Grocery Lines

Grocery Lines and Similar

- I was standing in line outside a grocery store, waiting to get in
- This was unusual…

Grocery Lines and Similar

- I was standing in line outside a grocery store, waiting to get in
- Like the “good old days”

Grocery Lines and Similar

- Or

Grocery Lines and Similar

- OK, the last pictures were from lines waiting for iPhones

- But, it is still unusual to wait to get ***into*** a grocery store
- Maybe to get out (***checkout***) there is a line

- But there are plenty of times one waits in lines

Grocery Lines and Similar

- But there are plenty of times one waits in lines

- Sports events, music concerts, restaurants,
- Doctors offices, Immigration and Customs,
- Put your own examples here…

Grocery Lines and Similar

- Anyway, I was in a grocery line
- (standing a couple of meters away from the person in front of me and the person behind me, who was trying to “crowd me”)

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- And I didn't want to wait longer than 30 minutes

- I could go to another store, or maybe just go hungry,
- But I didn't want to wait

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- As an aside, restaurants will always tell you the wait is 20 minutes,
- Even if it is longer, because you are willing to wait 20 minutes, and
- If it takes 35 minutes, you may complain (or may not notice it),

- But if they don't want you to stick around, then they tell you
- It is more than an hour wait…

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- So I was willing to wait 30 minutes

- Would I get into the store in 30 minutes?
- (I would give then a 20% margin, so really up to 36 minutes)

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- But I didn't want to wait for 30 or so minutes, and then ***still***
- not get in!

- So I wanted to estimate (calculate) my chances of getting ***into***
- the store in my time window

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- How should I do it?

Grocery Lines and Similar

- How should I do it?

- One method:
  - Count number of people in front of me (there were 18)
  - Calculate for the next N people how fast the line was moving
  - (That is, create a training set)
  - For N of 10, if the line moves 10 people in 30/18 x 10 minutes, I am OK

Grocery Lines and Similar

- How should I do it?

- One method:
  - So, I was calculating the average speed of entry into the grocery store
  - At about 1.5 minutes per person
  - And there were 18 people ahead of me
  - So, looks OK (?)

Grocery Lines and Similar

- How should I do it?

- One method:
  - BUT
  - Sometimes the line moved at a bit faster pace, sometimes a bit slower
  - Sometimes even slower, sometimes really quickly

Grocery Lines and Similar

- How should I do it?
- The ***variance*** or ***deviation*** of the pace of people entering the store was irregular
- OK, not surprising
- How many people (how much time) should I watch to get a reasonably accurate understanding?

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- How many people (how much time) should I watch to get a reasonably accurate understanding?
- If the differences (time) between people entering the store wasn’t too large, say (about) 1.5 minutes +/- 0.5 minutes, I could watch (time) maybe just 5 or 6 people
- If the differences were large, for example 1.5 minutes +/- 5 minutes
- (that is, from 0 to 6.5 minutes, since there is no “negative” time pace)
- Then need to watch more people

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- Statistics might be useful, here…

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- But I had plenty of time, and I thought about building a model of this store

- How many people in line?
- How long would the line grow?
- How fast would people take to shop in the store?

- Would I be able to do this before I got into the store?

Grocery Lines and Similar

- Queueing Theory (sometimes spelled queuing, but the more common spelling queueing, is the only English word with 5 consecutive vowels)

- The mathematical models of queues

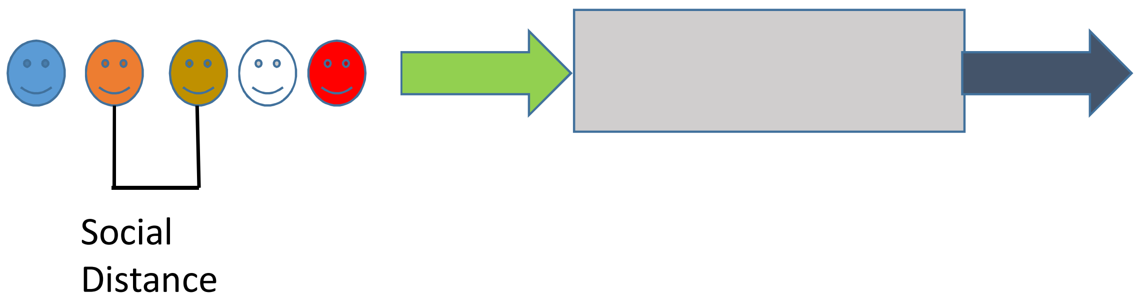
Grocery Lines and Similar

- Queueing Theory

- Useful to model and understand
  - Communication systems (Routers, switches, etc)
  - Industrial processes
  - Traffic
  - Offices
  - Hospitals
  - Many others

Grocery Lines and Similar

- Queueing Theory



Grocery Lines and Similar

- Queueing Theory

- There are many, many variations

- We will start simple
  - People arrive, get into a line (with some space between them)
  - They go into store, and shop,
  - They go into one of several lines, to get checked out
  - They leave the store

Grocery Lines and Similar

- Queueing Theory

- They arrive at some rate and get into the line
- In some systems (models), they arrive at a steady pace (such as a car assembly plant), in the grocery store case they arrive at an irregular pace, but with a pattern, an average time +/- a bit
- This is called the arrival rate or interarrival time

Grocery Lines and Similar

- Queueing Theory

- They shop and checkout, also with a time distribution model
- That is they take some time to shop, on average, +/- a bit,
- The checkout also takes some time, on average, +/- some

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- Queueing Theory
- "λ" is the arrival rate, that is the expected number of people arriving per unit time, for example 2 people per minute
- "μ" is the number of service completions per unit time, for example 3 people finished per minute
- "ρ" is λ / μ (normally) < 1
- "W" is the wait time, or time a person spends in the system
- "L" is the length of the line (queue), number of people in line

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- Queueing Theory

- Little’s Law
- $L = \lambda W$

- That is, The number of people in line is the arrival rate x the time
- spent in the system

Grocery Lines and Similar

- Queueing Theory

- Little's Law, and more
- $L = \lambda W$

- The (mean) response time = number of people in system/throughput
- Where throughput is people processed per time unit (for example, per minute)

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- Queueing Theory

- Little’s Law, and it variations can be applied within subsystems of the store, ie
- The line to get into the store
- The lines at multiple cashiers
- Maybe even the wait to get out of the parking lot

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- Queueing Theory

- There are various models of operation, (Kendall's notation)
- ArrivalTimeDistribution/ServiceTimeDistribution/NumberOfServers
- Common are:
- M/M/1 – where M is Markovian (Poisson), that is exponential time,
  - For the arrival time, the service time, and there is one server (cashier)

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- Queueing Theory

- ArrivalTimeDistribution/ServiceTimeDistribution/NumberOfServers
- M/M/2 or M/M/5 – where there are 2 or 5 servers
- M/D/1 –where D is fixed (deterministic) service time
- D/D/1, or D/D/5, and other variations

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- So the grocery store model, is to assume that people arrive at a “random” times, with a predictable mean arrival rate (about 40 people per hour), that while the line sometimes gets somewhat longer and sometimes a bit shorter, it is “about” steady state
- Using Little’s law, and observing people entering the store, and leaving the store
- Should be able to get into the store within 30 minutes
- (With possibly a small measurement error…)

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- So, I waited ("patiently") in line
- Got in to the store in 48 minutes…

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- https://en.wikipedia.org/wiki/Queueing_theory
- https://www0.gsb.columbia.edu/mygsb/faculty/research/pubfiles/5474/queueing%20theory%20and%20modeling.pdf
- http://people.brunel.ac.uk/~mastjjb/jeb/or/queue.html

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