

# Investigacion Operativa

## Coloreo Particionado de Grafos

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Integrante	LU	Correo electrónico
Martin Baigorria	575/14	martinbaigorria@gmail.com
Andrew Ab	???	???

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## 1. Modelo

Dado un grafo  $G(V, E)$  con  $n = |V|$  vertices y  $m = |E|$  aristas, un coloreo de  $G$  se define como una asignacion de un color o etiqueta a cada  $v \in V$  de forma tal que para todo par de vertices adyacentes  $(p, q) \in E$  poseen colores distintos. El clasico problema de *coloreo de grafos* consiste en encontrar un coloreo del grafo que utilice la menor cantidad de colores posibles.

En este trabajo resolveremos una variante de este problema, el *coloreo particionado de grafos*. A partir de un conjunto de vertices  $V$  que se encuentra particionado en  $V_1, \dots, V_k$ , el problema consiste en asignar un color  $c \in C$  a solo un vertice de cada particion de forma tal que dos vertices adyacentes no reciban el mismo color y minimizando la cantidad de colores utilizados.

Este problema se puede modelar con Programacion Lineal Entera. Para ello, definamos las siguientes variables:

$$x_{pj} = \begin{cases} 1 & \text{si el color } j \text{ es asignado al vertice } p \\ 0 & \text{en caso contrario} \end{cases}$$
$$w_j = \begin{cases} 1 & \text{si } x_{pj} = 1 \text{ para algun vertice } p \\ 0 & \text{en caso contrario} \end{cases}$$

### 1.1. Funcion objetivo

De esta forma la funcion objetivo del LP consiste en minimizar la cantidad de colores utilizados:

$$\min \sum_{j \in C} w_j \quad (1)$$

Notar que  $|C|$  esta acotado superiormente por la cantidad de particiones  $k$ .

### 1.2. Restricciones

Los vertices adyacentes no comparten color. Recordar que no necesariamente se le asigna un color a todo vertice.

$$x_{ij} + x_{kj} \leq 1 \quad \forall (i, k) \in E, \quad \forall j \in C \quad (2)$$

Solo se le asigna un color a un unico vertice de cada particion  $p \in P$ . Esto implica que cada vertice tiene a lo sumo solo un color.

$$\sum_{i \in V_p} \sum_{j \in C} x_{ij} = 1 \quad \forall p \in P \quad (3)$$

Si un nodo usa color  $j$ ,  $w_j = 1$ :

$$x_{ij} \leq w_j \quad \forall i \in V, \forall j \in C \quad (4)$$

Integralidad y positividad de las variables:

$$x_{ij} \in \{0, 1\} \quad \forall i \in V, \forall j \in C \quad (5)$$

$$w_j \in \{0, 1\} \quad \forall j \in C \quad (6)$$

## 2. Branch & Bound

La implementacion del modelo y del Branch & Bound se encuentran en el apendice.

## 3. Desigualdades

### 3.1. Desigualdad de Clique

Sea  $j_0 \in \{1, \dots, n\}$  y sea  $K$  una clique maximal de  $G$ . La desigualdad clique estan definida por:

$$\sum_{p \in K} x_{pj_0} \leq w_{j_0} \quad (7)$$

**Demostración** Para esta demostracion utilizaremos las desigualdades Chvátal-Gomory sobre las restricciones del LP planteado en la seccion 1.2 e induccion. A priori el teorema es bastante intuitivo. Si pinto algun vertice de una clique, no puedo pintar ninguno adyacente del mismo color sin importar la forma en la que particione los vertices del grafo. Sea  $n$  el tamaño de la clique maximal.

#### Casos Base

1.  $n = 1$ : Si en la clique maximal tengo solo un vertice, no existe arista que contenga este vertice, caso contrario la clique tendria dos elementos. Por lo tanto, este vertice puede estar pintado o no dentro de la particion. Es decir, se cumple la ecuacion que queremos probar.
2.  $n = 2$ : Si la clique maximal tiene dos elementos, por definicion son conexos. Por la restriccion que indica que los vertices adyacentes no comparten color, aqui hay 2 opciones. La primera opcion es que a ningun vertice se le asigna un color  $j_0$ . La otra opcion es que dada la estructura de particiones, se le asigne solo a uno de ellos el color  $j_0$ . Por lo tanto la desigualdad para  $n = 2$  vale.
3.  $n = 3$ : Este es el caso mas interesante en el que utilizamos la desigualdad de Chvátal-Gomory. Si la clique tiene 3 vertices, hay tres desigualdades que se deben cumplir:

$$\begin{aligned} \blacksquare x_{1j_0} + x_{2j_0} &\leq 1 \\ \blacksquare x_{2j_0} + x_{3j_0} &\leq 1 \\ \blacksquare x_{1j_0} + x_{3j_0} &\leq 1 \end{aligned}$$

Multiplicando todas estas desigualdades por  $1/3$  y sumando entonces:

$$1/3(x_{1j_0} + x_{2j_0}) + 1/3(x_{2j_0} + x_{3j_0}) + 1/3(x_{1j_0} + x_{3j_0}) \leq 1$$

Como  $x_{ij}$  toma valores enteros, entonces:  $1/3(x_{1j_0} + x_{2j_0}) + 1/3(x_{2j_0} + x_{3j_0}) + 1/3(x_{1j_0} + x_{3j_0}) \leq 1$

Simplificando:  $x_{1j_0} + x_{2j_0} + x_{3j_0} \leq 1$ .

Utilizando la definicion de  $w_j$  entonces:  $x_{1j_0} + x_{2j_0} + x_{3j_0} \leq w_{j_0}$

Por lo tanto la desigualdad vale para  $n = 3$ .

**Paso Inductivo:**  $P(n-1) \implies P(n)$

Como vale la hipotesis inductiva, sabemos que:

$$\sum_{p \in K-n} x_{pj_0} \leq w_{j_0}$$

Al agregar un vertice a la clique, agregamos  $n-1$  aristas:

$$x_{1j_0} + x_{nj_0} \leq 1, x_{2j_0} + x_{nj_0} \leq 1, \dots, x_{(n-1)j_0} + x_{nj_0} \leq 1$$

Utilizando esto, podemos ver que:

$$x_{nj_0} + \sum_{p \in K-n} x_{pj_0} \leq w_{j_0}$$

Esto es claramente equivalente a lo que queremos demostrar y se puede justificar a partir de dos casos:

- Si al vertice  $x_{nj_0}$  se le asigna un color, por las restricciones de las aristas que agregamos al resto de los vertices de la clique no se le puede asignar el color  $j_0$ .
- Si al vertice  $x_{nj_0}$  no se le asigna un color o se le asigna un color diferente a  $j_0$ , por hipotesis inductiva sabemos que lo que queremos probar vale.  $\square$

### 3.2. Desigualdad de Aujero Impar

Sea  $j_0 \in \{1, \dots, n\}$  y sea  $C_{2k+1} = v_1, \dots, v_{2k+1}$ ,  $k \geq 2$ , un agujero de longitud impar. La desigualdad esta definida por:

$$\sum_{p \in C_{2k+1}} x_{pj_0} \leq kw_{j_0} \quad (8)$$

**Demostración** Por teoremas de coloreo (que se prueban en general por induccion), sabemos que el numero cromatico  $\chi(C) = 3$ . En el peor de los casos, cada vertice del agujero estara en una particion diferente. Aqui nuevamente tenemos dos casos:

- Si no se asigna el color  $j_0$  a algun vertice del agujero, la desigualdad vale.
- Si se asigna el color  $j_0$ , en el peor de los casos el mismo sera utilizado por a lo sumo  $(|C| - 1)/2$  vertices. Como  $|C| = 2k + 1$ ,  $(2k + 1 - 1)/2 = k$ . Por lo tanto vale la desigualdad.  $\square$

### 3.3. Planos de Corte

Luego de relajar el PLEM, los algoritmos de separacion buscan acotar el espacio de busqueda para que se parezca mas a la capsula convexa. Existen algoritmos de separacion exactos y heuristicos. Los algoritmos heuristicos, luego de resolver la relajacion del problema entero y encontrar una solucion optima  $x^*$ , retornan una o mas desigualdades de la clase violadas por alguna familia de desigualdades. Por ser un algoritmo heuristico, es posible que exista una desigualdad de la clase violada aunque el procedimiento no sea capaz de encontrarla. Si se encuentra una desigualdad que es violada por la solucion optima de la relajacion, se agrega esta nueva restriccion y se vuelve a resolver el programa lineal. Este procedimiento se conoce como algoritmo de plano de corte. Si una solucion optima al problema existe, este tipo de algoritmo no necesariamente la encuentra. Por ejemplo, las heuristicas que encuentran desigualdades validas pueden fallar y el algoritmo no puede continuar.

#### 3.3.1. Heuristica de Separacion para Clique

#### 3.3.2. Heuristica de Separacion para Aujero Impar

## 4. Cut & Branch

## 5. Experimentacion

## 6. Conclusion

## 7. Apéndice A: Código

### 7.1. coloring.cpp

---

```
1 #include <ilcplex/ilcplex.h>
2 #include <ilcplex/cplex.h>
3
4 #include <stdlib.h>
5
6 #include <string>
7 #include <vector>
8
9 #define TOL 1e-05
10
11 ILOSTLBEGIN // macro to define namespace
12
13 struct edge {
14     int from;
15     int to;
16
17     edge(int a, int b) {
18         from = a;
19         to = b;
20     }
21 };
22
23 int getVertexIndex(int id, int color, int partition_size);
24 int loadObjectiveFunction(CPXENVptr& env, CPXLPptr& lp, int vertex_size, int
    partition_size);
25 int loadAdjacencyColorRestriction(CPXENVptr& env, CPXLPptr& lp, vector<edge>& edges,
    int edge_size, int partition_size);
26 int loadSingleColorInPartitionRestriction(CPXENVptr& env, CPXLPptr& lp, vector<vector
    <int>>& partitions, int partition_size);
27 int loadAdjacencyColorRestriction(CPXENVptr& env, CPXLPptr& lp, int vertex_size, int
    partition_size);
28 int solveLP(CPXENVptr& env, CPXLPptr& lp, int edge_size, int vertex_size, int
    partition_size);
29 int setBranchAndBoundConfig(CPXENVptr& env);
30
31 // colors array!
32 const char* colors[] = {"Blue", "Red", "Green", "Yellow", "Grey", "Green", "Pink", "
    AliceBlue", "AntiqueWhite", "Aqua", "Aquamarine", "Azure", "Beige",
33 "Bisque", "Black", "BlanchedAlmond", "BlueViolet", "Brown", "BurlyWood", "CadetBlue", "
    Chartreuse", "Chocolate", "Coral", "CornflowerBlue",
34 "Cornsilk", "Crimson", "Cyan", "DarkBlue", "DarkCyan", "DarkGoldenRod", "DarkGray", "
    DarkGrey", "DarkGreen", "DarkKhaki", "DarkMagenta", "DarkOliveGreen",
35 "Darkorange", "DarkOrchid", "DarkRed", "DarkSalmon", "DarkSeaGreen", "DarkSlateBlue", "
    DarkSlateGray", "DarkSlateGrey", "DarkTurquoise",
36 "DarkViolet", "DeepPink", "DeepSkyBlue", "DimGray", "DimGrey", "DodgerBlue", "FireBrick", "
    FloralWhite", "ForestGreen", "Fuchsia",
37 "Gainsboro", "GhostWhite", "Gold", "GoldenRod", "Gray", "GreenYellow", "HoneyDew", "HotPink",
    "IndianRed", "Indigo",
38 "Ivory", "Khaki", "Lavender", "LavenderBlush", "LawnGreen", "LemonChiffon", "LightBlue", "
    LightCoral", "LightCyan", "LightGoldenRodYellow",
39 "LightGray", "LightGrey", "LightGreen", "LightPink", "LightSalmon", "LightSeaGreen", "
    LightSkyBlue", "LightSlateGray", "LightSlateGrey",
```



```

40 "LightSteelBlue", "LightYellow", "Lime", "LimeGreen", "Linen", "Magenta", "Maroon", "
    MediumAquaMarine", "MediumBlue", "MediumOrchid",
41 "MediumPurple", "MediumSeaGreen", "MediumSlateBlue", "MediumSpringGreen", "
    MediumTurquoise", "MediumVioletRed", "MidnightBlue",
42 "MintCream", "MistyRose", "Moccasin", "NavajoWhite", "Navy", "OldLace", "Olive", "OliveDrab",
    "Orange", "OrangeRed", "Orchid",
43 "PaleGoldenRod", "PaleGreen", "PaleTurquoise", "PaleVioletRed", "PapayaWhip", "PeachPuff",
    "Peru", "Plum", "PowderBlue",
44 "Purple", "RosyBrown", "RoyalBlue", "SaddleBrown", "Salmon", "SandyBrown", "SeaGreen", "
    SeaShell", "Sienna", "Silver", "SkyBlue",
45 "SlateBlue", "SlateGray", "SlateGrey", "Snow", "SpringGreen", "SteelBlue", "Tan", "Teal", "
    Thistle", "Tomato", "Turquoise", "Violet",
46 "Wheat", "White", "WhiteSmoke", "YellowGreen"};
47
48 int main(int argc, char **argv) {
49
50     if (argc != 2) {
51         printf("Usage: %s inputFile\n", argv[0]);
52         exit(1);
53     }
54
55     /* read graph input file
56     * format: http://mat.gsia.cmu.edu/COLOR/instances.html
57     * graph representation chosen in order to load the LP easily.
58     * - vector of edges
59     * - vector of partitions
60     */
61     FILE* fp = fopen(argv[1], "r");
62
63     if (fp == NULL) {
64         printf("Invalid input file. \n");
65         exit(1);
66     }
67
68     char buf[100];
69     int vertex_size, edge_size;
70
71     vector<edge> edges;
72
73     while (fgets(buf, sizeof(buf), fp) != NULL) {
74         if (buf[0] == 'c') continue;
75         else if (buf[0] == 'p') {
76             sscanf(&buf[7], "%d %d", &vertex_size, &edge_size);
77             // printf("vertex_size: %d, edge_size: %d \n", vertex_size, edge_size);
78             // printf("Adding edges! \n");
79         }
80         else if (buf[0] == 'e') {
81             int from, to;
82             sscanf(&buf[2], "%d %d", &from, &to);
83             // printf("Edge: (%d,%d) \n", from, to);
84             edges.push_back(edge(from, to));
85         }
86     }
87
88     // set random seed
89     srand(time(NULL));
90
91     // assign every vertex to a partition
92     int partition_size = rand() % vertex_size;

```

```

93
94     vector<vector<int>> > partitions(partition_size , vector<int>());
95
96     // warning: this procedure doesn't guarantee every partition will have an element
97
98     for (int i = 1; i <= vertex_size; ++i) {
99         int assign_partition = rand() % partition_size;
100         partitions[assign_partition].push_back(i);
101     }
102
103     // update partition_size
104     for (std::vector<vector<int>> >::iterator it = partitions.begin(); it !=
105         partitions.end(); ++it) {
106         if (it->size() == 0) --partition_size;
107     }
108
109     // start loading LP using CPLEX
110     int status;
111     CPXENVptr env; // pointer to enviroment
112     CPXLPptr lp;   // pointer to the lp.
113
114     env = CPXopenCPLEX(&status); // create enviroment
115
116     if (env == NULL) {
117         printf("Error creating enviroment.\n");
118         exit(1);
119     }
120
121     // create LP
122     lp = CPXcreateprob(env, &status, "Instance of partitioned graph coloring.");
123
124     if (lp == NULL) {
125         printf("Error creating the LP.\n");
126         exit(1);
127     }
128
129     loadObjectiveFunction(env, lp, vertex_size, partition_size);
130     loadAdyacencyColorRestriction(env, lp, edges, edge_size, partition_size);
131     loadSingleColorInPartitionRestriction(env, lp, partitions, partition_size);
132     loadAdyacencyColorRestriction(env, lp, vertex_size, partition_size);
133
134     // write LP formulation to file , great to debug.
135     status = CPXwriteprob(env, lp, "graph.lp", NULL);
136
137     if (status) {
138         printf("Problem writing LP problem to file.");
139         exit(1);
140     }
141
142     setBranchAndBoundConfig(env);
143     solveLP(env, lp, edge_size, vertex_size, partition_size);
144
145     return 0;
146 }
147
148 int getVertexIndex(int id, int color, int partition_size) {
149     return partition_size + ((id-1)*partition_size) + (color-1);
150 }

```

```

150 int loadObjectiveFunction(CPXENVptr& env, CPXLPptr& lp, int vertex_size, int
    partition_size) {
151
152     // load objective function
153     int n = partition_size + (vertex_size*partition_size);
154     double *objfun = new double[n];
155     char *ctype = new char[n];
156     char **colnames = new char*[n];
157
158     for (int i = 0; i < partition_size; ++i) {
159         objfun[i] = 1;
160         ctype[i] = CPX_BINARY;
161         colnames[i] = new char[10];
162         sprintf(colnames[i], "w-%d", (i+1));
163     }
164
165     for (int id = 1; id <= vertex_size; ++id) {
166         for (int color = 1; color <= partition_size; ++color) {
167             int index = getVertexIndex(id, color, partition_size);
168             objfun[index] = 0;
169             ctype[index] = CPX_BINARY;
170             colnames[index] = new char[10];
171             sprintf(colnames[index], "x-%d%d", id, color);
172         }
173     }
174
175     int status = CPXnewcols(env, lp, n, objfun, NULL, NULL, ctype, colnames);
176
177     if (status) {
178         printf("Problem adding variables with CPXnewcols.\n");
179         exit(1);
180     }
181
182     // free memory
183     for (int i = 0; i < n; ++i) {
184         delete [] colnames[i];
185     }
186
187     delete [] objfun;
188     delete [] ctype;
189     delete [] colnames;
190
191     return 0;
192 }
193
194 int loadAdjacencyColorRestriction(CPXENVptr& env, CPXLPptr& lp, vector<edge>& edges,
    int edge_size, int partition_size) {
195
196     // load first restriction
197     int ccnt = 0; // new columns being added.
198     int rcnt = edge_size * partition_size; // new rows being added.
199     int nzcnt = rcnt*2; // nonzero constraint coefficients being
    added.
200
201     double *rhs = new double[rcnt]; // independent term in restrictions.
202     char *sense = new char[rcnt]; // sense of restriction inequality.
203
204     int *matbeg = new int[rcnt]; // array position where each restriction
    starts in matind and matval.

```

```

205     int *matind = new int[rcnt*2];           // index of variables != 0 in restriction
        (each var has an index defined above)
206     double *matval = new double[rcnt*2];    // value corresponding to index in
        restriction.
207     char **rownames = new char*[rcnt];      // row labels.
208
209     int i = 0;
210     for (std::vector<edge>::iterator it = edges.begin(); it != edges.end(); ++it) {
211         int from = it->from;
212         int to   = it->to;
213         for (int color = 1; color <= partition_size; ++color) {
214             matbeg[i] = i*2;
215
216             matind[i*2] = getVertexIndex(from, color, partition_size);
217             matind[i*2+1] = getVertexIndex(to, color, partition_size);
218
219             matval[i*2] = 1;
220             matval[i*2+1] = 1;
221
222             rhs[i] = 1;
223             sense[i] = 'L';
224             rownames[i] = new char[40];
225             sprintf(rownames[i], "%s", colors[color-1]);
226
227             ++i;
228         }
229     }
230
231     // debug flag
232     // status = CPXsetintparam(env, CPXPARAMDATACHECK, CPX_ON);
233
234     // add restriction
235     int status = CPXaddrows(env, lp, ccnt, rcnt, nzcnt, rhs, sense, matbeg, matind,
        matval, NULL, rownames);
236
237     if (status) {
238         printf("Problem adding restriction with CPXaddrows.\n");
239         exit(1);
240     }
241
242     // free memory
243     for (int i = 0; i < rcnt; ++i) {
244         delete [] rownames[i];
245     }
246
247     delete [] rhs;
248     delete [] sense;
249     delete [] matbeg;
250     delete [] matind;
251     delete [] matval;
252     delete [] rownames;
253
254     return 0;
255 }
256
257
258 int loadSingleColorInPartitionRestriction(CPXENVptr& env, CPXLPptr& lp, vector<vector
    <int>>& partitions, int partition_size) {
259

```

```

260 // load second restriction
261 int p = 1;
262 for (std::vector<vector<int> >::iterator it = partitions.begin(); it !=
    partitions.end(); ++it) {
263
264     int size = it->size(); // current partition size.
265     if (size == 0) continue; // skip empty partitions.
266
267     int ccnt = 0; // new columns being added.
268     int rcnt = 1; // new rows being added.
269     int nzcnt = size*partition_size; // nonzero constraint coefficients
        being added.
270
271     double *rhs = new double[rcnt]; // independent term in restrictions.
272     char *sense = new char[rcnt]; // sense of restriction inequality.
273
274     int *matbeg = new int[rcnt]; // array position where each
        restriction starts in matind and matval.
275     int *matind = new int[nzcnt]; // index of variables != 0 in
        restriction (each var has an index defined above)
276     double *matval = new double[nzcnt]; // value corresponding to index in
        restriction.
277     char **rownames = new char*[rcnt]; // row labels.
278
279     matbeg[0] = 0;
280     sense[0] = 'E';
281     rhs[0] = 1;
282     rownames[0] = new char[40];
283     sprintf(rownames[0], "partition_ %d", p);
284
285     int i = 0;
286     for (std::vector<int>::iterator it2 = it->begin(); it2 != it->end(); ++it2) {
287         for (int color = 1; color <= partition_size; ++color) {
288             matind[i] = getVertexIndex(*it2, color, partition_size);
289             matval[i] = 1;
290             ++i;
291         }
292     }
293
294     // add restriction
295     int status = CPXaddrows(env, lp, ccnt, rcnt, nzcnt, rhs, sense, matbeg,
        matind, matval, NULL, rownames);
296
297     if (status) {
298         printf("Problem adding restriction with CPXaddrows.\n");
299         exit(1);
300     }
301
302     // free memory
303     delete[] rownames[0];
304     delete[] rhs;
305     delete[] sense;
306     delete[] matbeg;
307     delete[] matind;
308     delete[] matval;
309     delete[] rownames;
310
311     ++p;
312 }

```

```

313
314     return 0;
315 }
316
317 int loadAdjacencyColorRestriction(CPXENVptr& env, CPXLPtr& lp, int vertex_size, int
partition_size) {
318
319     // load third restriction
320     int ccnt = 0; // new columns being added.
321     int rcnt = vertex_size * partition_size; // new rows being added.
322     int nzcnt = rcnt*2; // nonzero constraint coefficients being
added.
323
324     double *rhs = new double[rcnt]; // independent term in restrictions.
325     char *sense = new char[rcnt]; // sense of restriction inequality.
326
327     int *matbeg = new int[rcnt]; // array position where each restriction
starts in matind and matval.
328     int *matind = new int[rcnt*2]; // index of variables != 0 in
restriction (each var has an index defined above)
329     double *matval = new double[rcnt*2]; // value corresponding to index in
restriction.
330     char **rownames = new char*[rcnt]; // row labels.
331
332     int i = 0;
333     for (int v = 1; v <= vertex_size; ++v) {
334         for (int color = 1; color <= partition_size; ++color) {
335             matbeg[i] = i*2;
336
337             matind[i*2] = getVertexIndex(v, color, partition_size);
338             matind[i*2+1] = color-1;
339
340             matval[i*2] = 1;
341             matval[i*2+1] = -1;
342
343             rhs[i] = 0;
344             sense[i] = 'L';
345             rownames[i] = new char[40];
346             sprintf(rownames[i], "color_res");
347
348             ++i;
349         }
350     }
351
352     // add restriction
353     int status = CPXaddrows(env, lp, ccnt, rcnt, nzcnt, rhs, sense, matbeg, matind,
matval, NULL, rownames);
354
355     if (status) {
356         printf("Problem adding restriction with CPXaddrows.\n");
357         exit(1);
358     }
359
360     // free memory
361     for (int i = 0; i < rcnt; ++i) {
362         delete [] rownames[i];
363     }
364
365     delete [] rhs;

```

```

366     delete[] sense;
367     delete[] matbeg;
368     delete[] matind;
369     delete[] matval;
370     delete[] rownames;
371
372     return 0;
373 }
374
375 int solveLP(CPXENVptr& env, CPXLPptr& lp, int edge_size, int vertex_size, int
partition_size) {
376
377     int n = partition_size + (vertex_size*partition_size); // amount of total
variables
378
379     // calculate runtime
380     double inittime, endtime;
381     int status = CPXgettime(env, &inittime);
382
383     // solve LP
384     status = CPXmipopt(env, lp);
385
386     status = CPXgettime(env, &endtime);
387
388     if (status) {
389         printf("Optimization failed.\n");
390         exit(1);
391     }
392
393     // check solution state
394     int solstat;
395     char statstring[510];
396     CPXCHARptr p;
397     solstat = CPXgetstat(env, lp);
398     p = CPXgetstatstring(env, solstat, statstring);
399     string statstr(statstring);
400     if (solstat != CPXMIP_OPTIMAL && solstat != CPXMIP_OPTIMAL_TOL &&
solstat != CPXMIP_NODE_LIM_FEAS && solstat != CPXMIP_TIME_LIM_FEAS) {
401         exit(1);
402     }
403 }
404
405     double objval;
406     status = CPXgetobjval(env, lp, &objval);
407
408     if (status) {
409         printf("Problem obtaining optimal solution.\n");
410         exit(1);
411     }
412
413     // get values of all solutions
414     double *sol = new double[n];
415     status = CPXgetx(env, lp, sol, 0, n - 1);
416
417     if (status) {
418         printf("Problem obtaining the solution of the LP.\n");
419         exit(1);
420     }
421
422     // write solutions to current window

```

```

423 cout << endl << "Optimization result: " << statstring << endl;
424 cout << "Runtime: " << (endtime - inittime) << endl;
425 printf("Graph: vertex_size: %d, edge_size: %d, partition_size: %d\n", vertex_size
    , edge_size, partition_size);
426 cout << "Colors used: " << objval << endl;
427 for (int color = 1; color <= partition_size; ++color) {
428     if (sol[color-1] == 1) {
429         cout << "w_" << color << " = " << sol[color-1] << " (" << colors[color-1]
            << ")" << endl;
430     }
431 }
432
433 for (int id = 1; id <= vertex_size; ++id) {
434     for (int color = 1; color <= partition_size; ++color) {
435         int index = getVertexIndex(id, color, partition_size);
436         if (sol[index] == 1) {
437             cout << "x_" << id << " = " << colors[color-1] << endl;
438         }
439     }
440 }
441
442 delete [] sol;
443
444 return 0;
445 }
446
447 int setBranchAndBoundConfig(CPXENVptr& env) {
448
449     // CPLEX config
450     // http://www-01.ibm.com/support/knowledgecenter/SSSA5P\_12.2.0/ilog.odms.cplex.
        help/Content/Optimization/Documentation/CPLEX/\_pubskel/CPLEX916.html
451
452     // maximize objective function
453     // CPXchgobjsen(env, lp, CPX_MAX);
454
455     // enable/disable screen output
456     CPXsetintparam(env, CPX_PARAM_SCRIND, CPX_OFF);
457
458     // set execution limit
459     CPXsetdblparam(env, CPX_PARAM_TILIM, 3600);
460
461     // disable presolve
462     // CPXsetintparam(env, CPX_PARAM_PREIND, CPX_OFF);
463
464     // enable traditional branch and bound
465     CPXsetintparam(env, CPX_PARAM_MIPSEARCH, CPX_MIPSEARCH_TRADITIONAL);
466
467     // use only one thread for experimentation
468     CPXsetintparam(env, CPX_PARAM_THREADS, 1);
469
470     // do not add cutting planes
471     CPXsetintparam(env, CPX_PARAM_EACHCUTLM, CPX_OFF);
472
473     // disable gomory fractional cuts
474     CPXsetintparam(env, CPX_PARAM_FRACCUTS, -1);
475
476     // measure time in CPU time
477     CPXsetintparam(env, CPX_PARAM_CLOCKTYPE, CPX_ON);
478

```



```
479     return 0;
480 }
```

---