MATH 450 Seminar in Proof

Prove that the set of odd negative integers is countable.

Proof. We know that a set X is countable if there exists a bijection in $f : \mathbb{N} \to X$. Let X be the set of negative odd integers. Let $f : \mathbb{N} \to X$ such that $f(n) = 1 - 2n$.

We first show that the function has a valid co-domain. For every $n \in \mathbb{N}$ we have $f(n) = 1 - 2n$. We know that $n > 0$ then $-2n < 0$ and is even. Thus, $1 - 2n$ is a negative odd integer. Now we will prove that $f(n)$ is bijective.

One-to-One:

Let $f(a) = f(b)$ where $a, b \in \mathbb{N}$. Then since, $f(a) = f(b)$ we know that $1 - 2a = 1 - 2b$. Thus $a = b$. Therefore f is one-to-one.

Onto:

Let $y \in -2\mathbb{Z} - 1$ then $y < 0$. Observe that $f\left(\frac{1-y}{2}\right) = 1 - 2\left(\frac{1-y}{2}\right) = y$. Since $y \leq -1$ then we know that $-y \geq 1$, which means $1 - y \geq 2$ and thus, $\frac{1-y}{2} \geq 1$. Also, since y is a negative odd integer, $1 - y$ is a positive even integer and thus $\frac{1-y}{2} \in \mathbb{N}$. Hence, the function is onto.

Thus $f : \mathbb{N} \to X$ with $f(n) = 1 - 2n$ is bijective. Thus the set of odd negative integers are countable. $\square$