Nice job on this one overall!

MATH 450 Seminar in Proof

is (subject is “the set”)

Prove that the set of all infinite binary strings are uncountable.

Proof. Let us proceed by contradiction. Let A be the set that represents the set of all possible infinite binary strings. Then we assume that A is countable, $i.e$ $|A| = |\mathbb{N}|$ then we can say that there is a bijection $f : \mathbb{N} \to A$. Since A is countable we can list the elements of A which are mapped from $\mathbb{N}$ through f.

$$\begin{aligned} f(1) &= a_{11}a_{12}a_{13}a_{14}... \\ f(2) &= a_{21}a_{22}a_{23}a_{24}... \\ f(3) &= a_{31}a_{32}a_{33}a_{34}... \\ &.... \\ &.... \\ &.... \end{aligned}$$

where a_{ij} is the j^{th} digit of $f(i)$ where $i, j \in \mathbb{N}$. Since f is a bijection, f is onto. Thus for all $a \in A$ there exits an $x \in \mathbb{N}$. Let us define digit d_i as

such that?

$$d_i = \begin{cases} 0 & \text{if } a_{ii} = 1 \\ 1 & \text{if } a_{ii} \neq 1 \end{cases}$$

Thus the infinite binary string d whose i^{th} digit is d_i is different from all $f(i)$. By our assumption $d \in A$, but there is no pre-image in $\mathbb{N}$ such that $f(i) = d$ for any i. Thus there is contradiction. Thus the set of all infinite binary is not countable. $\square$

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missing a word