MATH325: Discrete Mathematics II
Quiz q080201

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Open `main.tex` and enter answers (look for `answercode`, `answerbox`, `answerlong`). Turn the page for detailed instructions. To rebuild and view pdf, in bash shell execute `make`. To build a gzip-tar file, in bash shell execute `make s` and you'll get `submit.tar.gz`.

This is a 10-min quiz. If you cannot complete this quiz in 10 minutes, you are strongly advised to study my notes carefully.

For all question, you only need to write down the answers you have computed. Do not show your justification/computation. Some LaTeX for you:

- Product: `$a \cdot b$` gives you $a \cdot b$
- Superscript: `$a^{n}$` gives you a^n.
- Square root: `$\sqrt{a}$` gives you $\sqrt{a}$.
- Fraction: `$\frac{a}{b}$` gives you $\frac{a}{b}$ or `$a/b$` gives you a/b
- Binomial: `$\binom{a}{b}$` gives you $\binom{a}{b}$
- Large parentheses: `$\left( a \right)^b$` gives you $\left(a \right)^b$

It's a common practice to wrote $(-2)^n$ as $(-1)^n 2^n$, i.e., so that $(-1)^n$ is factored as a "sign". It's also a common practice to simplify binomial coefficents such as $\binom{n+20}{n+1}$ to $\binom{n+20}{19}$, i.e., no n in the bottom. This is from $\binom{n}{r} = \binom{n}{n-r}$.

Consider the recurrence

$$a_n = 4a_{n-1} + a_{n-2}$$

for $n \geq 1$ and $a_0 = 1, a_1 = 2$.

Q1. What is the characteristic equation for the a_n recurrence?
Answer:

$x^2 - 4x - 1 = 0$

(No explanation needed.)

Q2. What are the roots of the characteristic equation?
Answer:

$r_1 = 2 + \sqrt{5}, r_2 = 2 - \sqrt{5}$

(No explanation needed.)

Q3. What is the general form of the closed form for a_n?
Answer:

$a_n = C_1 \cdot (2+\sqrt{5})^n + C_2 \cdot (2-\sqrt{5})^n$, where C_1, C_2 are constants to be determined.

(No explanation needed.)

Q4. Using the method of characteristic equation, find a closed form for a_n.
Answer:

$a_n = \frac{1}{2} \cdot (2+\sqrt{5})^n + \frac{1}{2} \cdot (2-\sqrt{5})^n$

(No explanation needed.)

Note. Just because the method of characteristic equation makes solving recurrences easier, it does not mean that generating functions are useless. Generating functions *explains* why the roots of the characteristic equation can be used to find closed.

Instructions

In `main.tex` change the email address in

```
\renewcommand\AUTHOR{jdoe5@cougars.ccis.edu}
```

yours. In the bash shell, execute "`make`" to recompile `main.pdf`. Execute "`make v`" to view `main.pdf`. Execute "`make s`" to create `submit.tar.gz` for submission.

For each question, you'll see boxes for you to fill. You write your answers in `main.tex` file. For small boxes, if you see

```
1 + 1 = \answerbox{}.
```

you do this:

```
1 + 1 = \answerbox{2}.
```

`answerbox` will also appear in "true/false" and "multiple-choice" questions.

For longer answers that needs typewriter font, if you see

```
Write a C++ statement that declares an integer variable name x.
\begin{answercode}
\end{answercode}
```

you do this:

```
Write a C++ statement that declares an integer variable name x.
\begin{answercode}
int x;
\end{answercode}
```

`answercode` will appear in questions asking for code, algorithm, and program output. In this case, indentation and spacing is significant. For program output, I do look at spaces and newlines.

For long answers (not in typewriter font) if you see

```
What is the color of the sky?
\begin{answerlong}
\end{answerlong}
```

you can write

```
What is the color of the sky?
\begin{answerlong}
The color of the sky is blue.
\end{answerlong}
```

For students beyond 245: You can put LaTeX commands in `answerbox` and `answerlong`.

A question that begins with "T or F or M" requires you to identify whether it is true or false, or meaningless. "Meaningless" means something's wrong with the statement and it is not well-defined. Something like "$1+_2$" or "$\{2\}^{\{3\}}$" is not well-defined. Therefore a question such as "Is $42 = 1+_2$ true or false?" or "Is $42 = \{2\}^{\{3\}}$ true or false?" does not make sense. "Is $P(42) = \{42\}$ true or false?" is meaningless because $P(X)$ is only defined if X is a set. For "Is $1 + 2 + 3$ true or false?", "$1 + 2 + 3$" is well–defined but as a "numerical expression", not as a "proposition", i.e., it cannot be true or false. Therefore "Is $1 + 2 + 3$ true or false?" is also not a well-defined question.

When writing results of computations, make sure it's simplified. For instance write 2 instead of $1 + 1$. When you write down sets, if the answer is $\{1\}$, I do not want to see $\{1, 1\}$.

When writing a counterexample, always write the simplest.

Here are some examples (see `instructions.tex` for details):

1. T or F or M: $1 + 1 = 2$ ..... T
2. T or F or M: $1 + 1 = 3$ ..... F
3. T or F or M: $1+^2 =$ ..... M
4. $1 + 2 =$ 3
5. Write a C++ statement to declare an integer variable named `x`.

   ```
   int x;
   ```

6. Solve $x^2 - 1 = 0$.

   Since $x^2 - 1 = (x - 1)(x + 1)$, $x^2 - 1 = 0$ implies $(x - 1)(x + 1) = 0$. Therefore $x - 1 = 0$ or $x = -1$. Hence $x = 1$ or $x = -1$.

7. Which is true? C
   (A) $1 + 1 = 0$
   (B) $1 + 1 = 1$
   (C) $1 + 1 = 2$
   (D) $1 + 1 = 3$
   (E) $1 + 1 = 4$