Revisiting the EPR Paradox: A Qiskit Simulation of Bell's Inequality

Course: Advanced Quantum Mechanics - I

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1 Introduction

"There are more things in Heaven and Earth, Horatio,
Than are dreamt of in your philosophy."
— *William Shakespeare,* Hamlet *(Act I, Scene V)*

From the earliest days of quantum theory, physicists have wrestled with the unsettling realization that the microscopic world defies the certainties of classical thought. Nowhere is this tension more vivid than in the phenomenon of *quantum entanglement*, where two particles, once linked, appear to share a single physical reality, so that the act of measuring one instantly defines the state of the other, no matter how far apart they are.

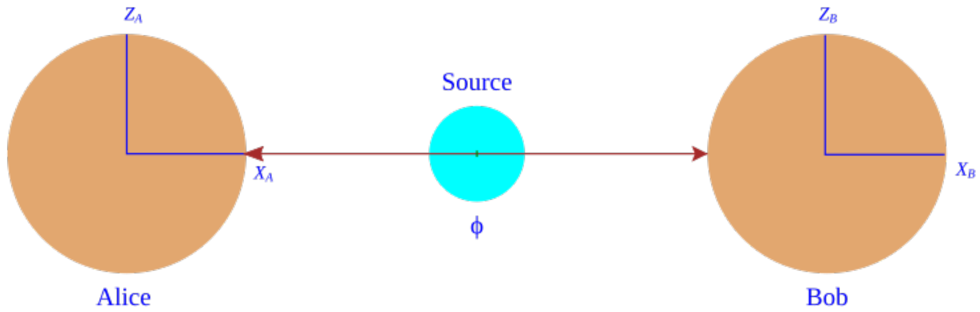
In 1935, Einstein, Podolsky, and Rosen introduced what is now known as the *EPR paradox*, questioning whether the quantum-mechanical description of physical reality could be considered complete [5]. David Bohm later reformulated this argument in terms of spin-entangled particles [3], leading to a clearer experimental framework for testing the nature of quantum correlations.

Three decades later, John Bell provided a mathematical criterion to distinguish quantum predictions from any local hidden-variable theory, formulating what is now known as *Bell's inequality* [2]. This theoretical foundation culminated in experimental verification, most notably through the optical experiments of Alain Aspect and collaborators [1], which demonstrated the violation of Bell inequalities in accordance with quantum mechanics. A more refined version of the CHSH formulation—was later proposed by Clauser, Horne, Shimony, and Holt [4], becoming the standard for studying quantum nonlocality.

In this work, the essence of the EPR–Bohm scenario and Bell's inequality test was reproduced using Qiskit simulations. Beginning with the construction of an entangled state, measurement correlations vary under different detector orientations was explored, and ultimately evaluate the CHSH parameter to verify the quantum violation of Bell's inequality, confirming the nonlocal nature of quantum mechanics [6].

2 Methodology

2.1 The EPR–Bohm Thought Experiment



The EPR–Bohm thought experiment [3] can be illustrated using electron–positron pairs. Consider a source that emits such pairs, where the electron is sent to a detector operated by Alice at location A, and the positron is sent to Bob at location B. According to quantum mechanics, the source can be configured so that each emitted pair occupies a *spin singlet state*, meaning that the two particles are *entangled*. This singlet state can be expressed as a quantum superposition of two possible configurations, which we may call *State I* and *State II*.

In State I, the electron has spin oriented upward along the z-axis ($+z$), and the positron has spin downward along the z-axis ($-z$). In State II, the electron has spin $-z$, and the positron has spin $+z$. Because the system exists in a superposition of these two states, it is impossible to determine the definite spin of either particle prior to measurement.[6]

When Alice measures the spin of her electron along the z-axis, she obtains one of two possible outcomes: $+z$ or $-z$. Suppose she measures $+z$; the quantum state of the system then *collapses* into State I. Consequently, if Bob subsequently measures the spin of his positron along the same z-axis, quantum mechanics predicts with 100% certainty that he will obtain $-z$. Similarly, if Alice's outcome is $-z$, Bob's result will be $+z$.

There is nothing special about the choice of the z-axis. The spin singlet state can be equally well expressed as a superposition of spin states defined along any axis, such as the x-axis. Regardless of which axis the spins are measured along, the results are always found to be perfectly anticorrelated.

In quantum mechanics, however, the spin components along the x and z directions are *incompatible observables*, meaning that the Heisenberg uncertainty principle prohibits a quantum system from having definite values for both simultaneously. Suppose Alice measures her electron's spin along the z-axis and obtains $+z$, collapsing the system into State I. If Bob then measures his positron's spin along the x-axis instead of the z-axis, the outcome is probabilistic: there is a 50% chance of obtaining $+x$ and a 50% chance of $-x$. The result cannot be known until the measurement is actually made.

Thus, while measurements along the same axis yield perfectly opposite results, measurements along different axes produce random outcomes consistent with quantum mechanical predictions. This counterintuitive behavior suggests that, upon Alice's measurement, information about her result appears to instantaneously influence Bob's distant particle—an effect that Einstein famously described as "spooky action at a distance."
The study was divided into two stages: (i) a preliminary demonstration of quantum entanglement, and (ii) a simulation of the Bell test experiment using the Qiskit framework.

2.2 Demonstration of Quantum Entanglement

A quantum circuit was designed with four bits — two quantum and two classical — to visualize basic quantum entanglement. A Controlled-NOT (CNOT) gate was applied, where the target qubit flips ($1 \to 0$ or $0 \to 1$) only if the control qubit is in the state $|1\rangle$. Thus, the system consists of four possible basis states:

$$|00\rangle, |01\rangle, |10\rangle, |11\rangle.$$

For an entangled pair, measurement results are expected to be correlated, i.e., the state collapses to either $|00\rangle$ or $|11\rangle$ with approximately equal probability. To verify this, the circuit was simulated 10^4 times, and the measurement outcomes were recorded. The resulting probability distribution was found to be nearly 50/50 between $|00\rangle$ and $|11\rangle$, confirming the formation of an entangled Bell state.This experiment was implemented using Qiskit.[1]

2.3 Bell Test Simulation

In the second stage, the Bell test[2] was simulated to verify the violation of Bell's inequality and demonstrate the presence of quantum correlations.

[1] https://qiskit.org/documentation/

Two qubits were assigned to represent the measurement parties, Alice and Bob. The circuit first generated an entangled Bell pair using a Hadamard and a CNOT gate. Measurement rotations were then applied to each qubit by angles θ_a and θ_b, corresponding to the respective measurement settings of Alice and Bob.

It is well known that the greatest conflict between quantum mechanical predictions and the Bell inequalities is expected for the set of orientations $(a, b) = (b, a') = (a', b') = 22.5°$ and $(a, b') = 67.5°$ as as per Aspect et al.[1]. In the simulation, the following corresponding angle pairs (in radians) were adopted:

$$\text{Angles} = \begin{cases} (a, b) = (0, \pi/8), \\ (a, b') = (0, 3\pi/8), \\ (a', b) = (\pi/4, \pi/8), \\ (a', b') = (\pi/4, 3\pi/8). \end{cases}$$

These choices align with the configuration that maximizes the expected quantum violation of the CHSH inequality [4].

For each combination of measurement settings (θ_a, θ_b), the circuit was executed 10^4 times to obtain the counts from the measurements. The expectation value $E(\theta_a, \theta_b)$ for each pair of settings was calculated using the normalized difference between correlated and anticorrelated results:

$$E(\theta_a, \theta_b) = \frac{N_{00} + N_{11} - N_{01} - N_{10}}{N_{00} + N_{11} + N_{01} + N_{10}},$$

where N_{ij} represents the number of measurement outcomes corresponding to the result $|ij\rangle$.

Finally, the Bell parameter S was evaluated using the CHSH (Clauser–Horne–Shimony–Holt)[4 formulation :

$$S = |E(\theta_a, \theta_b) - E(\theta_a, \theta_b') + E(\theta_a', \theta_b) + E(\theta_a', \theta_b')|.$$

According to local hidden variable theories, the upper bound for this parameter is $S \leq 2$. However, quantum mechanics predicts a maximum value of

$$S_{\max} = 2\sqrt{2},$$

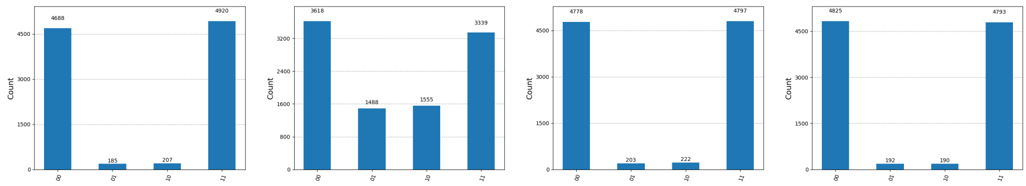
which indicates a clear violation of the classical limit, thereby confirming quantum entanglement between the qubits.

The complete code and data are available on GitHub.[2]

[2] https://github.com/Nil-b0/AQM_EPR_BellTest

3 Results and Discussion

3.1 Results



(a) (a, b) (b) (a, b') (c) (a', b) (d) (a', b')

Figure 1: Measurement outcome histograms for the four detector orientation pairs: (a, b), (a, b'), (a', b), and (a', b') from left to right.

The expectation value measurements are given below:

Table 1: Expectation values for the four detector orientation pairs used in the Bell test.

Pair	**Angles** (θ_a, θ_b)	**Expectation Value** $E(\theta_a, \theta_b)$
(a, b)	$(0, \pi/8)$	0.9216
(a, b')	$(0, 3\pi/8)$	0.3914
(a', b)	$(\pi/4, \pi/8)$	0.9150
(a', b')	$(\pi/4, 3\pi/8)$	0.9236

The corresponding CHSH combination is given by:

$$S = E(a, b) - E(a, b') + E(a', b) + E(a', b') = 2.3868,$$

which clearly violates the classical bound $|S| \leq 2$, consistent with quantum mechanical predictions.

3.2 Discussion

The calculated CHSH parameter 2.3868, clearly violates the classical bound of $|S| \leq 2$, confirming the presence of nonlocal correlations as predicted by quantum mechanics. Although this value is slightly below the theoretical maximum $2\sqrt{2} \approx 2.828$, the deviation can be attributed to finite sampling in the simulation and the specific conventions used for state preparation and measurement. The simulation results demonstrate that even with a finite

number of shots, the nonlocal correlations remain evident. This provides a clear computational verification of the predictions of quantum mechanics and a demonstration of the fundamental incompatibility with local hidden variable models, precisely the point highlighted by the EPR paradox.
From the perspective of the EPR argument, these results reinforce the nonlocal nature of entangled particles: the measurement outcome of one qubit is strongly correlated with that of its partner, in a way that cannot be explained by pre-existing local properties alone. This computational experiment thus provides a concrete illustration of "spooky action at a distance," highlighting the deep conceptual implications of quantum entanglement.

4 Conclusion

The simulation of Bell's inequality using Qiskit successfully demonstrates the violation of the CHSH bound, confirming the inherently nonlocal nature of quantum entanglement. The obtained value of $S = 2.3868$ exceeds the classical limit of 2, providing clear computational evidence against local realism and in agreement with quantum mechanical predictions. By linking the results to the original EPR paradox, this work highlights how modern quantum computing frameworks can reproduce and visualize foundational quantum phenomena. Future investigations could extend this approach to multi-qubit systems, analyze the effects of realistic noise, or perform the same experiment on actual quantum hardware to compare with the idealized simulation.

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