MATH 450 Seminar in Proof

is (subject is "the set")

Prove that the set of integers are countable.

You haven't actually said what the function is yet. So how can you say that it's well-defined?

Proof. We know that a set X is countable if there exists a bijection ~~in~~ $f : \mathbb{N} \to X$. We know that $f : \mathbb{N} \to \mathbb{Z}$ is a well defined function from the class lecture. So we will show that it is a bijection. Let

Note that this is where you define f.

$$f(n) = \begin{cases} -\left(\dfrac{n-1}{2}\right) & \text{if n is odd} \\ \dfrac{n}{2} & \text{if n is even} \end{cases}$$

One-to-One:

Let $f(a) = f(b)$ where $a, b \in \mathbb{N}$. Then since, $f(a) = f(b)$ we know that either $f(a) = -\left(\dfrac{a-1}{2}\right)$ and $f(b) = -\left(\dfrac{b-1}{2}\right)$ or $f(a) = \dfrac{a}{2}$ and $f(b) = \dfrac{b}{2}$. In both the cases we get $a = b$ if $f(a) = f(b)$. thus the function is one-to-one.

Why can't one use one expression and the other use the other one?

Onto:

This reasoning is backward - it's using what you're trying to prove. You don't "have" that y=n/2; that's what you're showing. Onto is an existence proof, meaning you produce the desired object and show it works, not assume that you already have it.

Let $y \in \mathbb{Z}$. If $y > 0$ then we have $f(n) = y = \dfrac{n}{2} = y \to n = 2y$. Since $y > 0 \to 2y > 0 \in \mathbb{N}$. Thus $2y = n \in \mathbb{N}$. If $y < 0$ then we have $y = -\left(\dfrac{n-1}{2}\right)$. Solving for n we get $n = -2y + 1$. Since $y < 0$, $-2y > 0$ and thus $-2y + 1 > 0$. Thus $-2y + 1 = n \in \mathbb{N}$. Hence, the function is onto.

none of these - write out

same issue

Thus the $f : \mathbb{N} \to \mathbb{Z}$ with

$$f(n) = \begin{cases} -\left(\dfrac{n-1}{2}\right) & \text{if n is odd} \\ \dfrac{n}{2} & \text{if n is even} \end{cases}$$

is well defined and is a bijection. Thus the set of integers are countable. $\square$