Topological Mechanics and Topological Phononic Phases

Shang Zhang

Department of Physics, University of Michigan

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Abstract

Electronic topological insulators have inspired the design of new mechanical systems that could soon find real-life applications.

I. INTRODUCTION

Creating artificial materials that control electrons, photons, sound waves or mechanical energy is a prime focus of modern materials science. In the past few years, a new paradigm is emerging with the application of ideas from topology, the mathematics describing properties that are unchanged by smooth deformations. People discovered interesting phenomenology of electrons in topological insulators, which are a kind of material with non-trivial topological order that behaves as an insulator in its interior but whose surface contains conducting states, meaning that electrons can only move along the surface of the material[1–3].

Following an explosion of studying in topological phases in hard condensed matter, this topological approach has recently been used from the quantum realm of electrons to the world of classical mechanical systems. Electronic topological insulators have inspired the design of new topological mechanical systems that could lead to new design principles for metamaterials[4]. In this review, we will try to establish the connections between topological electrons systems to classical mechanical systems based on recent research work, as well as perspectives in this field.

A. Analogy between Quantum and Newton's Mechanics

At first sight, electronic systems described by quantum mechanics seem totally different to mechanical modes or phonons described by Newton's equations of motion. But it is not always the case.

In 1985, it was discovered that similar geometric phases to Berry's phase [5] occurred in classical systems with single or few mechanical modes [6], which illustrates the appearing of geometric nature of phases in classical systems.

Then with the emergence of topological insulator, it turned out that the bulk and edge states of phononic systems can display the same unusual properties seen in the topological insulators[7, 8]. So it seems clearer to us that there's some analogy between quantum and newton's mechanics that introduces the similar topological properties in electronic and phononic systems.

To illustrate the analogy between quantum and Newton's mechanics, consider the equa-

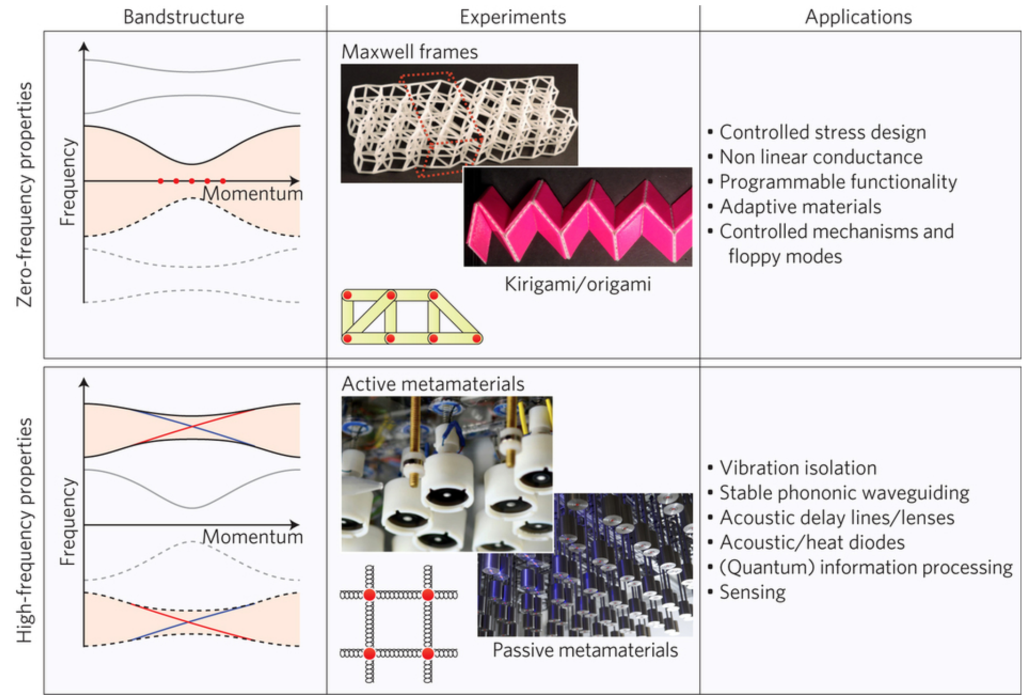

FIG. 1.

tion of motions in a set of coupled simple harmonic oscillators,

$$\ddot{x}_i = -D_{ij}x_j + A_{ij}\dot{x}_j \tag{1}$$

Where the dynamical matrix D is real and positive definite, and the skew-symmetric A describes the coupling between velocities and positions. Then one can rewrite equation (1) to,

$$i\frac{\partial}{\partial t}\begin{pmatrix}\sqrt{D}^T x \\ i\dot{x}\end{pmatrix} = \begin{pmatrix}0 & \sqrt{D}^T \\ \sqrt{D} & iA\end{pmatrix}\begin{pmatrix}\sqrt{D}^T x \\ i\dot{x}\end{pmatrix} \tag{2}$$

Equation (2) transfers Newton's equations into Hermitian eigenvalue equations, in similar form to Schrodinger equation.

Three routes to present topological mechanical systems,

- Capitalize on the intrinsic particlehole symmetry
- Engineer a topological dynamical matrix D directly
- A properly engineered A can give rise to systems with broken time-reversal symmetry, described by a Chern number

II. ZERO-FREQUENCY

III. HIGH-FREQUENCY

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