

MATH 450 Seminar in Proof

$$0.\bar{9} = 1$$

Proof. We know that:

$$\begin{aligned} 0.9 &= \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \frac{9}{100000} + \dots \\ &= 9\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{100000} + \dots\right) \\ &= 9 \sum_{n=1}^{\infty} \frac{1}{10^n} \\ &= 9\left(\frac{\frac{1}{10}}{1 - \frac{1}{10}}\right) \\ &= 9\left(\frac{\frac{1}{10}}{\frac{9}{10}}\right) &= 1 \end{aligned}$$

Thus proved.

□