

MATH 450 Seminar in Proof

Prove: If $f : A \rightarrow B$ be a function such that A and B are finite $|A| = |B|$, then f is one-to-one if only if it is onto.

missing some words or something in here

Proof.

\Leftarrow Let $f : A \rightarrow B$ be a function such that $|A| = |B|$ and f is onto. Then from the definition of onto, $\forall b \in B \exists a \in A$ such that, $f(a) = b$. Let f be not one-to-one. Then there exists $a_1, a_2 \in A, a_1 \neq a_2$ such that $f(a_1) = f(a_2) = b \in B$. This implies that there exists more elements in A than B . Thus saying, $|A| > |B|$, hence there is a contradiction $\rightarrow \Leftarrow$; and so, f is one-to-one. Why? That's exactly what you need to show.

\Rightarrow Let $f : A \rightarrow B$ be a function such that $|A| = |B|$ and f is one-to-one. Then from the definition of one-to-one, if $f(a_1) = f(a_2)$ then $a_1 = a_2$. Let f be not onto. Then there exists $b \in B$ such that, $\nexists a \in A$, where $f(a) = b$. This implies that there exists more elements in B than A . Thus saying, $|A| < |B|$, hence there is a contradiction $\rightarrow \Leftarrow$; and so, f has to be onto. same issue - why? \square