MATH 450 Seminar in Proof

Prove that the set of all infinite binary strings is uncountable.

Proof. Let us proceed by contradiction. Let A be the set that represents the set of all possible infinite binary strings. Then we assume that A is countable, *i.e* $|A| = |\mathbb{N}|$ then we can say that there is a bijection $f : \mathbb{N} \to A$. Since A is countable we can list the elements of A which are mapped from $\mathbb{N}$ through f.

$$\begin{aligned} f(1) &= a_{11}a_{12}a_{13}a_{14}... \\ f(2) &= a_{21}a_{22}a_{23}a_{24}... \\ f(3) &= a_{31}a_{32}a_{33}a_{34}... \\ &.... \\ &.... \\ &.... \end{aligned}$$

where a_{ij} is the j^{th} digit of $f(i)$ where $i, j \in \mathbb{N}$. Since f is a bijection, f is onto. Thus we will prove by contradiction that there exists a binary string that is not in $f(A)$ therefore proving that f is not onto and thus A is not countable. Let us define digit d_i as

You're not making a conclusion here

A isn't the domain of f

The existence of the string isn't proved by contradiction, but by construction. The original statement is proved by contradiction..

$$d_i = \begin{cases} 0 & \text{if } a_{ii} = 1 \\ 1 & \text{if } a_{ii} \neq 1 \end{cases}$$

Thus the infinite binary string d whose i^{th} digit is d_i is different from all $f(i)$. By our assumption $d \in A$, but there is no pre-image in $\mathbb{N}$ such that $f(i) = d$ for any i. Thus there is contradiction. Thus the set of all infinite binary strings is not countable. $\square$

redundant

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