Page No.
Date: / /

Logistic Regression Model

want $0 \le h_\theta(x) \le 1$

$$h_\theta(x) = g(\theta^T x)$$

$$g(z) = \frac{1}{1+e^{-z}}$$

↓

Sigmoid function
or
Logistic function

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$ (Hypothesis)

Training Set : $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), \ldots, (x^{(m)}, y^{(m)})\}$

m examples $x \in \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix}$

$x_0 = 1$, $y \in \{0, 1\}$

$$h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$$

Cost function is given by

$$J(\theta) = \frac{1}{m} \sum_{i=1}^{i=m} \frac{1}{2} \left(h_\theta x^{(i)} - y^{(i)}\right)^2$$

$$= \frac{1}{m} \sum_{i=1}^{i=m} \text{cost}\left(h_\theta x^{(i)}, y\right)$$

Where,

$$\text{Cost}(h_\theta x, y) = \frac{1}{2}\left(h_\theta(x) - y\right)^2$$

$$\text{cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y=1 \\ -\log(1-h_\theta(x)) & \text{if } y=0 \end{cases}$$

Note :- $y=0$ or $y=1$ always

$$\text{Cost}(h_\theta(x), y) = -y\log(h_\theta(x)) - (1-y)\log(1-h_\theta(x))$$

If $y=1$

$$\text{cos}(h_\theta(x), y) = -\log h_\theta(x)$$

If $y=0$

$$Cost(h_\theta(x), y) = -\log(1-h_\theta(x))$$

$$\therefore J(\theta) = -\frac{1}{m}\left[\sum_{i=1}^{i=m} y^{(i)} \log h_\theta(x^{(i)}) + (1-y^{(i)})\log(1-h_\theta(x^{(i)}))\right]$$

To fit parameters θ

$$\min_\theta J(\theta) \quad \text{get } \theta$$

To make a prediction given new x:

Output $h_\theta(x) = \frac{1}{1+e^{-\theta^T x}}$.

$$p(y=1 \mid x;\theta)$$

To minimize cost function

Apply gradient descent Algorithm

Repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial J}{\partial \theta_j}$$

}

Simultaneously update all θ_j

$$\frac{\partial J(\theta)}{\partial \theta_j} = \frac{1}{m} \sum_{i=1}^{i=m} \left(h_\theta x^{(i)} - y^{(i)} \right) x_j^{(i)}$$

$\therefore$ Repeat {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^{i=m} \left(h_\theta x^{(i)} - y^{(i)} \right) x_j^{(i)}$$

}

$$\theta = \begin{bmatrix} \theta_0 \\ \vdots \\ \theta_n \end{bmatrix}$$

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