Euler Circuits on DiGraph

Nilay Bhatt

Central Connecticut State University
Dr. Rachel Schwell

nilaybhatt@my.ccsu.edu

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Overview

Definitions

Definition (Graph)

A **graph** G consists of a non-empty finite set $V(G)$ of elements called **vertices**, and a finite 'family' $E(G)$ of unordered pairs of (not necessarily distinct) elements of $V(G)$ called **edges**.

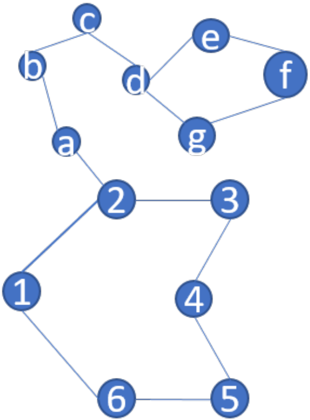
- Too formal? Think of a graph as a bunch of tennis balls connected with a thread.

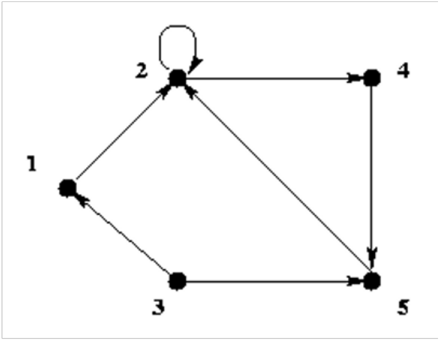
Definition (Digraph)

A **digraph** G consists of a non-empty finite set $V(G)$ of elements called **vertices**, and a finite 'family' $E(G)$ of ordered pairs of distinct elements of $V(G)$ called **directed edges**.

- NYC grid with the streets being one-way, which then restricts our direction of movement on the graph.

Graphs and Digraphs





Euler Paths v/s. Circuits

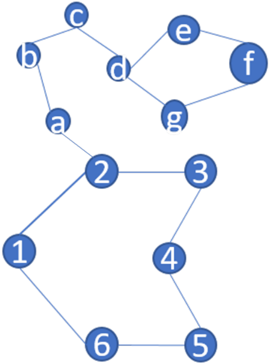
Definition (Euler Path)

An **Euler path** on a graph G is a special walk that uses each edge exactly once, and it starts and ends at **different** vertices.

Definition (Euler Circuit)

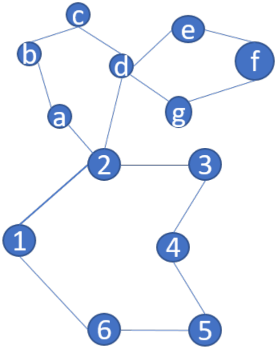
An **Euler circuit** on a graph G is a walk that uses each edge exactly once, and it starts and ends at the **same** vertex.

Euler Path



Path: d-e-f-g-d-c-b-a-2-3-4-5-6-1-2

Euler Circuit



Circuit: 2-3-4-5-6-1-2-a-b-c-d-e-f-g-d-2

Criterion for an Euler path or circuit on a given (di)graph

Euler Path

A given **graph** G has an Euler path if: **All but 2 vertices in the graph are of odd degree.**
A digraph D_g has an Euler path if: the above condition plus **"the** $|indegree - outdegree| = 1$ **for given two vertices in** D_g**.**

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Euler Circuit

A graph G has and Euler circuit if and only if all the vertices are of **even degree** and is connected.

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Euler Circuit

A graph G has and Euler circuit if and only if all the vertices are of **even degree** and is connected.

- What extra condition do you think will be needed on a digraph?

Criterion for an Euler path or circuit on a given (di)graph

Euler Circuit on digraphs

A graph G has and Euler circuit if and only if all the vertices are of **even degree** and is connected and **the indegree = outdegree for all vertices**

Criterion for an Euler path or circuit on a given (di)graph

Euler Circuit on digraphs

A graph G has and Euler circuit if and only if all the vertices are of **even degree** and is connected and **the indegree = outdegree for all vertices**

A theorem with proper proof done for the semester project in **Seminar in Proof** course.

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- Follow edges one at a time. If you have a choice between a bridge and a non-bridge, always choose the non-bridge. (If removal of a single edge from a connected graph can make it disconnected then such an edge is called a **bridge**.)

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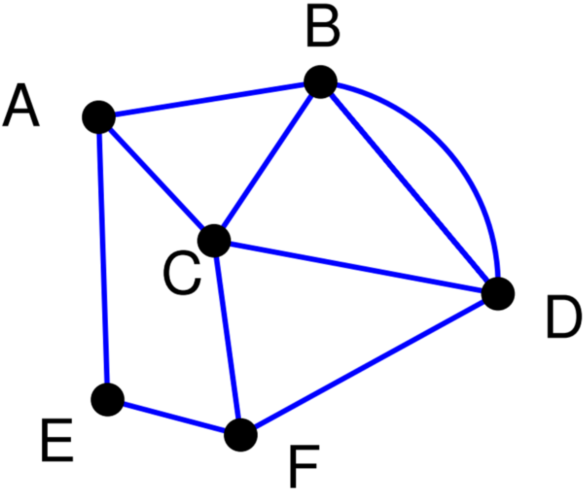
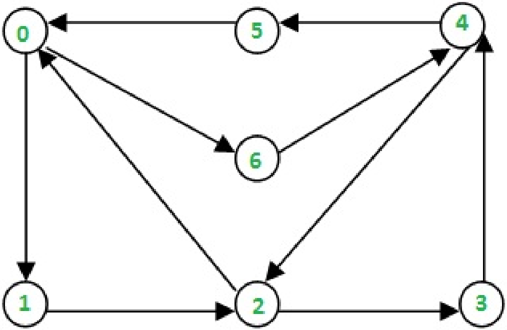
This is called **Fleury's Algorithm**

Find an Euler Path?



Euler Circuit on digraph



Euler Circuit : 0 -> 6 -> 4 -> 5 -> 0 -> 1 -> 2 -> 3 -> 4 -> 2 -> 0

What's next?

This is cool, but for all the applied mathematicians in the room and computer engineers like me the question lies:

How does that help me?

Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue. Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan dolor.

Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

References

John Smith (2012)
Title of the publication
Journal Name 12(3), 45 – 678.

The End