MATH325: Discrete Mathematics II
Quiz q0201

Name: nweadick1@cougars.ccis.edu Score: $\boxed{}$

Open `main.tex` and enter answers (look for `answercode`, `answerbox`, `answerlong`). Turn the page for detailed instructions. To rebuild and view pdf, in bash shell execute `make`. To build a gzip-tar file, in bash shell execute `make s` and you'll get `submit.tar.gz`.

When writing results of computations, make sure it's simplified. For instance when you write down sets, if the answer is $\{1\}$, I do not want to see $\{1, 1\}$.

When you write down an example or a counterexample, be elegant and write down the simplest. You only need to give a counterexample if asked.

$P(X)$ denotes the powerset of X.

In LaTeX math notation is enclosed by the `$` symbols. For instance `$x = a_{1} + b^{2}$` gives you $x = a_1 + b^2$.

(For more information about LaTeX go to my website http://bit.ly/yliow0, click on `Yes` you are one of my students, then look for latex.pdf.)

For the next few questions, let $A = \{3, 2, \pi, 4\}$, $B = \{2, 4, 1, 3, 2, 4\}$, $C = \{7, 5, 3\}$

Q1. $|A| = \boxed{4}$ (Please correct!)

Q2. $|B| = \boxed{4}$

Q3. What is $A \cup B$?

$\boxed{A \cup B = \{1, 2, 3, 4, pie\}}$

Q4. What is $A \cap B$?

$\boxed{A \cap B = \{2, 3, 4\}}$

Q5. What is $A - C = \{\square\}$?

$\boxed{A - C = \{2, pie, 4\}}$

Q6. What is $A \times C$?

$A \times C = \{(3,7),(3,5),(3,3),(2,7),(2,5),(2,3),(pi,7),(pi,5),(pi,3),(4,7),(4,5),(4,3)\}$

Q7. What is $P(C)$?

$P(C) = \{\emptyset, \{3\}, \{5\}, \{7\}, \{3,5\}, \{3,7\}, \{5,7\}, \{3,5,7\}\}$

For the next few questions, let X, Y, Z be sets (in the same universe).

Q8. T or F or M: $X \cup Y = Y \cup X$ .... T

Q9. T or F or M: $X \cup (Y \cap Z) = (X \cup Y) \cap Z$. .... F
If F, provide a counterexample

$X = \{1,2\}$, $Y = \{2,3\}$, $Z = \{3,4\}$
$X \cup (Y \cap Z) = \{1,2,3\}$ $(X \cup Y) \cap Z = \{3\}$

Q10. T or F or M: If $X \neq \emptyset$ and $Y \neq \emptyset$, then $X \cup Y \neq \emptyset$. .... T
If F, provide a counterexample

Q11. T or F or M: If $X \neq \emptyset$ and $Y \neq \emptyset$, then $X \cap Y \neq \emptyset$. .... F
If F, provide a counterexample

$X = \{1\}$, $Y = \{2\}$. $then X \cap Y = \emptyset$

Q12. T or F or M: $\emptyset \subseteq X$ for any set X. .... T

Q13. T or F or M: $\emptyset \in X$ for any set X. .... F

Q14. T or F or M: $\emptyset \times X = \emptyset$ .... T

Q15. Find the simplest set X and Y such that $X \in Y$ and $X \subseteq Y$.

$X = \{1\}$, $Y = \{1,2\}$.

Q16. T or F or M: $P(\emptyset) = \emptyset$ .... T

Q17. T or F or M: $X \subseteq Y \implies P(X) \subseteq P(Y)$. .... T
If F, provide a counterexample

Q18. T or F or M: $P(X \cup Y) = P(X) \cup P(Y)$. .... $\boxed{\text{F}}$
If F, provide a counterexample

$X = \{1\}, Y = \{2\}, P(X \cup Y) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$ $P(X) \cup P(Y) = \{\emptyset, \{1\}, \{2\}\}$

Q19. T or F or M: $P(X \cap Y) = P(X) \cap P(Y)$. .... $\boxed{\text{T}}$
If F, provide a counterexample

Q20. T or F or M: $P(X \times Y) = P(X) \times P(Y)$. .... $\boxed{\text{T}}$
If F, provide a counterexample

Q21. Hans Solo and Luke Skywalker are leading a team of 10 to attack an AT-AT and some stormtroopers. (The 10 includes Hans and Luke.) They have decided to split into two teams. The team attacking the AT-AT will have at least 2 members. How many ways are there to form such a team? (Explain your work with complete sentences.)

$2^10 - C(10,0) + C(10,1) = 1024 - 11 = 1013$

Q22. T or F or M. It is possible to construct sets W, X, Y, Z such that

$$|W \cap X| = |X \cap Y| = |Y \cap Z| = |Z \cap W| = 2$$

and

$$|W \cap X \cap Y| = |X \cap Y \cap Z| = |Y \cap Z \cap W| = |Z \cap W \cap X| = 0$$

.... $\boxed{\text{T}}$
If your answer is T, provide the simplest possible example.

$W = \{1,2,3,4\}, X = \{3,4,5,6\}, Y = \{5,6,7,8\}, Z = \{7,8,1,2\}$

Q23. T or F or M. It is possible to construct sets W, X, Y, Z such that

$$|W \cap X| = |X \cap Y| = |Y \cap Z| = |Z \cap W| = 2$$

and

$$|W \cap X \cap Y| = |X \cap Y \cap Z| = |Y \cap Z \cap W| = |Z \cap W \cap X| = 1$$

. . . . $\boxed{\text{T}}$

If your answer is T, provide the simplest possible example.

$W = \{1, 2, 3\}, X = \{2, 3, 4\}, Y = \{3, 4, 5\}, Z = \{1, 3, 4\}$

Q24. T or F or M: It is possible find a set X and a function $f : X \to X$ such that $|X| = 4$, $|f(X)| = 3$, $|f(f(X)| = 2$, $|f(f(f(X)))| = 1$ . . . . $\boxed{\text{T}}$

If your answer is T, provide the simplest possible example.

$X = \{1, 2, 3, 4\}$

Q25. T or F or M: If $f : X \to Y$ and $g : Y \to Z$ are onto (i.e., surjective) functions, then $g \circ f : X \to Z$ is also onto. . . . . $\boxed{\text{T}}$

If F, provide a counterexample.

Q26. T or F or M: If $f : X \to Y$ and $g : Y \to Z$ are 1–1 (i.e., injective) functions, then $g \circ f : X \to Z$ is also 1–1. . . . . $\boxed{\text{T}}$

If F, provide a counterexample.

Q27. T or F or M: Let $f : X \to Y$ and $g : Y \to Z$ be functions. If f is onto and g is not onto, then $g \circ f$ is onto. . . . . ☐

If F, provide a counterexample.

Q28. T or F or M: Let $f : X \to Y$ and $g : Y \to Z$ be functions. If f is not onto and g is onto, then $g \circ f$ is not onto. . . . . ☐

If F, provide a counterexample

Q29. T or F or M: Let $f : X \to Y$ and $g : Y \to Z$ be functions. If f is 1–1 and g is not 1–1, then $g \circ f$ is 1–1. . . . . ☐

If F, provide a counterexample

Q30. T or F or M: Let $f : X \to Y$ and $g : Y \to Z$ be functions. If f is not 1–1 and g is 1–1, then $g \circ f$ is 1–1. . . . . $\square$
If F, provide a counterexample

Q31. Find sets X and Y such that X and Y are countable and $X - Y$ is $\emptyset$.

$X = \{1\}, Y = \{1\}$

Q32. Given any positive integer n, find sets X and Y such that X and Y are countable and $|X - Y| = n$.

$X = \{1, ...2n\}$, $Y = \{n + 1, ...2n\}$

Q33. Find countable sets X and Y such that $X - Y$ is infinite and countable.

The set X is Natural Numbers and Set Y is all positive Even Numbers, then, (X - Y) = N - E+ = Positive Odd Integers, which is infinite and countable.

Q34. Compute the following matrix product:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

Q35. Find a matrix M with only 0s and 1s for entry such that $M^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$M = \begin{bmatrix} \square & \square \\ \square & \square \end{bmatrix}$$

(Hint: M is made up of only 0s and 1s.)

Instructions

In `main.tex` change the email address in

```
\renewcommand\AUTHOR{jdoe5@cougars.ccis.edu}
```

yours. In the bash shell, execute "`make`" to recompile `main.pdf`. Execute "`make v`" to view `main.pdf`. Execute "`make s`" to create `submit.tar.gz` for submission.

For each question, you'll see boxes for you to fill. You write your answers in `main.tex` file. For small boxes, if you see

```
1 + 1 = \answerbox{}.
```

you do this:

```
1 + 1 = \answerbox{2}.
```

`answerbox` will also appear in "true/false" and "multiple-choice" questions.

For longer answers that needs typewriter font, if you see

```
Write a C++ statement that declares an integer variable name x.
\begin{answercode}
\end{answercode}
```

you do this:

```
Write a C++ statement that declares an integer variable name x.
\begin{answercode}
int x;
\end{answercode}
```

`answercode` will appear in questions asking for code, algorithm, and program output. In this case, indentation and spacing is significant. For program output, I do look at spaces and newlines.

For long answers (not in typewriter font) if you see

```
What is the color of the sky?
\begin{answerlong}
\end{answerlong}
```

you can write

```
What is the color of the sky?
\begin{answerlong}
The color of the sky is blue.
\end{answerlong}
```

For students beyond 245: You can put LaTeX commands in `answerbox` and `answerlong`.

A question that begins with "T or F or M" requires you to identify whether it is true or false, or meaningless. "Meaningless" means something's wrong with the statement and it is not well-defined. Something like "1+$_2$" or "$\{2\}^{\{3\}}$" is not well-defined. Therefore a question such as "Is 42 = 1+$_2$ true or false?" or "Is $42 = \{2\}^{\{3\}}$ true or false?" does not make sense. "Is $P(42) = \{42\}$ true or false?" is meaningless because $P(X)$ is only defined if X is a set. For "Is 1 + 2 + 3 true or false?", "1 + 2 + 3" is well–defined but as a "numerical expression", not as a "proposition", i.e., it cannot be true or false. Therefore "Is 1 + 2 + 3 true or false?" is also not a well-defined question.

When writing results of computations, make sure it's simplified. For instance write 2 instead of $1 + 1$. When you write down sets, if the answer is $\{1\}$, I do not want to see $\{1, 1\}$.

When writing a counterexample, always write the simplest.

Here are some examples (see `instructions.tex` for details):

1. T or F or M: $1 + 1 = 2$ .... $\boxed{\text{T}}$

2. T or F or M: $1 + 1 = 3$ .... $\boxed{\text{F}}$

3. T or F or M: $1+^2 =$ .... $\boxed{\text{M}}$

4. $1 + 2 = \boxed{3}$

5. Write a C++ statement to declare an integer variable named `x`.

```
int x;
```

6. Solve $x^2 - 1 = 0$.

   Since $x^2 - 1 = (x - 1)(x + 1)$, $x^2 - 1 = 0$ implies $(x - 1)(x + 1) = 0$. Therefore $x - 1 = 0$ or $x = -1$. Hence $x = 1$ or $x = -1$.

7. Which is true? .... $\boxed{\text{C}}$

   (A) $1 + 1 = 0$
   (B) $1 + 1 = 1$
   (C) $1 + 1 = 2$
   (D) $1 + 1 = 3$
   (E) $1 + 1 = 4$