Univariate Linear Regression

Load the Data and Libraries

In [1]:
```
import numpy as np
import pandas as pd
import seaborn as sns

import matplotlib.pyplot as plt
plt.style.use('ggplot')
%matplotlib inline
```

In [2]:
```
df_bike = pd.read_csv("bike_sharing_data.txt")
df_bike.head()
```

Out[2]:

	Population	Profit
0	6.1101	17.5920
1	5.5277	9.1302
2	8.5186	13.6620
3	7.0032	11.8540
4	5.8598	6.8233

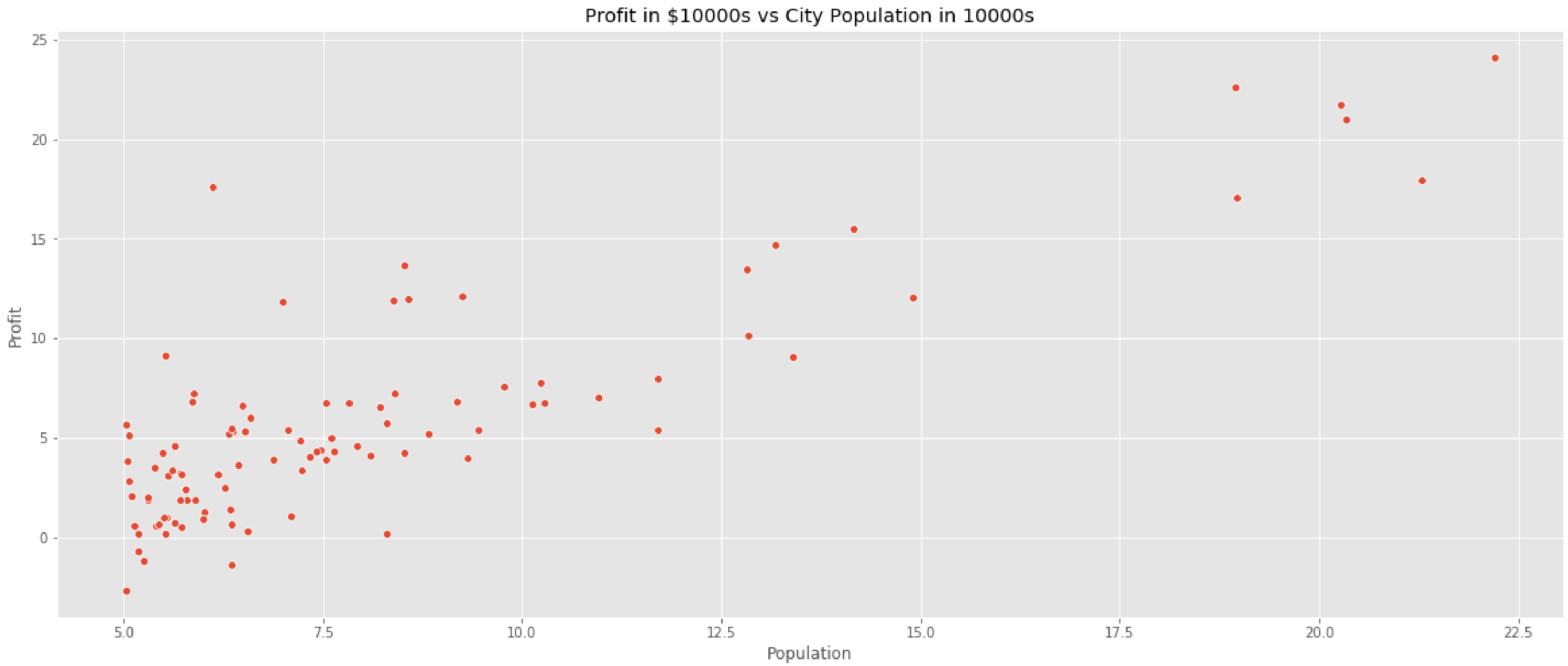
In [3]:
```
df_bike.shape
```

Out[3]: (97, 2)

Visualize the Data

In [4]:
```
fig = plt.figure(figsize = (20, 8))
ax = sns.scatterplot(x = "Population", y = "Profit", data = df_bike)
ax.set_title("Profit in $10000s vs City Population in 10000s")
```

Out[4]: <matplotlib.text.Text at 0x90961f0>



Compute the Cost

The objective of linear regression is to minimize the cost function

$$J(\theta) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

where $h_\theta(x)$ is the hypothesis and given by the linear model

$$h_\theta(x) = \theta^T x = \theta_0 + \theta_1 x_1$$

In [5]:
```
def cost_function(X, y, theta):
    m = len(y)
    y_pred = X.dot(theta)
    error = (y_pred - y) ** 2

    return 1 / (2 * m) * np.sum(error)
```

In [6]:
```
m = df_bike.Population.values.size
X = np.append(np.ones((m, 1)), df_bike.Population.values.reshape(m, 1), axis = 1)
y = df_bike.Profit.values.reshape(m, 1)
theta = np.zeros((2, 1))

cost_function(X, y, theta)
```

Out[6]: 32.072733877455676

Gradient Descent

Minimize the cost function $J(\theta)$ by updating the below equation and repeat unitil convergence

$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$ (simultaneously update θ_j for all j).

In [7]:
```
def gradient_descent(X, y, theta, alpha, iterations):
    m = len(y)
    costs = []
    for i in range(iterations):
        y_pred = X.dot(theta)
        error = np.dot(X.transpose(), (y_pred - y))
        theta -= alpha * 1/m * error
        costs.append(cost_function(X, y, theta))
    return theta, costs
```

In [8]:
```
theta, costs= gradient_descent(X, y, theta, alpha = 0.01, iterations = 2000)

print(" h(x) = {} + {}x1 ".format(str(round(theta[0, 0], 2)),
                                str(round(theta[1, 0], 2))))
```

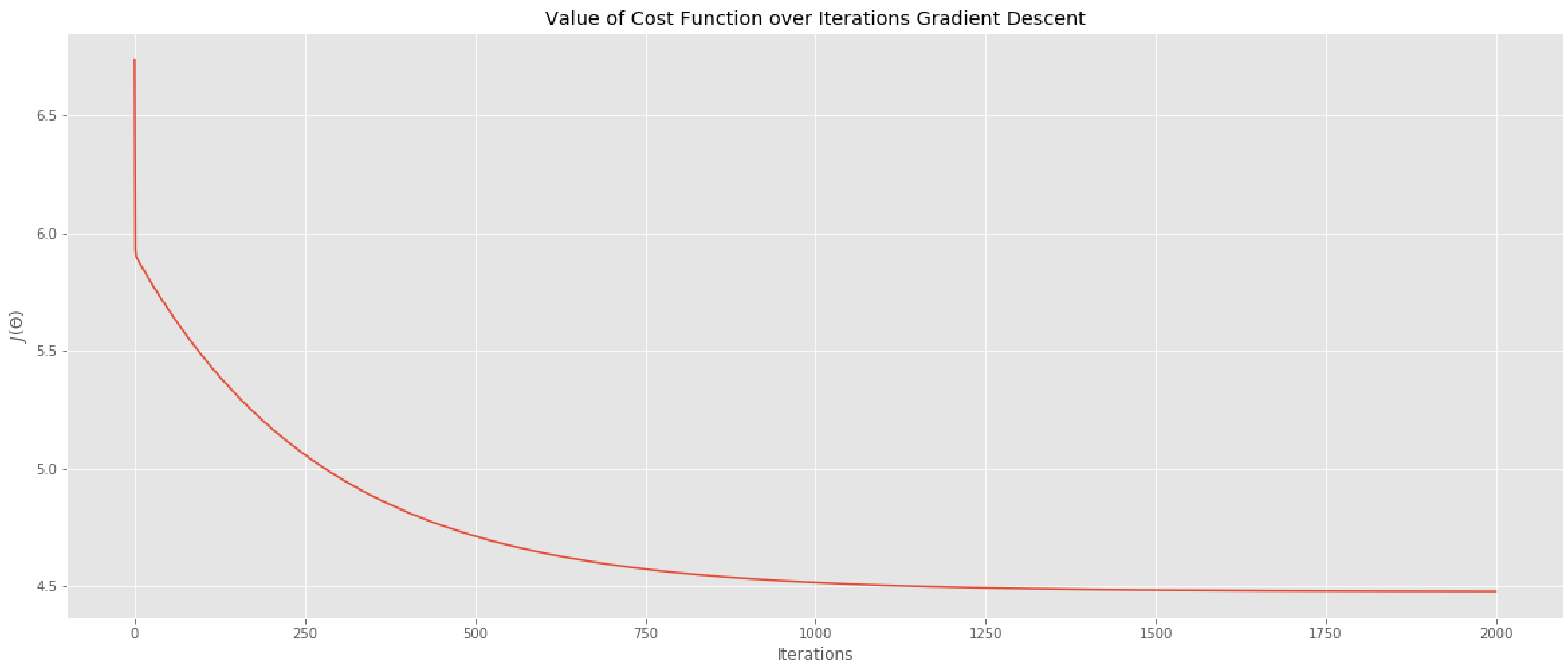
```
 h(x) = -3.79 + 1.18x1
```

Plotting the Convergence

Plot $J(\theta)$ against the number of iterations of gradient descent:

In [9]:
```
fig = plt.figure(figsize = (20, 8))
plt.plot(costs)
plt.xlabel("Iterations")
plt.ylabel("$J(\Theta)$")
plt.title("Value of Cost Function over Iterations Gradient Descent")
```

Out[9]: `<matplotlib.text.Text at 0x94890b0>`



Training Data with Linear Regression Fit

In [10]:
```
theta.shape
```

Out[10]: (2, 1)

In [11]:
```
theta
```

Out[11]:
```
array([[-3.78806857],
       [ 1.18221277]])
```

In [12]:
```
fig = plt.figure(figsize = (20, 8))
theta = np.squeeze(theta)
ax = sns.scatterplot(x = "Population", y = "Profit", data = df_bike)

x_value = [X for X in range(5, 25)]
y_value = [(X * theta[1] + theta[0]) for X in x_value]
sns.lineplot(x_value, y_value)

plt.xlabel("Population in 10,000s")
plt.ylabel("Profit in $10,000s")
plt.title("Linear Regression Fit")
```

Out[12]:
```
<matplotlib.text.Text at 0x9408110>
```

Inference using the optimized θ values

$h_\theta(x) = \theta^T x$

In [13]:
```
def predict(X, theta):
    y_pred = np.dot(theta.transpose(), X)
    return y_pred
```

In [14]:
```
y_pred_1 = predict(np.array([1, 4]), theta) * 10000
print("For a population of 40,000 people, the model predicts a profit of $" + str(round(y_pred_1, 0)))
```

```
For a population of 40,000 people, the model predicts a profit of $9408.0
```

In [15]:
```
y_pred_2 = predict(np.array([1, 8.3]), theta) * 10000
print("For a population of 83,000 people, the model predicts a profit of $" + str(round(y_pred_2, 0)))
```

```
For a population of 83,000 people, the model predicts a profit of $60243.0
```