MIT 18.03 Frequency Response

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Part 1

1.

Problem 1: A driven spring-mass-dashpot system is modeled by the DE

$$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$$

with $m = 1$, $c = 6$, and $k = 45$. $F_0 = 50$. Find the amplitude $A(\omega)$ of the response as a function of the input frequency ω and find the frequency which gives the largest system response. Is this a system for which 'practical resonance' occurs?

Putting the values of $m = 1; c = 6; k = 45; F_0 = 50$, we get

$$\ddot{x} + 6\dot{x} + 45x = 50\cos(\omega t)$$

Let

$$D = \frac{d}{dt} \implies D^2 = \frac{d^2}{dt^2}$$

Therefore, the given Equation becomes

$$(D^2 + 6D + 45)x = 50\cos(\omega t)$$

Let

$$L(D) = D^2 + 6D + 45$$

This gives us

$$L(D)x = 50\mathbf{Re}\Big[e^{i\omega t}\Big]$$

Frequency Response

$$H(i\omega) = \frac{50}{L(i\omega)} = \frac{50}{(i\omega)^2 + 6(i\omega) + 45} = \frac{50}{(45 - \omega^2) + i(6\omega)}$$

Gain

$$G(\omega) = \frac{50}{\sqrt{(45 - \omega^2)^2 + (6\omega)^2}}$$

Phase

$$\phi(\omega) = -\arctan\left(\frac{6\omega}{45-\omega^2}\right)$$

Therefore Response is Given by

$$x_P = G(\omega)\cos\Big(\omega t + \phi(\omega)\Big)$$

$$\implies x_P = \frac{50}{\sqrt{(45-\omega^2)^2+(6\omega)^2}}\cos\left(\omega t - \arctan\left(\frac{6\omega}{45-\omega^2}\right)\right)$$

Hence, Amplitude

$$A = \frac{50}{\sqrt{(45-\omega^2)^2+(6\omega)^2}}$$

For A to be maximum, $(45-\omega^2)^2+(6\omega)^2$ should be minimum.
Let

$$f(\omega) = (45-\omega^2)^2+(6\omega)^2$$

So,

$$\frac{df}{d\omega} = 72\omega - 2(45-\omega^2)2\omega = 0$$

$$\implies \omega = 3\sqrt{3}$$

Therefore, Maximum Amplitude is obtained at

$$\omega = 3\sqrt{3}$$

Maximum Amplitude

$$A_{MAX} = \frac{25}{18} \textbf{ i.e. Finite}$$

Hence the Practical Resonance Occurs for this System.