# MIT 18.03 Basic DE's and Separable Equations

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## PART 1

### Problem 7:

**Problem 7:** Early one morning it starts to snow. At 7AM a snowplow sets off to clear the road. By 8AM, it has gone 2 miles. It takes an additional 2 hours for the plow to go another 2 miles. Let $t = 0$ when it begins to snow, let $x$ denote the distance traveled by the plow at time $t$. Assuming the snowplow clears snow at a constant rate in cubic meters/hour:

a) Find the DE modeling the value of $x$.

b) When did it start snowing?

A snowplow picks up all the snow that is there on the road.
If we consider that the height of the snow at a place is $h$, distance traveled by the snowplow is $x$, width of the blade is $w$ and the volume of the snow picked up is $V$, we get
$\Delta V = wh\Delta x$
On Differentiation we get, $\frac{dV}{dt} = wh\frac{dx}{dt}$
So if we assume the height of the snow to be a function of time such that it linearly increases with time, then to keep the $\frac{dV}{dt}$ constant as per the question, we have to let $\frac{dx}{dt}$ to decrease with time. By reading the question we can see that the velocity of the snowplow is decreasing. It covered the first 2 miles in 1 hour and the next 2 miles in 2 hours. This proves that we can proceed with our assumption since it is correct.
If we consider, $h = h_o(t_i + t_o)$, where $t_i$ is the time passed after the snow plow came and $t_o$ is the time in between the events of starting of the snowfall and arrival of the snowplow. So total time of snowing is $t_i + t_o$.
Here, $t_i$ is the independent time variable with which we will represent $x$. $\frac{dx}{dt_i} = \frac{dx}{dt}$
So, we get
$\frac{dV}{dt_i} = wh\frac{dx}{dt_i} = wh_o(t_i + t_o)\frac{dx}{dt_i}$
Let constant $c = \frac{dV}{dt_i}\frac{1}{wh_o}$
So, $\frac{dx}{dt_i} = \frac{c}{(t_i+t_o)} => x = c \times ln(k/(t_i + t_o))$, $ln(k)$ is the constant of integration.

b)
At $t_i = 0$, $x = 0$. So, $0 = c \times ln(k/(t_o)) => k = t_o$
So we get the solution as, $x = c \times ln(t_o/(t_i + t_o))$
Putting the data from the question, we get,
$2 = c \times ln(t_o/(1 + t_o))$ and $4 = c \times ln(t_o/(3 + t_o))$
This gives us, $2 \times c \times ln(t_o/(1 + t_o)) = c \times ln(t_o/(3 + t_o))$
$t_o = 1$
Hence started snowing 1 hour before the snowplow arrived.
To be clear, "**It started snowing at 6 a.m.**"

a.
on solving, $2 = c \times ln(1/(1 + 1)) => c = 2/ln(1/(2))$
Now that we know the constants, we can safely write the solution for $x$
According to the question, $t = t_i + t_o$
So our solution becomes,
$x = \frac{2ln(1/(t_i+1))}{ln(1/2)}$

## Problem 8:

**Problem 8:** A tank holds 100 liters of water which contains 25 grams of salt initially. Pure water then flows into the tank, and salt water flows out of the tank, both at 5 liters/minute. The mixture is kept uniform at all times by stirring.

a) Write down the DE with IC for this situation.

b) How long will it take until only 1 gram of salt remains in the tank?

a.
Let $x(t)$ = amount of salt in the tank as a function of time t.
If 5L of salt water goes out per unit time (1 minute),keeping the initial volume of water (100 L) constant, then the amount of salt going out is given by $x \times \frac{5}{100} = \frac{x}{20}$.
Now $\frac{dx}{dt}$ will represent the rate of change of amount of salt in the tank. So $\frac{dx}{dt} = -\frac{x}{20}$. The minus sign indicates the decreasing of the amount of salt in the tank.
Hence our DE becomes $\frac{dx}{dt} + \frac{x}{20} = 0$
IC is **x(0)=25 grams**.

b.
The IF of the DE is $e^{\int \frac{1}{20} dt} = e^{\frac{t}{20}}$
Therefore the solution of our DE is $x(e^{\frac{t}{20}}) = \int 0 \times e^{\frac{t}{20}} dt = c$ where $c$ is the Integration constant.

So, $x = ce^{\frac{-t}{20}}$
Using the IC, we get $x(0) = c = 25$
Hence our solution is $x = 25e^{\frac{-t}{20}}$

Solving for $x(t) = 1 \Rightarrow 25e^{\frac{-t}{20}} = 1$
=> **t=20ln(25)**

# PART 2

## Problem 1:

**Problem 1:** [Natural growth, separable equations] In recitation a population model was studied in which the natural growth rate of the population of oryx was a constant $k > 0$, so that for small time intervals $\Delta t$ the population change $x(t + \Delta t) - x(t)$ is well approximated by $kx(t)\Delta t$. (You also studied the effect of hunting them, but in this problem we will leave that aside.) Measure time in years and the population in kilo-oryx (ko).

A mysterious virus infects the oryxes of the Tana River area in Kenya, which causes the growth rate to decrease as time goes on according to the formula $k(t) = k_0/(a+t)^2$ for $t \geq 0$, where $a$ and $k_0$ are certain positive constants.

**(a)** What are the units of the constant $a$ in "$a + t$," and of the constant $k_0$?

**(b)** Write down the differential equation modeling this situation.

**(c)** Write down the general solution to your differential equation. Don't restrict yourself to the values of $t$ and of $x$ that are relevant to the oryx problem; take care of all values of these variables. Points to be careful about: use absolute values in $\int \frac{dx}{x} = \ln|x| + c$ correctly, and don't forget about any "lost" solutions.

**(d)** Now suppose that at $t = 0$ there is a positive population $x_0$ of oryx. Does the progressive decline in growth rate cause the population stabilize for large time, or does it grow without bound? If it does stabilize, what is the limiting population as $t \to \infty$?

a.
Unit of $a$ is **years** since $a$ is added to $t$ and the question mentions to measure time in years.
In the question, $x(t+\Delta t) - x(t) = kx(t)\Delta t \Rightarrow kx = \frac{x(t+\Delta t)-x(t)}{\Delta t} = \frac{dx}{dt} \Rightarrow k = \frac{1}{x}\frac{dx}{dt}$ if $\Delta t$ is very small.
So unit of $k$ is year$^{-1}$.
This gives the unit of $k_o$ to be **years**

b.
Given $k = \frac{k_o}{(a+t)^2}$. So, $\frac{1}{x}\frac{dx}{dt} = \frac{k_o}{(a+t)^2}$
This gives us the DE as $\frac{dx}{dt} - \frac{k_o x}{(a+t)^2} = 0$

c.
IF of the DE is $e^{\int -\frac{k_o}{(a+t)^2}dt} = e^{\frac{k_o}{a+t}}$
So the solution is $xe^{\frac{k_o}{a+t}} = \int 0 \times e^{\frac{k_o}{a+t}}dt \Rightarrow x = ce^{\frac{-k_o}{a+t}}$ where $c$ is integration constant.

So the General Solution is $x = ce^{\frac{-k_o}{a+t}}$

d.
Using IC, $x(0) = x_o = ce^{\frac{-k_o}{a}} \Rightarrow c = x_o e^{\frac{k_o}{a}}$
Therefore our solution becomes, $x = x_o e^{\frac{k_o}{a}} e^{\frac{-k_o}{a+t}}$
As the time $t \to \infty$, $x \to x_o e^{\frac{k_o}{a}}$.
So our solution does not blow up at infinity.
**Hence, the population remains stable over a long time.**