<u>Statistics and Hypothesis testing Assignment</u>

Question 1:

The quality assurance checks on the previous batches of drugs found that — it is 4 times more likely that a drug is able to produce a satisfactory result than not. Given a small sample of 10 drugs, you are required to find the theoretical probability that at most, 3 drugs are not able to do a satisfactory job.

a.) Propose the type of probability distribution that would accurately portray the above scenario, and list out the three conditions that this distribution follows. b.) Calculate the required probability.

Answer:

a) Binomial probability distribution is ideal as it satisfies all the three conditions,

1) Total no of trials is fixed at n:
2) Only two possible outcomes success and failure as each trial is binary
3) Probability of success is same in all the trials

In the above case:

- Fixed trial n = 10
- 2 possible outcomes: Success – drug was able to do a satisfactory job and Failure – drug was not able to a satisfactory job.
- Probability of success (p) is same in all the trials, where p = 5/10 (0.2)

b) Calculation of required Probability:

(Q1.)

Calculating required probability :-

(b.) No. of Samples (n) = 10.

No. of unsuccessful drug trials (r) = 3.

probability of unsuccessful (p) = 0.2

(Hx more likely the drug is able to produce a Satisfactory result or not)

formula.

$$P(X = r) = {}^{n}C_{r}\,(p)^{r}\,(1-p)^{n-r}$$

a) $P(x = 0) = {}^{10}C_0\,(0.2)^0\,(1-0.2)^{10}$

$= 1\,(0.2)^0\,(0.8)^{10}$

$= 1 \times 1 \times 0.1073$

$= 10.73.$

b) $P(x = 1) = {}^{10}C_1\,(0.2)^1\,(1-0.2)^9$

$= 10.\,(0.2)\,(0.8)^9$

$= 10\,(0.2)\,(0.1342)$

$= 0.2684$

$= 26.84.$

c) $P(x = 2) = {}^{10}C_2\,(0.2)^2\,(1-0.2)^8$

$= 45\,(0.04)\,(0.1677)$

$= 0.30186$

$= 30.18$

d) $P(x = 3) = {}^{10}C_3\,(0.2)^3\,(1-0.2)^7$

$= 120\,(0.008)\,(0.2097)$

$= 0.2013$

$= 20.13.$

Computing the above results:-

$P(x \le 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= 10.73 + 26.84 + 30.18 + 20.13$

$= \underline{87.88}.$

So the probability that atmost 3 drugs were not able to do a Satisfactory job is

$P(x \le 3) = 87.88.$

Question 2:

For the effectiveness test, a sample of 100 drugs was taken. The mean time of effect was 207 seconds, with the standard deviation coming to 65 seconds. Using this information, you are required to estimate the range in which the population mean might lie — with a 95% confidence level.

a.) Discuss the main methodology using which you will approach this problem. State all the properties of the required method. Limit your answer to 150 words.

b.) Find the required range.

Answer:

a) I will go with Central Limit Theorem (CLT) methodology for this problem.

Following are the properties of the same:

1) The mean is equal to the population mean i.e.

$$\mu_{\bar{X}} = \mu$$

2) Sampling distribution std deviation aka std. error given by

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$

3) Sampling distribution becomes a normal distribution for large n (usually n >30)

b) Solution for finding required range:

Q2) finding the required range:-

Solution:-

Mean ($\bar{x}$) = 207

Std. deviation (σ) = 65

Sample Space (n) = 100.

Confidence level (γ) = 95%

Z Associated with Confidence level (95%) = ±1.96

Confidence Interval = $\left(\bar{x} - \frac{z \times \sigma}{\sqrt{n}}, \bar{x} + \frac{z \times \sigma}{\sqrt{n}}\right)$

$= \left(207 - \frac{1.96 \times 65}{\sqrt{100}}, 207 + \frac{1.96 \times 65}{\sqrt{100}}\right)$

$= \left(207 - \frac{1.96 \times 65}{10}, 207 + \frac{1.96 \times 65}{10}\right)$

$= (194.26, 219.74)$

Therefore Estimated range the population mean lie is (194.26, 219.74).

Question 3:

a) The painkiller drug needs to have a time of effect of at most 200 seconds to be considered as having done a satisfactory job. Given the same sample data (size, mean, and standard deviation) of the previous question, test the claim that the newer batch produces a satisfactory result and passes the quality assurance test. Utilize 2 hypothesis testing methods to make your decision. Take the significance level at 5 %. Clearly specify the hypotheses, the calculated test statistics, and the final decision that should be made for each method.

b) You know that two types of errors can occur during hypothesis testing — namely Type-I and Type-II errors — whose probabilities are denoted by α and β respectively. For the current sample conditions (sample size, mean, and standard deviation), the value of α and β come out to be 0.05 and 0.45 respectively.

Now, a different sampling procedure(with different sample size, mean, and standard deviation) is proposed so that when the same hypothesis test is conducted, the values of α and β are controlled at 0.15 each. Explain under what conditions would either method be more preferred than the other, i.e. give an example of a situation where conducting a hypothesis test having α and β as 0.05 and 0.45 respectively would be preferred over having them both at 0.15. Similarly, give an example for the reverse scenario - a situation where conducting the hypothesis test with both α and β values fixed at 0.15 would be preferred over having them at 0.05 and 0.45 respectively. Also, provide suitable reasons for your choice(Assume that only the values of α and β as mentioned above are provided to you and no other information is available).

Answer for Q3 (a):

1) Stating Null and Alternate Hypothesis:

Null Hypothesis: H0: $\mu \leq 200$ seconds, that the drug has done a satisfactory job.

Alternate Hypothesis: H1: $\mu > 200$ seconds, time effect that the drug has not done a satisfactory job

2) Type of the test - It will be an upper tailed test based on the sign in the Alternate Hypothesis and position of critical region

> in H_1 → Upper-tailed test → Rejection region on right side of distribution

3) Critical value and p-value method are the two hypothesis test we will be using to derive at a decision.

Critical Value method:

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Q3 (a) Critical value Method.

$H_0: \mu \leq 200$ → Null hypothesis.
$H_1: \mu > 200$ → Alternat hypothesis

So ~~here~~ here, based on the sign of Alternate hypothesis Upper tailed test will be Conducted.

$\mu = 200$
$n = 100$
$\mu \bar{x} = 65$
$\alpha = 5\% = 0.05$

Cumulative Probability = 1-0.005
Acceptance region = 0.95

Z Score for 0.95
As 0.95 is not present in the z table.
Z Score of 0.9495 = 1.64.
Z Score of 0.9505 = 1.65.

Average = 1.64 (Averge of two z scores)
Z Score (0.95) = 1.645

formula :-

$\sigma_{\bar{x}} = \sigma/\sqrt{n} = 65/\sqrt{100} = 6.5$

$UCV = \mu + (Z_c \times \sigma_{\bar{x}})$
$= 200 + 1.645 \times 6.5$
$= 210.69$

Conclusion :- fail to Reject the null hypothesis
since the Sample mean does not lie
in Critical region (UCV)
i.e $\mu_{\bar{x}} < UCV$

Conclusion: We fail to reject the null hypothesis since the sample mean doesn't lie in the critical region by using Critical Value Method.

p-value method:

Conclusion :- fail to Reject the null hypothesis
since the Sample mean does not lie
in Critical region (UCV)
i.e $\mu_{\bar{x}} < UCV$

(b) P- value Method :-

1) Z Score for Sample mean distribution.

$$Z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma/\sqrt{n}}$$

$$= \frac{207-200}{65/\sqrt{100}}$$

$$= \frac{207-200}{65/10} = 7/6.5$$

$$= \boxed{1.076}$$

2.) Calculating the P-value from Cumulative Probabilit
for the given Z score using Z table.

Cumulative Probability of Sample point = 0.8577

for one tailed test (upper tailed):

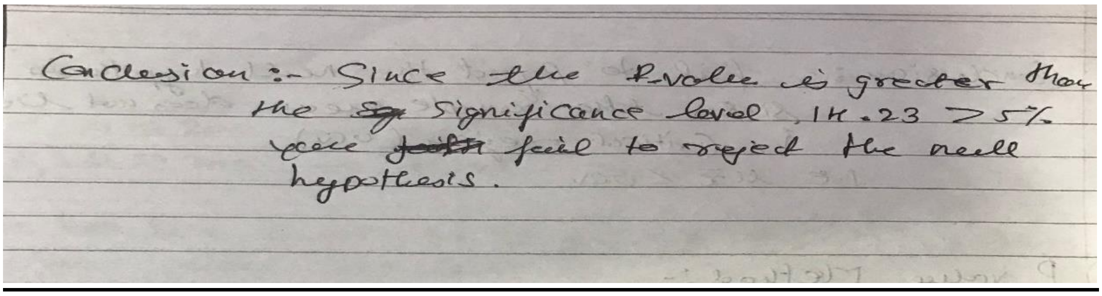
$$P = 1 - \text{Cumulative probability.}$$

$$= 1 - 0.8577$$

$$= 0.1423$$

$$= 14.23\%$$

Significance level (α) = 5% i.e, 0.05


Conclusion :- Since the P-value is greater than the ~~sig~~ significance level, 14.23 > 5% we ~~faith~~ fail to reject the null hypothesis.

Conclusion: Since the p-value is an upper tailed test and is greater than the significance level, we fail to reject the null hypothesis by using P-value Method

Answer to Q3 (b):

Types of errors:

1) Type I - reject the null hypothesis when it is true denoted by α
2) Type II - fail to reject the null hypothesis when it is False denoted by β

Null and Alternate Hypothesis:

- Null: H0 = Drug produces satisfactory results
- Alternate: H1 = Drug does not produce Satisfactory results

Consequences of errors:

- Type I error: we reject the null hypothesis that the drug produces satisfactory result, whereas it does produce satisfactory result.
- Type II error: we fail to reject the null hypothesis that the drug produces satisfactory result, whereas in reality it doesn't produce a satisfactory result.

Error that can have dangerous impact on company and consumer health:

- Type II error is more dangerous as compare to Type I because we fail to reject the null hypothesis which may cause severe health issues to the consumers. It will also impact the reputation and goodwill of the company once consumers start filing legal cases.
- Whereas, Type I error will be hazardous to consumer and company and may only require less efforts to manufacture a new batch of painkillers and test them for quality purpose.

Conclusion for Case I vs Case II:

- Case I: In order to avoid the dangerous effect of Type II error β we can increase the probability of $\alpha = 0.05$ to $\alpha = 0.10$ as both the probability α & β are inversely proportional and would help in reducing the chances of committing type II error.

- Case II: As per the above case discussed we should prefer α & $\beta = 0.15$ over $\alpha = 0.05$ & $\beta = 0.45$ as the company controls both types of errors to 15% of significance level. unlike the Case I where decrease in β is required which in return increases the type I error α and that may cost the company to invest more in manufacturing drugs again and testing for its effectiveness.

Question 4:

Now, once the batch has passed all the quality tests and is ready to be launched in the market, the marketing team needs to plan an effective online ad campaign to attract new customers. Two taglines were proposed for the campaign, and the team is currently divided on which option to use.

Explain why and how A/B testing can be used to decide which option is more effective. Give a stepwise procedure for the test that needs to be conducted.

Answer:

A/B testing:

- An A/B test is a comparison between two versions of the same marketing asset, such as a web page or email, that you expose to equal halves of your audience. Here, in this case it is taglines.
- And based on the conversion rate or other metrics, you can decide which one performs better or appeals more to the audience.

Approach/Procedure to be followed in the above case:

1) As mentioned above we first need to expose these two taglines to two set of audience.

2) Then frame or define the null hypothesis and Alternate Hypothesis considering the conversion rate.

3) Collect the behaviour reports/data and perform the Hypothesis test in order to calculate P-value.

4) Then decide on the basis of P-value and significance level. (For eg. P-value less than 0.05 (typically ≤ 0.05) is statistically significant and P-value higher than 0.05 (> 0.05) is not statistically significant and indicates weak evidence against the null hypothesis and this means we fail to reject the null hypothesis and cannot accept the alternative hypothesis).