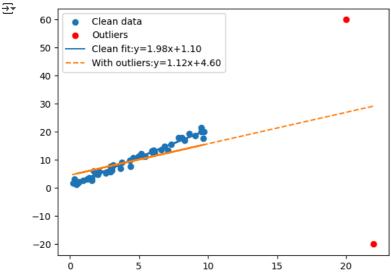
```
# Python simulation
import numpy as np
import matplotlib.pyplot as plt
from sklearn.linear_model import LinearRegression
np.random.seed(42)
X = np.random.uniform(0, 10, 50).reshape(-1,1)
y = 2 * X.flatten() + 1 + np.random.normal(0, 1, 50)
model_clean = LinearRegression().fit(X, y)
X_{out} = np.append(X, [[20], [22]], axis=0)
y_out = np.append(y, [60, -20])
model_out = LinearRegression().fit(X_out, y_out)
plt.scatter(X, y, label="Clean data")
plt.scatter(X_out[-2:], y_out[-2:], color='red', label="Outliers")
\verb|plt.plot(X, model_clean.predict(X), label=f"Clean fit:y=\{model_clean.coef_[\emptyset]:.2f\}x+\{model_clean.intercept_:.2f\}")|
plt.plot(X\_out, model\_out.predict(X\_out), linestyle='--', label=f"With outliers:y=\{model\_out.coef\_[0]:.2f\}x+\{model\_out.intercetation for the context of th
plt.legend()
plt.show()
 →*
                        60
                                                         Clean data
                                                         Outliers
```



```
# Python simulation
import pandas as pd
import statsmodels.api as sm
from itertools import combinations
# Load data
data = pd.read_csv('FE-GWP1_model_selecxtion_1.csv')
df = pd.DataFrame(data)
X = df.drop(columns='Y')
y = df['Y']
# Forward Selection
def forward_selection(X, y):
 remaining = list(X.columns)
 selected = []
 current_score, best_new_score = 0, 0
 while remaining:
   scores_with_candidates = []
    for candidate in remaining:
      model = sm.OLS(y, sm.add_constant(df[selected + [candidate]])).fit()
      scores_with_candidates.append((model.rsquared_adj, candidate))
    scores_with_candidates.sort(reverse=True)
   best_new_score, best_candidate = scores_with_candidates[0]
    if best_new_score > current_score:
      remaining.remove(best_candidate)
      selected.append(best_candidate)
      current_score = best_new_score
    else:
     break
  return selected, current_score
# Backward Elimination using BIC
def backward_elimination(X, y):
  selected = list(X.columns)
 current_model = sm.OLS(y, sm.add_constant(X[selected])).fit()
 current_bic = current_model.bic
 while True:
   bics = []
    for candidate in selected
                              ♦ What can I help you build?
                                                                                         ⊕ ⊳
      reduced = selected.copy
      reduced.remove(candidate)
      model = sm.OLS(y, sm.add_constant(X[reduced])).fit()
      bics.append((model.bic, candidate))
```

```
min_bic, worst = min(bics)
    if min_bic < current_bic:</pre>
       selected.remove(worst)
       current_bic = min_bic
    else:
       break
  return selected, current_bic
# Run selections
forward_vars, adj_r2 = forward_selection(X, y)
backward_vars, final_bic = backward_elimination(X, y)
print("Forward Selection (Adj. R²):", forward_vars, "| Score:", round(adj_r2, 4))
print("Backward Elimination (BIC):", backward_vars, "| BIC:", round(final_bic, 2))
     Forward Selection (Adj. R^2): ['X4', 'X3', 'X2', 'X5'] | Score: 0.634 Backward Elimination (BIC): ['X2', 'X3', 'X4', 'X5'] | BIC: 273.64
# Python simulation:
import numpy as np
import pandas as pd
import statsmodels.api as sm
# Set seed for reproducibility
np.random.seed(42)
# Create time, X, and error terms
t = np.arange(1, 21)
X = np.random.normal(0, 1, 20)
error = np.random.normal(0, 0.5, 20)
# Create Y with a slope change at t = 10
Y = np.where(t \le 10, 1.0 * X, 2.0 * X) + error
# Create dummy variable: 0 for t \le 10, 1 for t > 10
D = (t > 10).astype(int)
\# Interaction term: dummy \times X
X_{interaction} = D * X
# Build regression data
df = pd.DataFrame({'Y': Y, 'X': X, 'X_interaction': X_interaction})
X_model = sm.add_constant(df[['X', 'X_interaction']])
model = sm.OLS(df['Y'], X_model).fit()
# Output the regression summary
print(model.summary())
                                     OLS Regression Results
     Dep. Variable:
                                                   R-squared:
     Model:
                                            0LS
                                                   Adj. R-squared:
                                                                                         0.895
                                Least Squares
     Method:
                                                   F-statistic:
                                                                                         82.16
                             Sun, 15 Jun 2025
21:38:37
                                                   Prob (F-statistic):
                                                                                      1.83e - 09
     Date:
                                                   Loa-Likelihood:
                                                                                       -13.083
     Time:
     No. Observations:
                                             20
                                                   AIC:
                                                                                          32.17
     Df Residuals:
                                             17
                                                   BIC:
                                                                                         35.15
     Df Model:
                                               2
     Covariance Type:
                                     nonrobust
                                                                              [0.025
                                                                                            0.975]
                            coef
                                     std err
                                                                 P>|t|
     const
                         -0.1292
                                        0.149
                                                    -0.869
                                                                 0.397
                                                                              -0.443
                                                                                             0.185
                          0.8873
                                                     4.057
                                                                               0.426
                                                                                             1.349
                                        0.219
                                                                 0.001
                                                                               0.386
                                                                                             1.730
     X interaction
                          1.0583
                                                                 0.004
                                        0.318
                                                    3.324
     _____
                                                                                          =====
     Omnibus:
                                                   Durbin-Watson:
                                                                                          2.142
                                          1.143
     Prob(Omnibus):
                                          0.565
                                                   Jarque-Bera (JB):
                                                                                          0.678
                                          0.445
                                                   Prob(JB):
                                                                                          0.713
     Skew:
     Kurtosis:
                                          2.854
                                                    Cond. No.
                                                                                           4.28
```

## Notes

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.