

8.012 Problem Set 3

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Problem 5

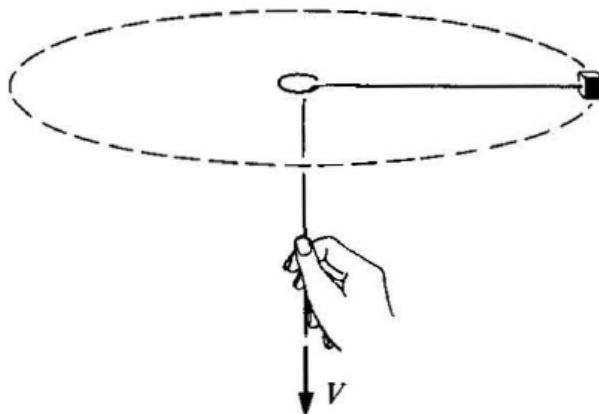
(Kleppner & Kolenkow, Problem 2.34)

2.34. A mass m whirls around on a string which passes through a ring, as shown. Neglect gravity. Initially the mass is distance r_0 from the center and is revolving at angular velocity ω_0 . The string is pulled with constant velocity V starting at $t = 0$ so that the radial distance to the mass decreases. Draw a force diagram and obtain a differential equation for ω . This equation is quite simple and can be solved either by inspection or by formal integration. Find

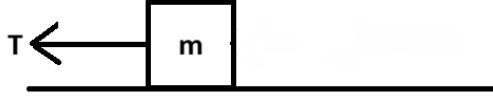
a. $\omega(t)$.

Ans. clue. For $Vt = r_0/2$, $\omega = 4\omega_0$

b. The force needed to pull the string.



The Force Diagram is given by:



Here T is the Tension in the string.

a.

Radial Distance at a time is given by

$$r(t) = r_0 - Vt$$

$$\implies \dot{r}(t) = -V$$

In this System, there is no Force which have any Tangential Component on the Mass. Therefore we have

Torque = 0 \implies Angular Momentum is constant

$$\implies m(\omega(t)r(t))r(t) = m(\omega_0 r_0)r_0$$

$$\implies \omega(t) = \omega_0 r_0^2 \frac{1}{(r(t))^2}$$

$$\implies \omega(t) = \omega_0 r_0^2 (r_0 - Vt)^{-2}$$

The corresponding Differential Equation is given by

$m(\omega(t)r(t))r(t)$ is constant

$$\implies \frac{d}{dt}(m(\omega(t)r(t))r(t)) = 0$$

$$\implies \frac{d}{dt}(\omega(t)(r(t))^2) = 0$$

$$\implies \dot{\omega}r^2 + 2r\dot{r}\omega = 0$$

$$\implies (r_0 - Vt)^2 \dot{\omega} - 2V(r_0 - Vt)\omega = 0$$

Solving this Differential Equation, we get,

$$(r_0 - Vt)^2 \dot{\omega} - 2V(r_0 - Vt)\omega = 0$$

$$\implies \frac{d}{dt}[(r_0 - Vt)^2 \omega] = 0$$

$$\implies (r_0 - Vt)^2 \omega = c \text{ [} c \text{ is Constant of Integration]}$$

Checking with $\omega(0) = \omega_0$ we get,

$$c = \omega_0 r_0^2$$

This gives us

$$\omega = \omega_0 r_0^2 (r_0 - Vt)^{-2}$$

b.

The string is pulled inward with constant speed V . The applied force therefore adjusts dynamically to provide the required centripetal acceleration of the mass.

$$\begin{aligned} F &= m\omega^2 r \\ &= m(\omega_0 r_0^2 (r_0 - Vt)^{-2})^2 (r_0 - Vt) \end{aligned}$$

Hence,

$$F = \frac{m\omega_0^2 r_0^4}{(r_0 - Vt)^3}$$