

## MATH 450 Seminar in Proof

Prove that  $f : A \rightarrow P(A)$  is not onto (Where  $P(A)$  is the power set of  $A$ ). Hint: consider the set  $\{a \in A : a \notin f(a)\}$

*Proof.* Let us proceed by contradiction. Let  $f : A \rightarrow P(A)$  be onto. Then  $f[A] = P(A)$ . Let  $B = \{a \in A : a \notin f(a)\}$ . Thus  $B \subset A$  and also  $B \in f[A]$ . Since  $f$  is onto, we know that  $B$  has a pre-image in  $A$ . Let  $f(a) = B$ , where  $a$  be an element in  $A$ . There are two possibilities  $a \in B$  or  $a \notin B$ . If  $a \in B$  then by definition  $a \notin f(a)$  but then since  $f(a) = B$  there is a contradiction. If  $a \notin B$  then by definition  $a \in f(a)$  but then since  $f(a) = B$  then again there is a contradiction. Thus  $f$  is not onto.  $\square$

Overall good job!