MIT 18.06SC Problem Set 17

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Problem 17.2

Problem 17.2: (4.4 #18) Given the vectors **a**, **b** and **c** listed below, use the Gram-Schmidt process to find orthogonal vectors **A**, **B**, and **C** that span the same space.

$$\mathbf{a} = (1, -1, 0, 0), \mathbf{b} = (0, 1, -1, 0), \mathbf{c} = (0, 0, 1, -1).$$

Show that $\{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$ and $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ are bases for the space of vectors perpendicular to $\mathbf{d} = (1, 1, 1, 1)$.

We are given the vectors

$$a = (1, -1, 0, 0), \quad b = (0, 1, -1, 0), \quad c = (0, 0, 1, -1),$$

and the vector

$$d = (1, 1, 1, 1).$$

Applying Gram–Schmidt

Define

$$A = a$$

Next, remove from b its component along A:

$$B = b - \frac{b \cdot A}{A \cdot A} A.$$

Since

$$b \cdot A = -1, \quad A \cdot A = 2,$$

we obtain

$$B = b + \frac{1}{2} A = \left(\tfrac{1}{2}, \tfrac{1}{2}, -1, 0\right).$$

Now remove from c its components along both A and B:

$$C = c - \frac{c \cdot A}{A \cdot A} A - \frac{c \cdot B}{B \cdot B} B.$$

Here,

$$c \cdot A = 0, \quad c \cdot B = -1, \quad B \cdot B = \tfrac{3}{2},$$

so

$$C = c + \frac{2}{3} B = \left(\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, -1\right).$$

Thus $\{A, B, C\}$ is an orthogonal basis spanning the same subspace as $\{a, b, c\}$, given by

$$\boxed{A = (1, -1, 0, 0)}$$

$$\boxed{B = (\tfrac{1}{2}, \tfrac{1}{2}, -1, 0)}$$

$$\boxed{C = (\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}, -1)}$$

Orthogonality to d

We verify that each vector is perpendicular to d:

$$A \cdot d = 1 - 1 + 0 + 0 = 0,$$

$$B \cdot d = \tfrac{1}{2} + \tfrac{1}{2} - 1 + 0 = 0,$$
$$C \cdot d = \tfrac{1}{3} + \tfrac{1}{3} + \tfrac{1}{3} - 1 = 0.$$

Therefore,

$$\boxed{\operatorname{span}\{A, B, C\} = \operatorname{span}\{a, b, c\} = \{x \in \mathbb{R}^4 \mid x \cdot d = 0\}}$$

This subspace is a three-dimensional hyperplane in $\mathbb{R}^4$ obtained by imposing the linear constraint $x_1 + x_2 + x_3 + x_4 = 0$