de Rham Cohomology & Stokes' Theorem

Math 412 – Final Project

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Overview

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We have Stokes' theorem:

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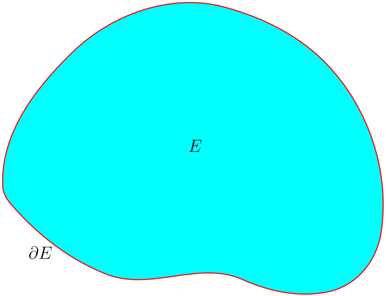

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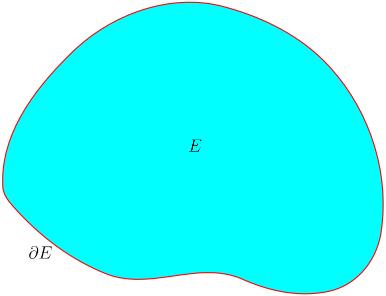

But what happens when Stokes' theorem fails, or when it cannot be used?

Overview & Idea

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- Introducing the de Rham cohomology as a tool to measure holes
- Applications (work on these)

Cells

Definition (k-Cell)

A k-cell on a space $U \subseteq \mathbb{R}^n$ is a continuous map from $\alpha : [0,1]^k \to U$.

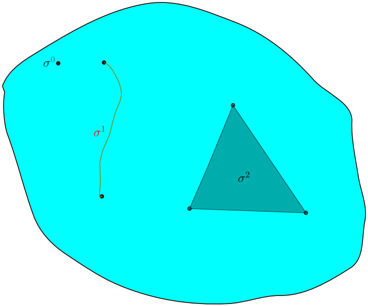

Figure: Some cells: A 0-cell σ^0, 1-cell σ^1, and a 2-cell σ^2