

MATH 450 Seminar in Proof

Let $f : \mathbb{Z} \rightarrow 2\mathbb{Z}$ be defined by $f(x) = 2x - 6$. Prove that f is a bijection.

Proof. Let f be the function defined as in the question.

One-to-One: Let $f(x_1) = f(x_2)$, then

$$2x_1 - 6 = 2x_2 - 6 \quad (1)$$

$$2x_1 = 2x_2 \quad (2)$$

$$x_1 = x_2 \quad (3)$$

This means that if $f(x_1) = f(x_2)$ then, $x_1 = x_2$ thus f is ^{hyphenate}one to one.

Onto: Let $x \in \mathbb{Z}$ and $y \in 2\mathbb{Z}$ such that, $x = \frac{y+6}{2}$.

$$f(x) = 2x - 6$$

You can't choose x twice. Once it's chosen as $(y+6)/2$, you have to *show* it's in \mathbb{Z} .

$$= 2 \left(\frac{y+6}{2} \right) - 6 \quad (4)$$

$$= y + 6 - 6$$

$$f(x) = y$$

This means that for every $y \in 2\mathbb{Z}$ there exists an $x \in \mathbb{Z}$ ^{such that...}. Thus f is a bijecction. \square