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MATH 450 Seminar in Proof

read this back to yourself - doesn't read right missing a word? missing a word?

Prove: If $f : A \to B$ be a function such that A and B are finite $|A| = |B|$, then f is one-to-one if only if it is onto.

Proof.
$\Longleftarrow$ Let $f : A \to B$ be a function such that $|A| = |B|$ and f is onto. Then from the definition of onto, for all $b \in B$ there exists $a \in A$ such that, $f(a) = b$. Let f be not one-to-one. Then there exists $a_1, a_2 \in A, a_1 \neq a_2$ such that $f(a_1) = f(a_2) = b \in B$. Since f is onto, every $b \in B$ has a pre-image in A. Also, since f is well defined, each $a \in A$ has only one image in B. But by our assumption f is not one-to-one and so there exists $a_1, a_2 \in A$ where $a_1 \neq a_2$ such that they both have the same image. Also, by our assumption since $|A| = |B|$ we will have an element in B which does not have a pre-image in A making it not onto. Hence there is a contradiction $\to\leftarrow$; and so, f is one-to-one.

These things in green are just repeating things you've already said

Why? You basically restated your assumptions and then said it gives you your conclusion, but you didn't justify why.

$\Longrightarrow$ Let $f : A \to B$ be a function such that $|A| = |B|$ and f is one-to-one. Then from the definition of one-to-one, if $f(a_1) = f(a_2)$ then $a_1 = a_2$. Also note that since f is one-to-one $|A| = |f[A]|$. Let f be not onto. Then there exist a $b \in B$ such that there does not exist any $a \in A$ where $f(a) = b$. This is a contradiction because, according to our assumption $|A| = |B|$ but then if f is not onto it implies that there exists more elements in B than there are in A. Also, since the cardinality of A and B is same then two elements in A map to one element in B thus making it not one-to-one. This is where our contradiction lies. Therefore f has to be onto. $\square$

exists

To what does this refer?

This is again kind of just restating your assumptions and jumping to your conclusion

You're on the right track but how does this relate to f[A] and your statement above?