CF Estimation

ESTIMATING CHARACTERISTIC FUNCTION FROM EXPERIMENTS

Introduction

- Characteristic functionals are one of the main analytical tools used to quantify the statistical properties of random functions.
- Random function is the correct model for the ensemble of objects being imaged by a given imaging system.
- Examples:
  - In x-ray computed tomography, this function gives the x-ray attenuation at each point
  - Position emission tomography and single photon emission computed tomography imaging, this function gives the activity distribution inside the body
- While there are anatomical structures that are common to all patients, there are also unpredictable variations in these structures throughout the patient population.

Intro. Cont.

- Any given attenuation map or activity distribution can be regarded as a realization of a random function
- There will be unpredictable variations in the function being imaged. Otherwise the object function would be known.

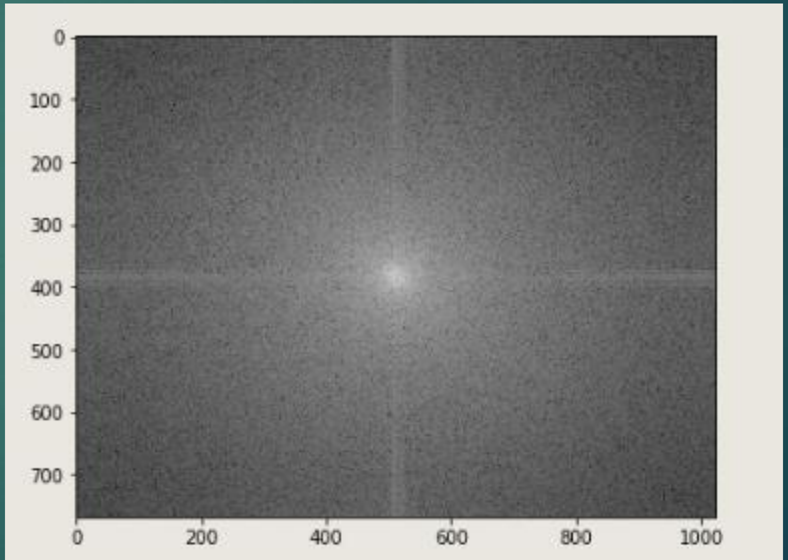
Frequency Response

- $L\{x(t)\} = X(s) = \int x(t)e^{-st}dt$ $\xrightarrow{s = j\omega}$ $F\{x(t)\} = X(jw) = \int x(t)e^{-jwt}dt$
- Transfer function $\xrightarrow{s = j\omega}$ Frequency response

- $H(jw) = |H(jw)|.e^{j\angle H(jw)} = amplitude.e^{phase}$
- What is amplitude and phase in case of images?
  - $amplitude = \sqrt{real^2 + img^2}$
  - $phase = arctan2\left(\frac{img}{real}\right)$

Frequency Response cont.

- The response of each image is our experiments
- We are looking for a generalized distribution that can represent the entire samples from experiments for a describe a particular object.



Frequency Response cont.

- If we assume that a MLP is the optimal function approximation tool, then target will be like this:
- CIFAR10 dataset and its corresponding weight visualizations as the distribution that can represent each class

Frequency Response cont.

- From depictions in the previous slide, we can see that the corresponding model of each class is very similar to average of all images(responses) of that particular class.
- We assume that the random function is specified by a parameter vector θ
- Our goal is produce an estimate for θ from the data generated by $f(r|\theta)$ where $X_r = f(r, \omega)$ is a random variable for each r and ω in sample set S and Ω respectively.

Frequency Response cont.

- Consider a sample $\{f_1(r|\theta), f_2(r|\theta), \dots, f_n(r|\theta)\}$ of independent realizations and the corresponding data matrix $G = \{g_1, g_2, \dots, g_n\}$
- We can form an estimate $\widehat{\Phi_g}$ of Φ_g by using a sample mean:

$$\widehat{\Phi_g}(\zeta|\theta) = \frac{1}{N}\sum_{n=1}^{\infty} f_n$$

Reference

- http://www.ee.ic.ac.uk/pcheung/teaching/DE2_EE/Lecture%208%20-%20Frequency%20Responses%20(x1).pdf
- Clarkson, E., & Barrett, H. H. (2016). Characteristic functionals in imaging and image-quality assessment: tutorial. *Journal of the Optical Society of America. A, Optics, image science, and vision*, *33*(8), 1464–1475. https://doi.org/10.1364/JOSAA.33.001464