Midterm Report

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Abstract

We introduce the notions of collections of identical particles, and their associated symmetries, following which we jump into phenomenology associated with superconductors. We use the London Equations to set up a model to explain these, figure out its constraints, and in the process discover Ginzburg-Landau theory to better explain some phenomenon. A digression on the Ginzburg Criterion and the applications of this theory is in order. Before moving on to BCS theory, we delve into two topics: a study of statistical mechanics, and a study of response functions (though we do not cover these here).

Chapter 1

Introduction

1.1 Quantum Mechanics of Identical Particles

Think of a world where there is no force, between particles, just an external field which is governed by some potential. In this world, consider we have a collection of identical particles, which are indistinguishable from each other. We can't tell them apart, and we can't tell them apart even if we exchange them. This is the world of quantum mechanics, and the particles are fermions or bosons.

One may suggest, just paint all the particles in different colors and then we could tell them apart. But this is not a good idea, because the particles are indistinguishable in the most fundamental sense, we cannot color them, or label them, or do anything to them to tell them apart.

The kinds of particles we are familiar with, electrons, protons, neutrons, are all fermions. They have half-integer spin, and obey the Pauli Exclusion Principle. The other kind of particles are bosons, which have integer spin, and do not obey the Pauli Exclusion Principle. All these notions have their roots in relativistic quantum mechanics, which we do not delve into. However, we do indulge ourselves in a discussion of what happens when we assume these to be true.

A discourse on this is presented in the Chapter 2.

1.2 Superconductivity

What makes a conductor *super*? The answer is simple, it is the fact that it conducts electricity without any resistance. This is a very useful property, and has many applications. However, it is not the only property of superconductors. They also exhibit the Meissner Effect, which is the expulsion of magnetic fields from the interior of the superconductor.

These notions together may seems contradictory. You may assume that zero resistance implies infinite conductivity, and by using the Maxwell Equations, you can show that infinite conductivity implies that the magnetic field is a constant inside the superconductor, not necessarily zero. However, the Meissner effect observes something else. It observes that the magnetic field decays from a finite value at the surface of the superconductor to zero inside it on a length scale called the London Penetration Depth (denoted by λ_L). This scale is of the order of a few hundred nanometers, and is a material property.

Of course, as physicists got obsessed with this brillian phenomenon, they started to discover more and more properties of superconductors. One of the most important ones is the fact that they have a critical temperature, below which they are superconducting, and above which they are not. This is called the superconducting phase transition, and is a second order phase transition.

It was soon discovered that magnetic fields can penetrate the superconducting state, but in quantums of a fundamental constant, called the flux quantum, $\phi_L = \frac{hc}{2e}$.

The theory of superconductivity is a very rich one, and has many facets. We will be studying the phenomenology and the theory of superconductivity in Chapter 3.

Chapter 2

QM of Identical Particles

2.1 Permutation Symmetry

We define a permutation operator for a system of two particles as:

$$P_{12}|k\rangle|k'\rangle = |k'\rangle|k\rangle \tag{2.1}$$

where the first state is the state of the first particle, and the second state is the state of the second particle. On applying permutation, we are effectively exchanging the two particles. It is easy to see that $P_{12}^2 = 1$, and that P_{12} is unitary.

From these, we obtain that the eigenvalues of P_{12} are ± 1, and the eigenstates are:

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}\left(|k\rangle|k'\rangle \pm |k'\rangle|k\rangle\right) \tag{2.2}$$

Consider these two particles to have some interaction potential $V_{int}(\vec{r}_1, \vec{r}_2)$ and they exist in some external potential described by $V(\vec{r})$. Clearly, if the particles are indistinguishable, then the Hamiltonian must be invariant under the exchange of the two particles. This implies that the Hamiltonian commutes with the permutation operator. From the Heisenberg equations of motion, we obtain that if the system begins in some eigenstate of the permutation of operator, then it will remain in that eigenstate for all time.

This formalism can be extended to a system of N particles, and we can define the permutation operator as:

$$P_{ij}|\text{N identical bosons}\rangle = +|\text{N identical bosons}\rangle \tag{2.3}$$

$$P_{ij}|\text{N identical fermions}\rangle = -|\text{N identical fermions}\rangle \tag{2.4}$$

where P_{ij} is the permutation operator which exchanges the i^{th} and j^{th} particles, with i and j arbitrary. Mixed symmetry has not been observed in nature, and hence we do not consider it.

2.2 Spin Statistics Theorem

All particles with integer spin are bosons.
All particles with half-integer spin are fermions.

This theorem can be proved using relativistic quantum mechanics, and we do not prove it here. A simple consequence of this manifests as:

- Two fermions:

$$|\psi_-\rangle = \frac{1}{\sqrt{2}}\left(|k\rangle|k'\rangle - |k'\rangle|k\rangle\right) \tag{2.5}$$

  is the only possible state for two fermions. This is called the singlet state.

- Two bosons:

$$|\psi_+\rangle = \frac{1}{\sqrt{2}}\left(|k\rangle|k'\rangle + |k'\rangle|k\rangle\right) \tag{2.6}$$

$$|\psi_1\rangle = |k\rangle|k\rangle \tag{2.7}$$

$$|\psi_2\rangle = |k'\rangle|k'\rangle \tag{2.8}$$

  are the possible states for two bosons. This are called the triplet states.

which is in stark contrast to the classical case, where we have four possible states: $|k\rangle|k\rangle$, $|k\rangle|k'\rangle$, $|k'\rangle|k\rangle$, and $|k'\rangle|k'\rangle$.

The mutliparticle extensions of this theory are known as 1. Fermi-Dirac, 2. Bose-Einstein, and 3. Maxwell-Boltzmann statistics. The first two are used to describe fermions and bosons respectively, and the third one is used to describe classical particles.

Observe that, even in a two particle system, we see that bosons prefer to be in triplet states, and fermions prefer to be in singlet states, even more so than classical particles ('prefer' here refers to the fact that the probability of finding the particles in these states is higher than the other states). This seemingly mathematical fact has profound consequences in the real world, where we observe Bose-Einstein condensation of ^{4}He.

2.3 Two-Electron System

A particularly illuminating system to study is the two-electron system because it has two important symmetries: 1. Spin symmetry, and 2. Permutation symmetry. It will become clear how these symmetries are related to each other.

The combined state for these can be written as:

$$|\Psi\rangle = |\phi\rangle|\chi\rangle \tag{2.9}$$

where $|\phi\rangle$ is the spatial part of the wavefunction, and $|\chi\rangle$ is the spin part of the wavefunction. The only condition is that the total wavefunction must be antisymmetric under exchange of the two electrons. This implies that when exchange the particles, one of two cases must hold:

1. $|\phi\rangle$ is symmetric, and $|\chi\rangle$ is antisymmetric.
2. $|\phi\rangle$ is antisymmetric, and $|\chi\rangle$ is symmetric.

and no other case is possible.

It is interesting to note that nature of symmetry of the spatial part of the state directly corresponds to even angular momentum (symmetric) and odd angular momentum (antisymmetric). This follows from the symmetries of the spin part of the wavefunction.

2.4 Remarks

Generalizations and extensions of these notions are possible, and are used in the study of many-body systems. However, we do not delve into these notions in this report. Some further reading topics are:

- Solution of Helium Atom (application of Perturbation Theory and Variational Principle)
- Second Quantization: a way to proceed to QFT
- Degenerate Fermi Gas
- Quantization of the Electromagnetic Field
- Casimir Effect

These topics, though not covered in this report, are of prime import and guide the theory of superconductivity.

One important remark is directly copied from the book by Sakurai.

> *There is no need to antisymmetrize if the electrons are far apart and the overlap is negligible. This is* quite *gratifying. We never have to worry about the question of antisymmetrization with 10 billion electrons, nor is it necessary to take into account the antisymmetrization requirement between an electron in New York and an electron in Beijing.*

Chapter 3

Superconductivity

3.1 Phenomenology

3.1.1 Perfect Conductivity

Below a certain critical temperature T_c, the resistivity of a superconductor becomes zero for DC. The same is true for AC, but only for frequencies below a certain critical frequency $\omega_c = 2\Delta/\hbar$. (2 in ω_c is a result of the fact that the Cooper pairs have charge 2e)

3.1.2 Meissner Effect

1. *For a simply connected state of a Type-I superconductor with $T < T_c$*:
   Magnetic field is expelled from the interior of the superconductor. The magnetic field decays exponentially from a finite value at the surface of the superconductor to zero inside it on a length scale called the London Penetration Depth (denoted by λ_L). This scale is of the order of a few hundred nanometers, and is a material property.

2. *For a Type-I superconductor but with holes (not simply connected)*:
   Meissner phase flux of magnetic field can be trapped in holes in integral multiples of $\phi_L = \frac{hc}{2e}$ while the rest of the superconductor remains in the Meissner phase. There still exists a lengthscale around each hole over which the magnetic field decays exponentially to zero.

3.1.3 Persistent Currents

For case 2 of 3.1.2, if we now reduce the external magnetic field to zero, there still exists a magnetic field inside the holes which is now generated by currents flowing in the superconductor. These currents are called persistent currents, and they have been observed to exist in a stable form for astronomical timescales.

3.1.4 Critical Fields

In the case when we apply external magnetic field H to a superconducting state at some $T < T_c$ (Type-I), we observe that there exists a critical field H_c, above which the superconducting state is destroyed. This is called the critical field.

$$H_c(T) = H_c(0)\left(1 - \left(\frac{T}{T_c}\right)^2\right) \tag{3.1}$$

where $H_c(0)$ is the critical field at $T = 0$. This is a material property, and is different for different materials.

For Type-II superconductors, the situation changes. Instead of instant transition from superconductor to normal (like in Type-I), we get a mixed state, where the superconductor is in the Meissner phase, but there are holes in it where the magnetic field is trapped in integral multiples of ϕ_L. These holes are henceforth called vortices. Vortices start to begin to appear at a critical field H_{c1}, and the superconducting state is destroyed at a critical field H_{c2}. Clearly, for Type-II superconductors, $H_{c1} < H_{c2}$. Moreover, H_{c2} is a sort of measure of the overlap of the vortices, and it's measurement will give us information about some lengthscale associated with decay of field around the vortices.

Vortices interact with each other over some other lengthscale, and information about this can be obtained from the measurement of H_{c1}. Vortices exhibit disorder due to thermal fluctuations. This disorder can be pinned down by lowering the temperature. Mostly, vortices in an ordered state form a triangular lattice, called the Abrikosov Lattice but other lattices are also possible.

3.1.5 Specific Heat Capacity

Since the mechanism of electron interaction changes in a normal to superconducting phase transition, we expect the specific heat capacity to change. This is indeed the case, and we observe that the specific heat capacity of a superconductor is lower than that of a normal conductor.

3.2 Thermodynamic Considerations

We observe from 3.1.5 that thinking about the superconducting phase transition from a thermodynamic perspective is a good idea.

3.2.1 Gibbs Free Energy

We should have begun with Helmholtz Free Energy, but we observe in just the first step that we do not have any direct control over B (which is the conjugate variable to H). So, we switch to Gibbs Free Energy, which works with changes of H instead of B. Note that H is the external magnetizing field, and B is the magnetic field inside the superconductor.

After some elementary calculations, we obtain:

$$g_{\mathrm{s}}(T, H) = g_{\mathrm{n}}(T, H) + \frac{1}{8\pi}(H^2 - H_{\mathrm{c}}^2(T)) \tag{3.2}$$

We can take its higher order derivatives to obtain the following:

$$s_{\mathrm{s}}(T,H) = s_{\mathrm{n}}(T,H) + \frac{1}{4\pi} H_{\mathrm{c}} \frac{\partial H_{\mathrm{c}}}{\partial T} \tag{3.3}$$

$$\Delta c(T,H) = \frac{T}{8\pi} \frac{d^2}{dT^2} \left(\frac{H_{\mathrm{c}}^2(T)}{T} \right) \tag{3.4}$$

With the approximation of (3.1), we obtain:

$$\Delta c(T,H) \cong \frac{T H_{\mathrm{c}}^2(0)}{2\pi T_{\mathrm{c}}^2} \left(3 \left(\frac{T}{T_c} \right)^2 - 1 \right) \tag{3.5}$$

One can verify this works nicely in the T near 0 case.

3.3 London Theory

There are many approaches to derive London Theory.

1. *Charged Superfluid*:
   We can think of the electrons as a charged superfluid, and use the equations of motion (Lorentz force) to derive a relation between the supercurrent density and the magnetic field.

   $$\nabla \times \vec{J}_s = -\frac{n_s e^2}{m} \vec{B} \tag{3.6}$$

   This can further be simplified to obtain:

   $$\nabla^2 \vec{B} = \frac{1}{\lambda_L^2} \vec{B} \tag{3.7}$$

   We can in fact simplify (3.6) to obtain a relation between the supercurrent density and the vector potential. This will be possible in a certain gauge.

   More theory can be extracted from this, such as vortex solutions, flux quantisation derived through gauge transformations, etc.

2. *de Gennes' derivation*:
   We use the total free energy of the system as a functional of the supercurrent density and magnetic field. Using Maxwell's equations, substitute supercurrent density by curl of $\vec{B}$. We then minimize this functional with respect to $\vec{B}$ to obtain the London Equations.

Pippard extended this theory by observing that the supercurrent density seems to be perfectly yoked to the magnetic vector potential. He corrected this as is done in classical electrodynamics, and obtained the following:

$$j_{\mathrm{s}}^{\alpha}(\vec{x}) = -\frac{c}{4\pi\lambda_L^2} \int d^3 r K^{\alpha\beta}(\vec{r}) A_{\beta}(\vec{x}+\vec{r}) \tag{3.8}$$

with

$$K^{\alpha\beta}(\vec{r}) = \frac{3}{4\pi\xi} \frac{e^{-r/\xi}}{r^2} \hat{r}^{\alpha} \hat{r}^{\beta} \tag{3.9}$$

with certain gauge conditions.

3.4 Ginzburg-Landau Theory

Assume that God has given you a magical tool. This tool is the order parameter of the system. It is a function of space and time, and it is a complex number. It is a magical tool because we do not know where it came from but we do know what it can be used for. It can be used to describe the superconducting state of a system.

The order parameter is equivalent to a wavefunction which tells us the probability of finding a superconducting electron pair at a certain point in space and time. Of course, we must normalize it before we can use it, else we will have to deal with 10 million billion billion electrons at once.

$$|\Psi(\vec{x}, t)|^2 = n_s/n \tag{3.10}$$

where n_s is the superconducting electron density, and n is the total electron density. We assume that at $T = T_c$, the order parameter is zero everywhere. This is a reasonable assumption because we know that at $T = T_c$, the superconducting state is destroyed. We also assume that at $T = 0$, the order parameter is normalized.

3.4.1 Helmholtz and Gibbs Free Energy functionals, Equations of Motion

We can write the free energy of the system as a functional of the order parameter. We directly write in the presence of an external magnetic field.

$$F[\Psi, \Psi^*, \vec{A}] = \int d^dx \left(a|\Psi(x)|^2 + \frac{b}{2}|\Psi(x)|^4 + K|(\vec{\nabla} + \frac{ie^*}{\hbar c}\vec{A})\Psi(x)|^2 + \frac{1}{8\pi}(\vec{\nabla} \times \vec{A})^2 \right) \tag{3.11}$$

where $a < 0, b > 0, K > 0$ are material parameters, and e^* is the charge of the Cooper pair. Define a lengthscale $\xi = \sqrt{K/|a|}$ called *coherence length.*

It will be easier to scale these equations appropriately. We define the following:

$$\Psi = \sqrt{\frac{-a}{b}}\psi, \quad \vec{x} = \lambda_L \vec{r}, \quad \vec{A} = \sqrt{2}\lambda_L H_c \vec{a}, \quad \vec{h} = \frac{\vec{H}}{\sqrt{2}H_c}, \quad \kappa = \frac{\lambda_L}{\xi} \tag{3.12}$$

where $H_\mathrm{c} = \frac{\phi_\mathrm{L}}{\sqrt{8}\pi\xi\lambda_L}$

The scaled equations, in presence of an external (scaled) field $\vec{h}$, are:

$$G[\psi, \psi^*, \vec{a}] = \frac{H_\mathrm{c}^2\lambda_L^3}{4\pi} \int d^dr \left(-|\psi|^2 + \frac{1}{2}|\psi|^4 + |(\kappa^{-1}\nabla + i\vec{a})\psi|^2 + (\nabla \times \vec{a})^2 - 2\vec{h} \cdot \nabla \times \vec{a} \right) \tag{3.13}$$

Taking $\delta G = 0$ with respect to ψ^* and $\vec{a}$, we obtain the following equations:

$$\left(\kappa^{-1}\nabla + i\vec{a}\right)^2 \psi = \psi(|\psi|^2 - 1) \tag{3.14}$$

$$\nabla \times (\nabla \times \vec{a} - \vec{h}) + |\psi|^2\vec{a} = \frac{i}{2\kappa}(\psi^*\nabla\psi - \psi\nabla\psi^*) \tag{3.15}$$

with the boundary condition:

$$\hat{n} \cdot (\nabla + i\kappa\vec{a})\psi\Big|_{\partial\Omega} = it \tag{3.16}$$

with $\hat{n}$ being the normal to the boundary, and $\partial\Omega$ being the boundary of the region of interest, $t \in \mathbb{R}$. t represents transmission coefficient in case when the boundary is a junction, and is zero when the boundary is vacuum.

3.4.2 London Limit

Taking $\Psi = \sqrt{n}e^{i\varphi}$, that is, assuming that all the information about the superconducting state is contained in the phase of the order parameter, we obtain the following equations:

$$\vec{j} = -\frac{c}{4\pi\lambda_L^2}(\frac{\phi_L}{2\pi}\nabla\varphi + \vec{A}) \tag{3.17}$$

$$\nabla \times \vec{j} = \frac{c}{4\pi}\nabla \times \nabla \times \vec{B} \tag{3.18}$$

$$= -\frac{c}{4\pi\lambda_L^2}\vec{B} - \frac{c}{4\pi\lambda_L^2}\frac{\phi_L}{2\pi}\nabla \times \nabla\varphi \tag{3.19}$$

which reduces to:

$$\lambda_L^2\nabla^2\vec{B} = \vec{B} + \frac{\phi_L}{2\pi}\nabla \times \nabla\varphi \tag{3.20}$$

this is the London Equation, with provision to solve for vortex solution as well. As expected, the solution for one vortex shows that the magnetic field decays exponentially from a finite value at the center of the vortex to zero at a lengthscale of ξ, related by $\kappa = \lambda_L/\xi$.

3.4.3 Ginzburg Criterion

We want to check how close to T_c we can explain using Ginzburg-Landau theory. We do this by considering fluctuations in the order parameter with respect to the mean field. The obvious constraint is that we need the fluctuations to contribute very little to the free energy of the system as compared to the mean field. This is the Ginzburg Criterion.

We can calculate two-point correlation functions of the order parameter from an arbitrary point to the coherence length lengthscale; this is the fluctuation. Mean can be calculated by taking the expectation value of the order parameter over the volume of this coherence lengthscale.

$$\frac{\int_0^\xi d^dr\langle\Psi^*(\vec{r})\Psi(0)\rangle}{\int_0^\xi d^dr\langle|\Psi(\vec{r})|^2\rangle} \ll 1 \tag{3.21}$$

By algebraic manipulation, we obtain that $t_G \ll t$ with

$$t_G = \left(\frac{a}{R_*}\right)^{\frac{2d}{4-d}} \tag{3.22}$$

where $R_* = \sqrt{\frac{K}{\alpha}}$, with $a(T) = \alpha t$. This temperature difference is exceptionally small and is of the order of 10^{-18} for $d = 3$. Hence, we can use GL Theory to explain superconductivity very close to T_c.

3.5 Applications of GL Theory

We can use GL Theory to explain many phenomenon. We present some of them here, in the form of their formal setup and boundary conditions. We assume uniqueness of the solution to the GL equations, and hence we do not worry about the existence of the solution (these are guaranteed by Maxwell's Equations having unique solutions).

3.5.1 Critical Fields

Type-I Superconductors

Consider an extreme Type-I superconductor ($\kappa \ll 1$) with its finite dimension in $x = \pm d/2$. We apply an external magnetizing field $\vec{h}$ in the z direction. Let $\vec{a} = a(x)\hat{y}$, and $\psi = \psi(x)$. We have equations:

$$\frac{d^2 a}{dx^2} = a\psi^2 \tag{3.23}$$

$$\frac{1}{\kappa^2}\frac{d^2\psi}{dx^2} + (1-a^2)\psi - \psi^3 = 0 \tag{3.24}$$

with boundary conditions:

$$\psi'(x) = 0 \quad \text{at} \quad x = \pm\frac{d}{2} \tag{3.25}$$

$$\nabla \times \vec{a} - \vec{h} = 0 \quad \text{at} \quad x = \pm\frac{d}{2} \tag{3.26}$$

We obtain $\psi =$ const. to a first approximation. We get some solution, in which we have to apply perturbation theory to obtain the critical field.

The final approximate solutions are:

1. Thin Film: $h_c = \frac{2\sqrt{3}}{d}\sqrt{1-\psi_0^2} \cong \frac{2\sqrt{3}}{d}$
2. Thick Film: $h_c = \frac{1}{\sqrt{2}}(1 + d^{-1})$

Type-II Superconductors

Lower Critical Field: H_{c1}
The system is in Meissner phase, and just the first vortex line is about to appear. We figure out the solution for single vortex line, with respect to the magnetic field passing through it, and relate that to the critical field by making the energy penalty of vortex formation negative.

Taking $|\psi| \sim 1$ we have:

$$\frac{G_v - G_0}{L} = \frac{H_c^2\lambda_L^2}{4\pi}\int d^2\rho\left(\vec{b}\cdot[\vec{b} + \nabla\times(\nabla\times\vec{b})] - 2\vec{h}\cdot\vec{b}\right) \tag{3.27}$$

$$\int d^2\rho\,\vec{b} = 2\pi n\kappa^{-1}\hat{z} \tag{3.28}$$

with n = 1 for a vortex of topological index 1. From these, we find h_{c1}.

$$h_{c1} = \frac{1}{2}\kappa^{-1}\ln 2e^{-C}\kappa \tag{3.29}$$

$$H_{c1} = \frac{\phi_L}{4\pi\lambda_L^2}\ln 1.23\kappa \tag{3.30}$$

where C is the Euler-Mascheroni constant.

Upper Critical Field: H_{c2}

For this, we can consider the order parameter to be very small, and we can thus ignore higher order terms. Observe that this implies $\vec{b} = \vec{h}$ (using free energy relations as well).

$$-(\kappa^{-1}\nabla + i\vec{a})^2\psi = \psi \tag{3.31}$$

We can think of this as an eigenvalue equation, and we can solve for the situation when all eigenvalues must exist. This yields the following:

$$\epsilon_n(k_z) = \frac{k_z^2}{\kappa^2} + \left(n + \frac{1}{2}\right)\frac{2b}{\kappa} \tag{3.32}$$

If the minimum eigenvalue is 1, then we have a solution. This gives us the critical field

$$H_{c2} = \sqrt{2}\kappa H_c = \frac{\phi_L}{2\pi\xi^2} \tag{3.33}$$

3.5.2 Critical Currents

We can use GL Theory to calculate the critical current of a superconducting wire. This is the current above which the state turns normal even in abscence of external magnetic field.

3.5.3 Abrikosov Lattice

We can derive how the lattice of vortices is ordered in the cases when vortices just begin to form and when the superconductor is just about to go normal. We do this by solving the GL equations in the presence of an external magnetic field and considering free energy penalties in different lattices. Generally, this penalty is minimized when the vortices form a triangular lattice.

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