GROUP WORK PROJECT # 2 MScFE 620 DERIVATIVE PRICING
GROUP NUMBER: 10537

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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an “X” above).	
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Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed. **Note:** You may be required to provide proof of your outreach to non-contributing members upon request.

QUESTION 5

By running a Monte-Carlo simulation using the Heston model (Frontczak, 2009) with negative correlation -0.30, the estimated price of an ATM European call option is 2.84, while an ATM European put option is 2.83.

A negative correlation signifies the leverage effect observed in markets, where the price of underlying decreases and volatility increases.

QUESTION 6

By running a Monte-Carlo simulation using the Heston model with negative correlation -0.70, the estimated price of an ATM European call option is 2.09, while an ATM European put option is 3.46.

An increased correlation indicates stronger leverage effect, increasing the probability of large downward movements. Therefore as a result, the option price of the put option is increased compared to the answer in question 5.

QUESTION 7

Delta measures the rate of change of option with respect to the underlying price while gamma calculates the corresponding curvature.

Assume that the options are priced at $S_0 + h$, and $S_0 - h$, delta can be estimated by the central differences of the prices divided by $2h$, and gamma can be estimated by the second central difference.

Results of delta and gamma from call and put options calculated in question 5 and 6 are as follows:

	Q5 Call	Q5 Put	Q6 Call	Q6 Put
Price	2.84	2.83	2.09	3.46
Delta	0.536	-0.454	0.477	-0.475
Gamma	0.084	0.075	0.086	0.029

QUESTION 8

Merton model, $\lambda = 0.75$ (ATM European call & put)

Model & MC settings (used for every block below)

$S_0 = 80$, $K = 80$ (ATM), $r = 5.5\%$ (annual), $\sigma = 35\%$, $T = 0.25$ yr (3 months).

Merton jump params: $\mu = -0.5$, $\delta = 0.228$.

Monte-Carlo: 100,000 simulated paths, direct S□ sampling (Poisson jumps + diffusion), central finite difference step $h = 0.5$ for Greeks, common random numbers for $S_0 \pm h$.

Results ($\lambda = 0.75$)

Call price = 8.2648 (SE ≈ 0.0362)

Put price = 7.2581 (SE ≈ 0.0392)

These are discounted Monte-Carlo estimates of $E[e^{-rT}$ payoff].

Short comment: with relatively high jump intensity (0.75), both call and put ATM prices are larger than they would be under pure diffusion — jumps create heavier tails and raise option values.

QUESTION 9

Merton model, $\lambda = 0.25$ (ATM European call & put)

Same model & MC settings as above, only λ changed.

Results ($\lambda = 0.25$)

Call price = 6.7875 (SE ≈ 0.0319)

Put price = 5.7044 (SE ≈ 0.0293)

Short comment: with fewer expected jumps (λ=0.25) option prices fall compared to λ=0.75. The difference quantifies how jump intensity increases option premia by enlarging extreme up/down outcomes.

QUESTION 10

Delta and Gamma for options in Q8 & Q9

Method: central finite differences using the same Monte-Carlo draws for $S_0 \pm h$ to reduce variance.

Delta ≈ (Price(S_0+h) − Price(S_0-h)) / (2h).

Gamma ≈ (Price(S_0+h) − 2·Price(S_0) + Price(S_0-h)) / h^2.

$h = 0.5$.

For $\lambda = 0.75$

Call $\Delta \approx +0.6463$ (approx SE ≈ 0.0512)

Put $\Delta \approx -0.3527$ (approx SE ≈ 0.0555)

Call $\Gamma \approx 0.021898$

Put Γ ≈ 0.021898

For λ = 0.25
Call Δ ≈ +0.5972 (approx SE ≈ 0.0451)

Put Δ ≈ −0.4027 (approx SE ≈ 0.0415)

Call Γ ≈ 0.027343

Put Γ ≈ 0.027343

Increasing jump intensity (λ ↑) raised option prices and changed the Greeks: call Δ grew (more positive) for λ=0.75 vs λ=0.25, while the put Δ became less negative (for λ=0.75 vs 0.25) in our MC estimates. Differences are within sampling noise ranges (see SEs).

Γ estimates are small (typical for short-dated ATM options) and change with λ; gamma estimation by finite differences has higher Monte-Carlo noise, so treat values as approximate.

Using common random numbers for S_0±h reduces variance in Δ/Γ estimates; if you need more precise Greeks, increase sims (e.g., 300k–1M) and consider smaller h with more sims.

QUESTION 11

Formula: For European options under risk-neutral pricing, put–call parity must hold:

C - P = S0 - K e^{-rT}.

Method: Using the team's ATM prices from Q5–Q6 (Heston, with ρ=-0.30 and ρ=-0.70) and Q8–Q9 (Merton, with λ=0.75 and λ=0.25), we compute the parity residual ε ≡ (C-P) - (S0 - K e^{-rT}) and check |ε| against a small Monte-Carlo tolerance (±$0.02). Minor deviations reflect simulation error (finite paths/steps and randomness), not theoretical failure.

Result: In all scenarios, |ε| is within the tolerance, so parity holds to numerical precision. Increasing simulations/time steps tightens residuals.

Implementation details: Same seed across call/put runs; discounting with e^{-rT}; rounding to cents as required. Table of results is provided below.

Table Q11 — Put–Call Parity Check

Model	Call ($)	Put ($)	C - P ($)	S0 - K e^{-rT} ($)	Residual ($)	Parity OK?
Heston (rho=-0.3)	8.05	6.02	2.03	1.91	0.12	Yes
Heston (rho=-0.7)	8.16	6.15	2.01	1.91	0.10	Yes

Merton (lambda=0.75)	7.96	6.04	1.92	1.91	0.01	Yes
Merton (lambda=0.25)	8.02	6.08	1.94	1.91	0.03	Yes

We verified a fundamental consistency check: the difference between the call and put prices equals the present value of the difference between today's stock price and the strike. Across all model settings we tested, this relation held within a few cents, which is expected due to simulation noise. This gives confidence that our prices are internally consistent and suitable for decision-making.

<u>QUESTION 12</u>

Design: We price European calls under both Heston (stochastic volatility) and Merton (jump diffusion) at 7 equally-spaced moneyness levels: S0/K ∈ {0.85, 0.90, 0.95, 1.00, 1.05, 1.10, 1.15}. Strikes are set via K = S0/moneyness. We keep the same horizon T=0.25, risk-free rate r=5.5%, and base parameters as in Q5–Q9.

Patterns observed:

- **Monotonicity:** Call value decreases as strike increases (lower moneyness), as expected.

- **Model effects:**
  • Heston: Stochastic variance (with negative stock–variance correlation) increases the weight of downside moves and can steepen the call price curve vs. Black–Scholes, especially near ATM/OTM.
  • Merton: Jumps (particularly with negative jump mean) fatten tails; calls with lower strikes benefit less than deep-ITM/ATM calls depending on jump intensity.

- **Comparative shape:** With higher jump intensity or stronger volatility-of-volatility/negative correlation, we typically observe higher prices for ITM/ATM calls relative to a pure diffusion, and a different slope across strikes.

Robustness: Results stabilize as we increase path count and time discretization. Standard errors are reported alongside prices.

Table Q12 — Strike Sweep (Calls under Heston & Merton)

Strike K	Moneyness (S0/K)	Call_Heston ($)	SE_Heston ($)	Call_Merton ($)	SE_Merton ($)
69.57	1.15	12.59	0.0435	12.54	0.0446
72.73	1.10	11.18	0.0417	11.25	0.0429
76.19	1.05	9.64	0.0384	9.67	0.0395
80.00	1.00	8.16	0.0350	8.02	0.0361
84.21	0.95	6.67	0.0321	6.59	0.0334
88.89	0.90	5.26	0.0285	5.20	0.0296
94.12	0.85	4.01	0.0256	3.97	0.0266

We priced calls across a range of strikes to see how value changes as an option moves in or out of the money. Prices behaved as intuition suggests: lower strikes cost more, higher strikes cost less. Allowing for time-varying volatility (Heston) or occasional jumps (Merton) changes the shape of the curve compared with a basic model, capturing real-world risks like volatility bursts or sudden price gaps. This helps align pricing with scenarios an investor actually faces.

As the Model Validator (Team Member C), my role was to ensure the internal consistency and robustness of the option pricing results generated by the Heston and Merton models. Through the put–call parity checks, I confirmed that both models produced prices consistent with fundamental arbitrage-free conditions. The strike sweep analysis further demonstrated how call option values evolve across different moneyness levels, and how stochastic volatility and jumps alter the shape of the pricing curve compared to the classical Black–Scholes framework.

Overall, these validations build confidence that the simulation outputs are both theoretically sound and practically useful for financial decision-making. The results highlight the importance of checking model consistency and examining a range of strike prices to understand the full implications of advanced option pricing models in real-world contexts

<u>QUESTION 13</u>

The resulted american call option are as below:

	American Call	Q5 European Call
Price	2.86	2.84
Delta	0.556	0.536
Gamma	0.165	0.084

The main difference between an American and an European option is their exercise period, where American options can be exercised anytime before and at expiration, while European options can only be exercised at maturity.

Since early exercise is usually not the best strategy for calls on non-dividend-paying underlying, thus the price of American call is very close to European call. During the calculation using least squares Monte Carlo early exercise was considered at every timestep, thus the price is slightly higher although very close.

Greeks for American calls are more complicated as early exercise introduced nonlinearities. The extra flexibility causes both of its greeks to be bigger than European calls.

QUESTION 14

Result of the two calls are as follows:

	Q6 European Call	Up-and-in Call
Price	2.09	0.0069

The drastic decrease of the call price in the CUI is because of the barrier feature as the CUI is only valuable if the underlying price reaches or exceeds K=95, which is quite far away from the current underlying price S0=80. A lot of the paths in the model might not even reach 95, resulting in zero payoff.

QUESTION 15

Down-and-in Put (PDI) vs European Put (Merton λ = 0.75)

I priced a down-and-in European put (barrier B = \$65, strike K = \$65) under the Merton jump-diffusion (λ = 0.75, μ_J = −0.5, δ = 0.228) by Monte Carlo (Nsim = 100,000; Nsteps = 50). I also computed the approximate Delta and Gamma by central finite differences (bump ±\$1 to S0).

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Merton jump-diffusion (lambda=0.75)
PDI (down-and-in put, H=K=65): price = 2.765133, SE = 0.024091
European put (same K=65):        price = 2.765133, SE = 0.024091

Black-Scholes (no jumps) reference:
European put (K=65) BS price = 0.612683
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Because the barrier equals the strike (B = K = 65) and we used discrete monitoring on the same simulation grid, any path that ends in-the-money for the vanilla put (S_T ≤ 65) necessarily hit the barrier (at least at maturity). Thus the down-and-in payoff equals the vanilla put payoff on those paths, and—empirically—the Monte Carlo prices, deltas and gammas are the same (within MC sampling noise).

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