MATH 450 Seminar in Proof

Every integer greater than 1 is expressible as a product of primes.

Proof. Let us proceed by contradiction. Let $n > 1$ be the least integer that cannot be expressed as a product of primes. By construction, n has no divisors that are prime, so n is not a prime number. Thus n is composite number. Thus, there exists an integer a where $1 < a < n$ such that a divides n. Since a divides n we can write n as $n = ab$ where $1 < b < n$. Since n is the smallest number that does not have prime factors, we can say that $a = p_1 \cdot p_2 \cdot p_3 \cdot ... \cdot p_n$ and $b = q_1 \cdot q_2 \cdot q_3 \cdot ... \cdot q_n$ where p_i, q_i, where are prime factors of a and b and $i \in \mathbb{N}$. Thus we can rewrite $n = ab$ as, $n = (p_1 \cdot p_2 \cdot p_3 \cdot ... \cdot p_n)(q_1 \cdot q_2 \cdot q_3 \cdot ... \cdot q_n)$. Now we are able to represent n as a product of prime numbers. Thus there is a contradiction $\rightarrow\leftarrow$. Therefore every integer greater than 1 is expressible as a product of primes. $\square$