Lecture 1

Important Concepts In Set Theory

October 28, 2023

Sets

Definition (Sets)

Put simply, a set is simply a collection of objects. Namely, in sets

- The order of items does not matter
- The number of occurences of an item does not matter

For example,

- As sets, $\{1, 2, 3\}$ is the same set as $\{3, 1, 2\}$ and as $\{2, 1, 3\}$ since *order does not matter*
- Also, $\{1, 1, 2, 3\}$ is the same set as $\{1, 2, 3\}$ since the *number of occurences does not matter*

Representing Sets

How do we write out sets?

- If the set is finite, we may simply list out its elements

$$\{1, 2, 3, 4, 5, 6\}$$

- We may also do the same thing if the set is infinite

$$\{1, 2, 3, 4, 5, ...\}$$

But the latter risks being ambigious in certain situations.

Representing Sets

For infinite sets, we may avoid ambiguity by using set-builder notation.

Definition (Set-Builder Notation (Parametric Form))

We may write a set as

$$\{f(t) : t \in T\}$$

Here f, is some expression which depends on t while T is a set that contains all possible values of t.

Definition (Set-Builder Notation (Conditional Form))

Another way to use set builder notation is as

$$\{t \in T : P(t)\}$$

Here, t and T are the same as before, but now P is an *open sentence* in t. The set contains only the values of t which make $P(t)$ true.

An open sentence in t is just a statement whose truth value depends on the value of t.

Special Sets

Some sets are so common in math, they are given special symbols

- $\mathbb{N}$: the set of natural numbers

$$\mathbb{N} = \{1, 2, 3, 4, 5, ...\}$$

- $\mathbb{Z}$: the set of integers

$$\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

- $\mathbb{Q}$: the set of rational numbers

$$\mathbb{Q} = \left\{ \frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0 \right\}$$

- $\mathbb{R}$: the set of real numbers

Basic Notation & Terminology About Sets

Definition

Here are some basic definitions regarding sets

- A set need not contain any elements. The empty set is the unique set which contains no elements. It is often denoted by $\emptyset$ (commonly) or by $\{\}$ (not really common)
- A set which contains at least one element is said to be nonempty.
- If A is nonempty and x is an element of A, we say "x is a member of A" and write $x \in A$.
  - We may also write this as $A \ni x$
- Likewise, $x \notin A$ means "x is not a member of A"
  - We may also write $A \not\ni x$

Examples

Example

What is the set E defined as $E = \{2k : k \in \mathbb{Z}\}$?

- We have that $0 \in E$, $2 \in E$, and $-110 \in E$ for example
- However, $1 \notin E$, $-19 \notin E$, and $9 \notin E$

Example

What is the set T defined as

$$T = \left\{ x \in \mathbb{R} : x \neq \frac{a}{b} \text{ for all integers } a \text{ and } b, b \neq 0 \right\}$$

- We have $\pi \in T$ and $\sqrt{2} \in T$
- But $0 \notin T$, $-3 \notin T$, $\frac{3}{4} \notin T$