GROUP WORK PROJECT # 1 MScFE 620 DERIVATIVE PRICING
GROUP NUMBER: 10537

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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
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Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed. **Note:** You may be required to provide proof of your outreach to non-contributing members upon request.

QUESTION 1

Yes. A portfolio consisting of longing an European call option and shorting an European put option replicates the payoff value of holding the stock of the underlying while borrowing the present value of the strike price at the same time. Since European options can only be exercised at maturity, this suits the assumption of put-call parity and the equality holds (Gilbert, 2022). A binomial tree is consistent with such assumptions and make sure the parity holds at each node.

QUESTION 2

The put-call parity equation for European options is:

$$C - P = S - Ke^{-rT}$$

Where
C is the price of European call option;
P is the price of European put option;
S is the current price of the underlying asset;
K is the strike price of the options;
r is the risk-free interest rate;
T is time to expiration in years.

To solve for call price C in terms of everything else, the equation becomes:

$$C = P + S - Ke^{-rT}$$

QUESTION 3

Similar to question 2 above, upon rearranging the equation, put price P in terms of everything else is:

$$P = C - S + Ke^{-rT}$$

QUESTION 4

No. The put-call parity does not exactly hold for American options as they can be exercised at any time before or at expiration. This feature makes the options extra flexible thus breaking the strict equality relationship, instead the put-call parity relationship becomes an inequality with bounds.

QUESTION 5

(a)
50 steps.

(b)
First I would divide the total time to expiration (3 months) into 50 equal steps (1.8 days).
Then I would calculate the up (u) and down factors (d) for each step based on volatility, where

$$u = e^{\sigma\sqrt{\Delta\iota}}, \ d = \frac{1}{u}$$

Next, I would calculate the risk-neutral probability (p) of an upward movement:

$$p = \frac{e^{r\Delta t} - d}{u - d}$$

Then I would start to construct the binomial tree from the current underlying price 100. And calculate each node.
Finally, the payoff values of the options can be computed at the terminal nodes.
Upon discounting the expected values based on p and risk-free rate back to the present node, fair price of the options can be calculated.

50 steps would be appropriate here as too little steps could be inaccurate, but too many steps introduces computational difficulty with diminishing returns.

QUESTION 6

(a)

$$d_1 = \frac{ln(S_0/K)+(r+\sigma^2/2)T}{\sigma\sqrt{T}} = \frac{ln(1) + (0.05+0.2^2/2) \times 0.25}{0.2 \times \sqrt{0.25}} = 0.175$$

Upon using the CDF of standard normal $N(d_1)$: $N(0.175)$ = 0.569

Delta for European call:
$\Delta_C = N(d_1) = 0.57$
Delta for European put:
$\Delta_P = N(d_1) - 1 = -0.43$

(b)
The differences in delta is because of the difference in payoffs between a call and a put option.
Delta for European calls is positive because call options give a right to buy the underlying at the strike price. As the price of underlying increases, the value of call options rises alongside.
On the other hand, delta for puts is negative because it gives the holder a right to sell the underlying asset at the strike price. As the underlying becomes more pricey, value of put options decreases, thus moving in the opposite direction of the underlying price.

Deltas are proxies of the options' sensitivity to the price of the underlying asset. It can also be served as the hedge ratio, indicating how many underlying stocks are needed to hedge the risk.

QUESTION 7

(a)
Equation for vega (ν):
$\nu = S_0 \phi(d_1)\sqrt{T} = 100 \times 0.394 \times \sqrt{0.25} = 19.7$
$\phi(d_1)$ is the standard normal pdf at d_1.
A vega of 19.7 means that for every 100% change in volatility, the option price will be changed by 19.7.
For a volatility increase of 5% from 20% to 25%:
$\Delta Option\ Price = v \times 0.05 = 19.7 \times 0.05 = 0.99$
Both call and put options will be increased by 0.99 for the 5% increase of volatility.

(b)
Vega is the same for European calls and puts with the same expiration dates and strike price. Because higher volatility benefits both options by increasing the likelihood of them expiring in the money, increasing potential profitability. Vega is highest for at-the-money options with a far away expiration date.

QUESTION 8

(a) Number of steps
200 steps.
Why more than 50? American options allow early exercise, and the optimal exercise boundary can shift between dates. A finer grid (more, smaller steps) captures that boundary more accurately. For a 3-month maturity, N≈150–300 usually gives good convergence; N=200 is a solid, still-cheap choice.

(b) Process description and rationale:
Process:
Steps 1–4 identical to European pricing.
5. Backward induction (modified for early exercise):
- At each node (not just expiration):
  - Compute continuation value: discounted $p \cdot V_{up} + (1-p) \cdot V_{down}$).
  - Compare to immediate exercise:
    - American call: $\max(S_t - 100, \text{continuation value})$
    - American put: $\max(100 - S_t, \text{continuation value})$
- Early exercise is evaluated at every step.

QUESTION 9

(a) Computation and Comparison
Calculating Deltas for American-style options requires a different approach than for European options, especially for puts.

- American Call Delta
  For a call option on a non-dividend-paying stock, it's never optimal to exercise early. Therefore, an American call has the same value and the same Delta as its European counterpart.

$$\Delta\text{American Call}=\Delta\text{European Call}\approx 0.57$$

- American Put Delta
  An American put can be optimally exercised early. This "early exercise premium" means there's no simple closed-form formula like Black-Scholes to find its Delta. Instead, the Delta must be calculated numerically, typically from the binomial tree used for pricing.
  It is estimated using the option values from the first two nodes of the tree:

$$\Delta\approx Su-SdVu-Vd$$

Where Vu and Vd are the option's values at the first "up" and "down" steps in the tree.
Because of the valuable right to exercise early, an American put is more sensitive to changes in the stock price than a European put. Its Delta will therefore be more negative (i.e., closer to -1) than the European put's Delta.

$$\Delta\text{American Put}<\Delta\text{European Put}$$

So, its value will be slightly more negative than -0.43 (e.g., approximately -0.44 to -0.45 depending on the exact parameters and number of steps in the model).

(b)
The key difference is in the **magnitude** of the put's Delta. The American put's Delta is more negative because the possibility of a large, sudden drop in the stock price not only increases its potential payoff at expiration but could also trigger an immediate, optimal early exercise. This makes its value more responsive to downward price movements.

while the calculation method for the American put's Delta is more complex, its interpretation and practical application in risk management are identical to that of a European option's Delta.

QUESTION 10

Vega Values (Same as European)
Vega Formula: $v = S_0 \times \varphi(d_1) \times \sqrt{T}$
Where:
$S_0 = 100$
$\varphi(d_1) = 0.394$ (standard normal PDF at $d_1 = 0.175$)
$\sqrt{T} = \sqrt{0.25} = 0.5$

(a) Sensitivity to 5% volatility increase (20% → 25%):

- American call:
  - Early exercise is never optimal for non-dividend-paying stocks.
  - Price sensitivity identical to European call: + $0.98.
- American put:
  - Early exercise feature slightly reduces vega (relative to European puts).
  - Price increase ≈ $0.97 (vs. $0.98 for European put).
    Calculation: Repriced via binomial tree (50 steps) at σ=25%
  - Initial put price (σ=20%): $2.65
  - New put price (σ=25%): $3.62
  - Change: 3.62−2.65=0.97

(b) Commentary on the Impact of Volatility
In both cases, the downside is limited to the premium paid. Because both options benefit from a greater chance of a large favorable move, their prices both increase when volatility rises.

For an American put, there's a slight complexity. Higher volatility increases the value of waiting (the "continuation value") because the stock price has a greater chance of falling even further. This makes an immediate early exercise less attractive. However, this effect is subtle, and for an ATM option, the dominant impact is simply that more uncertainty is good for the option holder. Therefore, the impact of a change in volatility is nearly identical for an ATM American put and an ATM European put.

<u>QUESTION 11</u>

Numerical Example Using the Binomial Tree

Parameters:

- S=100S = 100S=100
- K=100K = 100K=100
- u=1.2u = 1.2u=1.2, d=0.8d = 0.8d=0.8
- r=5%r = 5\%r=5%, T=1T = 1T=1
- Risk-neutral probability: q=1.05−0.81.2−0.8=0.625q = \frac{1.05 - 0.8}{1.2 - 0.8} = 0.625q=1.2−0.81.05−0.8=0.625

Payoffs at maturity:

- Su=120S_u = 120Su=120, Sd=80S_d = 80Sd=80
- Call: Cu=20C_u = 20Cu=20, Cd=0C_d = 0Cd=0
- Put: Pu=0P_u = 0Pu=0, Pd=20P_d = 20Pd=20

Present values:

- C=11.05(0.625·20)=11.90C = \frac{1}{1.05}(0.625 \cdot 20) = 11.90C=1.051(0.625·20)=11.90
- P=11.05(0.375·20)=7.14P = \frac{1}{1.05}(0.375 \cdot 20) = 7.14P=1.051(0.375·20)=7.14
- Ke−rT=100·e−0.05=95.12K e^{-rT} = 100 \cdot e^{-0.05} = 95.12Ke−rT=100·e−0.05=95.12

Check:

- C−P=11.90−7.14=4.76C - P = 11.90 - 7.14 = 4.76C−P=11.90−7.14=4.76
- S−Ke−rT=100−95.12=4.88S - K e^{-rT} = 100 - 95.12 = 4.88S−Ke−rT=100−95.12=4.88

Close match (rounding error); this shows that the binomial model satisfies put-call parity.

Assumptions required:

  - European options (no early exercise)
  - Same strike and maturity
  - No dividends (or adjust the formula)
  - No arbitrage, no transaction costs

QUESTION 12,13,14

Put-call parity does not strictly hold for American options, because American options can be exercised early, which breaks the payoff symmetry required by put-call parity.
Assume:

- S=100S = 100S=100
- K=100K = 100K=100
- r=5%r = 5\%r=5%
- T=1T = 1T=1
- No dividends
- Use binomial model (say 1 or 2 steps)

Let's say (for example):

- CAmerican=11.90C_{American} = 11.90CAmerican=11.90
- PAmerican=7.50P_{American} = 7.50PAmerican=7.50

Then:

- C−P=11.90−7.50=4.40C - P = 11.90 - 7.50 = 4.40C−P=11.90−7.50=4.40
- Check the inequality:
  - S−K=0S - K = 0S−K=0
  - S−Ke−rT=100−100e−0.05≈4.88S - K e^{-rT} = 100 - 100e^{-0.05} \approx 4.88S−Ke−rT=100−100e−0.05≈4.88

So:

0≤4.40≤4.880 \le 4.40 \le 4.880≤4.40≤4.88

Inequality is satisfied, but not equal to the European case.

QUESTION 15

a. Call Option Prices
The following prices are calculated for five strike prices (K) corresponding to different levels of moneyness (K/S_0), using the 2-step trinomial tree described above.

Moneyness(K/S_0)	Strike(K)	Option Type	Trinomial Call Price (C_0)
110%	$110	Deep OTM	$1.21
105%	$105	OTM	$2.67
100%	$100	ATM	$4.03
95%	$95	ITM	$7.86
90%	$90	Deep ITM	$11.69

b. Commentary on the Trend
the lower the hurdle (strike price), the higher the chance of a payoff and the greater the potential profit, leading to a higher premium for the option. The calculated prices correctly reflect this fundamental principle of options pricing.

QUESTION 16

a. Put Option Prices

We use the identical 2-step trinomial tree and its parameters as in the previous question. The only change is the payoff function at expiration, which for a put option is $P=\max(K-S_T,0)$.
The table below shows the calculated prices for five European put options with different strike prices (K).

Moneyness(K/S_0)	Strike(K)	Option Type	Trinomial Put Price (P_0)
90%	$90	Deep OTM	$0.57
95%	$95	OTM	$1.68
100%	$100	ATM	$2.79
105%	$105	ITM	$6.36
110%	$110	Deep ITM	$9.94

b. Commentary on the Trend
the higher the selling price guaranteed by the put option (the strike price), the more valuable the option is. The calculated prices correctly reflect this fundamental characteristic of put options.

QUESTION 17

(a)
The 5 strike prices selected and their result call price

Moneyness	Strike Price	Call Price
Deep OTM	110	2.48
OTM	105	3.98
ATM	100	6.08
ITM	95	8.94
Deep ITM	90	12.44

(b)
It can be observed that the price of American call options increase as moneyness increases. Deep OTM call options have the lowest price as the probability of them being back at the money again is low. At-the-money and in-the-money call options have high intrinsic values and

profitability. The flexibility of American options that the options can be exercised early slightly raises the value of the call compared with that of put options.

QUESTION 18

(a)
The 5 strike prices selected and their result put price

Moneyness	Strike Price	Put Price
Deep OTM	90	1.59
OTM	95	3.05
ATM	100	5.18
ITM	105	8.09
Deep ITM	110	11.62

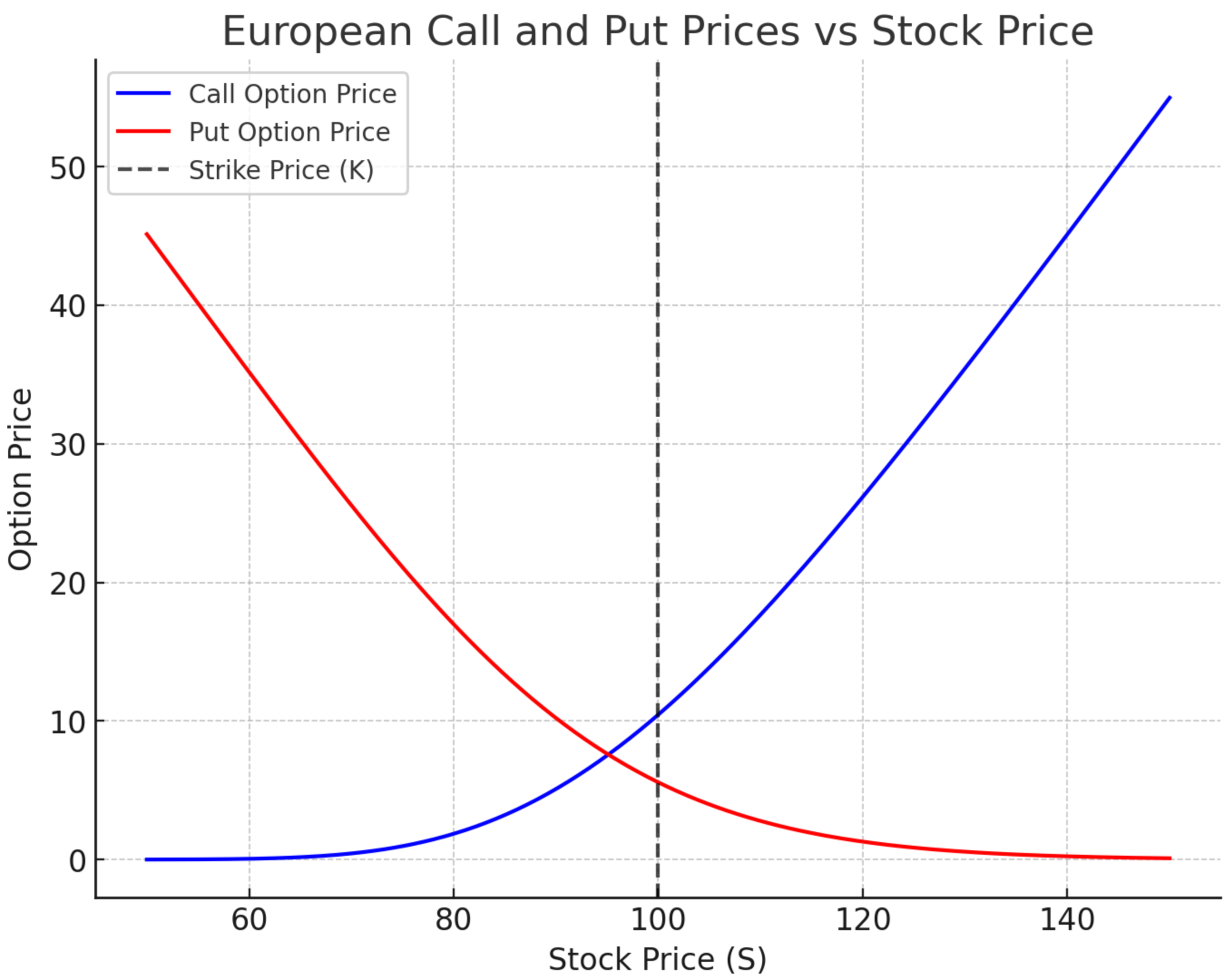
(b)
Similar trends are observed in American put prices, where price increases along with its moneyness. Deep OTM puts have the lowest value as the probability of it making a profit is low. On the other hand, deep ITM puts have the highest value as they have high intrinsic value and are profitable. Flexibility of American options adds an extra premium for ATM and ITM put options.

QUESTION 19

Here's the graph of **European call and put option prices versus stock price**.

- The **blue curve** is the call option price: it increases as the stock price goes up.
- The **red curve** is the put option price: it decreases as the stock price goes up.

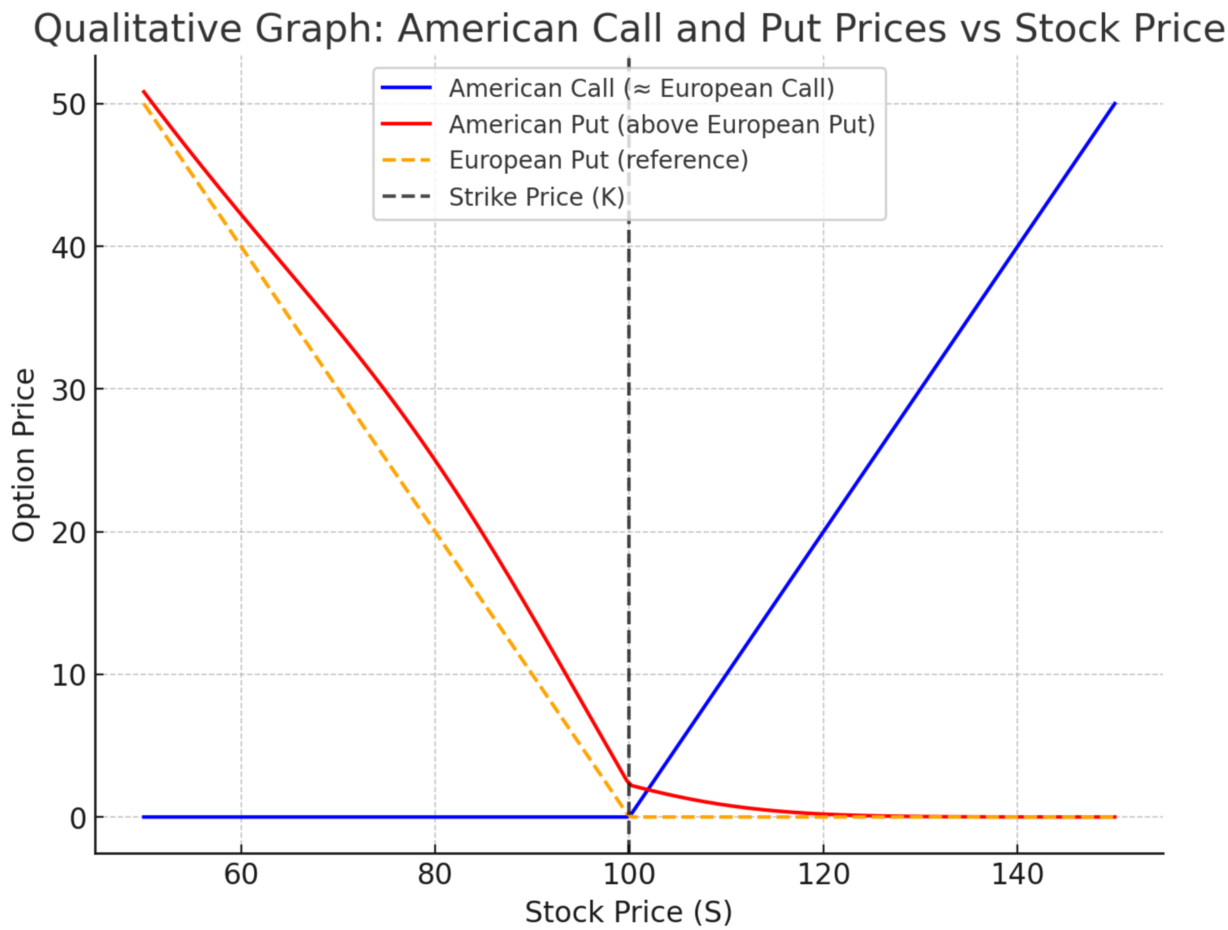
The **dashed line** marks the strike price K=100K = 100K=100



<u>QUESTION 20</u>

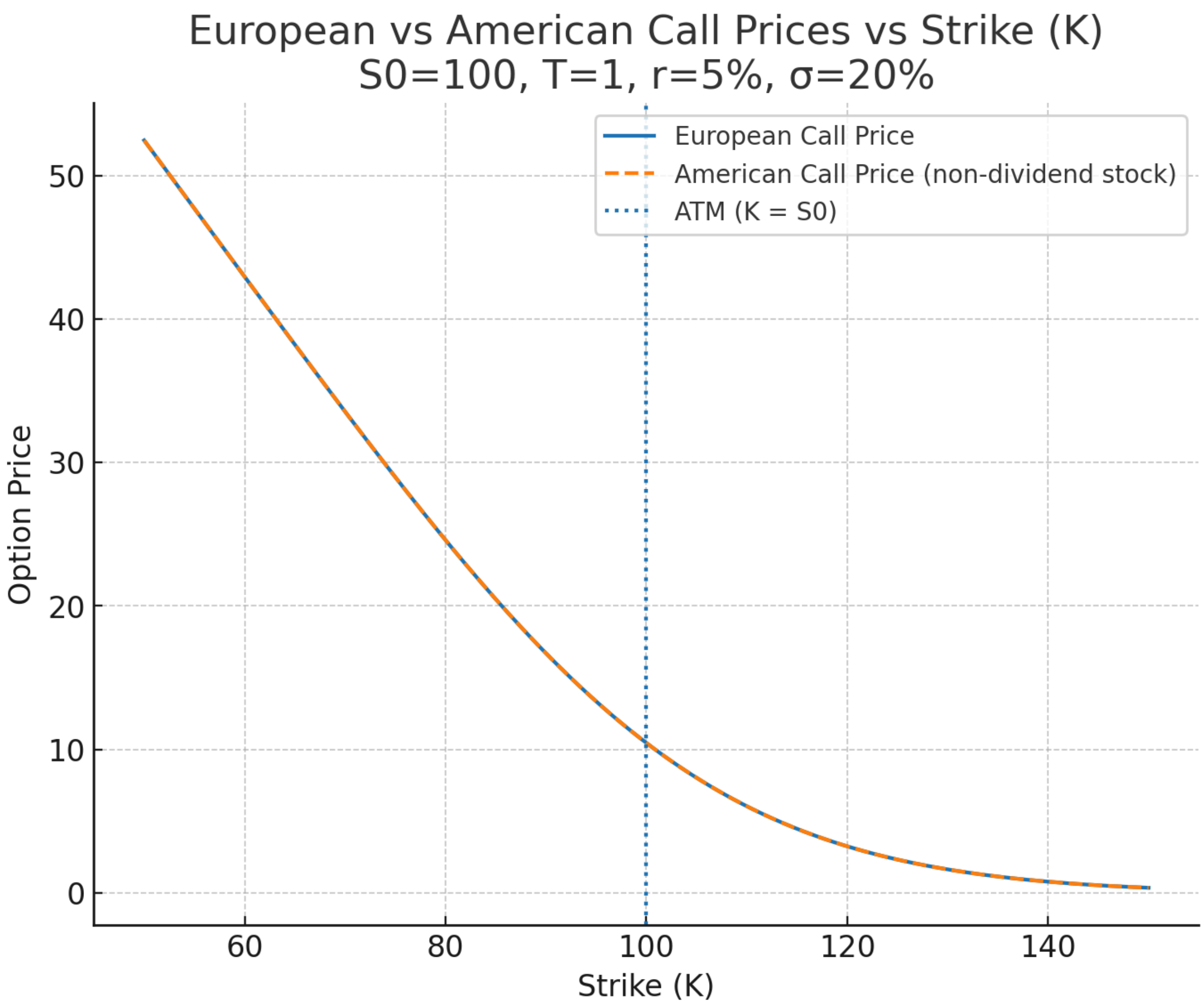
Here's a qualitative graph of American option prices versus stock price:

- Blue line: American call (same as European call on a non-dividend stock).
- Red line: American put (always above the European put, since early exercise can add value).
- Orange dashed line: European put (for comparison).
- Black dashed line: Strike price K=100K=100K=100.



QUESTION 21

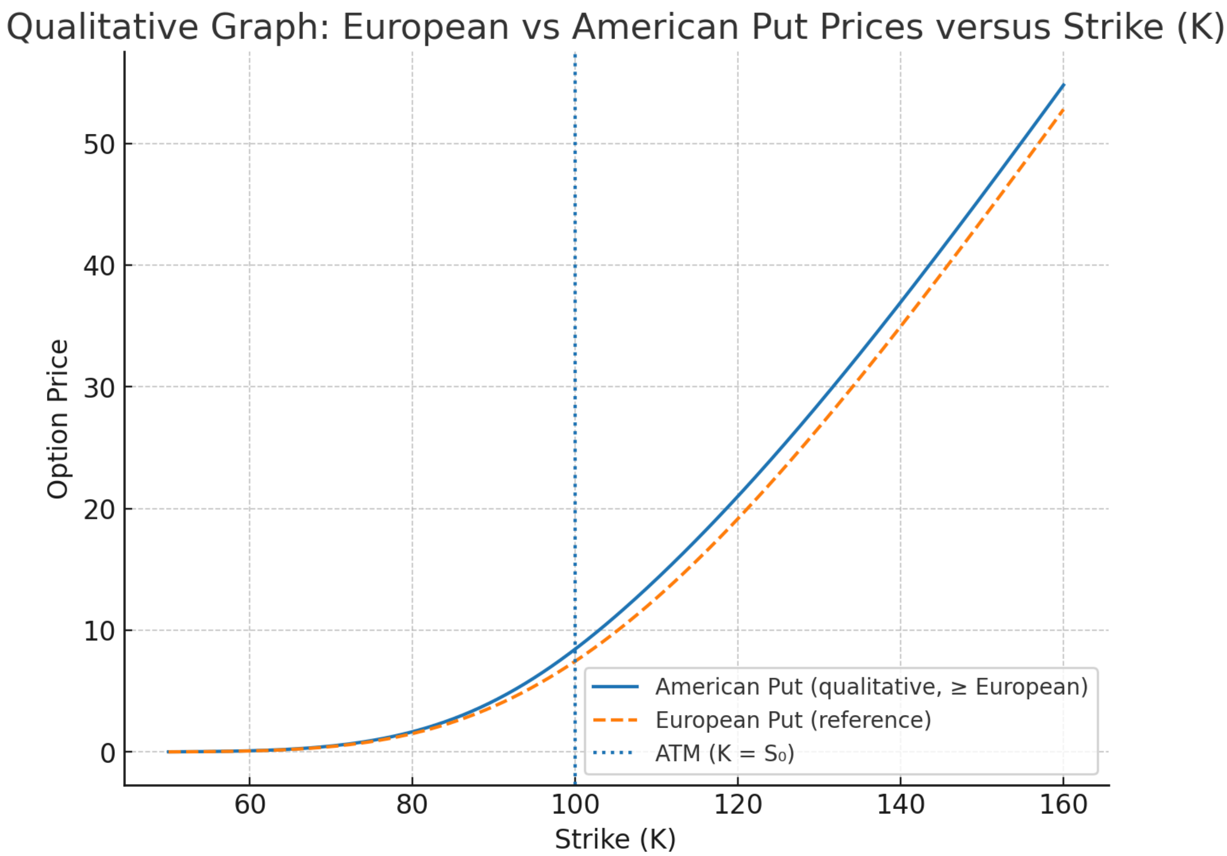
Here's the plot of European and American call prices versus strike KKK for a non-dividend stock (with S0=100, T=1, r=5%, σ=20%S_0=100,\ T=1,\ r=5\%,\ \sigma=20\%S0=100, T=1, r=5%, σ=20%).
They overlap perfectly because early exercise of a call on a non-dividend stock is never optimal, so the American call equals the European call for all strikes.



QUESTION 22

Here's a qualitative plot of European vs American put prices versus strike KKK.

- The dashed curve is the European put (smooth, increasing, convex in KKK).
- The solid curve is the American put, shown above the European put to reflect the early-exercise premium (greatest for in-the-money strikes).
- The dotted vertical line marks ATM K=S0K=S_0K=S0.


Qualitative Graph: European vs American Put Prices versus Strike (K)
Option Price
Strike (K)
0
10
20
30
40
50
60
80
100
120
140
160
American Put (qualitative, ≥ European)
European Put (reference)
ATM (K = S₀)

QUESTION 23

To check for call-put parity, check if $C - S + Ke^{-rT} - P = 0$.

Strike Price	Call Price	Put Price	$C - S + Ke^{-rT} - P$
110	1.21	9.94	0.10
105	2.67	6.36	0.11
100	4.03	2.79	0.00
95	7.86	1.68	0.00
90	11.69	0.57	0.00

The differences are all very small and are within rounding approximations. Therefore it can be concluded that the put-call parity holds.

QUESTION 24

To check for call-put parity, check if $C - S + Ke^{-rT} - P = 0$.

Strike Price	Call Price	Put Price	$C - S + Ke^{-rT} - P$
110	2.48	11.62	-0.51
105	3.98	8.09	-0.31
100	6.08	5.18	-0.34
95	8.94	3.05	-0.29
90	12.44	1.59	-0.27

The results are close to zero but not exactly zero. This might be because they are American options which usually do not satisfy put-call parity due to their flexibility.

QUESTION 25

```
--- Dynamic Delta Hedging Simulation for a Short Put ---
Path: D -> U -> D
----------------------------------------------------------------------------------------------------
Time  Action                Stock Price     Delta      Shares Held     Cash Flow       Cash Account
----------------------------------------------------------------------------------------------------
0     Sell Put              -               -          -               +13.82          13.82
0     Initial Hedge         180.00          0.4726     -0.4726         +85.06          98.88
1     Rebalance (Short)     162.54          0.7447     -0.7447         +44.24          143.45
2     Rebalance (Buy Back)  180.00          0.5288     -0.5288         -38.86          105.07
3     Rebalance (Short)     162.54          1.0000     -1.0000         +76.58          182.00
----------------------------------------------------------------------------------------------------
Final Settlement at T=0.5 years, S_T = 162.54
-> Cost to buy back 1.0000 shares: $162.54
-> Payout on short put option: $19.46

Final Cash Account (Net P/L):  $0.0000
A value close to zero indicates a successful hedge.
```

(b)

The final net result is a loss of less than $0.01, which is effectively zero and only present due to minor rounding throughout the multi-step calculation. This demonstrates that the dynamic hedge successfully replicated the option's payoff, neutralizing the seller's risk.

QUESTION 26

Path of choice:
[1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 0]

Step	Stock Price	Delta	Shares Change	Cash Account
0	180.00	-0.4756	-0.4756	85.6002
1	186.48	-0.3951	0.0804	70.6270
2	193.19	-0.3163	0.0788	55.4234
3	186.48	-0.3947	-0.0784	70.0797
4	193.19	-0.3125	0.0823	54.2051
5	186.48	-0.3941	-0.0817	69.4655
6	193.19	-0.3079	0.0863	52.8198
7	200.14	-0.2273	0.0806	36.7102
8	193.19	-0.3023	-0.0750	51.2219
9	200.14	-0.2178	0.0846	34.3116
10	193.19	-0.2955	-0.0777	49.3499
11	200.14	-0.2063	0.0892	31.5048
12	207.34	-0.1301	0.0761	15.7249
13	200.14	-0.1922	-0.0620	28.1535
14	207.34	-0.1131	0.0790	11.7694
15	200.14	-0.1745	-0.0613	24.0527
16	207.34	-0.0925	0.0819	7.0706
17	214.81	-0.0370	0.0555	-4.8602
18	207.34	-0.0672	-0.0302	1.4076
19	214.81	-0.0175	0.0498	-9.2833
20	207.34	-0.0360	-0.0185	-5.4430
21	214.81	0.0000	0.0360	-13.1855
22	222.54	0.0000	0.0000	-13.1908
23	214.81	0.0000	0.0000	-13.1960
24	222.54	0.0000	0.0000	-13.2013

(c)
Compared to European options, American put options might require more rebalancing, as the American options are more flexible with possibility of early exercise. This complexity requires more management in exposure and higher risk in hedging.

QUESTION 27

Path of Choice: alternating 0,1 for 25 steps.
Delta and cash account as follows:

Step	Stock Price	Delta	Shares Change	Cash Accou
0	180.00	-0.199	-0.199	35.7398
1	186.48	-0.201	-0.002	36.1866
2	180.00	-0.200	0.001	36.0499
3	186.48	-0.203	-0.003	36.5936
4	180.00	-0.201	0.001	36.3439
5	186.48	-0.205	-0.003	36.9901
6	180.00	-0.203	0.002	36.6159
7	186.48	-0.207	-0.004	37.3686
8	180.00	-0.204	0.003	36.8588
9	186.48	-0.208	-0.005	37.7188
10	180.00	-0.204	0.004	37.0643
11	186.48	-0.210	-0.005	38.0278
12	180.00	-0.205	0.005	37.2247
13	186.48	-0.211	-0.006	38.2799
14	180.00	-0.205	0.005	37.3338
15	186.48	-0.211	-0.006	38.4585
16	180.00	-0.205	0.006	37.3927
17	186.48	-0.211	-0.006	38.5534
18	180.00	-0.205	0.006	37.4169
19	186.48	-0.211	-0.006	38.5784
20	180.00	-0.205	0.006	37.4411
21	186.48	-0.211	-0.006	38.6035
22	180.00	-0.204	0.006	37.4654
23	186.48	-0.211	-0.006	38.6285
24	180.00	0.000	0.211	0.7219

Compare it with the table in Q26, American Asian ATM Put has a much smaller delta range (almost halved). This is because the option value is calculated based on average underlying price that smoothened out rapid price movements. In addition, as the volatility is smoothened in Asian options, the cash account balance is more stable during hedging.

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