MATH 450 Seminar in Proof

Let $f : \mathbb{Z} \to 2\mathbb{Z}$ be defined by $f(x) = 2x - 6$. Prove that f is a bijection.

Proof. Let f be the function defined as in the question.
One-to-One: Let $f(x_1) = f(x_2)$, then

$$2x_1 - 6 = 2x_2 - 6 \tag{1}$$

$$2x_1 = 2x_2 \tag{2}$$

$$x_1 = x_1 \tag{3}$$

This means that if $f(x_1) = f(x_2)$ then, $x_1 = x_1$ thus f is one-to-one.

Wrong word choice because x is not a property of y. Y is defined independently of x. Letting x be this expression is a separate assumption.

Onto: Let $y \in 2\mathbb{Z}$ such that, $x = \dfrac{y+6}{2}$. We will show that $x \in \mathbb{Z}$ period

Seems like the wrong place for this since this isn't where you show it.

$$\begin{aligned} f(x) &= 2x - 6 \\ &= 2\left(\frac{y+6}{2}\right) - 6 \\ &= y + 6 - 6 \\ f(x) &= y \end{aligned} \tag{4}$$

Also, since $y \in 2\mathbb{Z}$, and 6 is even we know that $y - 6 \in 2\mathbb{Z}$. Furthermore, $\frac{y-6}{2}$ can be even or odd, but more importantly it will be an integer. Thus, $x \in \mathbb{Z}$. This means that for every $y \in 2\mathbb{Z}$ there exists an $x \in \mathbb{Z}$ ~~and~~ making f onto. Thus f is bijective. $\square$

So then why do you need to say this?

such that? Just being called "x" doesn't automatically imply it has the relevant property.