

MATH 450 Seminar in Proof

Prove by induction that $|P(A)| = 2^n$ if $|A| = n$.

Proof. ^{capitalize}by induction.

Base Case: Let A be a set with 0 elements. Then ~~the~~ $P(A) = \{\emptyset\}$. Thus $|P(A)| = 1 = 2^0$.

Inductive Hypothesis: Assume if $|A| = n$ then $|P(A)| = 2^n$ is true.

Inductive Step:

We will prove that if $|A| = n + 1$ then, $|P(A)| = 2^{n+1}$. Now, ^{period}let the set $|A| = n + 1$, then let $B = A - \{a\}$ where $a \in A$ ~~be a specific element.~~ Then $|B| = n$. Thus $|P(B)| = 2^n$ from our hypothesis.

Also we can split ^{is this supposed to be "two" Or was it supposed to be "into"?} the subsets of A in ^{to} parts, namely ^{one} containing a and one that does not ^{contain} *i.e.* B . Note that $P(B)$ does not have any sets in it that ^{has} a . Let $B_1, B_2, B_3, \dots, B_{2^n}$ be the elements of $P(B)$. Then, $B_1 \cup \{a\} \in P(A)$, $B_2 \cup \{a\} \in P(A)$, $B_3 \cup \{a\} \in P(A)$, ..., $B_{2^n} \cup \{a\} \in P(A)$ ^{make more formal} gets us the subsets of A that contain the element a . Since ^{which one?} the union produces 2^n elements and $|P(B)| = 2^n$, then $|P(A)| = 2^n + 2^n = 2^n(1 + 1) = 2^{n+1}$. \square

Read this aloud - it doesn't parse correctly. It needs to read correctly.

But B is not "subsets" of A. It's one subset of A. And I assume you don't mean B is the only subset of A that doesn't contain a.

One what? One part that contains a? Wording is funny since you mean multiple sets.