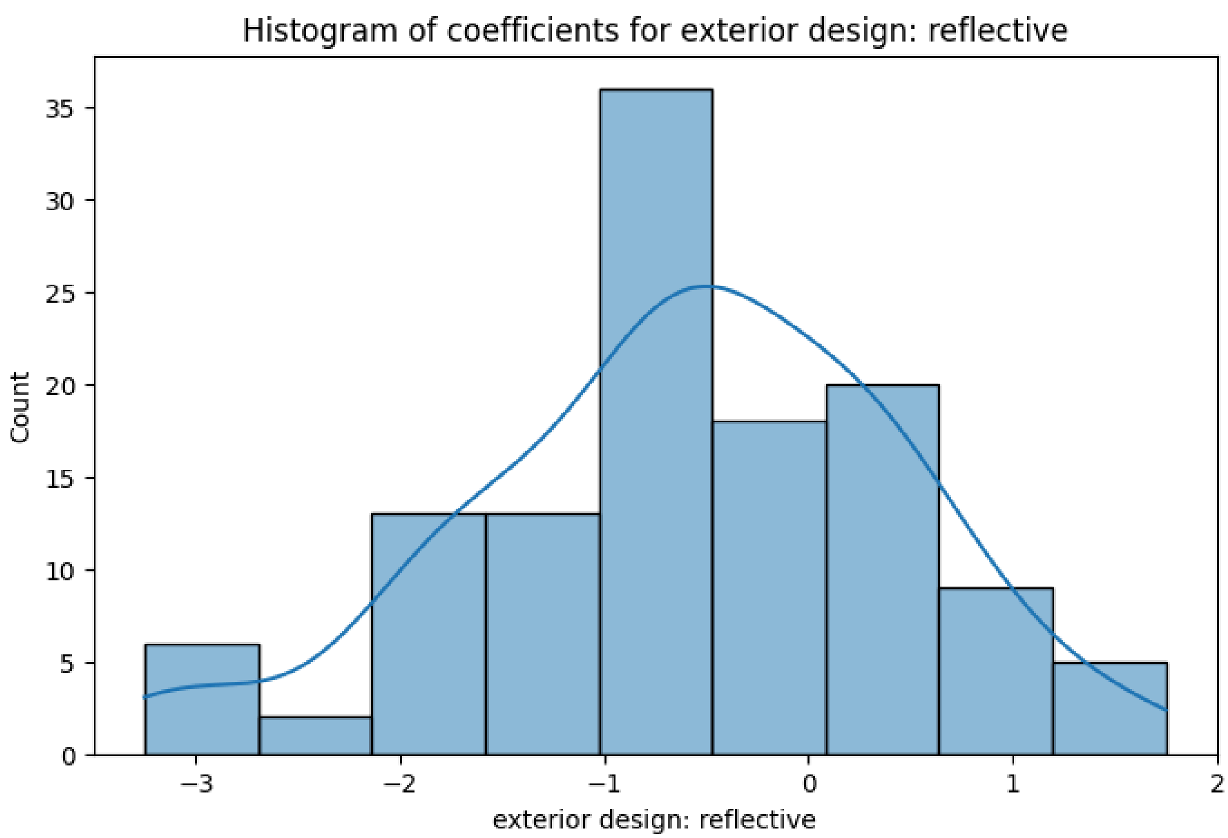
```python
# Individual_level_params is a table with coefficients for each respondent.
# Individual_level_std is a table with standard errors for each respondent's c

Individual_level_params_f = pd.DataFrame(index = respondents_f,
                                         columns = ['const'] + X_online.columns.

Individual_level_std_f = pd.DataFrame(index = respondents_f,
                                      columns = ['std_const'] + (['std_'] + X

for respondent in respondents_f:
    Individual_level_params_f.loc[respondent] = Individual_level_OLS_f[respond
    Individual_level_std_f.loc[respondent] = Individual_level_OLS_f[respondent
```

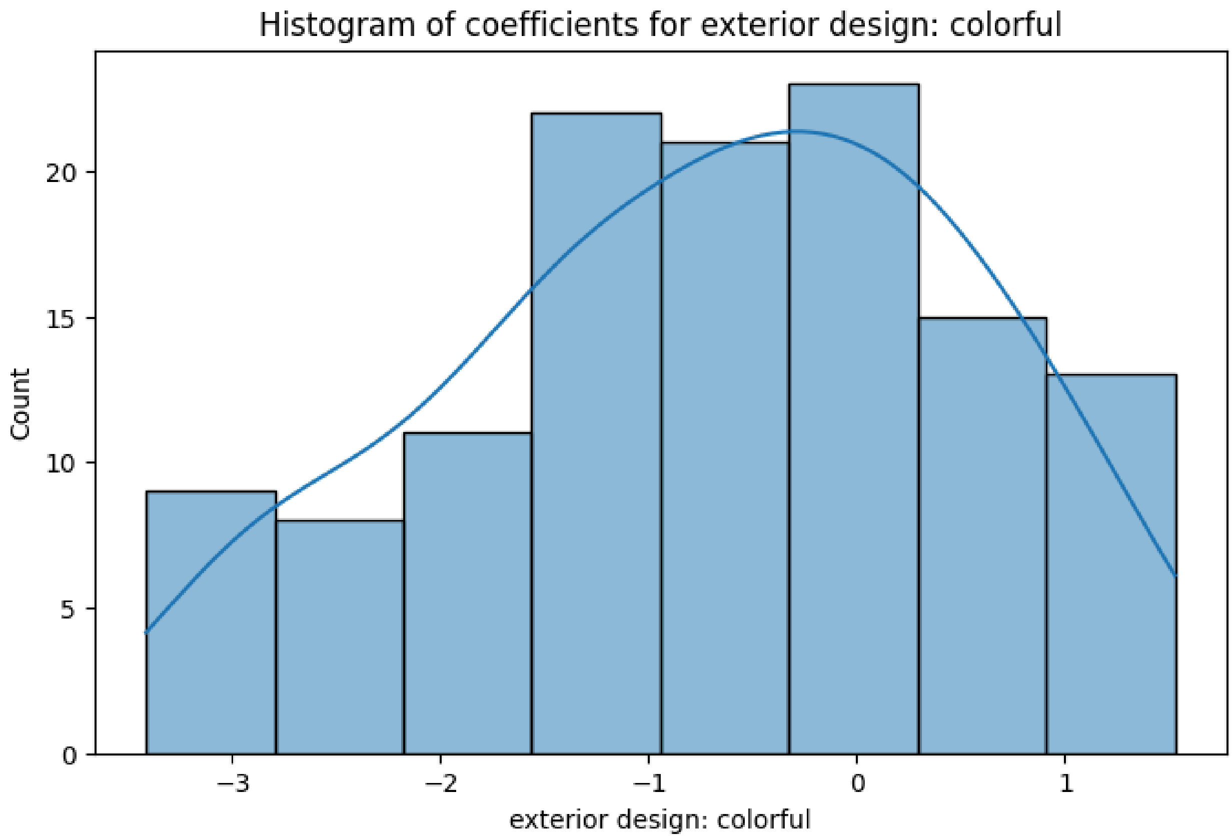
In [77]:

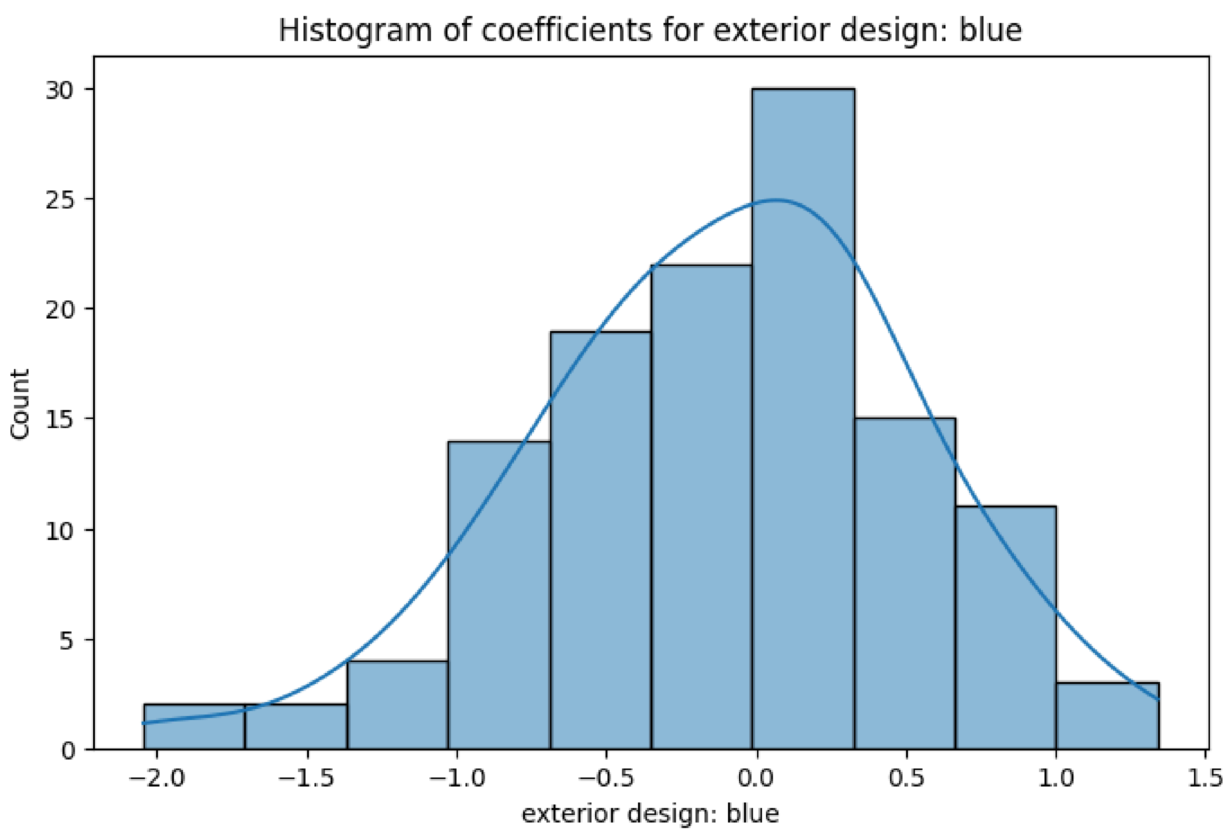
```python
for col in Individual_level_params_f.columns:
    sns.histplot(Individual_level_params_f[col], kde=True)
    plt.title(f'Histogram of coefficients for {col}')
    plt.show()
```

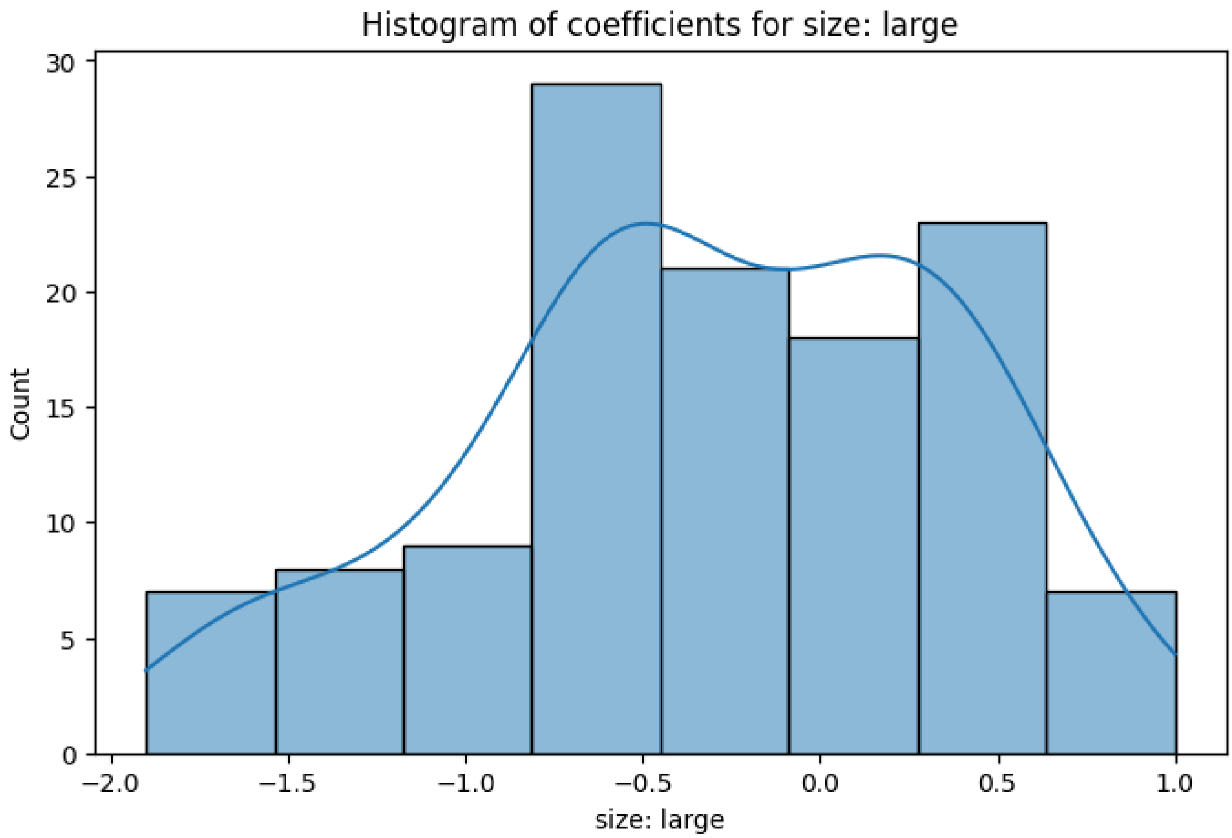


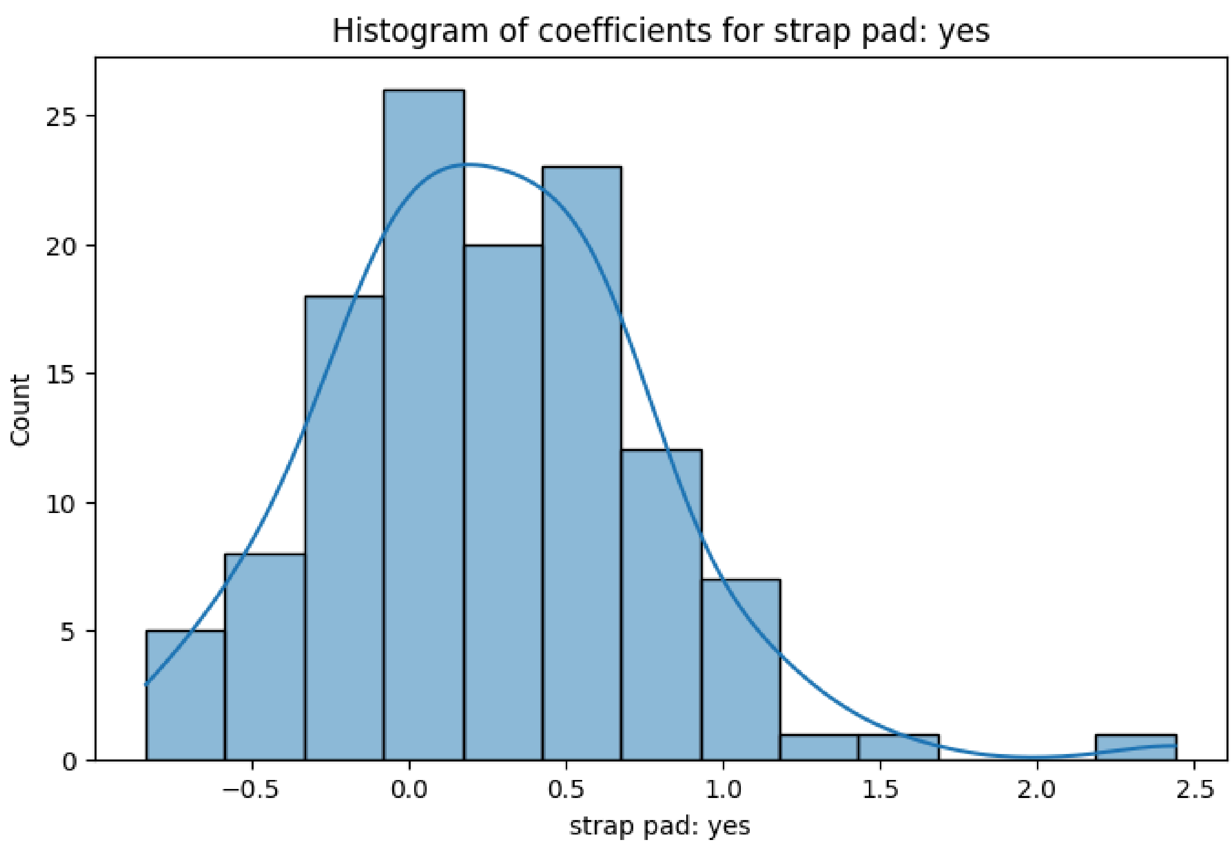

Histogram of coefficients for price
Count
price
30
25
20
15
10
5
0
−0.06
−0.04
−0.02
0.00
0.02

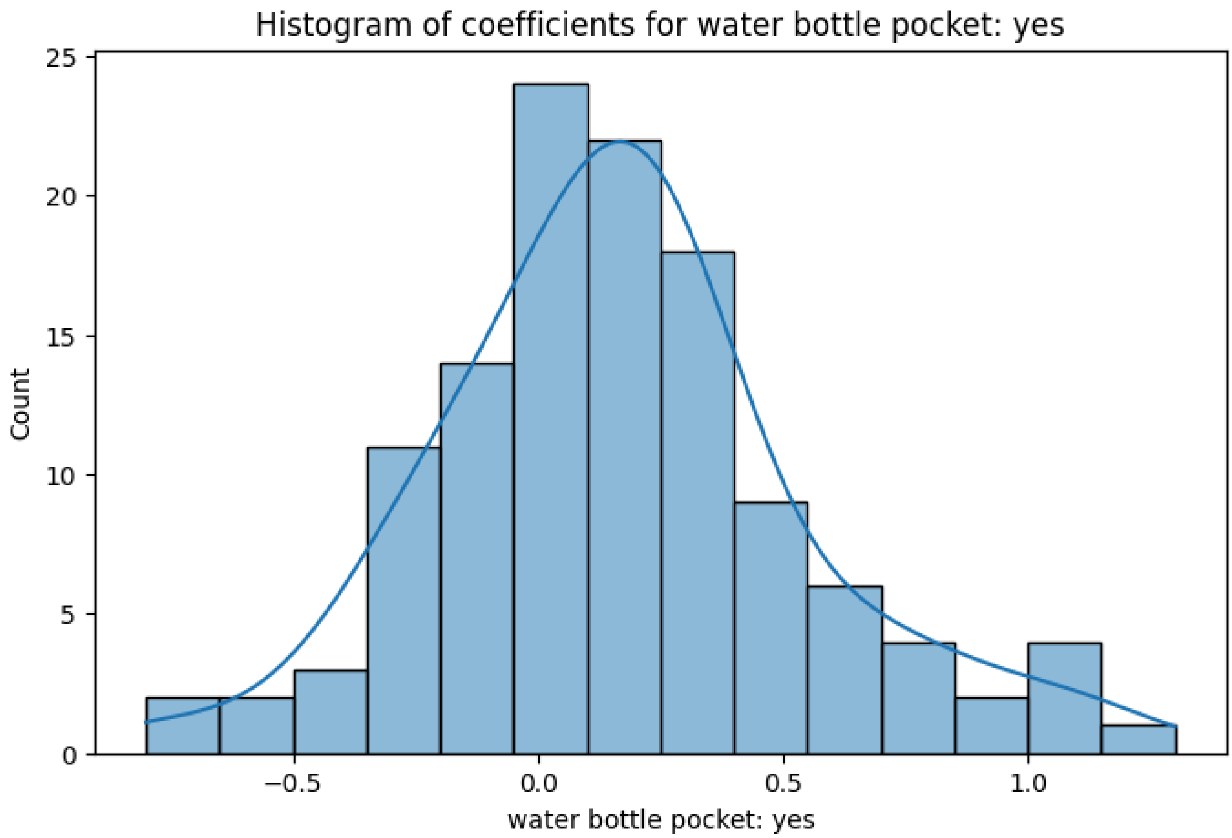

Histogram of coefficients for exterior design: reflective
Count
exterior design: reflective
35
30
25
20
15
10
5
0
−3
−2
−1
0
1
2

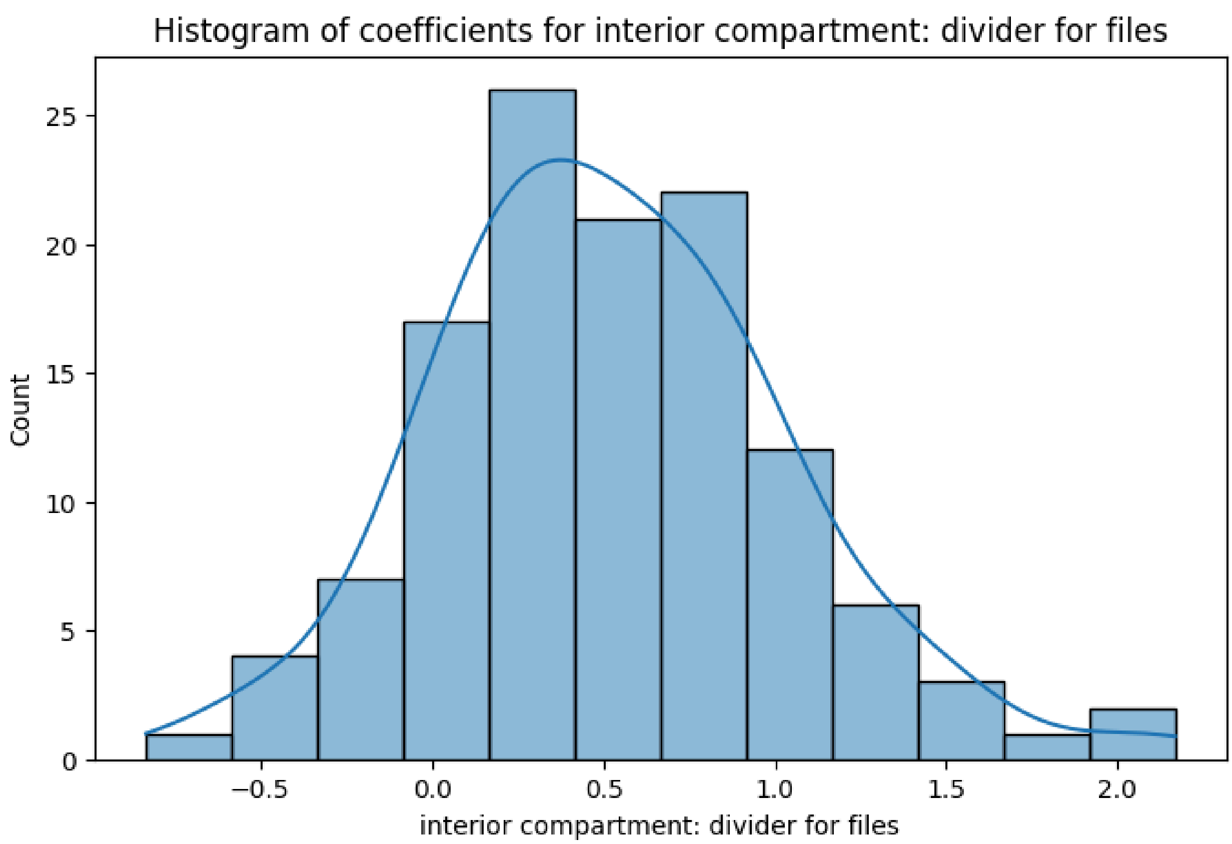

Histogram of coefficients for exterior design: colorful
Count
exterior design: colorful
0
5
10
15
20
−3
−2
−1
0
1

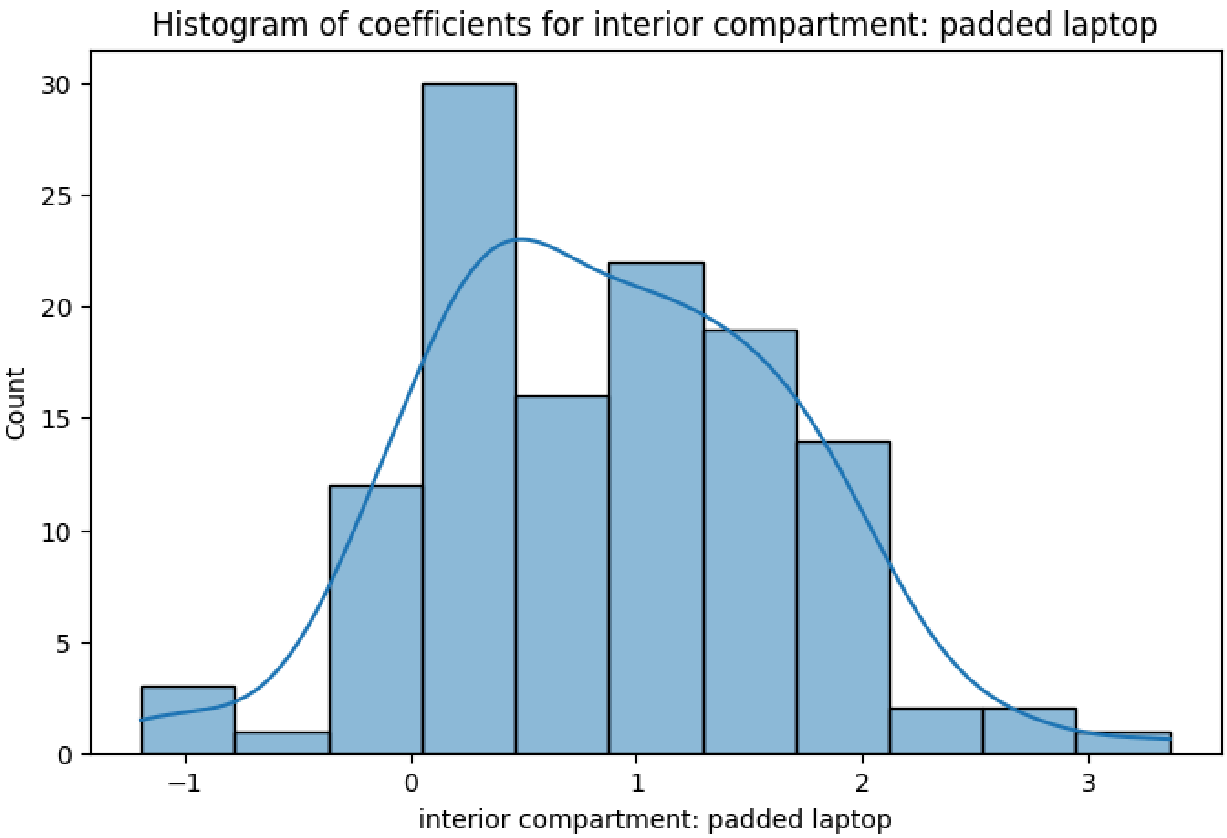

Histogram of coefficients for exterior design: blue
Count
exterior design: blue
0
5
10
15
20
25
30
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


Histogram of coefficients for size: large
Count
size: large
30
25
20
15
10
5
0
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0


Histogram of coefficients for strap pad: yes
Count
strap pad: yes
25
20
15
10
5
0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5


Histogram of coefficients for water bottle pocket: yes
Count
25
20
15
10
5
0
−0.5
0.0
0.5
1.0
water bottle pocket: yes


Histogram of coefficients for interior compartment: divider for files
Count
25
20
15
10
5
0
−0.5
0.0
0.5
1.0
1.5
2.0
interior compartment: divider for files



As we can see, the distribution here are more stretched that for the online case. Therefore, the difference betweeen preferences of the customers is larger for offline ones. The online users are more standardized because they get quite the same experience from the website with pre-defined by programmists points of interest. Also, for online customers only visual infirmation is available, which makes these people quite similar (in comparison with offline customers which get much more information about goods). Therefore, the preferences of offline customers seem to be more heterogeneous.

Revenue-maximizing strategies

In this simple case we assume that $u_{ij} = \hat{\beta}^{OLS}_i x_j$ - utility and that user i will buy the bag j if $\hat{\beta}^{OLS}_i x_j >= 3$.

In [78]:
```
all_bags = list(it.product([1, 2, 3, 4], [1, 2], [1, 2, 3, 4],
               [1, 2], [1, 2], [1, 2, 3]))
all_bags_df = pd.DataFrame(all_bags, columns=columns)
all_bags_full = sm.add_constant(get_dummies_and_price(all_bags_df))

index_offline, max_revenue_offline = max_revenue_bag(Individual_level_params_f
print(f'Max revenue for offline: {max_revenue_offline}')
print(all_bags_full.loc[index_offline])
```

```
Max revenue for offline: 14040
const                                             1.0
price                                           180.0
exterior design: reflective                       0.0
exterior design: colorful                         0.0
exterior design: blue                             0.0
size: large                                       0.0
strap pad: yes                                    1.0
water bottle pocket: yes                          1.0
interior compartment: divider for files           0.0
interior compartment: padded laptop               1.0
Name: 47, dtype: float64
```

Analyzing these results, we can notice that offline shoppers do not care about the price so much as online do. Therefore, the price is higher to maximize the profit. Moreover, large value is zero, according to the thoughts in the previous point where I compare online and offline features. However, the profit is smaller becuase online shopping is more affordable (using real-world sense) and that offline customers are more demanding (due to the fact that they estimate a good with more criterias such as smell, tactile sensations, and etc). And, finally, we can say that offline customers value the same additional options of bags as online customers do because the scope of application of this product is quite limited (in terms of comfort and convenient usage).