

MATH 450 Seminar in Proof

Prove that $f : A \rightarrow P(A)$ is not onto. Where $P(A)$ is the power set of A . Hint: consider the set $\{a \in A : a \notin f(a)\}$

Proof. Let us proceed by contradiction. Let f be onto. Then $f[A] = P(A)$. Let $B = \{a \in A : a \notin f(a)\}$. Thus $B \subset A$ and also $B \in P(A)$. Let a be a particular element in A . If $a \in B$ then $B \notin f(a)$ and if $a \in f(a)$ then $a \notin B$. This creates a contradiction as from our assumption since f is onto, $B \in f[A]$ but $B \notin f[A]$. So $f[A] \neq P(A)$. Thus f is not onto. \square