

MATH 450 Seminar in Proof

Prove that the set of integers are countable.

Proof. We know that a set X is countable if there exists a bijection ~~in~~ $f : \mathbb{N} \rightarrow X$. Let $f : \mathbb{N} \rightarrow \mathbb{Z}$ **b** defined as

$$f(n) = \begin{cases} -\left(\frac{n-1}{2}\right) & \text{if } \mathbf{n} \text{ is odd} \\ \frac{n}{2} & \text{if } \mathbf{n} \text{ is even} \end{cases}$$

No, that's not what we showed. We showed that \mathbb{Z} is a valid codomain.

We know that $f(n)$ is **a well defined function from the class lecture**. So we will show that it is a bijection.

One-to-One:

Let $f(a) = f(b)$ where $a, b \in \mathbb{N}$. Therefore, either $f(a) = -\left(\frac{a-1}{2}\right)$

and $f(b) = -\left(\frac{b-1}{2}\right)$ or $f(a) = \frac{a}{2}$ and $f(b) = \frac{b}{2}$. Note that if we

have $f(b) = -\left(\frac{b-1}{2}\right)$ and $f(a) = \frac{a}{2}$ with out loss of generality, we

will have $b = 1 - a$ which makes **$b \leq 0$** Why? Why can't a be <1? implies $b \notin \mathbb{N}$. Therefore both $f(a)$ and $f(b)$ will have the same structure.

In both the former cases we get $a = b$ if $f(a) = f(b)$. thus the function is one-to-one.

Onto:

Let $y \in \mathbb{Z}$. If $y > 0$ then we have $f(2y) = \frac{2y}{2} = y$. **Since $y > 0$** This doesn't read right. You're missing a word or something. **implies** $2y > 0$ and $2y \in \mathbb{N}$. Thus $2y = \mathbf{n}$ Do you need this? $\in \mathbb{N}$. If $y < 0$ then we have

$f(-2y + 1) = -\left(\frac{-2y + 1 - 1}{2}\right) = y$. Since $y < 0$, $-2y > 0$ and thus $-2y + 1 > 0$. Thus $-2y + 1 \in \mathbb{N}$. Hence, the function is onto.

Thus the $f : \mathbb{N} \rightarrow \mathbb{Z}$ with

$$f(n) = \begin{cases} -\left(\frac{n-1}{2}\right) & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even} \end{cases}$$

is well defined and is a bijection. Thus the set of integers are countable.

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