Introduction to Causality in Physics

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Back to the Future

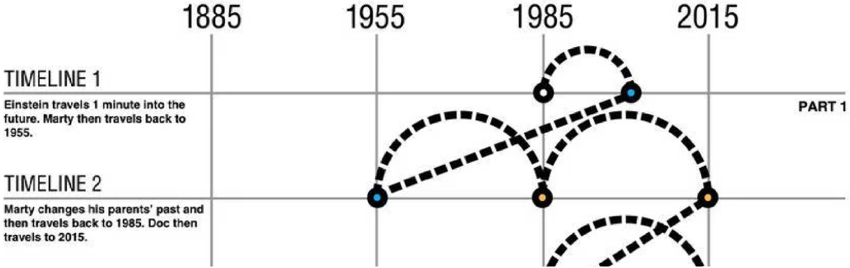


Figure: Sequence of events; Mushir Hoda

Causal Diagram

To represent the events in the movie, we can draw simple diagrams.

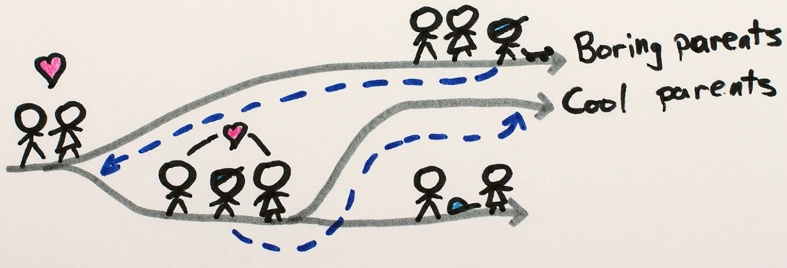
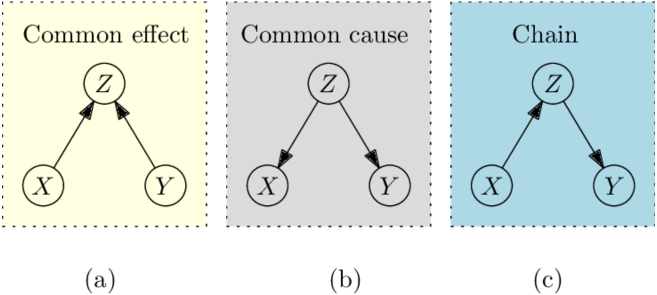


Figure: Representation of events; minutephysics

Basically, events affect each other in a causal manner.

Causal Models

Let there be a colection of events which can influence one another. We can represent this as a directed (acyclic?) graph, where the nodes are the events and the edges are the causal relations.



Y. Cao, B. Li, Q. Li, A. Stokes, D. Ingram and A. Kiprakis, "Reasoning Operational Decisions for Robots via Time Series Causal Inference," 2021 IEEE International Conference on Robotics and Automation (ICRA), Xi'an, China, 2021, pp. 6124-6131, doi: 10.1109/ICRA48506.2021.9561659.

Does the same happen in (3+1)-dimensional spacetime?

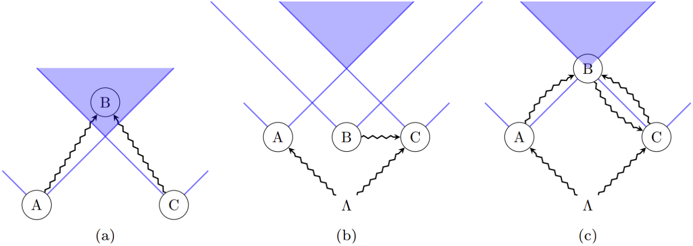
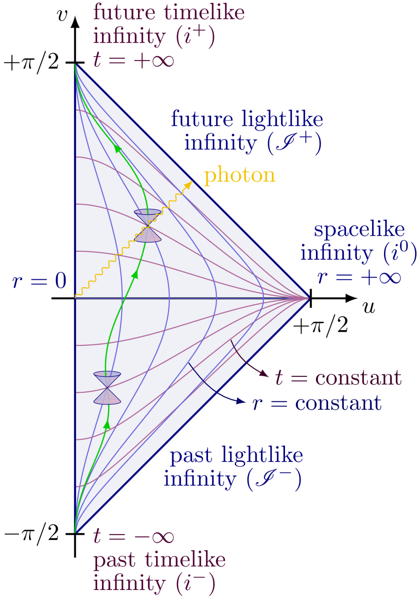


FIG. 1: **Three examples of causal models and their compatible embeddings in (1+1)-Minkowski space-time.** In each case, the operational causal structure associated with the model is given in black, circled variables are observed nodes, while uncircled ones are not and the black arrows denote causation. Space-time information is given in blue with time along the vertical and space along the horizontal axis. The solid lines represent light-like surfaces and the shaded region corresponds to the joint future of A and C in all cases.

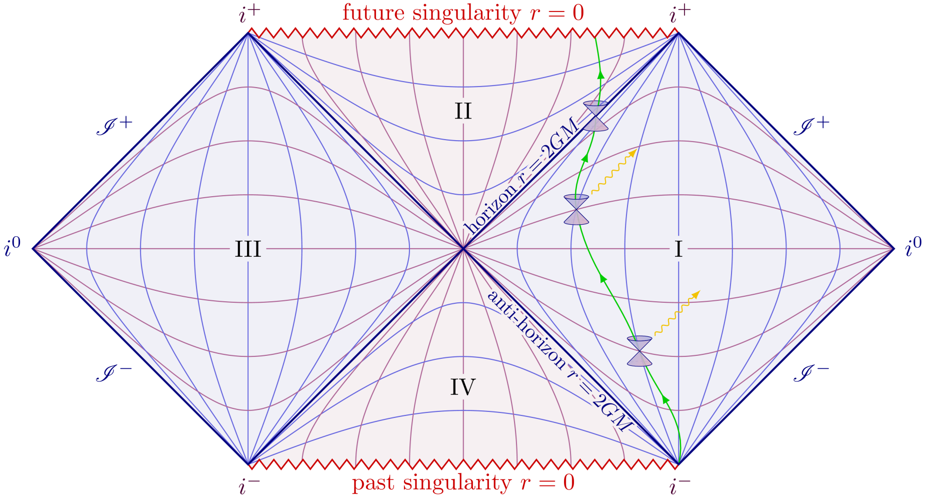
V Vilasini, Roger Colbeck, Physical Review Letters, 129, 110401 (2022), arXiv:2206.12887

Causal Structure of Spacetime



We have done a conformal transformation of the Minkowski spacetime. Coordinates have been changed from cartesian to polar. Compactified spacetime is represented by the Penrose diagram.

Black Holes, Cos Why Not?



Causal Sets

Definition 1 (Timelike relation). *Let $\langle \mathcal{M}, g_{ab} \rangle$ be a temporally oriented relativistic spacetime model. The binary relation $\ll$ of timelike separation is then defined as follows: $\forall p, q \in \mathcal{M}, p \ll q$ if and only if there is a smooth, future-directed timelike curve that runs from p to q.*

Definition 2 ($\ll$-isomorphism). *Let $\langle \mathcal{M}, g_{ab} \rangle$ and $\langle \mathcal{M}', g'_{ab} \rangle$ be temporally oriented relativistic spacetime models. A bijection $\varphi : \mathcal{M} \to \mathcal{M}'$ is a* $\ll$-isomorphism *if, for all $p, q \in \mathcal{M}, p \ll q$ if and only if $\varphi(p) \ll \varphi(q)$.*

Definition 6 (Future (past) distinguishing). *A spacetime $\langle \mathcal{M}, g_{ab} \rangle$ is* future distinguishing *iff, for all $p, q \in \mathcal{M}$,*

$$I^+(p) = I^+(q) \Rightarrow p = q$$

(and similarly for past distinguishing*).*

R. D. Sorkin, "Causal sets: Discrete gravity," in AIP Conference Proceedings, vol. 957, no. 1, pp. 150-173, 2007, doi: 10.1063/1.2821908.

Causal Sets

Theorem 1 (Hawking, King, and McCarthy 1976). *Let ϕ be a $\ll$-isomorphism between two temporally oriented spacetimes $\langle \mathcal{M}, g_{ab} \rangle$ and $\langle \mathcal{M}', g'_{ab} \rangle$. If ϕ is a homeomorphism, then it is a smooth conformal isometry.*

Homeomorphisms, i.e., continuous mappings that also have a continuous inverse mapping, are maps between topological spaces that preserve topological properties. Furthermore,

Definition 7 (Conformal isometry). *A $\ll$-isomorphism ϕ is a* conformal isometry *just in case it is a diffeomorphism and there exists a (non-vanishing) conformal factor $\Omega : \mathcal{M}' \to \mathbb{R}$ such that $\phi_*(g_{ab}) = \Omega^2 g'_{ab}$.*

Theorem 2 (Malament 1977). *Let ϕ be a $\ll$-isomorphism between two temporally oriented spacetimes $\langle \mathcal{M}, g_{ab} \rangle$ and $\langle \mathcal{M}', g'_{ab} \rangle$. If $\langle \mathcal{M}, g_{ab} \rangle$ and $\langle \mathcal{M}', g'_{ab} \rangle$ are distinguishing, then ϕ is a smooth conformal isometry.*

Causal Sets

It is important to note that neither future- nor past-distinguishability alone are sufficient to clinch the consequent (Malament 1977, 1402). The theorem thus establishes that, for a large class of spacetimes, causal isomorphisms also preserve the topological, differential, and conformal structure, and hence the metrical structure up to a conformal factor. In this sense, we can say that the causal structure of a relativistic spacetime in this large class 'determines' its geometry up to a conformal factor.

Thank You!

Paper Presentation

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