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MATH 450 Seminar in Proof

Prove: If $f : A \to B$ be a function such that A and B are finite $|A| = |B|$, then f is one-to-one if only if it is onto.

Proof.
$\Longleftarrow$ Let $f : A \to B$ be a function such that $|A| = |B|$ and f is onto. Then from the definition of onto, $\forall b \in B \exists a \in A$ such that, $f(a) = b$. Let f be not one-to-one. Then there exists $a_1, a_2 \in A, a_1 \neq a_2$ such that $f(a_1) = f(a_2) = b \in B$. This implies that there exists more elements in A than B. Thus saying, $|A| > |B|$, hence there is a contradiction $\rightarrow\leftarrow$; and so, f is one-to-one.
$\Longrightarrow$ Let $f : A \to B$ be a function such that $|A| = |B|$ and f is one-to-one. Then from the definition of one-to-one, if $f(a_1) = f(a_2)$ then $a_1 = a_2$. Let f be not onto. Then there exists $b \in B$ such that, $\nexists a \in A$, where $f(a) = b$ .This implies that there exists more elements in B than A. Thus saying, $|A| < |B|$, hence there is a contradiction $\rightarrow\leftarrow$; and so, f has to be onto. $\square$