FALL 2021 **STUDENT NAME: ______________________________**

MATH 503 NUMERICAL ANALYSIS **MID-TERM 1**

Instructions:

- Examination time: 2 h.
- This examination is open book.
- **Answer with an Expression/Formula/Text, write intermediate solutions, not just a final answer or suffer a penalty!** Clearly indicate the question number.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Total
2	4	2	2	2	2	4	2	20

1. (2 pts, 0.5 pts ea) Circle the correct answer(s).

1.1. Newton's Root finding method may fail when:
a) f(x) is negative
b) f'(x) is large
c) f'(x) is zero
d) The method never fails!

1.2. The number resulting from 0.01850 × 103 has ______ significant digits
a) 3 b) 4 c) 5 d) 6

1.3. The reason for using numerical methods is
a) one cannot always find exact solutions
b) numerical methods are more precise than analytical methods
c) one can easily experiment with model parameters.

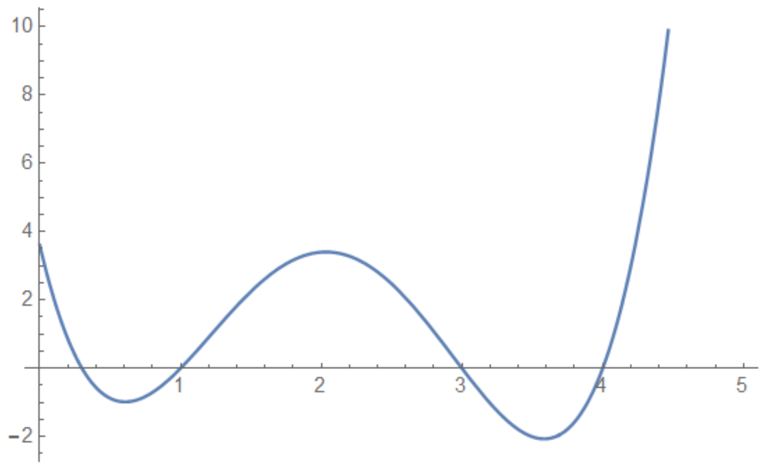
1.4. f(x) is a continuous function. $f(a)\cdot f(b) > 0$ for two real numbers a and b. Then,
a) At least one root of $f(x)$ is in interval $[a, b]$
b) No root of $f(x)$ lies in interval $[a, b]$
c) Either no root or an even number of roots lie in $[a, b]$
d) We must know whether the function is differentiable before stating something about number and existence of roots.

2. (4 pts, 1+1+2) The bisection method is applied to the function (graph is given below) $f(x) = (x-0.3)(x-1)(x-3)(x-4)$. The initial interval used for the method is [0, 5]. The function obviously has zeros at 0.3, 1, 3 and 4.

   a) Why is the interval [0, 5] not a satisfactory starting interval for bisection?

   b) If you start with [0.5, 5], which root will the method converge to (circle it on the graph)?

   c) Using as a starting interval [0, 0.32]. How many iterations will be required to attain an accuracy of 0.01?



3. (2 pts) Show that Newton's method applied to a linear function $f(x) = mx + b$, $m \neq 0$, will converge to a root on the first iteration.

4. (2 pts) The iteration scheme of Newton's method for root finding is

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}, \quad k = 0, 1, 2, \ldots, \quad x_0 \text{ is a given initial approximation.}$$

   Show that Newton's method is also an example of the fixed point method (express Newton's method in the iteration scheme of the fixed-point method).

5. (2 pts) For the equation $f(x) = x^2 - x - 2 = 0$ each of the following functions yields an equivalent fixed-point problem $x = g_i(x)$, where

$$g_1(x) = x^2 - 2,$$
$$g_2(x) = \sqrt{x+2}.$$

Analyze the convergence properties of **one** of the corresponding fixed-point iteration schemes for the root $x = 2$ by considering $|g_i'(2)|$.

6. (2 pts) A hypothetical algorithm for solution of nonlinear equation achieves the **cubic** convergence rate. During your testing you noticed that after the three initial iterations the error of the solution was 10^{-1}. How many **more** iterations do you need to reduce the error to better than 10^{-14}. Explain.

7. (4 pts) Suppose that $x_0 = 0, \quad x_1 = 1, \quad x_2 = 2$ and $x_3 = 3$.
   Find the coefficients a_0, a_1, a_2 and a_3 in the representation
   $$p(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + a_3(x - x_0)(x - x_1)(x - x_2)$$
   of the Newton's polynomial of degree 3 whose graph passes through the points:
   $$p(x_0) = 3, \quad p(x_1) = 1, \quad p(x_2) = 3 \text{ and } p(x_3) = 15.$$

8. (2 pts) The function values are given in 4 nodes: $f(0.1) = 0.665; \quad f(0.2) = 0.8; \quad f(0.3) = 1.8$ and $f(0.4) = 0.25$.

   Use the appropriate Lagrange interpolating polynomial of degree **two** $P_2(x)$ to approximate $f(0.22)$ (Note: You **do not need** to find the actual value of $f(0.22)$; instead, you only need to write down the interpolating polynomial. Do not simplify the expression !).