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Topological mechanics

Sebastian D. Huber

Electronic topological insulators have inspired the design of new mechanical systems that could soon find real-life applications.

Acoustic metamaterials are artificially designed mechanical structures with the purpose of obtaining emergent functionalities such as vibration isolation, acoustic cloaking or adaptive behaviour. In this Commentary, I review how a recently established bridge between the phenomenology of electrons in topological insulators[1–3] and the world of classical mechanical systems might lead to new design principles for such metamaterials.

At first sight, there seems to be an unbridgeable gap between the quantum mechanical description of electrons in solids on the one hand and Newton's equations of motion describing mechanical modes, or phonons, on the other. When it comes to geometric or topological properties, however, this need not be the case.

To understand this connection, let us remind ourselves of the Foucault pendulum. In 1851, Léon Foucault demonstrated the rotation of the Earth by showing that the plane of swing of a pendulum rotates throughout the day. This rotation obtained a beautiful geometric description with the introduction of the concept of parallel transport more than 60 years later[4]: the angle of rotation does not depend on the precise details of the pendulum, but only on the solid angle that the pendulum's point of support traces out in a day.

While the example of the Foucault pendulum is rooted in classical mechanics, research on geometric phases only really took off in the framework of quantum mechanics in the early 1980s. Sir Michael Berry demonstrated the geometric nature of phases appearing in adiabatic quantum evolution[5], David Thouless and co-workers[6] described the quantum Hall effect in terms of a topological invariant called the Chern number, and Barry Simon uncovered the mathematical connection between the two[7].

Only in 2005, with the prediction[8] of a 'topological insulator', was a whole new world of topology-dominated free-electron physics unravelled. But how is this world related to classical mechanics?

It had already been discovered in 1985 that cousins of Berry's phases also appear in classical systems[9]. Unlike in the context of metamaterials, which typically involve an extensive number of degrees of freedom, Hannay[9] focused on systems with single (or few) modes. Catalysed by the birth of the field of electronic topological insulators, topology found a new path back to classical systems[10–13].

To appreciate this new analogy, let us consider the equations of motion for a set of coupled oscillators.

$$\ddot{x}_i = -D_{ij}x_j + A_{ij}\dot{x}_j \tag{1}$$

Here, the real, symmetric and positive-definite dynamical matrix D encodes the forces between the oscillators x_i. The skew-symmetric matrix A describes the conservative (non-dissipative) coupling between positions and velocities. The second-order time derivative in Newton's equations seems to be incompatible with the Schrödinger equation. However, one can rewrite equation (1) in a form that offers more insight[14]

$$i\frac{\partial}{\partial t}\begin{pmatrix}\sqrt{D}^{\mathrm{T}}x\\ i\dot{x}\end{pmatrix} = \begin{pmatrix}0 & \sqrt{D}^{\mathrm{T}}\\ \sqrt{D} & iA\end{pmatrix}\begin{pmatrix}\sqrt{D}^{\mathrm{T}}x\\ i\dot{x}\end{pmatrix} \tag{2}$$

which casts Newton's equations into a Hermitian eigenvalue problem for the frequencies ω, akin to the Schrödinger equation. Besides being first-order in time, this equation makes one important symmetry explicit: owing to the reality of the coefficients in equation (1), for any solution of the equations of motion with frequency ω there is a corresponding solution with $-\omega$. The 'supersymmetric'[10] block form of the matrix in equation (2) encodes this structure in a 'particle–hole' symmetry usually known from the problem of superconductivity.

Given this set-up, three routes to topological mechanical systems present themselves. First, one can directly capitalize on the intrinsic particle–hole symmetry. In this case, topological boundary modes arise in the gap around zero frequency. This route is particularly appealing, as it seems to be the only way to connect topology to the thermodynamic or low-energy properties of a metamaterial (Fig. 1). (Remember that there is no Pauli principle for phonons to fill Bloch bands.)

A second route would be to engineer a topological dynamical matrix D directly. In this case, the stable surface states lie at finite frequencies (Fig. 1). Although they are not of

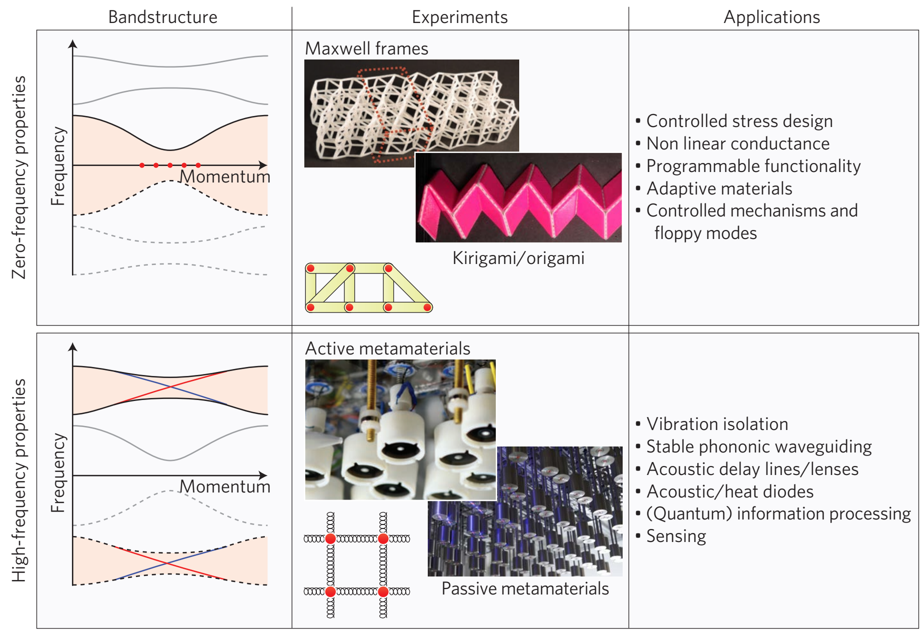

Figure 1 | Overview of the two different families of topological acoustic metamaterials (zero-frequency versus high-frequency). Left: frequency versus momentum for typical band structures of an acoustic crystal. Black lines denote bulk excitations and coloured lines or dots indicate localized boundary modes. Note that for active metamaterials one can obtain a situation in which edge modes are truly unidirectional: that is, only the red or blue edge modes are present. The gap for which the topological features are relevant is shaded in red. Middle: the schematics show the generic set-up ('hinges and rods' versus 'oscillating masses') for both families, together with examples of recent experiments. The red dashed box in the Maxwell frames image indicates a boundary between two topological sectors. Right: lists of potential applications. Figure reproduced from: Maxwell frames, ref. 21, National Academy of Sciences; kirigami/origami, ref. 22, APS; active metamaterials, ref. 28, National Academy of Sciences.

thermodynamic relevance, we argue below how one can put them to use in acoustic metamaterials. Note that owing to the reality of D, chiral systems with unidirectional edge modes are out of reach in this way. This is achieved through a third route, where a properly engineered A can give rise to systems with broken time-reversal symmetry, described by a Chern number.

Below, I review progress over the past few years along these three routes and assess how these developments might allow us to address open problems in the field of acoustic metamaterials. Clearly, in all of these efforts, the main goal is to harness the phenomenology of protected surface or defect modes for mechanical purposes.

Zero-frequency properties

In a seminal paper, Charles Kane and Tom Lubensky[10] introduced a topological characterization of mechanical systems based on the intrinsic particle–hole symmetry. Their focus was on Maxwell frames — systems of rods and hinges (Fig. 1) in which the total number of motional degrees of freedom of the hinges is exactly matched by the number of constraints imposed by the rods. In other words, they discussed mechanical systems at the brink of collapse. Although this scenario might appear fine-tuned and non-generic, it is relevant for critical systems such as granular materials at their jamming transition[15,16], or for various metamaterial designs as elaborated below.

For the case of such Maxwell frames, there is a natural way of taking the 'square root' of the dynamical matrix: it can be written[10] as $D = QQ^{\mathrm{T}}$, where Q^{T} connects the displacements of the hinges to stresses on the rods. Building on this Q-matrix, Kane and Lubensky[10] introduced a winding number that counts the number of localized topological modes.

Owing to the specific set-up of critically connected Maxwell frames, the topological modes encoded by the winding number come in two flavours. A positive winding number predicts zero-frequency modes in which parts of the system move freely. These modes are called 'mechanisms' or 'floppy' modes[15,17] (in which certain hinges can flop without resistance). A negative winding number, on the other hand, predicts a localized state of self-stress, where no forces act on the hinges despite the presence of finite stresses on the rods. Both of these flavours of topological states have recently been observed in experiments.

A one-dimensional Maxwell frame that maps to the famous Su–Schrieffer–Heeger (SSH) model known from the description of electrons in polyacetylene has recently been implemented[18,19]. In their simple and transparent set-up, the authors experimentally confirmed the predicted floppy modes of Kane and Lubensky. Moreover, they were able to show that these floppy modes can be transported through the bulk of the system by a nonlinear mechanism. In fact, this observation seems to be prototypical for mechanical topological metamaterials. Nonlinear effects are rather the standard than the exception. This might lead to exciting discoveries beyond the simple mapping from mechanics to non-interacting electrons of the topological flavour[20].

In another experimental endeavour, the dual partners of the floppy modes, the states of self-stress, were put to work[21]. A defect in a quasi-two-dimensional frame nucleates a localized state of self-stress. When put under load, the material will buckle in the places where the stresses on the rods are concentrated — in other words, around the states of self-stress. It is not much of a stretch to imagine how one could enslave mechanical, optical or electrical properties to the structural change in the selective buckling region. This in turn opens the way to the creation of metamaterials that display reliable adaptive behaviour controlled by external mechanical loads.

Finally, an interesting application of topological Maxwell frames to origami folding has recently been put forward[22]. Origami structures have been identified to be promising candidates for mechanical metamaterials before. However, a controlled strategy to predict folding patterns that support the desired functionality has been hard to come by. In a recent publication[22], researchers were able to show how topological floppy modes might do the job.

In all of the examples presented above, the topological properties arise from the gap in the middle of the particle–hole symmetric spectrum and hence display interesting zero-frequency properties. While useful for structural properties or thermodynamic quantities, they do not enable topologically protected transport by means of linear phonons. If we aim to achieve this, we have to address the high-frequency part of the phonon spectrum directly.

High-frequency properties

Two key challenges present themselves if we try to engineer topological boundary states at non-zero frequencies. First, a truly unidirectional transport channel requires the breaking of time-reversal symmetry. This necessitates the use of non-reciprocal[23–25] acoustic elements, which are not readily available. Second, if we resort to time-reversal-symmetric set-ups, we face another problem. For this case, the electronic topological insulators are conveniently tabulated[26,27] according to their symmetries. Yet these symmetries are natural to the quantum mechanical behaviour of electrons, and it is not clear *a priori* how they relate to classical problems. In particular, one often requires a time-reversal operator that squares to –1, which we typically associate with spin-1/2 particles and not classical phonons.

Several recent experiments give rise to the hope that we can meet these challenges with present-day technology. A Chern insulator with stable one-way edge channels was constructed using a two-dimensional array of hanging gyroscopes[28,29]. The spinning of the gyroscopes enables an effective breaking of time-reversal symmetry, which in turn leads to the observation[28] of strictly unidirectional transport of sound in a certain frequency range. Moreover, because this system is a counterpart to the quantum Hall effect, no fine-tuned symmetries are needed to stabilize the one-way waveguides at the surface.

The issue of a time-reversal symmetry that squares to –1 has also recently been addressed in an experiment. In my own group, we implemented a mechanical analogue of the quantum spin Hall effect and measured the helical edge spectrum (where the direction of propagation is linked to an internal degree of freedom) on a collection of coupled pendula[30] (Fig. 2). These results demonstrate that apparent spin-1/2 effects can be emulated classically. Moreover, we established a classification of all possible topological linear phonon models by adapting the table of topological insulators to the mechanical set-up[14].

Are these two experiments[28,30] all we need to claim that we have successfully built a useful bridge between the electronic and the phononic world? Unfortunately, there is an important shortcoming to both of these examples: the phonons are built from discrete local oscillators (gyroscopes or pendula). A practical material, however, is typically a continuous medium. Important steps in this direction have been taken in two publications last year. A continuous one-dimensional SSH model has been implemented, in which the localized modes at domain walls between different topological sectors have been observed[31]. Moreover, a two-dimensional topological

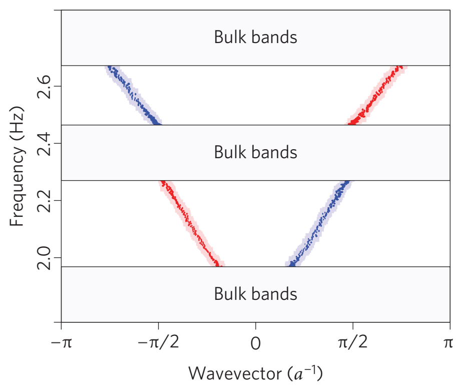

Figure 2 | Measured helical edge spectrum of a mechanical topological insulator. The red and blue colours represent different polarizations of the phonons. The wavevector is measured in units of the inverse lattice constant. Figure adapted from ref. 30, AAAS.

band structure has been measured in a cleverly engineered honeycomb lattice of metallic cylinders[32].

Perspectives

After all the impressive progress in electronic topological systems, one might ask why we also need a mechanical version of the same phenomena. This question is accentuated by recent progress in photonic systems (see Commentary by Lu *et al.*[33]), as these experiments already establish a classical version of the underlying quantum description of topological insulators.

The answer to this question lies in the feasibility of putting topological effects to work in an actual application. Owing to the relatively simple ingredients and the outstanding control over fabrication processes for mechanical systems, it would not be surprising if ideas based on the phenomenology of mechanical topological surface states were to find concrete applications in a rather short time. Specific examples beyond those mentioned here could be the combination of vibration isolation (owing to a bandgap) with a reduced reflection of sound (thanks to the surface states that guide energy around the gapped region). Another promising use of the edge channels is the construction of back-scattering-free waveguides for phonons of almost arbitrary shape. Moreover, if the bandgap is large enough, unidirectional channels act essentially as a heat diode.

Before we get there, however, there are still many exciting challenges to meet. We need a better handle on continuous systems. In particular, a modular way of translating discrete mass–spring models to continuous systems would be a great addition to the toolbox. This could then also aid in the much-needed step to three-dimensional systems. Given the large number of recent proposals for designs of topological phonons[34–40], we will certainly see many experiments leading the way to new applications. Finally, nonlinear effects, which are ubiquitous in mechanical systems, will surely surprise us with interesting features beyond the simple linear theory of topological phonons. ❐

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