MIT 18.06SC Problem Set 9

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Problem 9.2

Problem 9.2: (3.5 #20.) Find a basis for the plane $x - 2y + 3z = 0$ in $\mathbb{R}^3$. Then find a basis for the intersection of that plane with the xy plane. Then find a basis for all vectors perpendicular to the plane.

Basis of the Plane $x - 2y + 3z = 0$

A Plane has the Dimension 2. So any Basis of the Plane will have 2 Vectors. Note that there are 3 Variables in the Equation of Plane. So each Vector will have 3 Components.
So basically we have to represent $x - 2y + 3z = 0$ in terms of 2 Vectors i.e. 2 Parameters are allowed.

So let us set the Parameters as

$$y = u \text{ and } z = v$$

$$\implies x = 2u - 3v$$

Therefore the Plane Basically becomes

$$\begin{bmatrix} 2u - 3v \\ u \\ v \end{bmatrix}$$

We can decompose this Matrix like this :

$$\begin{bmatrix} 2u - 3v \\ u \\ v \end{bmatrix} = u \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + v \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

Note that $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$ are Independent.

Hence the Basis of the Plane is given by

$$\textbf{Basis} = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} ; \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Basis of the Intersection of $x - 2y + 3z = 0$ and xy Plane

Intersection of 2 Planes is always a Line. A Line has 1 Dimension. So any Basis will have 1 Vector i.e. we can have 1 Parameter.

Note that any Point on the xy Plane has

$$z = 0$$

So we can set the Parameters as

$$y = u$$

$$\implies x = 2(u) - 3(0) = 2u$$

So the Line comes out to be

$$\begin{bmatrix} 2u \\ u \\ 0 \end{bmatrix} = u \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

Hence the Basis of the Intersection of $x - 2y + 3z = 0$ and xy Plane is given by

$$\textbf{Basis} = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$$

Basis of Vectors Perpendicular to the Plane

Normal Vector to the Plane is given by

$$\vec{n} = c \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \text{ here } c \text{ is a Constant}$$

Hence the Basis of Vectors Perpendicular to the Plane $x - 2y + 3z = 0$ is given by

$$\textbf{Basis} = \left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \right\}$$