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Divisibility of Three

MATH 450 Seminar in Proof

A number is divisible by three if and only if the sum of the digits is divisible by 3.

C'mon….seriously? It's already defined. (Need to use a different word)

Proof. $\Rightarrow$ Let β be a whole number divisible by 3. We can then define β as:

I would define Beta and its expansion before you begin the first direction. That way the notation hold for both directions and you don't need to reintroduce it.

$$\beta = \beta_0 * 10^0 + \beta_1 * 10^1 + \beta_2 * 10^2 + ... + \beta_i * 10^i$$

where i is?

$$(\beta) \mod 3 = (\beta_0 * 10^0 + \beta_1 * 10^1 + \beta_2 * 10^2 + ... + \beta_i * 10^i) \mod 3$$

Spacing is off. It threw me off.

$$0 = (\beta_0 * 10^0 + \beta_1 * 10^1 + \beta_2 * 10^2 + ... + \beta_i * 10^i) \mod 3$$

Good! I'm so glad you were precise about using these results/properties!

$$= ((\beta_0 * 10^0) \mod 3 + (\beta_1 * 10^1) \mod 3 + (\beta_2 * 10^2) \mod 3 + ... + (\beta_i * 10^i) \mod 3) \mod 3$$

Can you mix up the brackets and use some square ones? I think it might be easier to parse.

$$= (((\beta_0) \mod 3 * (10^0) \mod 3) \mod 3 + ((\beta_1) \mod 3 * (10^1) \mod 3) \mod 3$$
$$+ ((\beta_2) \mod 3 * (10^2)) \mod 3) \mod 3 + ... + ((\beta_i) \mod 3 * (10^i) \mod 3) \mod 3) \mod 3$$
$$= (((\beta_0) \mod 3 * 1) \mod 3 + ((\beta_1) \mod 3 * 1) \mod 3 + ((\beta_2) \mod 3 * 1) \mod 3)$$
$$+ \mod 3 + ... + ((\beta_i) \mod 3 * 1) \mod 3) \mod 3$$

Why?

$$= ((\beta_0) \mod 3 + (\beta_1) \mod 3 + (\beta_2) \mod 3 + ... + (\beta_i) \mod 3) \mod 3$$
$$0 = (\beta_0 + \beta_1 + \beta_2 + ... + \beta_i) \mod 3$$

seriously? I would say this before you do the proof.

We are using the properties of modular arithmetic over here. We are using the following properties:

1. $(A + B) \mod C = ((A \mod C) + (B \mod C)) \mod C$
   and
2. $(A * B) \mod C = ((A \mod C) * (B \mod C)) \mod C$

is (sum is the subject)

Thus if a number is divisible by three then the sum if it's digits are also divisible by three.

space Don't need parentheses

$\Leftarrow$ Letβ be a whole number such that the sum of its digits $(\beta_0 + \beta_1 + \beta_2 + ... + \beta_i)$ is divisible by 3. Then we can say:

$$(\beta_0 + \beta_1 + \beta_2 + ... + \beta_i) \mod 3 = 0$$
$$((\beta_0) \mod 3 + (\beta_1) \mod 3 + (\beta_2) \mod 3 + ... + (\beta_i) \mod 3) \mod 3 =$$
$$(((\beta_0) \mod 3 * 1) \mod 3 + ((\beta_1) \mod 3 * 1) \mod 3 + ((\beta_2) \mod 3 * 1) \mod 3)+$$
$$\mod 3 + ... + ((\beta_i) \mod 3 * 1) \mod 3) \mod 3 =$$
$$(((\beta_0) \mod 3 * (10^0) \mod 3) \mod 3 + ((\beta_1) \mod 3 * (10^1) \mod 3) \mod 3+$$
$$((\beta_2) \mod 3 * (10^2)) \mod 3) \mod 3 + ... + ((\beta_i) \mod 3 * (10^i) \mod 3) \mod 3) \mod 3 =$$
$$((\beta_0 * 10^0) \mod 3 + (\beta_1 * 10^1) \mod 3 + (\beta_2 * 10^2) \mod 3 + ... + (\beta_i * 10^i) \mod 3) \mod 3 =$$
$$(\beta_0 * 10^0 + \beta_1 * 10^1 + \beta_2 * 10^2 + ... + \beta_i * 10^i) \mod 3 = 0$$
$$(\beta) \mod 3 = 0$$

Thus if the sum of the digits of a number are divisible by 3 then then number itself is divisible by 3.

$\square$

Actually, what I'd like to see you do is see how much of the two directions you can combine into one. Meaning, how much can you do before you split up into the two directions? Would be nice to make this as efficient as possible.