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Results about Euler's path and circuits

MATH 450 Seminar in Proof

I think you need to start with the definition of a graph!
The mathematical definition!

Definition 1.1: ***Euler Path:*** An *Euler Path* on a graph G is a path that traverses each edge exactly once.

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Definition 1.2: ***Euler Circuit/Cycle:*** An Euler circuit on a graph G is a Euler Path which starts and ends on the same node.

Results to be proven:

Remove this because you just said that the statements are to be proven

Euler or Eulerian? Just be consistent.

1. Show that any graph where the degree of every vertex is even iff it has an Eulerian circuit.

I think you need to define this

2. Show that if there are exactly two vertices a and b of odd degree, there is an Eulerian path on the graph.

math mode

Proof :

Okay, this (literally everything in green) is where you are going to have some good opportunity to push yourself to grow. How do we say all these things mathematically rigorously? You will want to engage the mathematical definition of graph you find/use.

1. $\Rightarrow$ Let G be a graph such that it has an Euler circuit. According to the definition of Euler Circuit on a graph, given a vertex v_1, we can traverse every other node only once and end at the vertex v_1. Thus every node/vertex has one entry point and one exit point. We know this because G has an Euler circuit. Thus every node/vertex has two edges attached to it thus making it a node of even degree.

   I would say "proceed" here, because the antecedent of "this" is technically the statement you just assumed.

   no comma

   $\Leftarrow$ Let every node in a graph G have an even degree. Let us prove this by induction. Assume, that there are only two nodes in G, each with two edges on them. Thus it is clear that you will end on the same node that you started with, thus making a Euler circuit on G.

   Okay, I think you know this is not really a proof.

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   Now,let G be graph of even degree with more than two nodes. Lets start with an arbitrary node v_1. Now since every node has an even degree, we can follow a trail from v_1 to the next node and repeat it for the next node we encounter. We know that this is possible because each time we enter a node v_k we know there is an unused edge adjacent to it for us to use to traverse. Thus using a unique path we will come back on v_1 since there are even number of edges, we will always find an untapped edge to let us continue our path back to v_1.

   need to define this

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2. Let G be a graph with Euler circuit. Thus every node/vertex has an even degree. now let us add one node say b and add an edge to a node a in the existing graph G. Note that before adding the edge from b to a, a in G had an even degree. Now if we start drawing our path from b, and since it has only one edge connecting to a we go to a now, if we hypothetically ignore the edge connecting a and b, the remainder of G has nodes with even edges, thus making it a Euler circuit. Therefore, the trail will end at a but since we have already used the edge connecting a and b, we stop at a. Thus we were able to traverse all the edges in $G + b$ exactly once, starting from b and ending at an edge a in G.

   I don't know what this is referring to.

So how can there be two edges between just two vertices?
This is where the definition of graph really matters.

Come on now, I know you know how to
prove by induction more rigorously than this.

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Wait, I'm confused. Why is this your first assumption? This doesn't seem to relate to the statement you said you were proving.

I think your process is going to be the opposite of many of your classmates' (which is fine, and actually really cool). Most other students are starting with a dense theorem whose heavy mathematical notation and rigor they didn't really process the first time around, and are trying to get a good mental picture of what's actually going on so they can understand and explain it. Your situation is definitely the opposite of that: you obviously have a good mental picture of what's going on. But it's pretty far from rigorous at this point (which I think you know, and which is exactly why you picked it).