MIT 18.02SC Problem Set 6

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Part 2

Problem 1

Problem 1 (4: 2,1,1)
Go to the 'Mathlet' **Lagrange Multipliers** (with link on the course webpage), and choose $f(x, y) = x^2 - y^2 \quad g(x, y) = x^2 + y^2$.
a) Solve by hand to find the two values of λ and the possiblities for the corresponding points (x, y) at which the gradients are proportional. Then check these possibities on the applet and verify the predicted proportionality on the graph.
b) Now take $b = 3$ and finish the solution of part(a) by hand to find the possible points which may give a relative extremum of f. Then return to the applet, set $b = 3$, move the f-levels until they makes contact with the $g = 3$ constraint curve, and read the values of f at the points of contact. Compare with the results found by hand; how close could you get?
c) What do the two values of λ correspond to in terms of the pairs of solution points? Do the gradients of f and g point in the same or the opposite direction at the contact points in the two different cases, and is this consistent with the signs of λ?

Solution:-

a.

We have

$$\nabla f =< 2x, -2y > \text{ and } \nabla g =< 2x, 2y >$$

From comparison we get,

$$(2x) = \lambda(2x) \text{ and } (-2y) = \lambda(2y)$$

Therefore we get two cases as a solution of the two equations.

- Case 1: $\lambda = 1$ and $y = 0$
- Case 2: $\lambda = -1$ and $x = 0$

From Case 1, we get,

$$g(x,0) = b \implies x = \pm\sqrt{b}$$

From Case 2, we get,

$$g(0,y) = b \implies y = \pm\sqrt{b}$$

Hence,

$\lambda = 1$ **gives the points** $(\pm\sqrt{b}, 0)$ **and** $\lambda = -1$ **gives the points** $(0, \pm\sqrt{b})$

b.

Putting, $b = 3$ we get,

$\lambda = 1$ **gives the points** $(\pm\sqrt{3}, 0)$ **and** $\lambda = -1$ **gives the points** $(0, \pm\sqrt{3})$

c.

From part (a) and (b), we obtained two values of the Lagrange multiplier:

$$\lambda = 1 \quad \text{and} \quad \lambda = -1.$$

Recall that the Lagrange multiplier condition is

$$\nabla f = \lambda \nabla g.$$

When $\lambda > 0$, the gradient vectors ∇f and ∇g point in the same direction, and when $\lambda < 0$, they point in opposite directions.

For $\lambda = 1$, the condition

$$\nabla f = \nabla g$$

occurs at the points $(\pm\sqrt{3}, 0)$, where

$$f = 3.$$

Hence these points correspond to the relative maxima of f on the constraint $x^2 + y^2 = 3$.

For $\lambda = -1$, the condition

$$\nabla f = -\nabla g$$

occurs at the points $(0, \pm\sqrt{3})$, where

$$f = -3.$$

Hence these points correspond to the relative minima of f on the constraint.

Thus the two values of the Lagrange multiplier represent the two different geometric situations: $\lambda > 0$ corresponds to maxima on the constraint curve, while $\lambda < 0$ corresponds to minima.