

MATH 450 Seminar in Proof

Let $f : \mathbb{Z} \rightarrow 2\mathbb{Z}$ be defined by $f(x) = 2x - 6$. Prove that f is a bijection.

Proof. Let f be the function defined as in the question.

One-to-One: Let $f(x_1) = f(x_2)$, then

$$2x_1 - 6 = 2x_2 - 6 \tag{1}$$

$$2x_1 = 2x_2 \tag{2}$$

$$x_1 = x_2 \tag{3}$$

This means that if $f(x_1) = f(x_2)$ then, $x_1 = x_2$ thus f is one-to-one.

Onto: Let $y \in 2\mathbb{Z}$. Let $x = \frac{y+6}{2}$.

$$\begin{aligned} f(x) &= 2x - 6 \\ &= 2 \left(\frac{y+6}{2} \right) - 6 \\ &= y + 6 - 6 \end{aligned} \tag{4}$$

$$f(x) = y$$

This means that for every $y \in 2\mathbb{Z}$ there exists an $x \in \mathbb{Z}$ such that $x = \frac{y+6}{2}$, making f onto. Thus f is bijective. \square