Nikshay Jain | MM21B044

DA5400: Assign 3

```
import os, csv
import numpy as np
from PIL import Image
import matplotlib.pyplot as plt
from datasets import load_dataset
```

Q1

Preparing dataset

```
dataset = load_dataset("mnist")
print(dataset)
```

```
DatasetDict({
    train: Dataset({
        features: ['image', 'label'],
        num_rows: 60000
    })
    test: Dataset({
        features: ['image', 'label'],
        num_rows: 10000
    })
})
```

```
train_data = dataset['train']
print(train_data)
```

```
Dataset({
    features: ['image', 'label'],
    num_rows: 60000
})
```

```
subset = {i: [] for i in range(10)}
for idx, data in enumerate(train_data):
    label = data['label']
    if len(subset[label]) < 100:
        subset[label].append((data['image'], label))
    if all(len(subset[i]) == 100 for i in range(10)):
        break

final_subset = []
```

```
for key in subset:
    final_subset.extend(subset[key])

np.random.shuffle(final_subset)

os.makedirs("data", exist_ok=True)
images = "data\images"
labels = "data\labels.csv"
os.makedirs(images, exist_ok=True)

with open(labels, "w", newline="") as f:
    writer = csv.writer(f)
    writer.writerow(["Image_Path", "Label"])
    for i, (image, label) in enumerate(final_subset):
        image_path = os.path.join(images, f"image_{i}.png")
        image.save(image_path)
        writer.writerow([image_path, label])
```

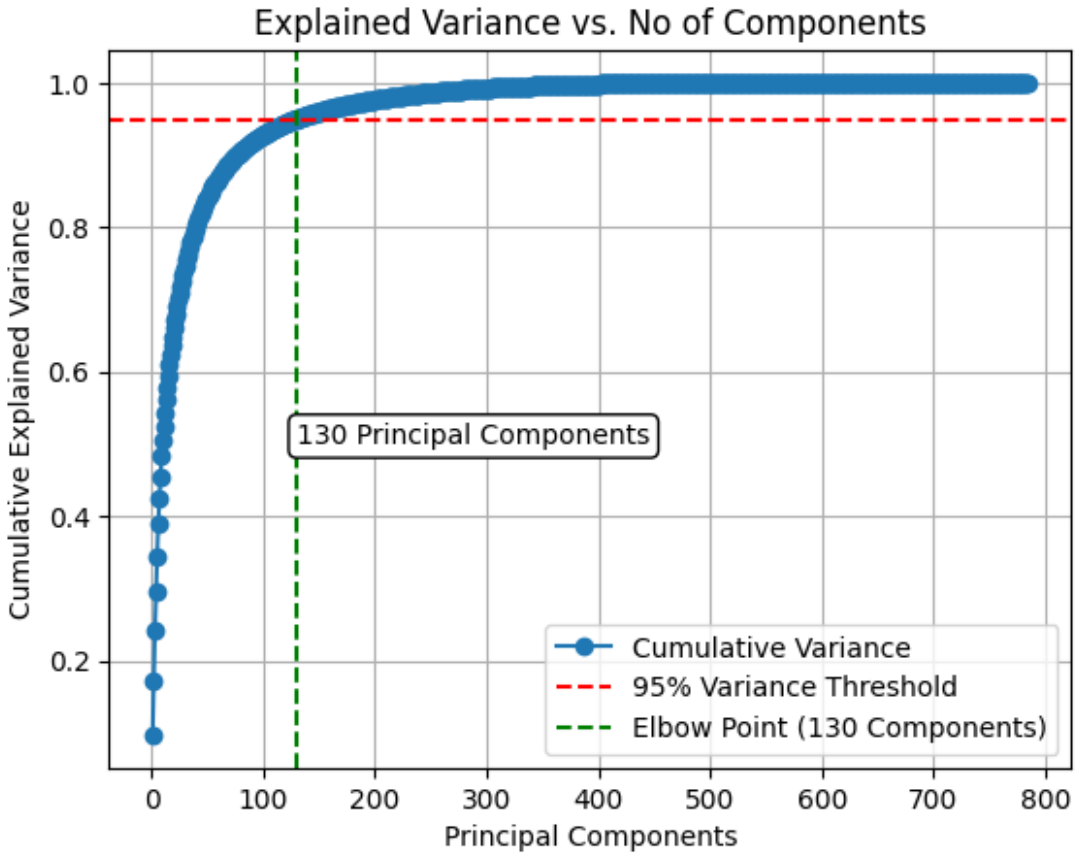
Part 1(i)

```
# Directory paths and parameters
image_dir = "./data/images"
labels_csv = "./data/labels.csv"

# Load the images and labels
def load_data(image_dir, labels_csv):
    images = []
    labels = []
    with open(labels_csv, "r") as f:
        rd = csv.reader(f)
        next(rd)      # Skip the header
        for row in rd:
            image_path = row[0]
            label = int(row[1])
            img = Image.open(image_path)
            images.append(np.array(img).flatten())       # Flatten the
image into a 1D vector
            labels.append(label)
    return np.array(images), np.array(labels)

# Load the data
images, labels = load_data(image_dir, labels_csv)

def pca(data):
    # Center the data
    mean_vec = np.mean(data, axis=0)
    centered_data = data - mean_vec

    cov_matr = np.cov(centered_data, rowvar=False)
```

```
    # Compute eigenvalues and eigenvectors
    eigenvals, eigenvecs = np.linalg.eigh(cov_matr)

    # Sort eigenvalues and eigenvectors in descending order
    sorted_idx = np.argsort(eigenvals)[::-1]
    eigenvals = eigenvals[sorted_idx]
    eigenvecs = eigenvecs[:, sorted_idx]

    # checking for direct linear dependecies in parameters
    print('No of eigenvals:', eigenvals.shape)       # gives (784, )
    print('Shape of eigenvecs:', eigenvecs.shape)    # gives (784,
784)

    # therefore, no redundancy in data as no of eigen values =
dimenstions of data

    return eigenvals, eigenvecs, centered_data

def analyse_pca(eigenvals, eigenvecs, centered_data):
    # Calculate explained variance
    tot_var = np.sum(eigenvals)
    expl_var_ratio = eigenvals / tot_var
    cumul_var_ratio = np.cumsum(expl_var_ratio)

    # Find the number of components to cover 95% variance
    n_comps = np.argmax(cumul_var_ratio >= 0.95) + 1

    # Plot the eigenvalues vs princ comps
    plt.plot(range(1, len(eigenvals) + 1), cumul_var_ratio,
marker='o', label="Cumulative Variance")

    plt.axhline(y = 0.95, color = 'r', linestyle = '--', label = "95%
Variance Threshold")
    plt.axvline(x = n_comps, color = 'g', linestyle = '--', label =
f"Elbow Point ({n_comps} Components)")
    plt.text(n_comps, 0.5, f"{n_comps} Principal Components",
color='black', fontsize=10,
             bbox=dict(facecolor='white', edgecolor='black',
boxstyle='round,pad=0.3'))

    plt.xlabel("Principal Components")
    plt.ylabel("Cumulative Explained Variance")
    plt.title("Explained Variance vs. No of Components")

    plt.legend()
    plt.grid()
    plt.show()

    print(f"No. of principal components (to cover 95% variance):
{n_comps}")
```

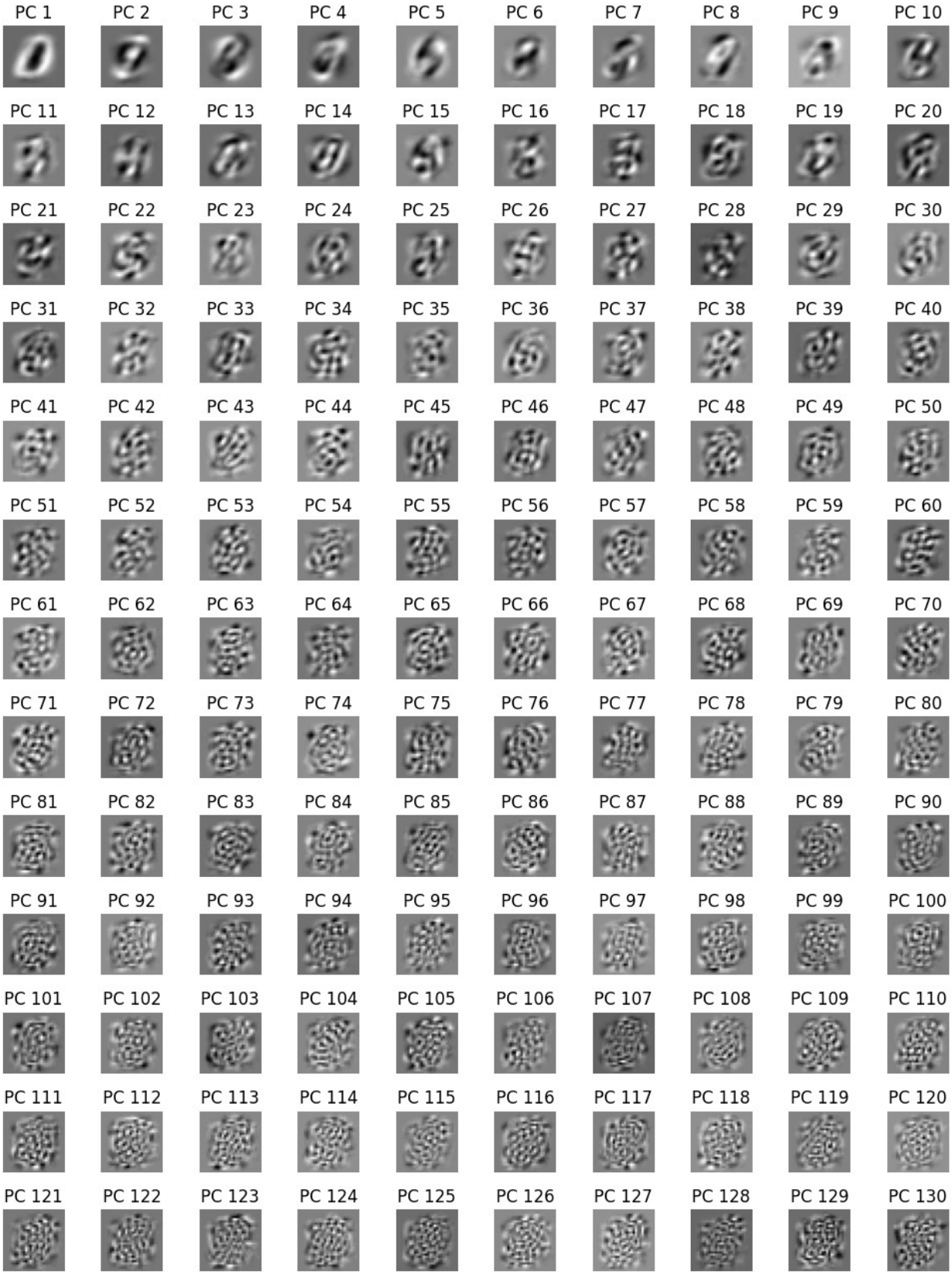
```
    # Select the top `n_comps` eigenvectors
    p_comps = eigenvecs[:, :n_comps]
    p_eigenvals = eigenvals[:n_comps]

    # Project data onto principal components
    project_data = np.dot(centered_data, p_comps)

    return p_comps, p_eigenvals, project_data, n_comps

image_shape = (28, 28)

# Run PCA
eigenvals, eigenvecs, centered_data = pca(images)

p_comps, p_eigenvals, project_data, n_comps = analyse_pca(eigenvals,
eigenvecs, centered_data)
```

```
No of eigenvals: (784,)
Shape of eigenvecs: (784, 784)
```




```
No. of principal components (to cover 95% variance): 130
```

```python
# Visualize principal components as images with reduced sizes
def visualize_p_comp(components, image_shape, n, grid_shape):
    plt.figure(figsize=(10, n // grid_shape[1]))

    for i in range(n):
        # Create a single subplot for each component
        plt.subplot(grid_shape[0], grid_shape[1], i + 1)

        pc_image = components[:, i].reshape(image_shape)
        plt.imshow(pc_image, cmap="gray")
        plt.title(f"PC {i+1}")
        plt.axis("off")

    plt.tight_layout()
    plt.show()

# Example usage
visualize_p_comp(p_comps, image_shape, n_comps, (n_comps // 10, 10))
```


PC 1
PC 2
PC 3
PC 4
PC 5
PC 6
PC 7
PC 8
PC 9
PC 10
PC 11
PC 12
PC 13
PC 14
PC 15
PC 16
PC 17
PC 18
PC 19
PC 20
PC 21
PC 22
PC 23
PC 24
PC 25
PC 26
PC 27
PC 28
PC 29
PC 30
PC 31
PC 32
PC 33
PC 34
PC 35
PC 36
PC 37
PC 38
PC 39
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PC 41
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PC 44
PC 45
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PC 48
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PC 50
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PC 104
PC 105
PC 106
PC 107
PC 108
PC 109
PC 110
PC 111
PC 112
PC 113
PC 114
PC 115
PC 116
PC 117
PC 118
PC 119
PC 120
PC 121
PC 122
PC 123
PC 124
PC 125
PC 126
PC 127
PC 128
PC 129
PC 130

```python
# Calc explained variance by each component
def explained_variance(lambdas):
    tot_variance = np.sum(lambdas)
    expl_vars = lambdas / tot_variance
    return expl_vars

expl_vars = explained_variance(p_eigenvals)

for i, var in enumerate(expl_vars[:n_comps]):
    print(f"Variance explained by Principal Component {i+1}:
{var*100:.3f}%")
print(f'\nTotal variance explained by all Principal Components:
{sum(expl_vars)*100}%')
```

```
Variance explained by Principal Component 1: 10.192%
Variance explained by Principal Component 2: 7.826%
Variance explained by Principal Component 3: 7.286%
Variance explained by Principal Component 4: 5.720%
Variance explained by Principal Component 5: 5.136%
Variance explained by Principal Component 6: 4.806%
Variance explained by Principal Component 7: 3.674%
Variance explained by Principal Component 8: 3.190%
Variance explained by Principal Component 9: 2.946%
Variance explained by Principal Component 10: 2.264%
Variance explained by Principal Component 11: 2.137%
Variance explained by Principal Component 12: 2.005%
Variance explained by Principal Component 13: 1.922%
Variance explained by Principal Component 14: 1.805%
Variance explained by Principal Component 15: 1.721%
Variance explained by Principal Component 16: 1.589%
Variance explained by Principal Component 17: 1.367%
Variance explained by Principal Component 18: 1.337%
Variance explained by Principal Component 19: 1.307%
Variance explained by Principal Component 20: 1.224%
Variance explained by Principal Component 21: 1.135%
Variance explained by Principal Component 22: 1.063%
Variance explained by Principal Component 23: 1.014%
Variance explained by Principal Component 24: 0.997%
Variance explained by Principal Component 25: 0.947%
Variance explained by Principal Component 26: 0.898%
Variance explained by Principal Component 27: 0.836%
Variance explained by Principal Component 28: 0.806%
Variance explained by Principal Component 29: 0.761%
Variance explained by Principal Component 30: 0.723%
Variance explained by Principal Component 31: 0.717%
Variance explained by Principal Component 32: 0.678%
Variance explained by Principal Component 33: 0.668%
Variance explained by Principal Component 34: 0.613%
Variance explained by Principal Component 35: 0.578%
Variance explained by Principal Component 36: 0.559%
```

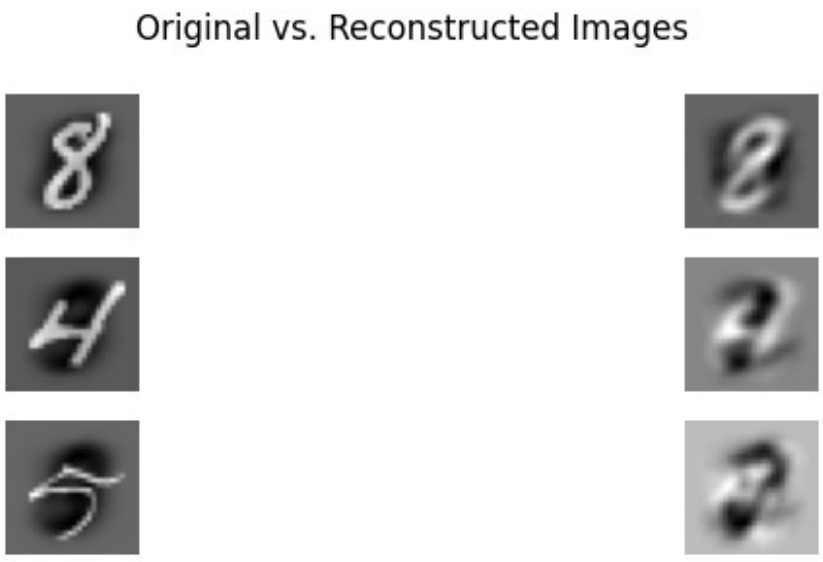
```
Variance explained by Principal Component 37: 0.543%
Variance explained by Principal Component 38: 0.538%
Variance explained by Principal Component 39: 0.502%
Variance explained by Principal Component 40: 0.494%
Variance explained by Principal Component 41: 0.463%
Variance explained by Principal Component 42: 0.456%
Variance explained by Principal Component 43: 0.437%
Variance explained by Principal Component 44: 0.415%
Variance explained by Principal Component 45: 0.401%
Variance explained by Principal Component 46: 0.387%
Variance explained by Principal Component 47: 0.381%
Variance explained by Principal Component 48: 0.366%
Variance explained by Principal Component 49: 0.360%
Variance explained by Principal Component 50: 0.341%
Variance explained by Principal Component 51: 0.326%
Variance explained by Principal Component 52: 0.319%
Variance explained by Principal Component 53: 0.312%
Variance explained by Principal Component 54: 0.305%
Variance explained by Principal Component 55: 0.294%
Variance explained by Principal Component 56: 0.285%
Variance explained by Principal Component 57: 0.274%
Variance explained by Principal Component 58: 0.268%
Variance explained by Principal Component 59: 0.255%
Variance explained by Principal Component 60: 0.252%
Variance explained by Principal Component 61: 0.242%
Variance explained by Principal Component 62: 0.240%
Variance explained by Principal Component 63: 0.223%
Variance explained by Principal Component 64: 0.220%
Variance explained by Principal Component 65: 0.216%
Variance explained by Principal Component 66: 0.214%
Variance explained by Principal Component 67: 0.211%
Variance explained by Principal Component 68: 0.206%
Variance explained by Principal Component 69: 0.201%
Variance explained by Principal Component 70: 0.192%
Variance explained by Principal Component 71: 0.188%
Variance explained by Principal Component 72: 0.183%
Variance explained by Principal Component 73: 0.177%
Variance explained by Principal Component 74: 0.175%
Variance explained by Principal Component 75: 0.172%
Variance explained by Principal Component 76: 0.165%
Variance explained by Principal Component 77: 0.163%
Variance explained by Principal Component 78: 0.158%
Variance explained by Principal Component 79: 0.158%
Variance explained by Principal Component 80: 0.149%
Variance explained by Principal Component 81: 0.148%
Variance explained by Principal Component 82: 0.143%
Variance explained by Principal Component 83: 0.140%
Variance explained by Principal Component 84: 0.135%
Variance explained by Principal Component 85: 0.132%
```

```
Variance explained by Principal Component 86: 0.131%
Variance explained by Principal Component 87: 0.128%
Variance explained by Principal Component 88: 0.126%
Variance explained by Principal Component 89: 0.125%
Variance explained by Principal Component 90: 0.121%
Variance explained by Principal Component 91: 0.117%
Variance explained by Principal Component 92: 0.116%
Variance explained by Principal Component 93: 0.113%
Variance explained by Principal Component 94: 0.111%
Variance explained by Principal Component 95: 0.110%
Variance explained by Principal Component 96: 0.105%
Variance explained by Principal Component 97: 0.103%
Variance explained by Principal Component 98: 0.102%
Variance explained by Principal Component 99: 0.101%
Variance explained by Principal Component 100: 0.099%
Variance explained by Principal Component 101: 0.097%
Variance explained by Principal Component 102: 0.097%
Variance explained by Principal Component 103: 0.093%
Variance explained by Principal Component 104: 0.092%
Variance explained by Principal Component 105: 0.091%
Variance explained by Principal Component 106: 0.089%
Variance explained by Principal Component 107: 0.087%
Variance explained by Principal Component 108: 0.086%
Variance explained by Principal Component 109: 0.086%
Variance explained by Principal Component 110: 0.084%
Variance explained by Principal Component 111: 0.082%
Variance explained by Principal Component 112: 0.080%
Variance explained by Principal Component 113: 0.079%
Variance explained by Principal Component 114: 0.078%
Variance explained by Principal Component 115: 0.077%
Variance explained by Principal Component 116: 0.076%
Variance explained by Principal Component 117: 0.074%
Variance explained by Principal Component 118: 0.073%
Variance explained by Principal Component 119: 0.072%
Variance explained by Principal Component 120: 0.071%
Variance explained by Principal Component 121: 0.069%
Variance explained by Principal Component 122: 0.069%
Variance explained by Principal Component 123: 0.068%
Variance explained by Principal Component 124: 0.067%
Variance explained by Principal Component 125: 0.066%
Variance explained by Principal Component 126: 0.065%
Variance explained by Principal Component 127: 0.064%
Variance explained by Principal Component 128: 0.063%
Variance explained by Principal Component 129: 0.062%
Variance explained by Principal Component 130: 0.061%

Total variance explained by all Principal Components:
99.99999999999989%
```

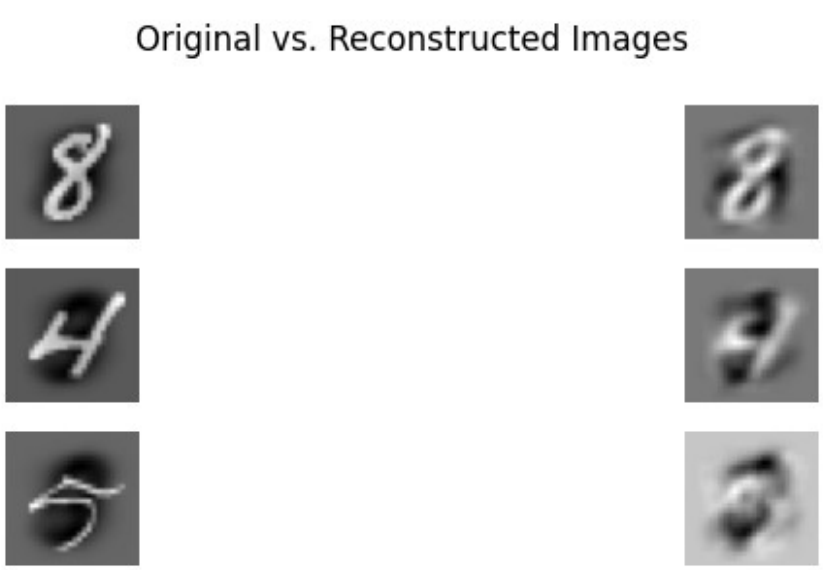
Part 1(ii)

```
def reconstruct_images(centred_data, eigenvecs, d):
    top_comps = eigenvecs[:, :d]                                    # Top
d eigenvectors (principal components)
    data_projected = np.dot(centred_data, top_comps)                #
Project data into lower-dimensional space
    data_reconstructed = np.dot(data_projected, top_comps.T)    #
Reconstruct data from lower dimensions
    return data_reconstructed

def visualize_reconstruction(data_original, data_reconstructed,
num_images=3):
    plt.figure(figsize=(10, num_images))
    plt.suptitle("Original vs. Reconstructed Images", fontsize=12)

    for i in range(num_images):
        # Original image
        plt.subplot(num_images, 2, 2 * i + 1)
        plt.imshow(data_original[i].reshape(28, 28), cmap='gray')  #
Assuming 28x28 images
        plt.axis("off")

        # Reconstructed image
        plt.subplot(num_images, 2, 2 * i + 2)
        plt.imshow(data_reconstructed[i].reshape(28, 28), cmap='gray')
# Assuming 28x28 images
        plt.axis("off")

    plt.tight_layout()
    plt.show()

# Reconstruct dataset with different dimensional representations
dim = [10, 20, 50, 100, 130]
recons_err = []

for d in dim:
    print(f"Reconstructing dataset using {d} dimensions...")
    data_reconstructed = reconstruct_images(centered_data, eigenvecs,
d)
    visualize_reconstruction(centered_data, data_reconstructed,
num_images=3)

    # Calculate reconstruction error (Root Mean Squared Error)
    recons_err.append(np.sqrt(np.mean((centered_data -
data_reconstructed) ** 2)))
```

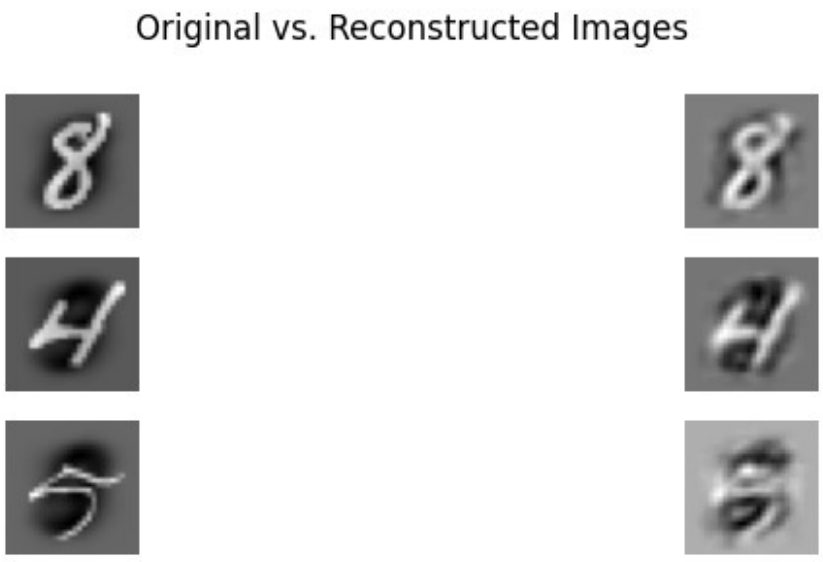
```
Reconstructing dataset using 10 dimensions...
```



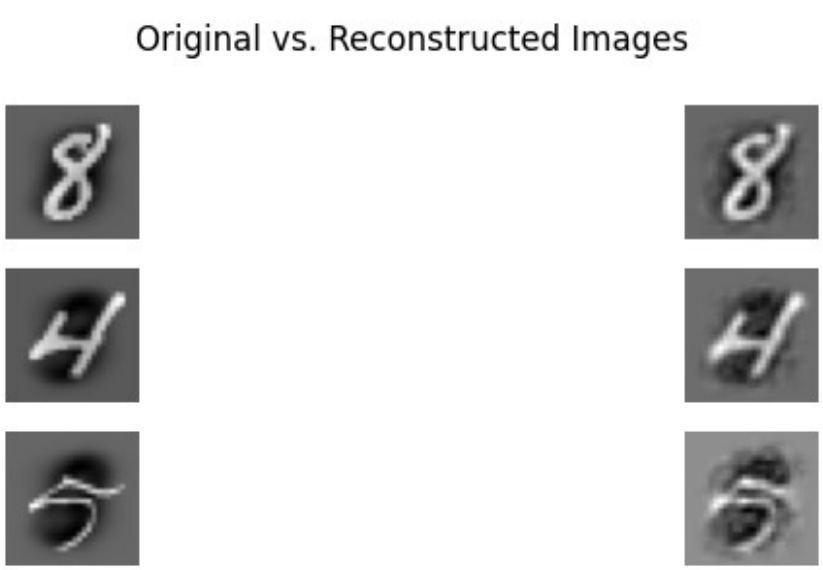

```
Reconstructing dataset using 20 dimensions...
```



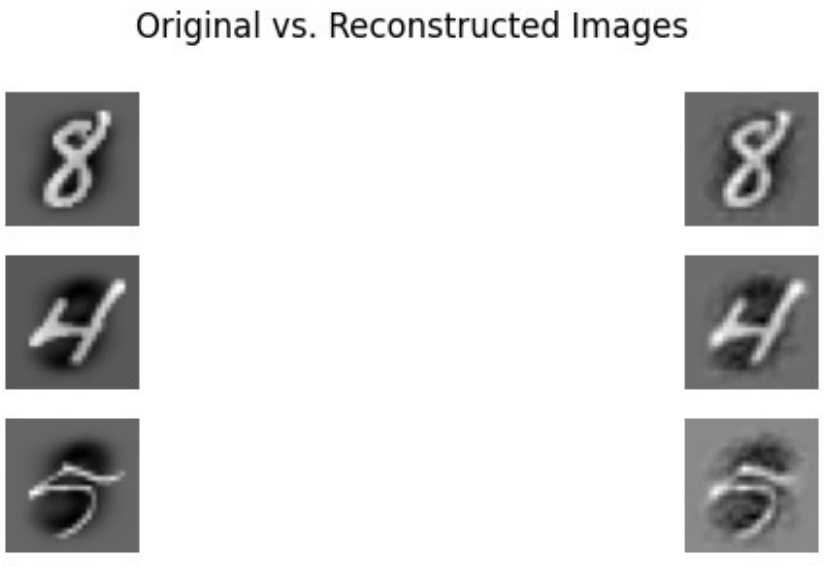

```
Reconstructing dataset using 50 dimensions...
```




```
Reconstructing dataset using 100 dimensions...
```




```
Reconstructing dataset using 130 dimensions...
```




```
print("Dimensions \t\tReconstr error")
for i in range(len(dim)):
    print(f"{dim[i]}\t\t\t{recons_err[i]:.4f}")
```

```
Dimensions            Reconstr error
10                46.0730
20                38.1439
50                26.0558
100               17.5086
130               14.5712
```

Conclusion:

As it is evident from above plots, we can clearly idetify that the **image contains a number with just top 20 principal components** and the numbers become clearly **distinguishable with top 50** of them.

To reconstruct the images reasonably well, we may go ahead by 50 PCs, but the Reconstruction error table above suggests that going by 130 PCs reduces the error to half of that by 50 PCs. This is also shown as the **elbow** on the graph plotted above which contains **>95% of the information** of the image and the rest components are just remaining 5%.

Therefore, it is safe to use **top 130 components** to get the images reconstructed.

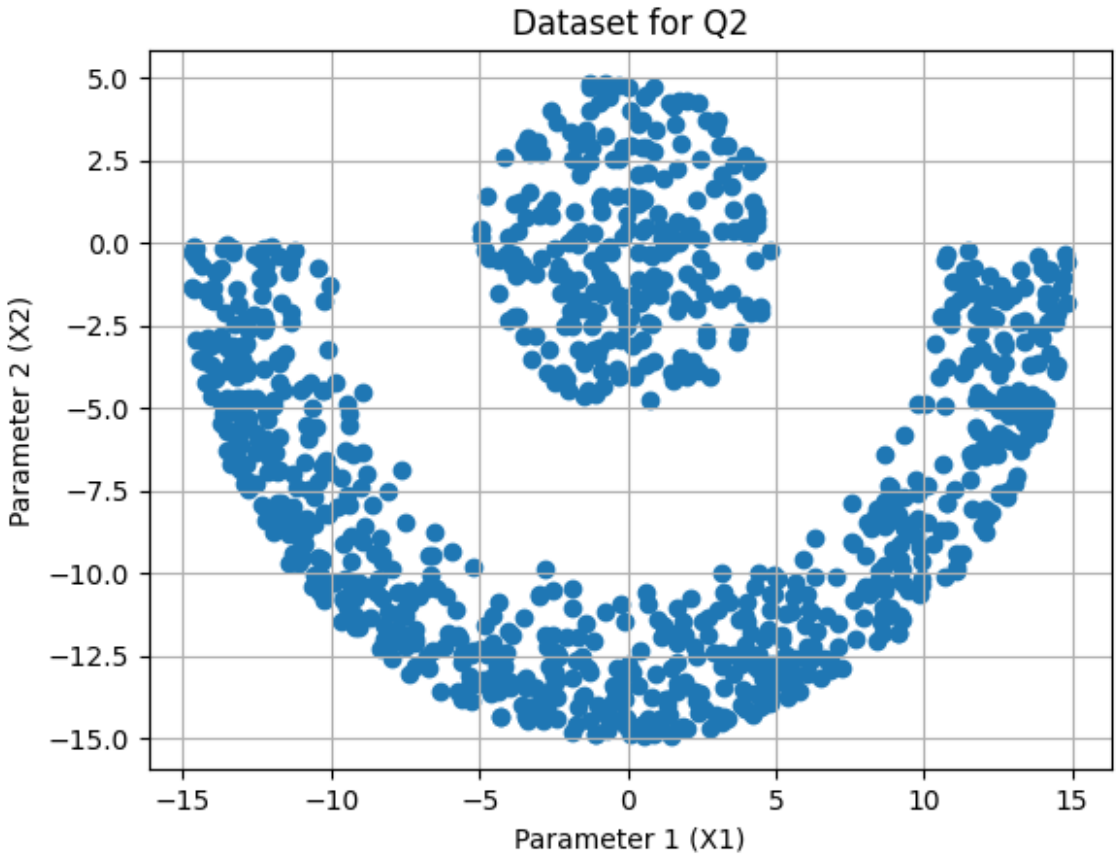
Q2

Preparing dataset

```
data = np.genfromtxt('cm_dataset_2.csv', delimiter=',')
print(data)
```

```
[[ -1.2061   -2.5268 ]
 [ -3.7038    0.33831]
 [  4.3444    0.96302]
 ...
 [ -2.8458  -14.469  ]
 [-11.173    -7.4088 ]
 [ -2.3857  -13.212  ]]
```

```
plt.scatter(data[:,0], data[:,1])
plt.title("Dataset for Q2")
plt.xlabel("Parameter 1 (X1)")
plt.ylabel("Parameter 2 (X2)")
plt.grid()
plt.show()
```

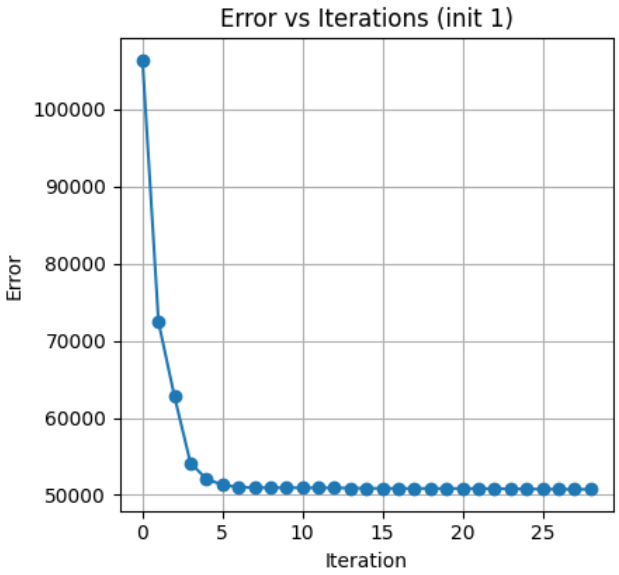


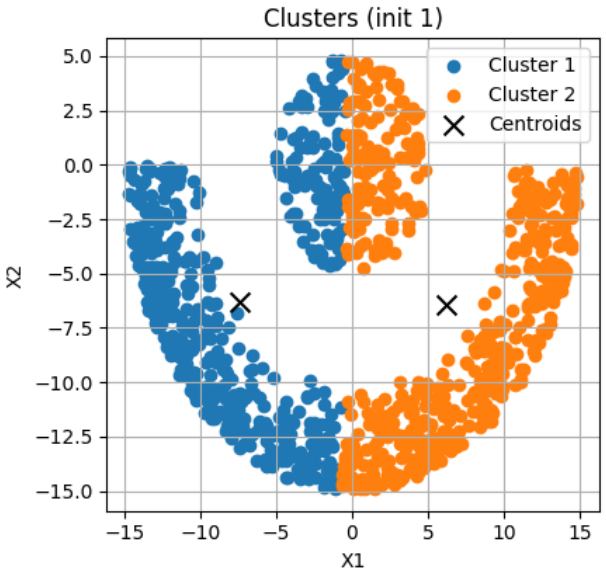
Part 2(i)

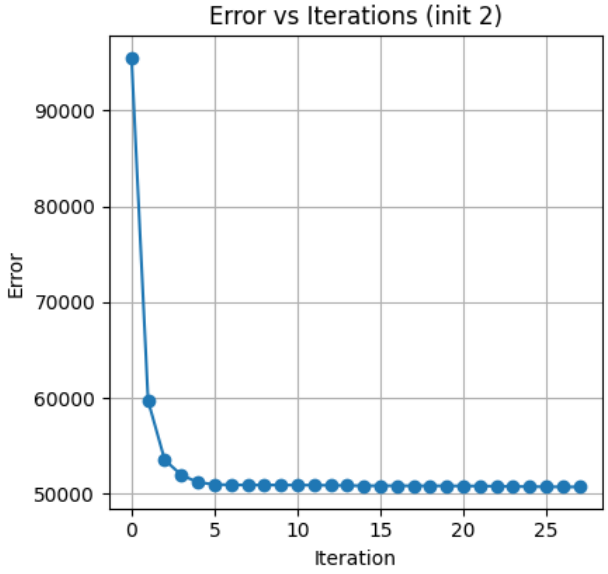
```
# Lloyd's Algorithm for K-Means
def k_means(data, k, max_iter=100):
    # Randomly initialize k cluster centroids
    n = data.shape[0]
    centroids = data[np.random.choice(n, k, replace=False)]
```

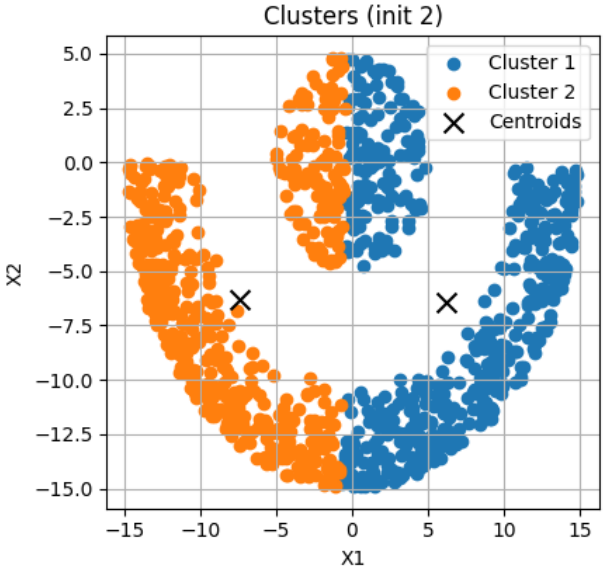
```
    error_history = []
    for _ in range(max_iter):
        # Assign each point to the nearest centroid
        distances = np.linalg.norm(data[:, np.newaxis] - centroids,
axis=2)
        cluster_labels = np.argmin(distances, axis=1)

        # Update centroids as the mean of assigned points
        new_centroids = np.array([data[cluster_labels ==
j].mean(axis=0) for j in range(k)])

        # Compute error (sum of squared distances to centroids)
        error = np.sum(np.min(distances, axis=1) ** 2)
        error_history.append(error)

        # Convergence check (if centroids don't change)
        if np.allclose(centroids, new_centroids, rtol=1e-6):
            break

        centroids = new_centroids

    return cluster_labels, centroids, error_history

# Run K-Means with 5 random initializations
k = 2
max_iter = 100
n = 5

plt.figure(figsize=(9, 21))

for i in range(n):
    cluster_labels, centroids, error_history = k_means(data, k,
max_iter=max_iter)

    # Plot error history
    plt.subplot(5, 2, 2 * i + 1)
    plt.plot(range(len(error_history)), error_history, marker='o')
    plt.xlabel('Iteration')
    plt.ylabel('Error')
    plt.title(f'Error vs Iterations (init {i + 1})')
    plt.grid()

    # Plot clusters
    plt.subplot(5, 2, 2 * i + 2)
    for j in range(k):
        cluster_points = data[cluster_labels == j]
        plt.scatter(cluster_points[:, 0], cluster_points[:, 1],
label=f'Cluster {j + 1}')
    plt.scatter(centroids[:, 0], centroids[:, 1], c='black',
marker='x', s=100, label='Centroids')
```

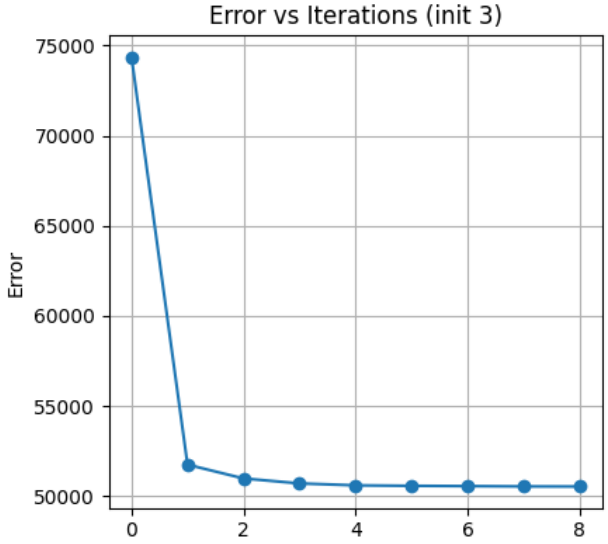
```
    plt.xlabel('X1')
    plt.ylabel('X2')
    plt.title(f'Clusters (init {i + 1})')
    plt.legend()
    plt.grid()

plt.tight_layout()
plt.show()
```

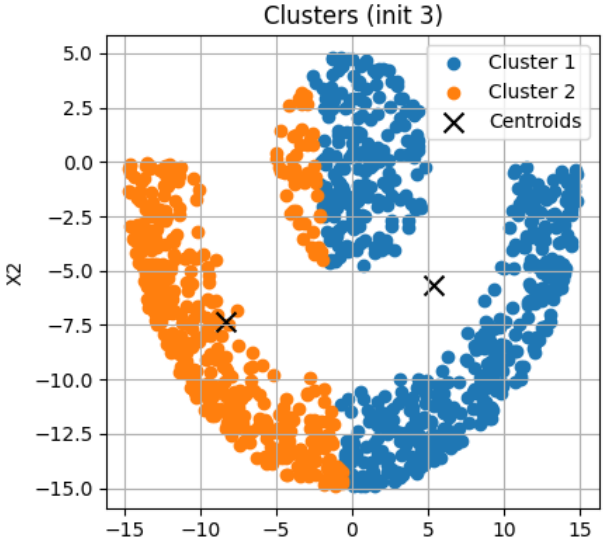

Error vs Iterations (init 1)
Error
Iteration


Clusters (init 1)
Cluster 1
Cluster 2
Centroids
X1
X2


Error vs Iterations (init 2)
Error
Iteration


Clusters (init 2)
Cluster 1
Cluster 2
Centroids
X1
X2

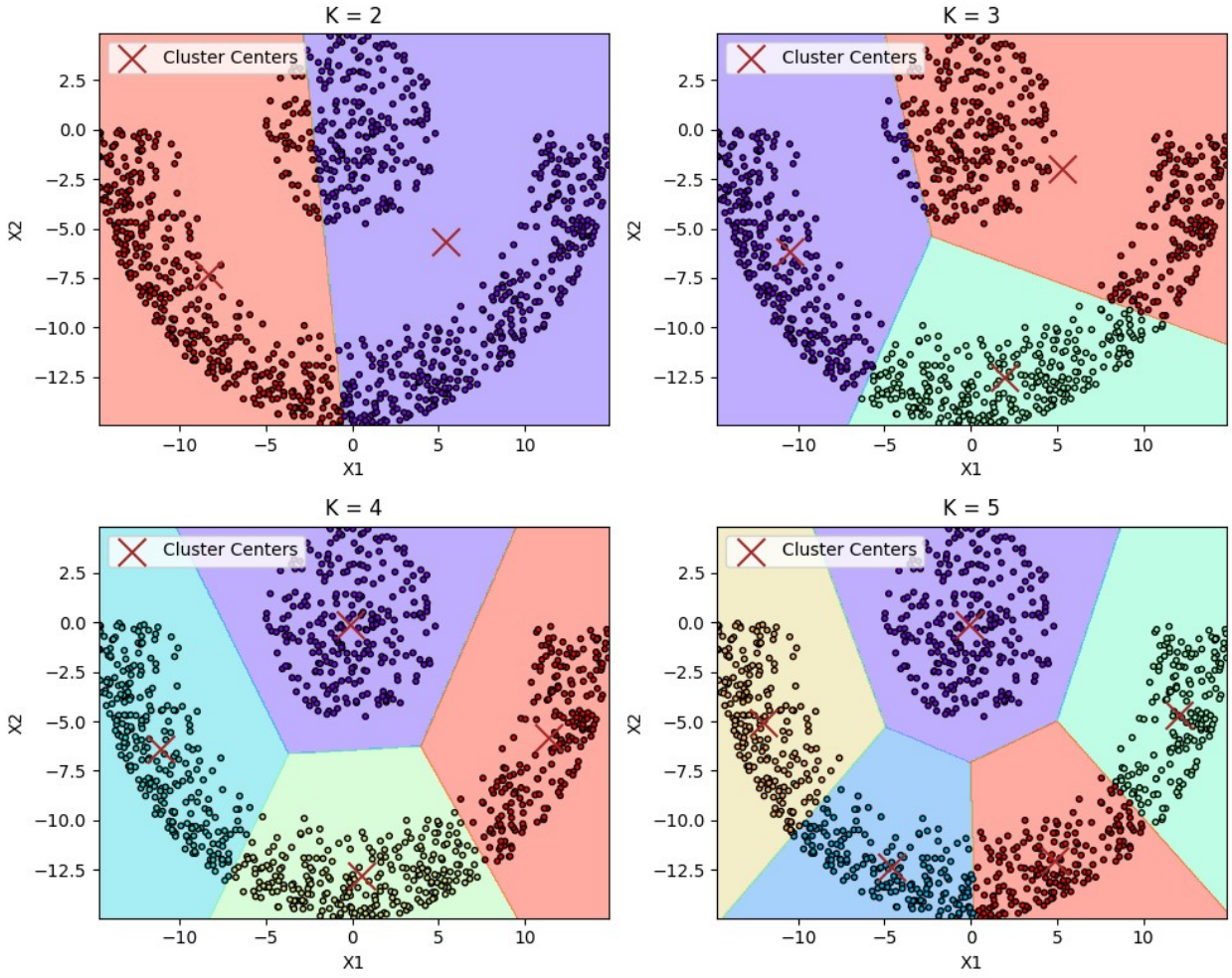

Error vs Iterations (init 3)
Error


Clusters (init 3)
Cluster 1
Cluster 2
Centroids
X2

Part 2(ii)

```
# K-means algorithm from scratch
def k_means_fixed_init(data, k, init_centers, max_iter=100, tol=1e-4):
    centers = init_centers.copy()
    for _ in range(max_iter):
        # Assign points to the nearest center
        distances = np.linalg.norm(data[:, np.newaxis, :] -
centers[np.newaxis, :, :], axis=2)
        labels = np.argmin(distances, axis=1)

        # Update cluster centers
        new_centers = np.array([data[labels == i].mean(axis=0) if
np.any(labels == i) else centers[i] for i in range(k)])

        # Check for convergence
        if np.linalg.norm(new_centers - centers) < tol:
            break
        centers = new_centers
    return centers, labels

# Voronoi regions computation
def compute_voronoi(data, centers, x_min, x_max, y_min, y_max, n=500):
    x = np.linspace(x_min, x_max, n)
    y = np.linspace(y_min, y_max, n)
    xx, yy = np.meshgrid(x, y)
    grid_points = np.c_[xx.ravel(), yy.ravel()]

    distances = np.linalg.norm(grid_points[:, np.newaxis, :] -
centers[np.newaxis, :, :], axis=2)
    voronoi_labels = np.argmin(distances, axis=1)
    return xx, yy, voronoi_labels.reshape(xx.shape)

# Main process for each K
k_vals = [2, 3, 4, 5]
x_min, x_max = data[:, 0].min(), data[:, 0].max()
y_min, y_max = data[:, 1].min(), data[:, 1].max()

plt.figure(figsize=(10, 8))
for idx, k in enumerate(k_vals):
    # Fixed initialization
    init_centers = data[np.random.choice(data.shape[0], k,
replace=False)]

    # Run K-means
    centers, labels = k_means_fixed_init(data, k, init_centers)

    # Compute Voronoi regions
    xx, yy, voronoi_labels = compute_voronoi(data, centers, x_min,
x_max, y_min, y_max)
```

```
    # Plot Voronoi diagram
    plt.subplot(2, 2, idx + 1)
    plt.contourf(xx, yy, voronoi_labels, alpha=0.4,
cmap=plt.cm.rainbow)
    plt.scatter(data[:, 0], data[:, 1], c=labels, s=10,
cmap=plt.cm.rainbow, edgecolor='k')
    plt.scatter(centers[:, 0], centers[:, 1], c='brown', marker='x',
s=250, label='Cluster Centers')
    plt.title(f"K = {k}")
    plt.xlabel('X1')
    plt.ylabel('X2')
    plt.legend()

plt.tight_layout()
plt.show()
```



Part 2(iii)

The Lloyd's algo, which is the standard implementation of the K-means algorithm, is **not** an ideal approach for clustering this datasets due to its non linear decision boundary.

Some important points to note are:

1. The lloyd's algo assigns clusters points by **2nd-norm** which creates **linear decision boundaries** for the voronoi region.
2. This algo assumes clusters are **spherical** in shape which is not the case in this dataset.

Alternatives for this dataset:

1. **Kernelised K-Means:** we can extend the given data to higher dimentions using kernel function, where the data becomes linearly seperable (a polynomial kernel of degree 2 would be suitable for this case). Then the standard k means algo would be safely applicable on this data, while still being computationally feasible.

2. **Spectral Clustering:** In spectral clustering, dimensionality reduction is carried out prior to clustering utilizing the eigenvalues of a similarity matrix that is created using data point distances or affinities. This works very well for non-convex or non-linear clusters as in this case.

3. **Gaussian Mixture Models (GMMs):** these are a probabilistic clustering approach that models the data as a mixture of Gaussian distributions. This data can also be modeled well using GMMs. It can handle overlapping clusters as well as non linear decision boundaries between the clusters.

Summary While Lloyd's algorithm is computationally simple and efficient, it is unsuitable for clustering this data due to its reliance on linear decision boundaries. **Kernel-based approaches** or **Spectral Clustering** should be preferred for such problems to effectively capture non-linear cluster structures.