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MATH 450 Seminar in Proof

$0.\bar{9} = 1$

Proof. We know that:

$$\begin{aligned} 0.9 &= \frac{9}{10} + \frac{9}{100} + \frac{9}{1000} + \frac{9}{100000} + ... \\ &= 9\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{100000} + ...\right) \\ &= 9\sum_{n=1}^{\infty} \frac{1}{10^n} \end{aligned} \tag{1}$$

Since this is a geometric series, and $\frac{1}{10} < 1$, we know that the above series diverges. (really? :)) Thus,

$$\begin{aligned} &= 9\left(\frac{\frac{1}{10}}{1 - \frac{1}{10}}\right) \\ &= 9\left(\frac{\frac{1}{10}}{\frac{9}{10}}\right) \\ &= 1 \end{aligned} \tag{2}$$

Thus proved.

□