**MATH 450 Seminar in Proof**

Let $f : \mathbb{Z} \to 2\mathbb{Z}$ be defined by $f(x) = 2x-6$. Prove that $f$ is a bijection.

*Proof.* Let $f$ be the function defined as in the question.
*One-to-One:* Let $f(x_1) = f(x_2)$, then

$$2x_1 - 6 = 2x_2 - 6 \tag{1}$$
$$2x_1 = 2x_2 \tag{2}$$
$$x_1 = x_1 \tag{3}$$

This means that if $f(x_1) = f(x_2)$ then, $x_1 = x_1$ thus $f$ is one-to-one.
*Onto:* Let $y \in 2\mathbb{Z}$ such that, $x = \dfrac{y+6}{2}$. We will show that $x \in \mathbb{Z}$

$$\begin{aligned} f(x) &= 2x - 6 \\ &= 2\left(\frac{y+6}{2}\right) - 6 \\ &= y + 6 - 6 \\ f(x) &= y \end{aligned} \tag{4}$$

Also, since $y \in 2\mathbb{Z}$, and 6 is even we know that $y-6 \in 2\mathbb{Z}$. Furthermore, $\frac{y-6}{2}$ can be even or odd, but more importantly it will be an integer. Thus, $x \in \mathbb{Z}$. This means that for every $y \in 2\mathbb{Z}$ there exists an $x \in \mathbb{Z}$ and making $f$ onto. Thus $f$ is bijective. $\square$