

# MIT 18.06SC Problem Set 3

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## Problem 3.2

**Problem 3.2:** (2.5 #24. *Introduction to Linear Algebra: Strang*) Use Gauss-Jordan elimination on  $[U \ I]$  to find the upper triangular  $U^{-1}$ :

$$UU^{-1} = I \quad \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

$$[U \mid I] = \left[ \begin{array}{ccc|ccc} 1 & a & b & 1 & 0 & 0 \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

**Step 1:**  $R'_2 \leftarrow R_2 - cR_3$

$$\left[ \begin{array}{ccc|ccc} 1 & a & b & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

**Step 2:**  $R'_1 \leftarrow R_1 - aR_2 - bR_3$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -a & ac-b \\ 0 & 1 & 0 & 0 & 1 & -c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

Thus the left block is  $I_3$  and the right block is  $U^{-1}$ , so

$$U^{-1} = \begin{bmatrix} 1 & -a & ac-b \\ 0 & 1 & -c \\ 0 & 0 & 1 \end{bmatrix}.$$