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Divisibility of Three

MATH 450 Seminar in Proof

A number is divisible by three if and only if the sum of the digits is divisible by 3.

What kind?

Proof. Let β be a number. We can then write β as:

$$\beta = \beta_0 * 10^0 + \beta_1 * 10^1 + \beta_2 * 10^2 + ... + \beta_i * 10^i$$

where beta_i are….

NOTE: We are using the following properties of modular arithmetic:

1. $(A + B) \bmod C = ((A \bmod C) + (B \bmod C)) \bmod C$
   and
2. $(A * B) \bmod C = ((A \bmod C) * (B \bmod C)) \bmod C$

put this before you start the proof (as lemmas)

where A, B, C are some numbers. either say what kind or don't say this at all (because it doesn't add any information really)

awkward

$\Rightarrow$ Let that β is divisible by 3. Then $\beta \bmod 3 = 0$, thus we can write the expansion as:

$$\begin{aligned}
(\beta) \bmod 3 &= (\beta_0 * 10^0 + \beta_1 * 10^1 + \beta_2 * 10^2 + ... + \beta_i * 10^i) \bmod 3 \\
0 &= (\beta_0 * 10^0 + \beta_1 * 10^1 + \beta_2 * 10^2 + ... + \beta_i * 10^i) \bmod 3 \\
&= [(\beta_0 * 10^0) \bmod 3 + (\beta_1 * 10^1) \bmod 3 + (\beta_2 * 10^2) \bmod 3 + ... + (\beta_i * 10^i) \bmod 3] \bmod 3 \\
&= [\{(\beta_0) \bmod 3 * (10^0) \bmod 3\} \bmod 3 + \{(\beta_1) \bmod 3 * (10^1) \bmod 3\} \bmod 3 \\
&+ \{(\beta_2) \bmod 3 * (10^2)) \bmod 3\} \bmod 3 + ... + \{(\beta_i) \bmod 3 * (10^i) \bmod 3\} \bmod 3] \bmod 3 \\
&= [\{(\beta_0) \bmod 3 * 1\} \bmod 3 + \{(\beta_1) \bmod 3 * 1\} \bmod 3 + \{(\beta_2) \bmod 3 * 1\} \bmod 3 \\
&+ ... + \{(\beta_i) \bmod 3 * 1\} \bmod 3] \bmod 3 \\
&= [(\beta_0) \bmod 3 + (\beta_1) \bmod 3 + (\beta_2) \bmod 3 + ... + (\beta_i) \bmod 3] \bmod 3 \\
0 &= [\beta_0 + \beta_1 + \beta_2 + ... + \beta_i] \bmod 3
\end{aligned}$$

don't need parentheses on this

I wouldn't use this type since it's for sets

these () aren't needed

Thus if a number is divisible by three then the sum if its digits is also divisible by three.
$\Leftarrow$ Let β be a number such that the sum of its digits $\beta_0 + \beta_1 + \beta_2 + ... + \beta_i$ is divisible by 3. From the algebra done in the equations above, and by the definition of equality, we can follow the last equation

$$[\beta_0 + \beta_1 + \beta_2 + ... + \beta_i] \bmod 3 = 0$$

from bottom up and we thus we can then say that β is also divisible by 3.
Thus if the sum of the digits of a number are divisible by 3 then then number itself is divisible by 3.

$\square$